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Agriculture in a Dynamic Cross-Sector Model. Carlos Arnade, Commercial Agriculture Division, Economic Research Service, U.S. Department of Agriculture, Staff Paper No. AGES 9618.

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## **Agriculture in a Dynamic Cross-Sector Model**

**Carlos Arnade**

**Keywords:** Cross-sector, Agriculture, Manufacturing, Services, Dynamics, Productivity

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### Abstract

The assumption of adjustment costs is used to specify a dynamic model of the U.S. economy. Output is divided into in three sectors: agriculture, manufacturing, and services. The advantage of this approach is that it can measure more factors that contribute to productivity growth than a static model. Output growth and productivity are measured using data and parameters from an estimated dynamic model. Parameters that are unique to dynamic models and a returns-to-scale measure make up part of the productivity calculations. Dynamic components of productivity are less than 5 percent of productivity growth for most years. This occurs because dynamic components of productivity are a function of returns to scale, and production is measured to be close to constant returns to scale. Elasticities from the estimated model show that both manufacturing and service prices have a major impact on agricultural output. Own-price elasticities are relatively small for agriculture.

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## Summary

Numerous studies that have measured productivity growth in the U.S. economy have found a productivity slowdown in the early 1970's. These studies often are based on static models that assume production functions are constant returns to scale. It has been shown that such assumptions do not take into account all the factors that can contribute to productivity growth. It also is evident that sector own-price and cross-price supply elasticities may be biased if a static, rather than dynamic, model is used to compute the elasticities.

A dynamic adjustment cost model is used in this paper to specify a model for the three major sectors of the U.S. economy: agriculture, manufacturing, and services. By assuming there are costs to adjusting their level of capital, producers make dynamic decisions in a forward-looking manner. In choosing a level of investment, producers influence the level of capital across a range of time periods. Producers, therefore, choose investment and the level of output of each good by maximizing profits across time.

The solution to the dynamic profit-maximizing decision is a value function. This function can be used to specify output supply and investment equations. A value function is specified and supply equations for each sector are derived and jointly estimated alongside a capital investment equation. Parameters obtained from the system of equations are used to measure the interaction between these three sectors of the U.S. economy. Own-price and cross-price elasticities are calculated. Also calculated are the total impact of price changes on each sector's output. Both manufacturing and services have a significant influence on aggregate agricultural output. Agriculture in turn has less of an impact on manufacturing and on service output. Own-price changes have a small effect on agricultural output. Generally, factors outside of agriculture appear to have had a greater effect on aggregate agricultural output than factors within the agricultural sector.

Parameter estimates from the supply and investment equations also can be used to obtain aggregate productivity estimates. Productivity changes arise from changes in inputs and outputs. When a dynamic model is considered, there are five possible sources of productivity growth: growth arising from technical change, growth coming from changes in variable inputs, growth coming from changes in fixed inputs, growth coming from changes in the marginal values of stocks of fixed factors, growth stemming from net physical investment. If production is constant returns to scale, technical change is the sole source of productivity growth.

Calculation of U.S. productivity changes show that productivity growth slowed significantly in the early 1970's. Productivity growth was 2.3 percent annually in the 1950's, 2.45 percent annually in the 1960's, and 1.56 percent in the 1970's. This slowdown is consistent with most models of U.S. productivity growth. However, this model shows productivity growth slowing down even more in the mid-1980's, dropping

from 1.52 percent in the early 1980's to 1.2 percent in the late 1980's. Dynamic factors contributed 5.8 percent of the productivity growth rate in the 1930's and 2.5 percent of the productivity growth rate in the 1980's. However, with continuous compounding, even this small contribution to productivity growth could become significant over time.



# Agriculture in a Cross-Sector Dynamic Model

Carlos A. Arnade<sup>1</sup>

## Introduction

The past 40 years has witnessed significant growth in U.S. agricultural productivity and output, accompanied by growth in agricultural capital, and a decline in agricultural labor. During this period, there have been many analyses of the response of agricultural production to agricultural prices both at the crop-specific level and the aggregate level (Ball, Griliches, Tweeten and Quance). However, there has been only limited analysis of the interaction between agriculture and other sectors of the economy outside of using fixed proportion production technology reflected in input-output tables. Examples of intersectoral linkages are provided in Arrow et al., Jorgerson et al., and a recent article by Gopinath and Roe.

In the study by Gopinath and Roe, nonparametric indices and econometric estimation measure the effect of output prices, productivity, and input growth on U.S. GDP and on sector-specific output. They find total factor productivity and input growth have had a major effect on the growth of outputs and GDP. Typically, productivity growth represents total output growth minus the effect input growth has on output growth. Economists in general believe that U.S. productivity growth in the past 20 years has been a critical factor in GDP growth (Jorgerson). Some economists have argued that, in contrast to the United States, input growth rather than productivity growth has been the cause of recent East Asian GDP growth (Krugman). This remains a point of contention, however, and others stress the importance of productivity growth for East Asian nations also (World Bank).

Gopinath and Roe also investigate cross-price effects between sectors. They find that agriculture has high cross-price elasticities with the manufacturing and service sectors. Both the econometric and nonparametric methods for measuring productivity and measuring cross-sector relationships assume that there exists a revenue function. A revenue function serves as a base for analysis since it has been shown that the GNP is equivalent to maximizing revenues at one point in time across all sectors of the economy. Dewiert and Morrison have shown that productivity can be measured as the ratio of two revenue functions, each using a different period's technology, but evaluated at the same prices and input levels. Thus, indices based on revenue functions provide the basis for productivity research.<sup>2</sup>

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<sup>1</sup>The author is an economist with the Trade Analysis Branch, Commercial Agriculture Division, Economic Research Service, USDA.

<sup>2</sup>Gopinath and Roe assume total revenues equal total costs so they implicitly assume production functions have constant returns to scale.



