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WELFARE MEASURES FOR A PRICE DISTORTION IN A MULTI-PRODUCT MULTI-FACTOR SETTING

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ABSTRACT

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MULTI-PRODUCT MULTI-FACTOR SETTING

This paper investigates welfare measures in an economy comprised of vertically related multi-product and multi-factor industries, where a particular market is subject to a price distortion. It is shown that total welfare changes can be obtained with the use only of general equilibrium prices and quantities in the distorted market.

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In recent research considerable attention has been focused on the desirability of farm policies from the standpoint of producer and consumer welfare. The tool of analysis has centered upon classical welfare measurements of producer and consumer surplus. Several different approaches for evaluating these surpluses are found in the literature. The first is a partial equilibrium approach offered by Mishan (1968), which showed that the area above a competitive supply curve derived by a set of fixed inputs measures returns or quasi rents to fixed production factors when all variable input supplies are perfectly elastic. More recently Just and Hueth (1979) demonstrated that consumer surplus in an input market measures quasi rents to producers who use that input. Just and Hueth also examine welfare measures arising from a price distortion in a competitive single-factor single-product vertical sector of the economy. They demonstrate that when a market price within the sector is forcibly altered, total change in sector welfare is given by the producer and consumer surplus change measured from the general equilibrium supply and demand functions of the altered industry level.

As a consequence of these results many questions have been raised regarding the relationship of surpluses when horizontal as well as vertical markets exist. Given that multi-product multi-factor firms represent a common situation in the economy, the interpretation of welfare measures in this context is certainly relevant. Indeed, it has been suggested by Harberger (1971) that possibilities may exist for measuring the distribution of welfare when markets are horizontally and vertically related. However, within the literature one finds little guidance as to how to proceed and interpret surpluses derived from horizontal and vertical markets. This paper is an attempt to resolve this issue. That is, the relationship of surpluses is examined at a price distortion when multi-product and multi-factor conditions occur in a vertical market framework.

The case considered in this paper is a price distortion of a sector of the economy which is comprised of a number of interdependent competitive industries, with each industry producing multiple outputs which are sold to other industries or to final consumers, and using a set of fixed inputs and multiple variable inputs purchased from other related industries or from the initial resource suppliers. In this context if a commodity price is forcibly altered, the actions of any industry within the sector are assumed to affect all prices and quantities in the sector.

General Equilibrium Welfare Measures

Consider a competitive industry, say the k^{th} industry, producing M_k outputs $(Y_{k,1}, \ldots Y_{k,M_k})$ and using M_{k-1} inputs $(Y_{k-1,1}, \ldots, Y_{k-1,M_{k-1}})$ with a technology characterized by the transformation function.

$$F_k(Y_{k,1}, \ldots, Y_{k,M_k}; Y_{k-1,1}, \ldots, Y_{k-1,M_{k-1}}) = 0; \; F_k \in C^2 \quad (1).$$

Since the specification of (1) is in implicit form it is general enough to account for joint or non-joint production.

Denote the profit maximizing output and input levels by $Y^*_{k,i}$ $(i=1, \ldots, M_k)$ and $Y^*_{k-1,i}$ $(i=1, \ldots, M_{k-1})$. Now, the indirect profit function (or quasi rent) for the k^{th} industry is,

$$\pi^*_k(P) = \sum_{i=1}^{M_k} P_{k,i}Y^*_{k,i}(P) - \sum_{i=1}^{M_{k-1}} P_{k-1,i}Y^*_{k-1,i}(P) \qquad (2)$$

where $P_{k,i}$ is the price of the i^{th} output and $P_{k-1,i}$ is the price of the i^{th} input and P is considered a parametric vector for the k^{th} industry.[1]

