THE LONG RUN RELATIONSHIP BETWEEN AVERAGE AND MARGINAL COST.

Anwarul Hoque and Adesoji Adelaja*

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With the advent of duality theory, it became apparent that the production function need not be estimated in order to analyze the characteristics of production. The application of duality theory became widespread as many studies utilized the translog cost function approach in analyzing factor substitution and derived demand for inputs (Binswanger, 1974a), the nature of technological change (Binswanger, 1974b), returns to scale (Ball and Chambers), agricultural policy (Antle and Aitah) and, more recently, the implication of size in production (Hoque, Adelaja and Ganguly).

The cost function approach is particularly attractive for many reasons. First, it is easy to apply since all that is needed to fit the cost function is data on input prices and factor shares. Second, it allows greater flexibility with respect to homotheticity, homogeneity and returns to scale in the specification of the dual cost function. Third,

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the cost function can be used to analyze factor substitution, factor demand, returns to scale and technological progress without explicit specification of the production function.

Of particular importance is the use of the cost function approach in the analysis returns to scale. Following Hanoch, most previous studies generally used the elasticity of scale measures obtained as the reciprocal of the elasticity of total cost estimated from the cost function, for analyzing returns to scale (Ball and Chambers). However, there is an alternative approach to obtain returns to scale which seems to be more straight forward and more powerful than tlhe elasticity of scale approach. This involves the use of the elasticity of total cost itself.

In addition to measures of returns to scale, the elasticity of total cost can provide measures of the long run average and marginal costs of production. Since the translog cost function is usually a long run total cost function, estimates of the long run average and marginal cost can be obtained via the elasticity of total cost obtained from the translog cost function. These estimates reflect production along the expansion path and thus are useful in the analysis of production efficiency and supply.

In this paper, the long run average and marginal costs of production are estimated from the translog cost function by applying the provisions of long run cost theory. In addition to the elasticity of total cost, the elasticities of average cost is also derived directly from the translog cost function and used to describe the nature of returns to scale. The suitability of the approach is further examined by subjecting it to an empirical test with data from a group of dairy farms from selected Northeastern states.

Analytical Model and Estimation Procedure.

Assuming that the aggregate production function can be expressed as

(1) $Q = g(X_i , t),$ $(i=1, 2, \ldots\ldots\ldots\ldots\ldots\ldots, n)$

where X_is are the inputs used in the production of output Q and t is a time variable used as a proxy for technical change, the reduced form of its dual cost function can be expressed as (Diewert)

(2) $C = h(Q , P_i , t)$ $(i=1, 2, \ldots\ldots\ldots\ldots\ldots\ldots, n),$

where C is the total cost of production and the P_is are the prices of respective inputs. To satisfy the requirements of duality, the cost function must be positive and non-decreasing in Q, linearly homogeneous, concave and continuous in input prices for all positive rates of output. It must also be twice differentiable with respect to input prices.

The generalized translog expansion of the cost function in (2) can be expressed as

(3)
$$\ln C = a_0 + a_Q \ln Q + \sum_i a_i \ln P_i + \tfrac{1}{2} G_{QQ} (\ln Q)^2$$
$$+ \tfrac{1}{2}\sum_i\sum_j B_{ij} \ln P_i \ln P_j + \sum_i G_{Qi} \ln Q \ln P_i$$
$$+ H_T t + \tfrac{1}{2} H_{TT} t^2 + H_{TQ} t \ln Q + \sum_i H_{Ti} t \ln P_i .$$

Linear homogeneity of the cost function in prices requires that

$$\sum_i a_i = 1, \quad \sum_i B_{ij} = \sum_j B_{ij} = \sum_i G_{Qi} = \sum_i H_{Ti} = 0.$$

Besides, the symmetry condition ($B_{ij} = B_{ji}$) must also be imposed.