Given the importance of productivity and intersectoral relationships, it is worthwhile to build upon the Gopinath and Roe paper by altering their assumptions. I relax two assumptions: static maximization and longrun constant returns to scale. Allowing for longrun nonconstant returns and dynamic decision-making should indicate how robust Gopinath and Roe's results are across a different set of assumptions and provide insights into the twin issues of intersectoral relationships and productivity.

In this paper, I assume that there are adjustment costs associated with capital investment. That is, when producers change the level of capital stocks, they incur a cost beyond the payments to capital. When there are adjustment costs, the timing of capital payments affects output, so producers must choose optimal levels of inputs across time. Thus, producers can be depicted as dynamic profit maximizers. Taking into account adjustment costs and dynamic decision-making alters the method for measuring productivity and can lead to different estimates of productivity.<sup>3</sup>

## Background

Many of the economic problems in the U.S. economy in the 1970's and 1980's were either attributed to or reflected in a decline in U.S. productivity growth that economists have observed. This decline was measured as beginning in the early 1970's and continuing until the early 1990's. In contrast to the overall economy, U.S. agriculture has enjoyed relatively high productivity growth. However the effects of high agricultural productivity need not always be beneficial to the sector. Gardner points out that high agricultural productivity growth coupled with a low income demand elasticity for food could depress farm prices and farm income. Such a situation is made even worse if the rate of income growth, at the national level, is falling due to declines in economy-wide productivity growth. This argument suggests that intersectoral relationships have major influences on the agricultural economy and that these relationships are influenced by productivity.

Gopinath and Roe's article empirically addresses both productivity and cross-sector relationships. They divide the economy into three sectors: agriculture, manufacturing, and services. They first calculate productivity using nonparametric techniques. Then they use an econometric model to investigate the impact technical change has on output and factor rental rates. This is important because technical change is a major component of productivity growth. In both the nonparametric and econometric analysis they calculate the impact relative prices have on output growth. Their primary aim is to provide a growth accounting framework for the U.S. economy. They decompose GDP growth into three components: growth arising from productivity gains, growth arising from greater input use, and growth arising from intersectoral price effects. An important additional finding is that cross-sector price relationships are important. Though

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<sup>3</sup>The adjustment cost model is only one way of allowing for a dynamic model. Modeling a learning-by-doing process can also lead to a dynamic optimization problem. Later, we test the commerce data base for the adjustment costs to determine if this is a valid assumption.

aggregating the various outputs to such a level eliminates the ability to analyze product-specific responses to price changes, such analysis provides measures of cross-sector influences that are typically ignored in more detailed models of the agricultural economy.

Luh and Stefanou (L&S) describe a procedure for measuring productivity in a dynamic adjustment cost model. This paper applies the L&S procedure to measure aggregate productivity in the United States using the same data that Gopinath and Roe employ in their productivity measurement. This is done by estimating a dynamic econometric model and using model parameters to calculate sources of growth in specific sectors of the economy.

In the following section, the adjustment cost model is presented. Following that, data and model specification are discussed. An empirical section presents elasticities and productivity estimates. A section is devoted to presenting the sources of growth of agricultural output and providing a comparison with other sectors in the economy. Details of the methodology are discussed in the appendix.

## Adjustment Cost Models

The adjustment cost assumption provides a basis for a great many dynamic models in the economic literature (Kamien and Schwartz, Epstein, Howard and Shumway, Stefanou, Vasavada and Chambers). A multi-output adjustment cost model assumes the existence of a transformation function of the form:

$$F(Y, X, I, K, t) = 0 \quad (1)$$

where  $Y$  is a vector of outputs,  $X$  a vector of variable inputs,  $K$  is a vector of quasi-fixed inputs,  $I$  is a vector representing new investment in the quasi-fixed input,  $t$  is time. The function  $F(\cdot)$  is named a transformation function because inputs,  $X$  and  $K$  are transformed into outputs.  $F$  is a continuous, twice differentiable function that is strictly increasing in  $Y$ , decreasing in  $X$  and  $K$ , and strictly increasing in  $I$ .<sup>4</sup>

The assumption that  $F(\cdot)$  is strictly increasing in  $I$  is itself based on the assumption of adjustment costs. As new investment is transformed into capital stock, it absorbs resources typically devoted to producing output. Therefore, in the short run, capital investment has the opposite influence on the function  $F(\cdot)$  as does the level of capital,  $K$ . If there were one output and the equation were explicitly solved for that output, then production would appear as decreasing in  $I$ . Investment activity diverts resources rendered toward production and temporarily decreases output. For example, production lines are often temporarily shut down when new equipment is installed.

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<sup>4</sup> $F$  also is convex in  $Y$ ,  $X$ ,  $K$  and  $I$ .

Profit-maximizing firms that have adjustment costs make dynamic decisions in a forward-looking manner. In choosing a level of investment, profit-maximizing producers influence output across a range of time periods. The following outline compares static optimization with dynamic optimization:

**Table 1 Comparison of Static and Dynamic Decisions**

Issue	Static	Dynamic
Output price	Firms instantly adjust capital in time $t$ based on prices $P_t$	Firms invest in time $t$ based on expected price $E(P_t) \dots E(P_n)$
Investment	Investment is a costless transaction	Investment cost is forgone output
Output response to capital stock	$\delta Y_t / \delta K_t > 0$	$\delta Y_t / \delta K_t > 0$
Profit decisions	Profit is maximized in $t$	Profit is maximized across time
Capital levels	Capital is always at its desired level	Capital slowly adjusts to its desired level implying there is disequilibrium

