1980

Policies and programs

MULTIPLE CROP SUPPLY COMPONENT OF THE WORLD GRAINS, OILSEEDS, AND LIVESTOCK MODEL

by

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Selected paper for the

American Agricultural Economics Association Meetings

University of Illinois, Urbana, Illinois

July 27-30, 1980

The helpful comments of Robert L. Thomspon and Leroy Quance are gratefully acknowledged.

ABSTRACT

MULTIPLE CROP SUPPLY COMPONENT OF THE WORLD GRAINS, OILSEEDS, AND LIVESTOCK MODEL

The purpose of this paper is to resolve conceptual problems in the crop supply component of the present GOL model and to develop the conceptual framework for a multiple-product production system of a multiple-commodity agricultural trade model. The major emphasis on the revision of the crop supply system is structural consistency in order to assure consistent acreage allocation among crops and to impose total arable land area constraints on supply.

Keywords: multiple crop supply, GOL model, elasticity, area, production.

MULTIPLE CROP SUPPLY COMPONENT OF THE WORLD GRAINS, OILSEEDS, AND LIVESTOCK MODEL*

I. Introduction

The world Grains, Oilseeds, and Livestock (GOL) model is the principal analytical tool for global long-run policy and program analysis of the International Economics Division, ESCS, USDA. It is a multiple-commodity and multiple-region model consisting of supply, demand and trade components of twelve commodities and twenty-eight regions. The equations in the model were developed to reflect: (1) the economic behavioral pattern of the grains-oilseeds-livestock economy, (2) important technical input-output relationships, and (3) institutional settings. The objective of the GOL model when built was to have the capability of making long-run projections of world food and agriculture under alternative scenarios. The model has also been used as a policy analysis tool in staff analysis work on a number of program and policy questions. However, the current structure of the GOL model still presents many limitations in using it as a policy analysis tool. Modeling efforts are a continuing process to improve the analytical capability of the GOL model. In this paper, we focus on conceptualizing and modifying the crop supply system of the GOL model.

The objective of this paper is to resolve conceptual problems in the crop supply component of the present GOL model and to develop the conceptual framework for a multiple crop supply and input demand module of an improved multiple-commodity agricultural trade model. The paper is organized as follows. The theoretical consideration for a multiple-product production system is presented in section II. In sections III and IV, the shortcomings in the current GOL crop supply equations are presented and justifications for the revisions described. And in the last section, longer-term considerations of the sector modeling approach are discussed.

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II. Theoretical Foundations

The framework for modeling a multiple output production system can be derived from the concept of production transformation and duality theory. Assuming the sector confronts downward sloping output demand and upward sloping factor supply schedules, and the goal is profit maximization, the general multiple product, multiple input production decision model for a competitive industry can be characterized as: 1/

$$[1] \quad \text{Max.} \quad \pi = P\,Y$$

$$\text{s.t.} \quad (Y;X)\varepsilon T$$

where Y = vector of outputs, $Y_1, Y_2, \ldots, Y_m$,
X = vector of n nonnegative inputs, $X_1, X_2, \ldots, X_n$,
T = the set of feasible inputs and outputs (production possibilities set),
P = vector of prices net of factor costs, $P_1, P_2, \ldots, P_m$.

Problem [1] uses the concept of transformation to describe the set of efficient input-output combinations. Conditions on the set T are defined as: (1) T must be a closed, nonempty, and convex set, (2) T has characteristics of diminishing marginal rates of transformation of output for inputs (i.e., decreasing returns to scale), increasing marginal rates of transformation of outputs for outputs, and diminishing marginal rates of substitution of inputs for inputs, (3) boundedness from above, and (4) free disposal. Under these conditions, one can show an equivalence between the set of production possibilities, a transformation function and a profit function.

The set of efficient input-output combinations may be described symmetrically as the set of (Y;X) which satisfy the equation $t^*(Y;X)=0$, where t^* is the transformation function for one output, for example, Y_1. This means that one output Y_1 can be singled out, and the efficient set can be described by $Y_1 = t(Y_2, Y_3, \ldots Y_m, X)$, where the transformation function t tells us what the maximum production of Y_1 is, given the vector of inputs

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1/ This part is based on Diewert.