Suppose that prices in all industries are related through competition at the industry level so that, as price $P_{n,1}$ is forcibly altered, all industry prices change monotonically following $P_{n,1}$.[2] Consider first the effects when $n < k$. Employing the envelop theorem on (2) results in,

$$\frac{\partial \pi_k}{\partial P_{n,1}} = \sum_{m=1}^{M_k} Y_{k,m} \frac{\partial P_{k,m}}{\partial P_{n,1}} - \sum_{m=1}^{M_{k-1}} Y_{k-1,m} \frac{\partial P_{k-1,m}}{\partial P_{n,1}} \tag{3}$$

Integration for a specific change from $P^0_{n,1}$ to $P^1_{n,1}$ implies,

$$\Delta\pi_k = \int_{P^0_{n,1}}^{P^1_{n,1}} \frac{\partial \pi_k}{\partial P_{n,1}} dP_{n,1} = \sum_{m=1}^{M_k} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m} \frac{\partial P_{k,m}}{\partial P_{n,1}}$$

$$- \sum_{m=1}^{M_{k-1}} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{n-1,m} \frac{\partial P_{k-1,m}}{\partial P_{n,1}} dP_{n,1}, \tag{4}$$

where $\Delta\pi_k$ represents the change in quasi rents for vertical industry k. In order to interpret (4) one should note that integrations are along equilibrium quantities. Hence, even though both general equilibrium supplies and demands may shift we are integrating along the path of equilibrium quantities supplied and demanded. Thus, following Just and Hueth's notation, we will define the area along the path of integration as consumer surplus for all industries beyond n, and define the area as producer surplus for all industries before n, and including n. Thus, we note that when $n < k$ integration in (4) is along equilibrium quantities demanded as the supply curve, influenced by $P_{n,1}$ shifts. Therefore the first set of terms in (4) can be expressed as,

$$\sum_{m=1}^{M_k} \Delta CS_{k,m} \equiv - \sum_{m=1}^{M_k} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k,m}(P) \frac{\partial P_{k,m}}{\partial P_{n,1}} dP_{n,1}$$

$$= - \sum_{m=1}^{M_k} \int_{P_{k,m}(P^0_{n,1})}^{P_{k,m}(P^1_{n,1})} Y_{k,m}(P) dP_{k,m} \tag{5}$$

Similarly since $n < k$ the second set of terms in (4) are expressed as,

$$\sum_{m=1}^{M_{k-1}} \Delta CS_{k-1,m} \equiv - \sum_{m=1}^{M_{k-1}} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m}(P) \frac{\partial P_{k-1,m}}{\partial P_{n,1}} dP_{n,1}$$

$$= - \sum_{m=1}^{M_{k-1}} \int_{P_{k-1,m}(P^0_{n,1})}^{P_{k-1,m}(P^1_{n,1})} Y_{k-1,m}(P) dP_{k-1,m} \tag{6}$$

Using (5) and (6) implies that (4) can alternatively be expressed as,

$$\Delta\pi_k = \sum_{m=1}^{M_{k-1}} \Delta CS_{k-1,m} - \sum_{m=1}^{M_k} \Delta CS_{k,m} \qquad k = n + 1, \ldots, K, \qquad (7)$$

which reveals upon solving the difference equation in (7) for $\Delta CS_{n,m}$, that

$$\sum_{m=1}^{M_n} \Delta CS_{n,m} = \sum_{k=n+1}^{K} \Delta\pi_k + \sum_{m=1}^{M} \Delta CS_{K,m} \qquad (8)$$

where $\Delta CS_{K,m}$ represents the changes in final consumer surpluses of the last M_K industry products. Thus, the sum of consumer surpluses in industry n associated with an alteration of one of the prices $P_{n,1}$ in industry n, measures the sum of final consumer surpluses plus all industry rents involved in transforming the commodities traded at industry n into their final consumption form.