Luh and Stefanou show that under the assumptions of an adjustment cost model, a measure of multiple output growth, represented by  $\hat{Y}$ , is:

$$\hat{Y} = \hat{A} + (TSC/TR) (\hat{F}_v + \hat{F}_{q1} + \hat{F}_{q2} + \hat{F}_{ss}) \quad (2)$$

where  $\hat{\cdot}$  denotes rate of growth

$\hat{A}$  represents the shift in output resulting from technical change or from gains in production efficiency. TSC is total longrun shadow costs and consists of variable costs plus additional dynamic terms and TR is total revenues.<sup>5</sup> The TSC/TR ratio represents the longrun returns to

<sup>5</sup>Total longrun shadow costs (TSC) consist of shortrun variable costs plus additional terms which reflect the cost of adjusting the quasi-fixed factor. TSC is formally defined in the appendix. For the moment, readers should consider TSC/TR as only a measure of the degree of scale in production.

scale. Constant returns to scale occurs where revenue TR equals costs (TSC). The term  $\hat{F}_v$  represents the proportional rate of growth in the use of variable inputs,  $\hat{F}_{q1}$  represents the proportional rate of growth in net physical investment,  $\hat{F}_{q2}$  represents the proportional rate of change in the marginal values of stocks of capital.  $\hat{F}_{ss}$  represents the proportional rate of growth in capital at longrun equilibrium levels. In equilibrium, net investment is zero and the marginal value of capital stocks is constant. Therefore the two middle terms in parentheses ( $\hat{F}_{q1}$  and  $\hat{F}_{q2}$ ) reflect the disequilibrium inherent in adjustment cost models.

In a static model with constant returns to scale, the above equation collapses to:

$$\hat{Y} = \hat{F}_v + \hat{F}_{ss} + \hat{A} \quad (3)$$

In equation 3, growth arises only from factor growth and technical change. These are the two primary sources of output growth in most models. The differences between equations 2 and 3 show that dynamics impose additional terms to measurement of output growth and thus productivity growth.

From equation 2, total factor productivity growth ( $T\hat{F}P$ ) can be calculated as:

$$T\hat{F}P = ((TSC/TR) - 1) (F_v + \hat{F}_{q1} + \hat{F}_{q2} + \hat{F}_{ss}) + \hat{A} \quad (4)$$

With longrun constant returns to scale, TSC equals TR, and productivity growth collapses to  $\hat{A}$ , which represents growth arising from technical change.<sup>6</sup> Luh and Stefanou provide formulas for calculating each term in equation 4.<sup>7</sup> These formulas are provided in the appendix. The significant point to be made here is that estimation of supply and investment equations can be used to obtain parameters that, when combined with data, can be used to estimate productivity growth.

The following section presents parameters of supply and investment equations estimated from data obtained from the Department of Commerce. Also presented are elasticities, a discussion of

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<sup>6</sup>The measure of returns to scale in the long run is not the same as the measure of shortrun returns to scale. In this paper, shortrun constant returns to scale are assumed so that annual revenues equal shortrun costs. However the degree of scale in the long run is estimated.

<sup>7</sup>These terms are derived from first-order conditions of a dynamic profit maximization problem. In these formulas, some terms represent derivatives of a value function, which represents the solution to a dynamic maximization problem. If dynamic profit maximization is assumed, a value function can be specified. Later we show that the parameters of the value function in turn can be obtained from estimated supply, demand, and investment equations. The appendix also presents the dynamic optimization problem and the well-established relationship between the derivatives of a dynamic-dual value function and supply and investment equations.

sources of growth of each output, and estimates of aggregate productivity growth.

## Data and Specification

The commerce data for estimating the GDP has three outputs and two inputs. In the following model we represent labor as a variable input and capital as a quasi-fixed factor. Details of the data are described in Gopinath and Roe.

Epstein shows that the solution to a dynamic optimization problem is a function of prices and levels of the quasi-fixed inputs. In this paper capital serves as a quasi-fixed input. Epstein describes the properties of this function, which he calls the value function. He shows that relationships exist between a value function, output supply equations, input demand functions, and investment equations. In brief, once a value function is specified, it can be used to derive output supply and investment demand equations that are consistent with the theory of dynamic optimization. This is discussed in the appendix.

I also have chosen the convention of specifying a value function. Below, a value function is specified as a normalized quadratic function. The function has linear terms in normalized prices (for example  $P_i$ ), interaction terms in normalized prices ( $P_i P_j$ ) and quadratic terms in normalized prices ( $P_i^2$ ). It also includes interaction terms between prices and the level of capital ( $P_i K_1$ ) and prices and technology ( $P_i T$ ). This function is written as:

$$\begin{aligned}
 J(P, c, K, T) = & a_0 + \sum_{i=1}^3 a_i * P_i + a_4 * c + 1/2 * \left( \sum_{i=1}^3 \sum_{j=i}^3 * a_{ij} * P_i * P_j \right. \\
 & \left. + \sum_{i=1}^3 * a_{i4} * P_i * c + \sum_{j=1}^3 a_{4j} * c * P_j + a_{44} * c^2 \right) \\
 & + \sum_{i=1}^3 b_i * K_1 * P_i + b_4 * K_1 * c + \sum_{i=1}^3 d_i * T * P_i + d_4 * c * T
 \end{aligned} \tag{5}$$

where:  $P_1$ =the price of services,  $P_2$ =the price of agricultural goods,  $P_3$ =the price of industrial goods,  $c$ =the price of capital goods.  $K_1$  is the level of capital and  $T$  is a time variable that is meant to represent technical change. All prices in the value function were normalized on the price of labor, hence the name normalized quadratic.

Using well-established relationships between the value function and supply and investment equations (see appendix), three output supply equations and one investment equation were derived from equation 5. The estimated equation representing the supply of services is:

$$\begin{aligned}
 Y_1 = & d_1 + r \left( a_1 + \sum_{j=1}^3 a_{1j} * P_j + a_{14} * c + b_1 * K_1 \right) \\
 & - b_1 * (I_1 - \alpha K_1) + d_1 * T
 \end{aligned} \tag{6}$$

where  $I_1$  is investment in capital  $K_1$  and  $\alpha$  represents the rate of depreciation. A similar equation can be specified for the agricultural and manufacturing output.