X, and the vector of other outputs $Y^0 = (Y_2, Y_3, \ldots Y_m)$ to produce. Thus, the outcome of problem [1] is equivalent to the outcome of revenue maximization subject to the transformation function for Y_1, i.e.

$$[2]\quad \text{Max. } \pi_1 = PY$$
$$\text{s.t. } Y_1 = t(Y_2, \ldots, Y_m; X)$$

The production possibilities set T which corresponds to the transformation function t for output Y_1, is $T = [(Y_1, Y^0; X): Y_1 \leq t(Y^0; X), Y_1 \geq 0, Y^0 \geq 0_{m-1}, X \geq 0_n)]$. This means that given the same bundle of inputs X to produce multiple outputs Y_1 and $Y^0 (= Y_2, Y_3, \ldots, Y_m)$, the corresponding production possibility frontier for the sector is T, the efficient input-output combinatons of Y_1, Y^0 and X.

One can also show that another equivalent parameterization of the industry's technology can be obtained by means of the profit function. Given a vector of output prices $P = (P_1, P_2, \ldots, P_m)$, and a vector of input prices $W = (W_1, W_2, \ldots, W_n)$, with all prices positive, and a production possibility set T, then the profit function is defined by $\pi(P;W) = P'Y - W'X$. For a given vector of prices (P,W), the producer is assumed to choose a feasible production plan $(Y;X) \varepsilon T$ which maximizes the profit. From this, one can use the profit function to geneate a production possibility set, $T = [(Y;X): P'Y - W'X \leq \pi(P;W)$ for every $(P,W) > (0,0)$ and $(Y;X) \geq (0,0)]$. This relationship establishes a duality between transformation function and profit function. And, from the well known Hotelling's Lemma, we can generate the profit maximizing derived input demand and output supply functions by straight forward differentiation. The derivation of output supply and input demand functions can be shown as follows. If a profit function $\pi(P;W)$ is differentiable with respect to output and input prices at the point $(P^*, W^*) > 0$, then we can obtain $\frac{\partial \pi(P^*;W^*)}{\partial P_m} = Y_m(P^*, W^*) = Y$ for $m = 1, 2, \ldots, m$, and $\frac{\partial \pi(P^*;W^*)}{\partial W_n} = X_n(P^*;W^*) = X$, for $n = 1, 2, \ldots, n$.

This leads to an explicit formulation of the input demand and output supply functions which can be estimated by econometric techniques. Conditions such as homogeneity of degree zero in all prices, symmetry and boundedness from above can be imposed on these equations (Keyzer and Clements).

The above discussion is applicable to "long-run" profit maximization when all output and factor prices are exogenous to the sector. Duality theory is also applicable to situation when additional exogenous variables or additional information is incorporated in the analysis. That is, the dual relationships between quantities and prices are unaffected by the existence of additional variables which affect the production technology. Because of the flexibility of duality theory, exogenous variables such as the index of technology change and fixed inputs can be explicitly incorporated into the dual structure. The incorporation of an index of technological change is probably essential to the construction of an operational model of industry supply. Incorporating fixed inputs in the model is primarily applicable to "short-run" profit maximization or cost minimization.

The static structure outlined in the foregoing pages implicitly assumes that the response of decisionmakers to changes in prices is instantaneous, i.e. changes in prices may change the choice of output and input mix simultaneously. However, in the agricultural production process, biological lag in supply response and uncertainty in prices and weather have important impacts on factor allocation and resource mix. Lags in adoption and diffusion of technology also have impacts on factor mix and factor demand. Assuming the farmers' sole goal is profit maximization, the production decision can be thought to be a two stage process. In the first stage, based on expected prices (often represented by lagged prices) and other determining factors (such as government farm programs), farmers allocate the available land area among crops and decide on the total amount of current inputs such as labor, capital and fertilizer. In the second stage, these current inputs are allocated to various crops so as to maximize expected gross revenue minus variable costs. Since price received and yields (because of weather uncertainty) cannot be foreseen perfectly at the time the decisions are made, expected values for both of these variables can be assumed. The producer duality theory carried out above with price certainty is equally applicable to the case of expected profit maximization in deriving the "optimal" output supply and input demand to depend on expected prices. In order to assure that output is constrained by resource availability such as allocating available land area among crops, the firm's output supply and input demand functions can be parameterized in terms of proportional changes in the share of resources devoted to the product in question. By this way, the adding up condition can be satisfied and the sector can be ensured to stay on (move around) its transformation surface (Clements).

Similarly, the effects of government policies on multiple-product supply response may be many fold. For example, government acreage control policies place restrictions on land use, such policy may lead to changes in the mix of outputs or inputs. The implications may be explained by a concept based on a generalization of Hick's measure on the bias of technological change (Weaver). Just as technological change may shift the production possibility surface in many different ways, the changes in the level of any restricted or fixed inputs will shift the production possibility surface which trace optimal combinations of variable inputs and outputs. In either case, changes in technology or effects of government policies may shift production possibility curves and lead to changes in the choice of outputs and input mix. In an empirical context, numerous supply response studies have attempted to measure government program effects on crop supply response. For example, Houck, et. al., incorporated policy variables such as "effective" support price and "effective" diversion payment in a series of acreage response studies of the U.S. major grains and soybeans.