The welfare significance of $\Delta\pi_k$ is the same as in Mishan (1968), only in this case, $\Delta\pi_k$ measures the rents associated with multi-product and multi-factor production. That is, $\Delta\pi_k$ is the measure of quasi rents to all of the $Y_{k,m}$ products. The welfare significance of $\Delta CS_{K,m}$ is more complicated since more than one price may change at the final consumption level. In this case, if $\Delta CS_{K,m}$ is measured along a compensated demand curve, it provides an exact measure of the change in consumer welfare with the proper Hicksian welfare significance. If ordinary rather compensated demand curves are used, then one must know the path of prices to implement Willig's (1973) results to evaluate the approximation of welfare measures.

In order to examine the relationships of producer surpluses when multi-product and multi-factor conditions occur, consider the effects of a similar alteration of price $P_{n,1}$, when $n \geq k$. In this case, demands rather than supplies in industry k are affected so that integration of (4) is along equilibrium quantities supplied as demands are being shifted.

Thus when $n \geq k$ the first set of integrations for industry k can be written as,

$$\sum_{m=1}^{M_k} \Delta PS_{k,m} \equiv \sum_{m=1}^{M_k} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k,m} \frac{\partial P_{k,m}}{\partial P_{n,1}} dP_{n,1} = \sum_{m=1}^{M_k} \int_{P_{k,m}(P^0_{n,1})}^{P_{k,m}(P^1_{n,1})} dP_{k,m}, \quad (9)$$

where $\Delta PS_{k,m}$ represents producer surpluses at the k^{th} industry level, for the commodities $m=1, \ldots, M_k$. Furthermore, the remaining set of integrations in (4) when $n \geq k$ measure producer surpluses in industry k-1. Hence,

$$\sum_{m=1}^{M_{k-1}} \Delta PS_{k-1,m} \equiv \sum_{m=1}^{M_{k-1}} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m} \frac{\partial P_{k-1,m}}{\partial P_{n,1}} dP_{n,1}$$

$$= \sum_{m=1}^{M_{k-1}} \int_{P_{k-1,m}(P^0_{n,1})}^{P_{k-1,m}(P^1_{1,n})} Y_{k-1,m} dP_{k-1,m} \quad (10)$$

Substituting (9) and (10) into (4) gives the difference equation,

$$\Delta\pi_k = \sum_{m=1}^{M_k} \Delta PS_{k,m} - \sum_{m=1}^{M_{k-1}} \Delta PS_{k-1,m} \qquad k = 1, \ldots, n. \quad (11)$$

Solving (11) obtains,

$$\sum_{m=1}^{M_n} \Delta PS_{n,m} = \sum_{k=1}^{n} \Delta\pi_k + \sum_{m=1}^{M_0} \Delta PS_{0,m} \quad (12)$$

where $\Delta PS_{0,m}$ represents the change in the initial resource suppliers surpluses of the $m=1, \ldots, M_0$ initial factors. Thus, summing the changes along equilibrium quantities in industry n associated with a price change $P_{n,1}$ measures the sum of the initial resource supplier surpluses plus all industry rents involved in transforming the initial resources into their present form at industry n.

Summing the consumer and producer surpluses measures (7) and (12) at the n^{th} industry level obtains,

$$\sum_{m=1}^{M_n} \Delta CS_{n.m} + \sum_{m=1}^{M_n} \Delta PS_{n,m} = \sum_{m=1}^{M_0} \Delta CS_{K,m} \sum_{m=1}^{M_0} \Delta PS_{0,m} + \sum_{k=1}^{K} \Delta\pi_k . \qquad (13)$$

From (13) it is tempting to argue that for a price distortion all one has to do is sum the producer and consumer surpluses at the n^{th} altered level to obtain the total welfare effect. However, a closer examination of (13) reveals that the producer and consumer surpluses of the commodities $Y_{n,2}$, . . ., Y_n, M_n are measured along the same path of integration. This occurs because both general equilibrium supplies and demands shift for the commodities $Y_{n,m}$ (m=1, . . ., M_n). Furthermore, since $\Delta CS_{n,m}$ (m=2, . . ., M_n) are defined as the negative of the area along the equilibrium path and $\Delta PS_{n,m}$ (m=2, . . ., M_n) are defined as the positive of the area along the identical path then

$$\sum_{m=2}^{M_n} \Delta CS_{n,m} + \sum_{m=2}^{M_n} \Delta PS_{n,m} = 0.$$

Hence (13) reduces to,

$$\Delta CS_{n,1} + \Delta PS_{n,1} = \sum_{m=1}^{M_k} \Delta CS_{K,m} + \Delta PS_{0,m} + \sum_{k=1}^{K} \Delta\pi_k \qquad (14)$$