Since Shephard's Lemma implies that the cost minimization input demand (X_i) is equal to the derivative of the cost function with respect to input price (dC/dP_i), the cost shares of input i (S_i) can be expressed as

$$(4) \qquad \frac{d \ln C}{d \ln P_i} = \frac{d\,C}{d\,P_i} \cdot \frac{P_i}{C} \cdot \frac{X_i . P_i}{C} = S_i \ .$$

From the translog cost function in (3), the cost shares can be obtained as

$$(5) \qquad \frac{d \ln C}{d \ln P_i} = S_i = a_i + \sum_j B_{ij} \ln P_j + G_{Qi} \ln Q + H_{Ti}\, t.$$

Production along the expansion path can be studied by examining the elasticity of total cost (E_{CQ}) which is equal to the ratio of marginal to average cost when long run cost minimization is assumed (Ferguson). This relationship is expressed as follows:

$$(6) \qquad E_{CQ} = \frac{d \ln C}{d \ln Q} = \frac{d\,C\ .\ Q}{d\,Q\ .\ C} = \frac{C'}{\overline{C}}\ ,$$

where C' and $\overline{C}$ are the long run marginal and average cost of production, respectively. It should be noted that when $E_{CQ} = 1$, the total cost and output grow at the same rate and, therefore, the marginal cost is equal to the average cost. This implies constant returns to scale. However, when $E_{CQ} > 1$, the total cost is growing faster than output and therefore, marginal cost is greater than the average cost. This suggests decreasing returns to scale. But when $E_{CQ} < 1$, there is increasing returns to scale. Moreover, the relationship in equation (6) is not affected by input price changes in the long run (Ferguson).

Assuming that the cost function (3) is a long run total cost function, the elasticity of total cost can be obtained as

(7) $$E_{CQ} = \frac{d \ln C}{d \ln Q} = \frac{C'}{\overline{C}} = a_Q + G_{QQ} \ln Q + \sum_i G_{Qi} \ln P_i + H_{TQ} t.$$

It can be seen from (7) that the ratio of marginal to average cost can be obtained from coefficients of the cost function.

The elasticity of average cost ($E_{\overline{C}Q}$) is also obtained as

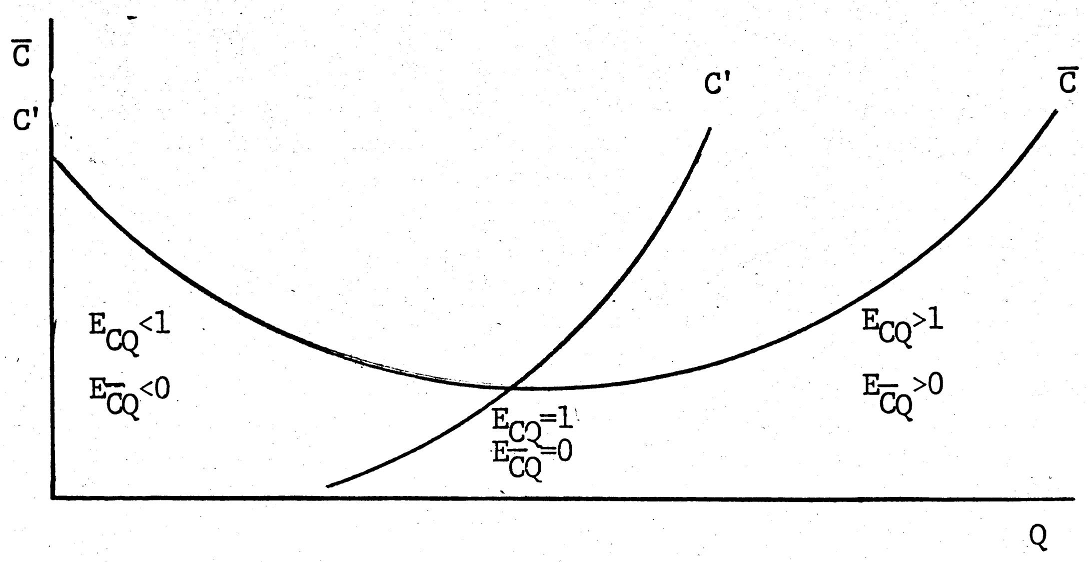
(8) $$E_{\overline{C}Q} = \frac{d\,\overline{C}}{d\,Q} \cdot \frac{Q}{\overline{C}} = \frac{d\,(C/Q)}{d\,Q} \cdot \frac{Q}{(C/Q)} = \frac{Q\,.\,d\,C - C\,.\,d\,Q}{Q^2} \cdot \frac{Q}{d\,Q} \cdot \frac{Q}{C}$$

$$= \frac{Q\,.\,d\,C - C\,.\,d\,Q}{C\,.\,d\,Q} = \frac{d\,C}{d\,Q} \cdot \frac{Q}{C} - 1$$