III. Specification of Crop Supply in Present GOL

The supply block for both grains and oilseeds in a typical GOL model region includes area equations for total crop area and individual crops, and a production equation to represent yield for each crop. Typical crop supply equations in the present version of the world GOL model are shown as follows:

Total crop area equation for each region

$$[3]\quad HAT = A_0 + \sum_{i=1}^{m} b_i P_i + a_1 ZI, \quad i = 1, 2, \ldots, m$$

Area equation for each individual crop i

$$[4]\quad HA_i = A_{i0} + b_i P_i + \sum_{\substack{j=1 \\ j \neq i}}^{m-1} b_j P_j + b_{m+1} HAT$$

Production equation for each individual crop i

$$[5]\quad QS_i = A_{si0} + b_{si1} HA_i + b_{si2} P_i + a_{si1} T + a_{si2} ZI$$

where HAT = total crop area in a region,
i,j = indexes for the major crops in a region, $i=1, \ldots, m$, $j=1, \ldots, m$,
P_i = domestic price of crop i in each region,
P_j = domestic price of crop j, crop j is the competing crop for crop i,
HA_i = area planted for individual crop i,
QS_i = production quantity of individual crop i,
ZI = index of cost of physical inputs in a region (exogenous variable),
T = time trend variable (exogenous variable).

In these crop supply equations, all the relationships are synthesized. Total crop area is defined as a function of the prices of major crops in the region and the index of cost of physical inputs. Individual crop area is a function of prices of own crop and competing crops and total crop area in the region. Individual crops compete for total area based on the historical share and relative crop prices. Production is a function of individual crop area, own price, technological trend and an index of cost of physical inputs.

The crop supply system in the present GOL model, recognizing the multiple-product environment, has attempted to include the relevant simultaneities, i.e., own and cross-price effects among crops and implicitly imposed homogeneity condition in the area equations. However, the current model structure still presents the following problems in use as a policy analysis tool:

(1) The model is a system of linear equations. There is no guarantee against negative production and/or consumption values in the model solution. Because of the linear specification, a non-linearity such as production=yield*area, is not feasible for the solution algorithm used to solve the system of linear equations. So no yield equation is specified.

(2) Although individual crop areas are assumed to sum to total area, this restriction is not explicitly imposed in the system of supply equations. Also, no maximum total crop land area restriction in each region is imposed and no consistent acreage allocation among crops is assured. Consistency of resource allocation and availability (such as crop land) should be ensured in a multiple product production system (to ensure the sector stays on (moves around) its transformation surface) (Clements).

(3) Policy variables reflecting the effects of government farm policies on crop supply response are not included in the supply equations. From both theoretical and empirical grounds, the effects of government policies have important implications on multiple crop supply response.

(4) Elasticity estimates are outdated. For a multiple-commodity agricultural trade model, it is essential to have adequate estimates of all relevant cross price effects in the model. This appears to be particularly important in grains and oilseeds sectors, where important interactions and substitution possibilities exist on both the supply and demand sides of the market. As Thompson has pointed out, omission of relevant variables can lead to biased estimates of the own price term, and if significant cross-price effects among commodities exist, simulation of the effects of policy

changes with the model can lead to erroneous conclusions. This further emphasizes the importance of reviewing and updating the relevant elasticity estimates (Thompson).

(5) There are no linkages between product and factor markets. It is important to include factor market adjustments into multiple commodity trade model because the factor market adjustments have important effects on cost structure and the positions of the supply schedules in each country.

IV. Planned Modifications in GOL Crop Supply System

Based on the theoretical consideration of a multiple product supply system discussed above and the shortcomings of the current GOL model, planned modifications in GOL crop supply component include (i) introducing nonlinearities into the model with constant elasticity supply schedules (to overcome the problem of negative solution values, and to represent the economic behavioral relationships properly), (ii) imposing total crop land area constraints on supply and to assure consistent acreage allocation among crops in each region, and (iii) specifying crop yield and acreage functions for each region; production is then derived by multiplying yield by acreage. A system of equations (in non-linear form) is specified as follows:

Total crop area equation for each region:

$$[6] \quad HAT = A_0 \prod_{i=1}^{m} P_i^{bi} \; T^{b_{m+1}}$$

Total crop land area constraint

$$[7] \quad \sum_{i=1}^{m} HA_i \leq \overline{HAT} \max$$

Area equation for the individual crop i:

$$[8] \quad HA_i = A_{i0}\, HAT^{b_{i1}}\, P_i^{b_{i2}} \prod_{\substack{j=1 \\ j \neq i}}^{m-1} Pj^{b_{ij}}\, T^{b_{i3}} \cdot GP^{b_{i4}}$$