Taking the appropriate derivatives of the value function (see appendix), the equation representing changes in capital ( $K_1$ ) is:

$$\dot{K} = \frac{d_4 + K_1 + (r(a_4 + \sum_{j=1}^3 a_{4j} * P_j + a_{44} * c + b_4 * K_1 + d_4 * T))}{b_4} \quad (7)$$

Although supply and variable input demand equations are linear in parameters, the capital investment equation is not. However, the investment equation can be rearranged and estimated in linear form. For example, rewrite equation 7 as:

$$\dot{K} - rK_1 = g_4 + \theta_4 * K_1 + \gamma_4 * r + \sum_{j=1}^3 \gamma_{4j} * P_j * r + \gamma_{44} * c * r + g_4 * T$$

where

$$\begin{aligned} \gamma_4 &= a_4 / b_4 \\ \gamma_{4j} &= a_{4j} / b_4 \\ \theta_4 &= 1 / b_4 \\ g_4 &= d_4 / b_4 \end{aligned} \quad (8)$$

Equations 6 and 8 form a system of equations that can be estimated using linear estimation techniques. Simplifying the investment equation to ensure that it is linear in the parameters, as in equation 8, can save considerable effort when it comes to estimation. When a system of equations is nonlinear in parameters, iterative search routines are used to obtain parameter estimates. Often in nonlinear estimation, final parameter estimates are sensitive to an analyst's estimate of starting values, sensitive to the algorithm chosen, sensitive to the step size used over a grid search, and sensitive to the convergence criteria. Often, parameter estimates from nonlinear models are not robust. It was assumed that the cost of simplifying the model is justified to avoid nonlinear estimation techniques.

### Empirical Evidence

Parameters of the system of three supply equations and one investment equation were estimated by Seemingly Unrelated Regression (SUR) using data from 1930 to 1992. Parameter values and t statistics are provided in table 2. Reduced-form estimates are provided for the investment equation. Following conventional practice (Vasavada and Chambers, Howard and Shumway) changes in capital were represented by first differences of annual data. Right-hand side values of  $K$  were represented by capital lagged one period.

The estimation technique (SUR) is based on the assumption that right-hand side variables are exogenous. Investment decisions are made prior to output decisions so that right-hand side investment terms are exogenous. Thus the system is recursive. Furthermore, the equations represent the sum of price-taking producers so right-hand side prices are exogenous. If the data represented the sum of colluding producers, then prices would be endogenous.

A similarly specified system of equations possibly could be devised from a model where aggregate quantities influenced aggregate prices. In using SUR to estimate the model, I assume this is not the case so that, in effect, quantity decisions among numerous producers are not determined in concert. In industries where there is collusion or producers influence each other's price, I assume the sum of U.S. producers take world market prices as a given.

### Elasticities

Table 2 presents elasticities calculated from the means of the data. These elasticities represent the current period percentage change in the dependent variable from a change in the independent variable. Thus table 3 elasticities are analogous to impact elasticities. Of the three outputs, agricultural output has the lowest own-price elasticity (0.04) but has significantly high cross-price elasticities. Agricultural output is a substitute for manufacturing output with a cross-price elasticity of -0.476 but is a complement of service output with a cross-price elasticity of 0.604. The influence of changes in manufacturing and service prices on agricultural output are also strong in the Gopinath and Roe article. Though the precision of the estimates could be debated, it is clear from the results here and the Gopinath and Roe article that prices outside the agricultural sector have an even greater impact on agricultural output than prices within the agricultural sectors.

Services has the highest own-price elasticity, 0.33.<sup>8</sup> Yet even in this sector, the cross-price elasticities tend to be higher than the own-price elasticity. Manufacturing tends to have a low own-price elasticity (0.1) but this is significantly higher than the own-price elasticity in agriculture. Services and manufacturing are complements and have strong cross-price effects. Though manufacturing and agriculture are substitutes, the effect of agricultural prices in elasticity terms (-0.003) is far lower than the effect of manufacturing prices on agriculture in elasticity terms (-0.476).

Notice that all three sectors have a low percentage output response to a change in agricultural prices. Manufacturing may not respond strongly to agriculture because this sector contains few

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<sup>8</sup>Each price in the model was normalized on the price of labor. This does not influence elasticities. Own-price and relative price elasticities are the same. When discussing sources of growth, readers should note that prices are represented relative to the price of labor.



Table 2--Estimated Model 1930-1992

**Service Supply Equation  $Q(21)^1 = 33.92$**

	<i>Variable</i>	<i>Estimate</i>	<i>T statistic</i>
Constant	d1+a1	-1192.46	-5.36
Price Services	a11	8.18	4.28
Price Agriculture	a12	0.63	5.18
Price Manufacture	a13	7.54	5.62
Price Capital	a14	-11875.25	-4.03
Capital Terms	b1	-0.0008	-2.66
Trend	d1	49.69	22.41
WARDUM <sup>2</sup>	v1	457.12	10.36

**Agricultural Supply Equation  $Q(21) = 14.19$**

	<i>Variable</i>	<i>Estimate</i>	<i>T statistic</i>
Constant	d2+a2	10.90	1.10
Price Services	a21	0.63	5.18
Price Agriculture	a22	0.02	1.07
Price Manufacture	a23	-0.37	-4.33
Price Capital	a24	595.88	3.63
Capital Terms	b2	-0.0001	-8.97
TREND	d2	0.559	5.79