$$= E_{CQ} - 1.$$

Equation 8 shows that the elasticity of average cost is one less than the elasticity of total cost which is obtained from equation (7). When $E_{CQ} > 1$, $E_{\overline{C}Q} > 0$, the average cost is increasing, marginal cost must, therefore, be greater than average cost. In this region, output is expanded only at a higher unit cost and so, diseconomies of scale result. When $E_{CQ} < 1$, $E_{\overline{C}Q} < 0$, the average cost is decreasing, marginal cost is lower than average cost and there is increasing returns to scale. However, when $E_{CQ} = 1$, $E_{\overline{C}Q} = 0$, the average cost function is constant and marginal cost is increasing, resulting in constant returns to scale. These can be seen in Figure 1. Thus, the nature of returns to scale can be obtained either from the elasticity of average cost or from the elasticity of total cost, both of which can be estimated by the use of the translog cost function.

Figure 1. The Long Run Average and Marginal Cost Curves



While equations (7) and (8) can be used to derive the elasticity of total cost, elasticity of average cost and the ratio between average and marginal cost along the expansion path, these equations do not provide the actual values of average and marginal cost. This problem can be circumvented by generating annual unit costs of production using the following equations.

Since

(9) $$E_{\bar{C}Q} = \frac{d \ln \bar{C}}{d \ln Q} = \frac{d\,\bar{C}}{\bar{C}} \Big/ \frac{d\,Q}{Q}$$

and

(10) $$(d\,\bar{C} / \bar{C}) = (d\,Q / Q) \,.\, (E_{\bar{C}Q}) \,,$$

the growth in average cost can be given by

(11) $$\dot{\bar{C}}_t = \frac{d\,\bar{C}}{\bar{C}.d\,t} = \frac{d\,Q}{Q.d\,t} \,.\, (E_{\bar{C}Q}) \,,$$

where $\dot{\bar{C}}_t$ is the growth in average cost in year t.

The average cost in any one year is equal to the average cost in the previous year plus the growth in average cost. That is to say that

(12) $\overline{C}_{t+1} = \overline{C}_t(1 + \dot{\overline{C}}_t) = \overline{C}_t(1 + (d\,Q\,/\,Q.d\,t).(E_{\overline{C}Q}))$.

Because

(13) $$\frac{d\,Q}{Q\,d\,t} = \frac{Q_{t+1} - Q_t}{Q_t},$$

the following is true.

(14) $$\overline{C}_{t+1} = (\,1 + (\,\frac{Q_{t+1} - Q_t}{Q_t}\,)(E_{\overline{C}Q}))\;.\;\overline{C}_t\,.$$

Assuming that production in any one year is consistent with expansion path behavior, we can generate a series of average cost for all the other years via equation (14). The approach described above provide us with an analytical tool with which to observe not only the relationship between the marginal and average cost, but also approximations of the actual values of average cost. Therefore it allows us to make more conclusive observations on production costs.

Since some of the parameters needed to determine the elasticity of total cost can not be obtained by estimating only the cost share equations alone, the cost function in (3) and the share equations in (5) must be estimated simultaneously. In order to estimate the equations in (3) and (5), error terms are added to them which are assumed to be independent and normally distributed with mean zero and non-singular variance-covariance matrix.

Furthermore, following Barten, since the share equations sum to unity, one equation can be dropped from the system of equations in (5) if a maximum likelihood technique that would provide independent estimates, irrespective of which equation is dropped, is applied. Kmenta and Gilbert, and Ruble argued that estimates from the iterative Zellner's procedure are equivalent to maximum likelihood estimates. Furthermore, Berndt and Christensen suggest that the iterative Zellner's procedure is equivalent to the iterative three Stage Least Squares (I3SLS) procedure. On that basis, we dropped one of the equations in the system in (5), imposed the symmetry-homogeneity restrictions, and estimated the coefficients using I3SLS procedure contained in the SAS computer package. The estimated coefficients were then used to calculate $E_{\overline{C}Q}$ as in equation 8.

The Data

The model was applied to dairy farm data furnished by the ELFAC system for the 1967 to 1981 period. ELFAC is a farm record keeping program organized by the University of Vermont which operates through farmers' voluntary participation in a number of Northeastern states of which West Virginia, Connecticut, New Hampshire, Vermont and Maine are prominent. The total number of farms included in the program in the early years were 217, which increased to 303 in 1981. Each year, ELFAC publishes a report providing costs and return data for dairy farms in each state. Thus, the yearly activity of the average farm in each state was taken to represent an observation point. When the statewise time series data obtained from ELFAC were pooled, they provided enough observations to fulfil the degrees of freedom requirement for estimating the large number of coefficients contained in the translog cost function.