Land market clearing condition:

$$[9] \quad \sum_{i=1}^{m} HA_i = HAT$$

Crop yield equation for each individual crop i:

$$[10] \quad Y_i = Y_{i0}\, P_i^{b'_{i1}}\, P_f^{b'_{i2}}\, HA_i^{b'_{i3}} \cdot (1+r_i)^t$$

Production identity for each individual crop i:

$$[11] \quad QS_i = HA_i \cdot Y_i$$

where HAT = total crop land area in a region,
HA_i = individual crop area in each region, for crop i, i=1, 2, ...,m,
m = number of major crops produced in a region,
$\overline{HAT}_{max}$ = maximum total crop land available in a region,
Y_i = average yield for crop i,
QS_i = production quantity for crop i,
P_i = producer price of crop i,
P_j = producer price of competing crop(s), $j \neq i$, j = 1, 2, ...,m-1
P_f = price of variable inputs, such as fertilizer
GP = government policy variable such as "effective" support price, diversion payment, etc.
T = time trend,
r_i = yield growth rate for crop i,
t = time period, and
b_{ij}'s and b'_{ij}'s = elasticities used in different equations.

This crop supply system can solve for total land area, individual crop area, production quantity, yield and prices simultaneously for each region. Land area allocated to each crop in a region is related to total cropland availability, and the expansion of available crop area is constrained by maximum cropland availability in each region. The total cropland area equation is defined as a function of the prices of major crops in a region, a time trend variable to reflect the expansion of available cropland, and government policy variables. Condition of homogeneity of degree zero in all prices is assumed. Land area used to produce each crop in each region is related to total available cropland, prices of own and competing crops and technological change. Total land area constraint and land market clearing conditions are defined to ensure consistency of cropland allocation and cropland availability, and thus to impose total arable land area constraints on supply. Crop yields are dependent upon prices of own crop and variable inputs, individual crop area, and yield growth rate. Production is then derived by multiplying yield by area. The relevant own and cross-price elasticity estimates will be reviewed and updated. The base period for the model will be rebased to 1975-77.

V. Longer-Term Modifications in the GOL Model Crop Supply Structure

Modifications in the GOL model crop supply system to be undertaken over the next year or so primarily emphasize structural consistency to assure consistent acreage allocation among crops and to impose total arable land area constraints on supply. Because of resource limitations and time constraints, the model parameters will continue to be synthesized from existing studies, analysis and expert opinion, rather than econometric estimates. Not much improvement in the quality of the empirical content of the model can be attained in the near-term activities aside from updating the base period. As data and resources permit, we will turn our attention to obtaining the best structural cross-price effects to improve the quality of the empirical content of the model.

Since the major objectives for the GOL model development are to provide projections of the long-run world food and agriculture under alternative scenarios and to provide an intermediate-run policy analysis tool, the model must include significant simultaneties among subsectors (such as important cross-price effects among commodities) as well as linkage between factor and product markets in the system. A multiple-output production system should describe reactions of factor markets, production and market supply in response to a variety of predetermined variables and policy influences. For modeling a multiple-output supply system, given the assumption of producers' profit maximization, and based on the concept of the profit function, production transformation set and duality theory (by assuming specific functional forms for the profit and production transformation functions), we can estimate the input demand and output supply functions. Recent development in empirical general equilibrium analysis such as the work of Laitinen and Theil on the supply and demand of the multi-product firm and Clements' aggregate multi-product supply model can be applied to estimate the supply responses of a set of multiple-products. Other possibilities might be to try the supply-side analogue to a linear expenditure system (or some such budget share technique), or to use TOBIT analysis in which the dependent variable in each acreage allocation equation is the proportion of the total land in production which is planted to that crop (to ensure the sector stays on (moves around) its production possibility frontier) (Thompson).

Econometric supply models are useful in providing structural estimates of relationships in the agricultural sector. But for evaluating effects of long-run resource adjustments, technological change, or government policies upon a sector, econometric models are not always adequate. Sectoral programming models (either linear programming or quadratic programming) have proven to be a very useful tool (Lattimore and Thompson). Based upon production possibilities, output demand, and factor supply, sectoral programming models can be used to derive output supply response, input demand and factor substitution (McCarl and Spreen). So, over the longer-run, some type of mathematical programming sector model should be considered to model crop supply systems.

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