**Manufacturing Supply Equation  $Q(21) = 40.10$**

	<i>Variable</i>	<i>Estimate</i>	<i>T statistic</i>
Constant	d3+a3	-656.16	-3.16
Price Services	a31	7.54	5.62
Price Agriculture	a32	-0.37	-4.33
Price Manufacture	a33	2.03	1.67
Price Capital	a34	2452.41	0.92
Capital Terms	b3	0.00058	1.99
Trend	d3	41.92	19.77
WRDUM	v3	288.64	6.74

**Capital Investment Equation  $Q(21) = 27.36$**

	<i>Variable</i>	<i>Estimate</i>	<i>T statistic</i>
Constant	d4/b4+a/b4	219580.3	1.62
Capital	1/b4	-0.0422	-2.56
Price Services	a41/b4	-3452.18	-1.65
Price Agriculture	a42/b4	249.69	1.22
Price Manufacturing	a43/b4	956.64	0.96
Price Capital	a44/b4	-5435210.0	-3.60
Trend	d4/b4	2113.99	0.82
WRDUM	v3/b4	-40536.45	-1.63

<sup>1</sup>Q is a Chi squared statistic derived from an autocorrelation function of the errors of the estimated equations. If Q is less than the number in parenthesis, the error terms are white noise.

<sup>2</sup>WRDUM represents a dummy variable for War years (1941-45). In this period, price controls and production boards overrode private decisions in the service and manufacturing sectors.

Table 3--Elasticities

Quantities	Prices					
	Service price	Agriculture	Manufacturing price	Capital price price	Capital	Investment price
qs <sup>1</sup>	0.332	0.051	0.401	-0.198	0.062	-0.072
qag	0.604	0.040	-0.476	0.248	0.168	-0.207
qmft	0.287	-0.003	0.101	0.038	0.041	-0.047
investment	-11.971	1.707	4.342	-7.749	-8.513	

<sup>1</sup>qs is output of services, qag agricultural output, and qmft is manufacturing output. The quantities are dependent variables. For example the elasticity of agricultural output with respect to a change in service prices is .604.

inputs that can be bid away into agricultural uses. For example, agricultural labor may migrate to manufacturing, but it is less likely that the converse is true. Agriculture's relatively low own-price elasticity may reflect the impact of government programs. A change in agricultural prices, within a certain range, should have no effect on output since programs make up for producer revenues lost to a price that falls below support levels. However these elasticities were estimated over periods where there was no program support, where program support was tied to output, and where program support was not tied to output. Therefore agricultural programs may not be fully responsible for the low elasticities. Manufacturing and services have no such programs.

In the adjustment cost model, the level of capital has a positive effect on output while the flow of capital (investment) has a negative effect. The capital and investment elasticities have the expected sign and are highest in the agricultural sector but tend to be low in general. In dynamic models, the effect of the capital price on output can be any sign although intuitively a capital price that has a negative effect on output is more appealing. However, in equations 2 and 3, the price of capital also has a positive effect on manufacturing and agriculture.<sup>9</sup>

The elasticities from the investment equation are particularly high.<sup>10</sup> Investment decisions are typically more optional for producers than output decisions. Investment elasticities should be expected to be high. The price of manufactured goods has a positive effect on capital investment as expected, but the t-statistic is not significant. The price of services has a negative effect on investment. As expected, investment is particularly sensitive to the price of capital.

<sup>9</sup>The data were tested for unit roots and cointegration. These tests showed the one case of cointegration that occurred between agricultural output and the price of capital.

<sup>10</sup>These elasticities actually represent the changes in first differences of capital with respect to a change in the exogenous variable.

Investment elasticities stemming from a dynamic model with static price expectations are quite different from elasticities derived from a static model. In a static model, investment arising from a change in the price of capital represents investment that otherwise would not have taken place. In a dynamic model, with the possibility of intertemporal substitution of investment, investment in a specific period arising from a change in the price of capital could come at the expense of future investment. Thus, investment elasticities from static and dynamic models measure distinct phenomena and should not be compared. The above elasticities do not represent only additional investment arising from a change in capital prices. Rather they represent both intertemporal substitution of investment and additional investment resulting from a change in capital prices.

### Adjustment Costs and Productivity

The adjustment cost model also is based on the assumption that capital does not instantly adjust. If capital were to instantly adjust to its optimal level, then adjustment costs would be zero. This is equivalent to the parameter  $1/b_4$  being equal to  $-(1+r)$  where  $r$  is the real rate of interest. In this model, real interest rates were set equal to 3 percent, which is close to the mean rate over the whole data set. A chi square statistic rejected instant adjustment at the 99 percent confidence level, thus indicating that the adjustment-cost assumption is valid. The calculated rate of adjustment of capital is equal to  $(1/b_4)+r$  or 1.2 percent a year, indicating sluggish adjustment.

Table 4 presents aggregate productivity estimates derived from model parameters and equation 10. Even when dynamics and nonconstant returns to scale are accounted for, productivity growth is shown to decline significantly in the 1970's. Productivity growth rises to 2.3 percent annually in the 1950's and again rises to almost 2.5 percent in the 1960's. However it again falls significantly in the 1970's. Furthermore, it falls again, from 1.5 percent to 1.2 percent in the late 1980's. This second slowdown in productivity growth is not as well documented among economists as the first.

It is evident that technical change is the major component of aggregate productivity growth. The contributions to productivity that arise from dynamic factors are small but notable. Dynamic effects contribute from almost 6 percent of productivity growth in 1930-40 to between 2 percent and 3 percent of productivity growth in all other decades. These effects are not large and demonstrate that, with the Department of Commerce data base, the twin assumptions of CRS and static decision-making do not seriously distort productivity measurement.