Only data on output, input cost shares and input prices are required for fitting the cost function and the cost share equations. Output was measured in pounds of milk produced as presented by the ELFAC reports and the production inputs were classified under seven categories; labor (X_1), feed (X_2), utility (X_3), fuel oil (X_4), machinery (X_5), capital (X_6), and miscellaneous inputs (X_7). Input shares were calculated from the ELFAC data on production expenses on each input. In order to make all dairy production costs variable to reflect the long run, investment in land, capital, machinery and buildings were depreciated at the rate of 9 percent per year. The depreciation was added to the operating capital expenses so that it reflected the annual flow of investment inputs.

Input prices were obtained as follows. Farm labor wage rates, prices of gasoline, and prices of electricity were used as proxies for P_1, P_4 and P_3. They were obtained from the Agricultural Statistics, (U.S.D.A.). The prices of feed, prices of machinery and implements, interest on indebtedness of farm real estate and prices of farm and other supplies were obtained in index form from the Agricultural Price Summaries of the U.S. Crop Reporting Board. They were used as proxies for P_2, P_5, P_6 and P_7, respectively.

A word of caution must, however, be given regarding the interpretation of the results. Since ELFAC data came from a group of dairy farms who neither were randomly selected nor large enough in size, the results should not be taken as being fully representative of the dairy industry. The data has been used primarily for model testing purpose and the results therefore should be seen in that light only.

Empirical Results

The estimated coefficients of the translog cost function, the corresponding standard errors and the R^2 value of the function are presented in Table 1. The estimated cost function was found to be monotonic and concave in input prices and therefore it is considered well behaved[1]. The estimated coefficients shown in Table 1 were used to calculate the elasticities of total cost and the elasticities of average cost presented in Table 2. Table 3 shows the long run average cost obtained by the use of equation (14) and the corresponding marginal costs.

The estimated elasticity of total cost (E_{CQ}) for the sample declined over the years from 1.0420 in 1968 to about 0.9908 in 1981. Similarly, the elasticity of average cost ($E_{\bar{C}Q}$) fell from 0.0420 to -0.0092 over the same period. Since our estimates of the elasticity of total cost were very close to unity and those of the elasticity of average cost were very close to zero, we conclude that there existed constant returns to scale in dairy farming through most of the period of our study. This means that by definition, the farms have achieved the most cost efficient capacity. Since the range of average farm output over which the elasticity of average cost is close to zero is very wide, it appears that the envelop curve is relatively flat over a wide range.

Estimates of Average and Marginal Cost.

The observed constancy in returns to scale also has implications for the long run marginal cost. Theoretically, in the neighborhood where the long run average cost is constant, the long run marginal cost must be equal to it. By using the approximation technique depicted in equation (14), we can observe not only the shape of the envelop curve and thus returns to scale, but also approximations of actual values of the long run average and marginal costs. These values represent the potential cost of production for the efficient farms and are useful in observing production efficiency. In obtaining these values it was assumed that the cost per unit in 1968 fell along the envelop curve since most economic indicators suggest that it was a rather stable year in terms of prices and production. According to our estimates, the average cost and the marginal cost estimated by equation (14) were equal, as expected, at about 6 cents per pound between 1968 and 1981.

Since evidence from the industry suggests that many farmers experienced production costs well above the 6 cents per pound observed through the total cost elasticity, for comparative purposes, we calculated a series of annual average cost directly from the data by dividing the total cost for the average farm in each year by the output for that year. The values of average cost we obtained by using this method are quite different from those obtained through equation (14). This method suggests an average cost which increased from 5.76 cents per pound in 1968 to 14.93 cents per pound by 1981. Furthermore, the largest increases in these measures of average cost occured in 1973 and 1979, the years of the oil embargo and the oil price deregulation, respectively.

The difference in the estimates of average cost obtained by equation (14) and the unit cost obtained directly from the data perhaps needs to be explained. While on the one hand, equation (14) yields estimates of average cost along the expansion path, the average cost obtained from the data are actually a series of short run average cost measures. These costs are much higher because they reflect the economic environment of dairy production. In the short run, farmers' production decisions are affected by industry factors such as changes in input and output prices, inflation, weather and all other factors beyond the influence of farmers. These factors result in much higher production cost than is suggested by the costs obtained by equation (14). The large increases in unit cost following the oil embargo of 1973 and oil price deregulation in 1979 illustrate this point. On the other hand, the average costs obtained through equation (14) reflect production at or near the optimal level of capacity utilization. Such cost structure does not account for short run factors which affect day to day production decisions.