This can be more clearly seen by examining the n+1 and n industry. When k=n+1 the industry indirect profit function is given by,

$$\pi_{n+1} = \sum_{m=1}^{M_{n+1}} P_{n+1,m} Y_{n+1,m} - \sum_{m=1}^{M_n} P_{n,m} Y_{n,m} \qquad (15)$$

Altering (15) for a price change of $P_{n,1}$ one can obtain from the envelope theorem,

$$\frac{\partial \pi_{n+1}}{\partial P_{n,1}} = \sum_{m=1}^{M_{n+1}} Y_{n+1,m} \frac{\partial P_{n+1,m}}{\partial P_{n,1}} - Y_{n,1} - \sum_{m=2}^{M_n} Y_{n,m} \frac{\partial P_{n,m}}{\partial P_{n,1}} \qquad (16)$$

Integrating (16) along equilibrium quantities for a specific price change from $P^0_{n,1}$ to $P^1_{n,1}$ yields,

$$\Delta\pi_{n+1} = \sum_{m=1}^{M_{n+1}} \int_{P_{n+1,m}(P^0_{n,1})}^{P_{n+1,m}(P^1_{n,1})} Y_{n+1,m}\,dP_{n+1,m} - \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{n,1}\,dP_{n,1} - \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m}\,dP_{n,m} \tag{18}$$

Now for the directly affected industry n the indirect profit function is given by,

$$\pi_n = \sum_{m=1}^{M_n} P_{n,m} Y_{n,m} - \sum_{m=1}^{M_{n-1}} P_{n-1,m} Y_{n-1,m} \tag{19}$$

Forcibly altering $P_{n,1}$ one can also obtain from the envelope theorem,

$$\frac{\partial \pi_n}{\partial P_{n,1}} = Y_{n,1} + \sum_{m=2}^{M_n} Y_{n,m} \frac{\partial P_{n,m}}{\partial P_{n,1}} - \sum_{m=1}^{M_{n-1}} Y_{n-1,m} \frac{\partial P_{n-1,m}}{\partial P_{n-1}} \tag{20}$$

Integrating (20) along equilibrium quantities for a price change from $P^0_{n,1}$ to $P^1_{n,1}$ one finds

$$\Delta\pi_n = \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{n,1}\,dP_{n,1} + \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m}\,dP_{n,m} - \sum_{m=1}^{M_{n-1}} \int_{P_{n-1,m}(P^0_{n,1})}^{P_{n-1,m}(P^1_{n,1})} Y_{n-1,m}\,dP_{n-1,m} \tag{21}$$

Now, since n < n + 1, the last set of integrations in (18) are defined as consumer surpluses. Furthermore, at the altered industry n the second set of integrations in (21) are defined as producer surpluses. However, since the path of integration is along equilibrium quantities the paths of integration are the same for the last set of terms in (18) and the second set of terms in (21). Hence, from (18),

$$\sum_{m=2}^{M_n} \Delta CS_{n,m} \equiv - \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m}\,dP_{n,m} \tag{22}$$

and from (21)

$$\sum_{m=2}^{M_n} \Delta PS_{n,m} \equiv \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m} dP_{n,m} \qquad (23)$$

Adding (22) and (23) one finds

$$\sum_{m=2}^{M_n} \Delta PS_{n,m} + \Delta CS_{n,m} = \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m} dP_{n,m} - \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m} dP_{n,m} = 0 \qquad (24)$$

Hence (13) reduces to (14). Thus, where industry 0 is an initial resource industry and industry k a final consumption industry, the sum of producer and consumer surplus of the altered commodity $Y_{n,1}$ measures the change in total sector welfare. Notice that this result is a generalization of the Just and Hueth result (single-product and single-factor industries) to include multi-product and multi-factor industries. In both cases the relevant total welfare measure of a price distortion is to sum the producer and consumer surplus of the distorted commodity.