Dynamic decisions models can capture additional components of productivity. The empirical results indicate that these components are small, but at some point in the future they may have a greater influence on productivity. Moreover, over a significant time period, even slight contributions to productivity growth can have a strong cumulative effect. One important conclusion is that applying the L&S procedure is worthwhile if nothing more than to remove

Table 4--Productivity Growth Estimates

Period	Total factor productivity <sup>1</sup>	Technical change	Dynamic contributions to productivity
1930/40	2.074	1.955	0.119
1941/50	2.250	2.237	0.046
1951/60	2.327	2.273	0.054
1961/70	2.457	2.388	0.069
1971/80	1.567	1.511	0.057
1981/86	1.516	1.478	0.037
1987/92	1.192	1.162	0.030

	Technical change	Dynamic
<i>Percent of Productivity</i>		
1930/40	0.942	0.058
1941/50	0.994	0.020
1951/60	0.977	0.023
1961/70	0.972	0.028
1971/80	0.964	0.036
1981/86	0.975	0.025
1987/92	0.975	0.025

<sup>1</sup>For example, productivity of the aggregate U.S. output grew approximately 2.07% from 1930 to 1940. Dynamic factors contributed only 5.8% to that growth rate.

doubt as to the validity of productivity estimates based on the more restrictive assumptions of static decision-making and CRS.

Elasticities measure quantity responses in percentage terms. It is also of interest to employ parameter values to measure output change arising from size changes in exogenous variables. Such an exercise provides a historical review of sources of output growth in each sector. These results can differ greatly from the relative magnitudes of elasticities. For example, agriculture has a low own-price elasticity and a high manufacturing price elasticity. If the magnitude of change in agricultural prices were significantly greater than the magnitude of change in manufacturing prices, then even with a relatively low elasticity, agricultural price changes could have a greater impact on agricultural output than changes in manufacturing prices.

Parameter estimates of each exogenous variable were multiplied by the changes in the exogenous variable itself. When the error term is also included as a source of change, then the sum of explanatory sources of output will sum to the change in the endogenous variable, i.e., output. The results of this exercise are presented in graphical form in figures 1-4. The influence of each factor on output is represented relative to the change in each output. Therefore, in the following figures, the sum of effects equals 100.

The height of each bar in figures 1-4 represents the amount that the endogenous variable would have changed had all other variables in the equation been constant; for example, the service price variable equals 62 in the service equation. This indicates that the change in the quantity of services would have been 62 percent of the observed change in the quantity of services had all other variables in the model not changed at all.<sup>11</sup>

As with the elasticity tables, the following figures demonstrate that cross-sector influences are important. For example, relative increases in manufacturing prices have had a strong negative influence on agricultural output. Not surprisingly, resources devoted to agriculture are bid away to manufacturing when manufacturing prices rise. In contrast, a relative rise in service prices has had a positive impact on agricultural output. One reason for this may be that restaurants and other food services are a significant component of the service price index.

The level of capital and the price of capital have had a low but positive impact on agricultural output. Investment has a negative impact on output as predicted by adjustment cost models. This impact is relatively small. (Keep in mind that investment is a flow and capital levels are a stock, and the two have distinctly different effects on output.) The trend variable explains a large component of growth in agricultural output. However, the effect of trend is significantly smaller in agriculture than trend effects in manufacturing and services.

Economists use trends to serve as a proxy for technical change and efficiency improvements that occur over time. Despite obvious shortfalls in this interpretation of a trend variable, there has been little acceptance of alternative explanations of trend influences. Technical change obviously has had a major impact on agriculture and manufacturing.

Figures 1-4 indicate that trend effects have had the greatest impact on manufacturing output and output of services. This may reflect significant gains in production efficiency as well as technical change. Increases in output due to increases in efficiency often represent a learning process that comes from increased production experience. The service industry may be particularly prone to this effect. However, it is likely that trends also represent the cumulative experience of agents in a sector and thus have a learning-by-doing effect.

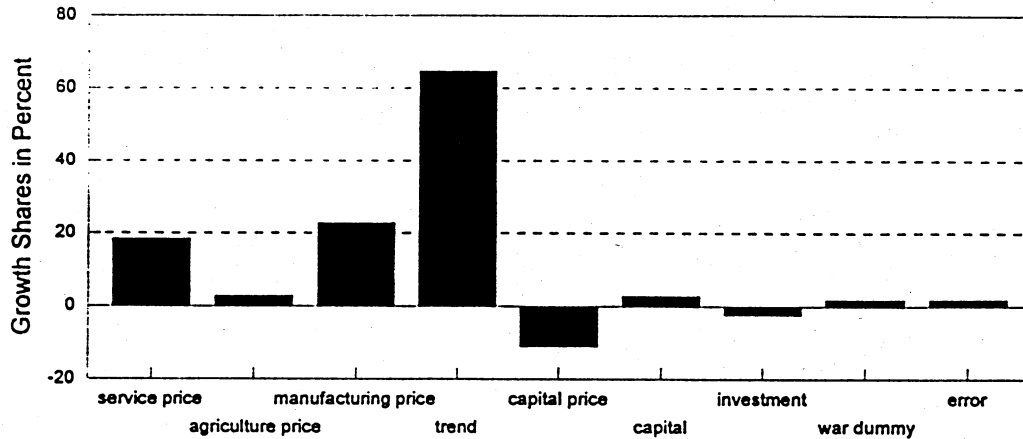
If trend effects on output are influenced by a learning-by-doing effect, then it may be that the trend variable has less influence on agricultural output than it has on output in other sectors. Though the rate of change in technology in agriculture has been rapid, the rate of change in cumulative experience in agricultural production is less rapid than in other sectors. Because of this, it is unlikely agriculture has experienced much gain in efficiency from a learning-by-doing process.

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<sup>11</sup>The positive sum of explanatory influences adds up to more than 1 since some variables have a reducing influence on the endogenous variable. Negative influences are indicated by negative numbers.