Summary and Conclusions.

The elasticity of scale derived from translog cost function estimates of production technology has been widely used to study returns to scale. The elasticity of total cost, which is an alternative way to measure returns to scale provides additional information about the shape of the envelop curve as well as estimates of the long run average cost and the long run marginal cost. Because these additional information are useful in the analysis of production efficiency, the elasticity of total cost concept appears to be more useful and more informative in production analysis. When used in the analysis of returns to scale in a sample of dairy farms from the Northeast, constant returns to scale was observed.

However, our estimates of short run average cost are considerably higher than those for the long run average cost. The differences are attributed to external factors which affect dairy production in the short run but do not affect production structure along the expansion path.

FOOTNOTES

[1]Monotonicity is tested for by fitting the cost share equations with estimates to check if they are positive at each annual observation. Concavity of the cost function is satisfied if the Hessian matrix based on the parameter estimates is negative semidefinite.

References

Antle, John M. and Ali S. Aitah, "Rice Technology, Farmer Rationality, and Agricultural Policy in Egypt" Amer. J. Agr. Econ., 65(1983):667-674.

Ball, V. Eldon and Robert O. Chambers, "An Economic Analysis of Technology in the Meat Products Industry" Amer. J. Ag. Econ., 64(1982):699-709.

Barten, A.P. "Maximum Likelihood Estimation of a Complete System of Consumer Demand Equations" Europ. Econ. Rev., 1(1967):7-73.

Berndt, E.R. and L. R. Christensen, "The Translog Function and the Substitution of Equipment, Structures and Labor in U.S. Manufacturing, 1929-68" J. Econometrics, 1(1973): 81-114.

Berndt, E. R. and D. O. Wood "Technology, Prices, and the Derived Demands for Energy" Rev. Econ. Stat., 57(1975):259-68.

Binswanger, H. P. "A Cost Function Approach to the Measurement of Factor Demand and Elasticities of Substitution" Amer. J. Agr. Econ., 56(1974): 377-86.

Binswanger, H. P. "The Measurement of Technical Change Prices with Many Factors of Production" Amer. Econ. Rev., 64(1974)964-76.

Buxton, M. Boyd and Herald R. Jensen, Economies of Size in Minnesota Dairy Farming, University of Minnesota Agr. Exp. Sta. Bull. 488, 1968.

Christensen, L.R., D. W. Jorgenson, and L. Lau, "Transcendental Logarithmic Production Frontiers" Rev. Econ. Stat., 55(1973):28-45.

Diewert, W.E. "An Application of Shephard Duality Theorem: A Generalized Leontiel Production Function" J. Polit. Econ., 79(1971): 481-507.

Ferguson, C. I. The Neoclassical Theory of Production and Distribution, Cambridge: Cambridge University Press 1969.

Griffin, J.M., and P. R. Gregory. "An Inter Country Translog Model of Energy Substitution Responses" Amer. Econ. Rev., 66(1976): 845-857.

Hanoch, G. "The Elasticity of Scale and the Shape of Average Costs" Amer. Econ. Rev., 55 (1965): 492-97.

Hoque, Anwarul, Adesoji Adelaja and Pradeep Ganguly "Substitution of Energy Inputs, Returns to Scale and Size in Milk Production" Paper presented at the Sixteenth Meeting of the Atlantic Economic Society, Philadelphia, Oct 6-9 1983.

Humphrey D. and J. Moroney "Substitution Among Capital, Labor and Natural Resource Products in American Manufacturing" J. Polit. Econ., 83(1975): 57-82.

Kmenta, J. and R. Gilbert, "Small Sample Properties of Alternative Estimators of Seemingly Unrelated Regressions" J. Amer. Statis. Assoc., 63(1968):1180-2000.

Matulich, Scott C. "Efficiencies in Large-Scale Dairying: Incentives for Future Structural Change" Amer. Jour. Agr. Econ., 60(1978): 642-647.

Ruble, W.L. Improving the Computations of Simultaneous Stochastic Linear Equation Estimates, PhD. Dissertation. Michigan State University, 1968.

Tremblay, Raymond H. ELFAC Dairy Farm Business Analysis, University of Vermont Coop. Ext. Serv. for North eastern States, Annual Report Series, 1967 to 1981.

U.S. Department of Agriculture, Agricultural Statistics, Washington D.C. 1982.