Empirical Implications

The results in previous sections imply that when all welfare measures are taken along general equilibrium functions (i.e., all quantities and prices in the sector are allowed to monotonically adjust) equation (14) provides a convenient way to evaluate the total change in welfare. For example, consider a large scale econometric model giving a representation of an economy (or of a sector if this sector is facing fixed prices from other sectors of the economy). If the general equilibrium supply and demand curves in the distorted industry are linear[3], then the producer surplus calculations, for the altered industry for a price change from $P^0_{n,1}$ to $P^1_{n,1}$

$$\Delta PS_{n,1} = \tfrac{1}{2}[P^1_{n,1} - P^0_{n,1}][Y^s_{n,1}(P^0_{n,1}) + Y^s_{n,1}(P^1_{n,1})]. \tag{25}$$

Similarly, the consumer surplus calculations can be represented by,

$$\Delta CS_{n,1} = -\tfrac{1}{2}[P^1_{n,1} - P^0_{n,1}][Y^d_{n,1}\ (P^0_{n,1}) + Y^d_{n,1}\ (P^1_{n,1})]. \tag{26}$$

Then, by summing (25) and (26) one can obtain the change in total welfare for the sector. Thus, the only information required to evaluate the change in welfare in the sector is the set of general equilibrium prices and quantities in the distorted market before and after the price policy change. These results can usually be estimated fairly easily from econometric models or linear programming. In this context, there is no need to have measurement in other industries of the economy as long as the objective of the researcher is to evaluate the total change in welfare of the sector. Furthermore, these results appear to have important implications for empirical welfare analysis since they provide a simple and practical approach to studying welfare in an economy comprised of horizontally and vertically related markets.

If one is interested in the distribution of the welfare change, then there is a need to disaggregate the total welfare effect into impacts on individual industries. In a general equilibrium framework, this amounts to subtracting consumer surpluses using equation (7) or producer surpluses using equation (11). In a partial equilibrium framework, ordinary supply and demand curves can be used, following Just and Hueth for single-product single-factor industries.

Notice also, that the supply equations $Y^s_{k,m}$ in (25) and the demand equations $Y^d_{k,m}$ in (26) do not necessarily have to be general equilibrium. That is, since one is only interested in the initial and final vectors of prices and quantities these can be found from partial equilibrium supply and demand functions or any alternative specifications.[4]

Conclusion

This paper has investigated welfare measures in an economy comprised of vertically related multi-product and multi-factor industries, where a particular market is subject to a price distortion. Generalizing Just and Hueth's results, it is shown that total welfare changes can be obtained in the distorted industry, where all measurements are made in a general equilibrium framework. This provides a practical way of evaluating the welfare impact of a price distortion. Given the extent of multi-product firms and vertical market chains, and the concern about future governmental price intervention, these results appear to have empirical applicability in a wide variety of cases.

FOOTNOTES

[1]For notational convenience the i and (P) will be dropped.

[2]The industry price is assumed to be altered by governmental action such that the new price $P^1_{n,1}$ is an effective price floor. One possible technique which the government could employ would be to purchase the excess supply.

[3]If the general equilibrium functions are non-linear, then equations (25) and (26) provide only an approximated welfare measure. These approximated welfare measures will differ from the true welfare measures by the area difference between the non-linear function and a linear line between the initial and final prices and quantity.

[4]The exclusion of supplies and demands from being general equilibrium does not prevent the researcher from solving the set of equations for general equilibrium prices and quantities.

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