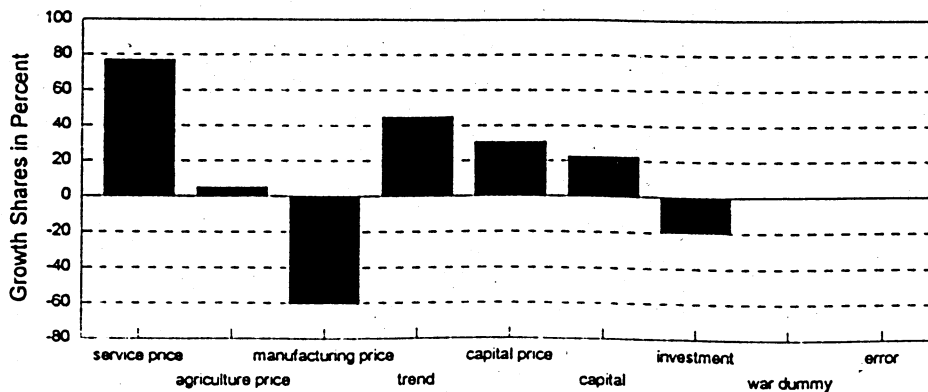
**Figure 1: Prices count but trend is the most important factor for services**

Growth of Services, All Sources  
Average 1930-1992



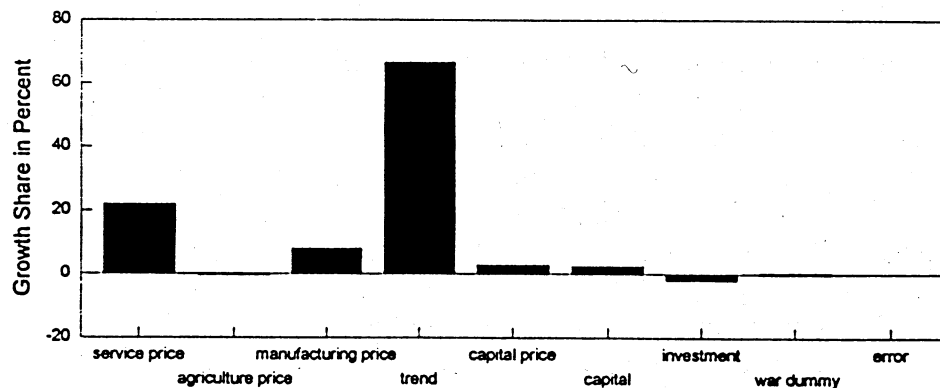
**Figure 2: Prices outside the sector dominate agricultural growth**

Growth of Agriculture, All Sources  
Average 1930-1992



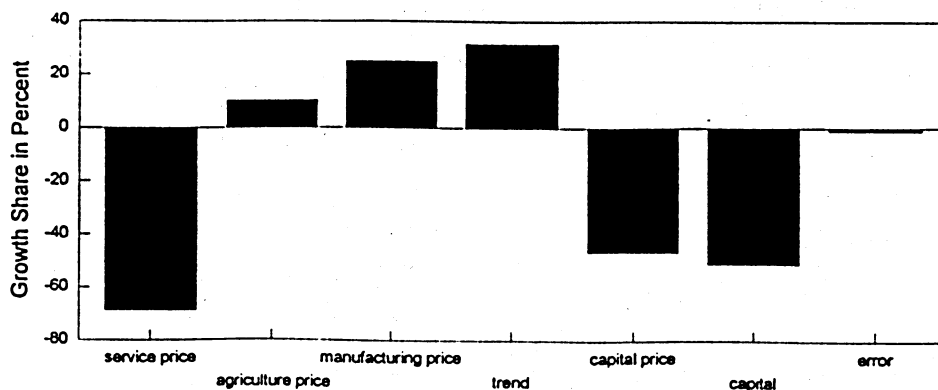
**Figure 3: Trend is dominant growth factor for manufacturing**

**Growth of Manufacturing, All Sources**  
Average 1930-1992



**Figure 4: Prices determine investment**

**Growth of Investment, All Sources**  
Average 1930-1992





## Conclusion

This paper presents the results obtained from estimating a dynamic adjustment cost model of the U.S. economy. The effect of prices, technology, investment, and levels of capital on service output, agricultural output, and manufacturing output are explored. This paper provides a brief overview of Luh and Stefanou's method for measuring productivity in a dynamic model. In this method, a dynamic dual value function is used to specify supply and investment equations. Estimation methods consistent with such a technique are applied. The Luh and Stefanou model is applied to a multisector (multioutput) framework to account for cross-industry effects.

The results show that the price of agricultural goods has only a minor influence on agricultural supply. The aggregate nature of our measure of agriculture is one reason for this low response. Second, the model in this paper does not include specific agricultural capital. A model that captures the movement of capital between sectors may exhibit a similar shortrun elasticity but may also produce higher longrun elasticity. However, service prices have a significant and positive influence on agricultural supply, and manufacturing prices have a significant and negative effect on agricultural supply. Similarly, cross-sector effects are seen in the manufacturing and service supply equations. The influence of capital, investment, and capital prices on output is significant but not nearly as large as the effect of the trend variable.

Tests of the investment equation show that it is possible to reject the assumption that there is no cost in adjusting the level of capital. Therefore, the adjustment cost model can be seen as a valid depiction of aggregate decision-making. Output and capital prices have a considerable influence on capital investment. Rises in service price and capital price slightly reduce investment, while rises in agricultural and manufacturing prices slightly increase investment.

Though testing shows that the adjustment cost assumption is valid, application of Luh and Stefanou's procedure shows that dynamics (and allowance for nonconstant returns to scale) has a small effect on aggregate productivity estimates. Therefore, with the commerce database, the joint assumption of CRS and static decision-making does not seriously distort productivity estimates.