U.S. Department of Agriculture, Crop Reporting Board; Agricultural Price Summaries, Washington D.C. 1982.

U.S. Department of Commerce, Bureau of the Census; United States Census of Agriculture, Washington D.C. 1969, 1974, 1979.

Zellner, A. "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias" J. Amer. Stat. Assoc., 57(1962): 585-612.

Table 1. Estimated Coefficients of the Translog Cost Function

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
a_0	1.5825	1.6900	B_{34}	-0.0044	0.0026
a_1	-0.2707	0.0391	B_{35}	-0.0280	0.0067
a_2	0.1420	0.0379	B_{36}	-0.0238	0.0088
a_3	0.0046	0.0074	B_{37}	0.0019	0.0304
a_4	0.0382	0.0111	B_{44}	0.0174	0.0047
a_5	0.0506	0.0279	B_{45}	-0.0082	0.0091
a_6	0.6381	0.0604	B_{46}	-0.0289	0.0138
a_7	0.3972	--	B_{47}	-0.0125	0.0510
B_{11}	0.0162	0.0335	B_{55}	0.0393	0.0303
B_{12}	0.0119	0.0142	B_{56}	-0.1228	0.0352
B_{13}	0.0339	0.0057	B_{57}	0.0146	0.1053
B_{14}	-0.0036	0.0084	B_{66}	0.0818	0.0660
B_{15}	0.0330	0.0201	B_{67}	0.4335	0.1116
B_{16}	0.0485	0.0361	B_{77}	-0.5679	--
B_{17}	-0.1729	0.0818	G_{Q1}	0.0486	0.0029
B_{22}	0.1143	0.0172	G_{Q2}	0.0220	0.0039
B_{23}	-0.0165	0.0026	G_{Q3}	-0.0037	0.0005
B_{24}	-0.0049	0.0039	G_{Q4}	-.0047	0.0007
B_{25}	0.0255	0.0102	G_{Q5}	-0.0114	0.0019
B_{26}	-0.0766	0.0192	G_{Q6}	-0.0430	0.0042
B_{27}	-0.1311	0.0370	G_{Q7}	-0.0078	--
B_{33}	0.0005	0.0029	G_{QQ}	0.0882	0.0441

cont.

Table 1. Estimated Coefficients of the Translog Cost Function (Cont.)

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
H_{T1}	-0.0061	0.0027	H_{T5}	0.0118	0.0020
H_{T2}	0.0059	0.0015	H_{T6}	0.0006	0.0044
H_{T3}	0.0042	0.0005	H_{T7}	-0.0192	--
H_{T4}	0.0027	0.0008	H_{TQ}	-0.0030	0.0026

$R^2 = 0.9872$

Table 2. Estimated Elasticities of Total and Average Cost.

Year	E_{CQ}	$E_{\bar{C}Q}$
1968	1.0420	0.0420
1969	1.0390	0.0390
1970	1.0380	0.0380
1971	1.0408	0.0408
1972	1.0382	0.0382
1973	1.0382	0.0382
1974	1.0306	0.0306
1975	1.0196	0.0198
1976	1.0154	0.0154
1977	1.0104	0.0104
1978	1.0082	0.0082
1979	0.9983	-0.0017
1980	0.9981	-0.0019
1981	0.9908	-0.0092
Avg.	1.0230	0.0230

Table 3. Output Average and Marginal Cost Along the Expansion Path, and Calculated Average Cost of the Average Farm.

		Estimated Production Cost Along the Expansion Path		Production Cost as Calculated from Data	
Year	Output (Million Pounds)	Average Cost (¢ per Pound)	Marginal Cost (¢ per Pound)	Average Cost (¢ per Pound)	% Increases in avg. cost
1968	.688	5.760	6.254	5.760	--
1969	.769	5.786	6.012	6.059	5
1970	.778	5.789	6.009	6.340	5
1971	.838	5.807	6.044	6.627	5
1972	.884	5.819	6.041	6.969	5
1973	.874	5.816	6.038	8.283	19
1974	.883	5.819	5.997	9.384	13
1975	.919	5.823	5.937	9.662	3
1976	.963	5.828	5.918	10.442	8
1977	.948	5.824	5.885	10.608	2
1978	1.034	5.829	5.877	11.508	8
1979	1.050	5.829	5.819	13.098	14
1980	1.054	5.829	5.818	14.312	9
1981	1.022	5.830	5.776	14.927	4