The results in the paper, from the elasticities to the productivity estimates, are conditional on the data used and the specification chosen for the value function. The issue of whether these results are robust across a different set of data or different specification is beyond the scope of the current paper. However it should be apparent that this paper's results are not meant as a final statement on cross-sector elasticities and productivity but only part of an ongoing exploration of these issues.

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## Appendix

### Part A

If producers are depicted as facing adjustment costs, investment has an effect on output. Profit-maximizing decisions that involve investment become dynamic, and investment in one year influences production and profits for extended periods. Profit-maximizing investors can be represented as solving:

$$\begin{aligned}
 & \text{Max}_{Y, I, X} \int_0^{\infty} e^{-rt} [P^T Y - W^T X - C^T K] dt \\
 & \quad \text{st} \\
 & \quad \dot{K} = I - \delta K \\
 & \quad F(Y, X, I, K, T) = 0
 \end{aligned} \tag{9}$$

where  $r$  is the discount rate.  $W$  represents a vector of input prices,  $C$  represents the rental price of the quasi-fixed inputs,  $\delta$  represents a matrix of depreciation rates of the quasi-fixed input.  $P$  represents a vector of output prices. The superscript  $T$  refers to the transpose of a vector. Our model uses Gopinath and Roe's data, which have two inputs: labor and capital. Since there is only one variable input, labor, and one quasi-fixed factor, capital, the terms  $W$ ,  $X$ ,  $C$ ,  $K$ , and  $I$  reduce to scalars.

Numerous textbooks (for example, Kamien and Schwartz) show that a dynamic choice problem can be converted into a static problem to be solved at point  $t$ . The static representation of the problem in equation 10 is called the Hamilton-Jacobi-Bellman (HJB) equation:<sup>12</sup>

$$J(P, W, C, K, T) = \text{Max}_{Y, X, I} [ (PY - WX - CK) ] + J_k * (I - \delta K) +. \tag{10}$$

Empirical applications of dynamic models increased after Epstein showed duality could be applied to dynamics. Epstein's key insight was that once the dynamic model has been converted into its static equivalent, the HJB equation (as in equation 10), standard duality methods can be used to develop an indirect objective function with well-defined properties. The value function  $J(P, W, K, c, T)$  represents the indirect objective function for the problem in equation 10. Its

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<sup>12</sup>The HJB equation is derived by taking the derivative of equation 9 with respect to time, using Leibnitz's rule to eliminate terms to get equation 10 (see Kamien and Schwartz).

properties are described in Epstein. The dynamic dual approach has been applied to agricultural issues by Vasavada and Chambers (V&C), Howard and Shumway (H&S), Luh and Stefanou, Stefanou, and others. This approach has been criticized for imposing unnecessary restrictions on dynamic models (Freisen). Critics point out that the adjustment costs assumption is ad hoc. However, V&C provide parametric restrictions that test for adjustment costs, thus allowing the data to determine if the assumption is true.

Investment equations can be derived from equation 10 in the same fashion as Epstein, H&S, and C&V. Take the derivative of J with respect to c and use the first-order conditions to eliminate indirect terms (the envelope theorem) to obtain:

$$\dot{K} = J_{ck}^{-1} (rJ_c + K - J_{tc}) \quad (11)$$

Typically, inversion of a  $J_{ck}$  matrix is required to estimate the capital stock equation. With only one factor assumed to impose adjustment costs,  $J_{ck}$  reduces to a scalar which is easily inverted.

Envelope properties are used to obtain supply functions and variable input demands, which are represented as (see H&S, V&C, Epstein):

$$\begin{aligned} Y(0) &= rJ_p - J_{kp} * (I - \delta * K_0) - J_{tp} \\ &\quad \text{a n d} \\ X(0) &= -rJ_w + J_{kw} * (I - \delta * K_0) + J_{tw} \end{aligned} \quad (12)$$

In the text, we represent the value function ( $J(P,c,K,T)$ ) as a normalized quadratic function. The resulting supply and demand functions that were estimated utilize the relationship between the value function and the above derivatives.

## Part B

Equation 4 divides productivity growth into five components: technical change  $\hat{A}$ , the proportional rate of growth stemming from variable input use  $\hat{F}_v$ , the proportional rate of output growth stemming from growth in net physical investment  $\hat{F}_{q1}$ , the proportional rate of output growth arising from changes in the marginal values of stocks in the quasi-fixed factor  $\hat{F}_{q2}$ , and the proportional rate of growth in quasi-fixed factors at longrun equilibrium levels  $\hat{F}_{ss}$ . Luh and Stefanou derive these terms from the first-order conditions for profit maximization. After numerous substitutions they arrive at:

$$\hat{F}_v = \left[ \frac{\sum_p w_p X_p}{TSC} \right] \hat{X}_p$$

$$\hat{F}_{q2} = - \frac{\sum_j J_{kj} \dot{K}_j}{TSC} \hat{J}_{kj}$$

$$\hat{F}_{ss} = \sum_j \frac{(rJ_{kj} + c_j * k_j)}{TSC} \hat{K}_j$$

$$\hat{F}_{q1} = - \left[ \sum_j J_{kj} \frac{\dot{K}_j}{TSC} \right] \hat{K}_j$$

where terms with hats represent the proportional rate of growth and terms with dots represent the rate of change.

The Luh and Stefanou formula for total shadow costs are:

$$TSC = \sum w_i X_i + \sum c_j * k_j - \dot{K} * J_k - J_t \quad (14)$$

Note the first two right-hand terms in equation 14 represent variable costs.  $J_k$  represents a change in the shadow value of capital and  $J_t$  represents in the change in value function with respect to time. The  $J_k$  and  $J_t$  terms are obtained by taking the derivatives of the value function in equation 5. For example,  $J_t = d1 * P1 + d2 * P2 + d3 * P3 + d4 * c$ .



