MEASURING THE COVARIANCE MATRICES OF RESTRICTED REDUCED FORM COEFFICIENTS WITH SMALL SIZE COMPUTERS

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Kuo S. Huang

National Economics Division
Economic Research Service
U.S. Department of Agriculture
Washington, D.C. 20250

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The restricted reduced form equation system derived from a simultaneous structural model is the system used for forecasting and of immediate concern to policy decisionmakers and forecasters. For verifying the significance of restricted reduced form coefficients and for measuring the confidence of forecasts, the covariance matrices of these coefficients are required. However, the computation of the covariance matrix of restricted reduced form coefficients involves a heavy workload for a large size model, and the computer software packages for this computation are usually not available.

This paper formulates an operational procedure which can be easily implemented by applied econometricians for computing such a matrix. In particular, the procedure, which computes the covariance matrix in a sequential block by block fashion, is computationally efficient and suitable for programming in computers without larger core capacity. In fact, the limitation of core capacity is a critical factor to consider for programming with the increasing use of personal computers. Besides, as discussed later, the estimation procedure could be more accurate than that of the pioneer work developed by Goldberger, et al.(1961).

I. The Procedures

A structural econometric model, containing M jointly dependent endogenous variables vector y, K predetermined variables vector x, and a corresponding vector of residuals u, can be expressed as

$$y' = y' B + x' C + u' \tag{1}$$

where B and C are matrices of structural parameters. Given $\tilde{B}$ and $\tilde{C}$ as consistent estimates of B and C, the consistent estimates of the restricted reduced form coefficient matrix, say $\tilde{\Pi}$, can be derived by

$$\tilde{\Pi} = \tilde{C} (I - \tilde{B})^{-1} \tag{2}$$

The estimated reduced form coefficient matrix, which indicates the estimated effect of a change in predetermined variables on endogenous variables after taking account of the interdependence among the current endogenous variables, is of primary interest in many applications. Each derived reduced form coefficient in $\tilde{\Pi}$ is in general a nonlinear function of many structural coefficients in $\tilde{B}$ and $\tilde{C}$. Now the problem is to obtain the asymptotic variances and covariances of the restricted reduced form coefficient estimates in terms of the asymptotic variances and covariances of the structural coefficient estimates.

The asymptotic covariance matrix of restricted reduced form coefficients was first derived by Goldberger, et al.(1961). Basically they used the first order Taylor expansion to approximate the error relationships between reduced form and structural form estimators as

$$(\tilde{\Pi} - \Pi) = F \begin{bmatrix} \tilde{B} - B \\ \tilde{C} - C \end{bmatrix} \tag{3}$$

in which F is a matrix of first order partial derivatives of elements in $\tilde{\Pi}$ with respect to elements in $\tilde{B}$ and $\tilde{C}$ evaluated at their corresponding structural parameters. By proper arrangement of the reduced form and the structural form errors in column vectors, the covariance matrix of reduced form coefficients is then derived.

The derivation of error relationships by Goldberger, et al. as shown in (3) is obviously not precise because of neglecting the terms contained in the higher powers of the expansion. Dhrymes(1973) proposed an alternative expression of the error relationships by a simple rearrangement of the components in the reduced form equation. Although his main concern was to derive and to compare the asymptotic efficiency of various reduced form estimators, his conceptual procedure in deriving the covariance matrix can be further transformed into an operational form as developed below. To facilitate practical measurements, the focus of this paper is on coding and computing the consistent estimates of the covariance matrix in an efficient way.

Defining $D = (I - B)^{-1}$, the exact error relationships can be expressed as

$$\begin{aligned} (\tilde{\Pi} - \Pi) &= (\tilde{\Pi} - \tilde{C} D + \tilde{C} D - C D) \\ &= [\, \tilde{\Pi} (I - B) - \tilde{\Pi} (I - \tilde{B}) + \tilde{C} - C \,] D \\ &= (\tilde{\Pi}, I) \begin{bmatrix} \tilde{B} - B \\ \tilde{C} - C \end{bmatrix} D \end{aligned} \tag{4}$$

By rearranging the errors of reduced form and structural form coefficients in vectors and using the fact that $\text{vec}(ABC) = (C' \otimes A)\,\text{vec}(B)$, one can transform equation (4) into

$$\begin{aligned} \text{vec}(\tilde{\Pi} - \Pi) &= [\, D' \otimes (\tilde{\Pi}, I) \,]\ \text{vec} \begin{bmatrix} \tilde{B} - B \\ \tilde{C} - C \end{bmatrix} \\ &= (D' \otimes I_K)[\, I_M \otimes (\tilde{\Pi}, I)]\ \text{vec} \begin{bmatrix} \tilde{B} - B \\ \tilde{C} - C \end{bmatrix} \end{aligned} \tag{5}$$

By further rearranging of the errors of structural coefficients in equation order, while excluding zero entries in $(\tilde{B} - B)$ and $(\tilde{C} - C)$, equation (5) becomes

$$(\tilde{P} - P) = (D' \otimes I_K) \tilde{G} (\tilde{\beta} - \beta), \tag{6}$$

where

$(\tilde{P} - P) = [(\tilde{\Pi}_{.1} - \Pi_{.1})', \ldots\ldots, (\tilde{\Pi}_{.M} - \Pi_{.M})']'$ which is a KM x 1 vector of reduced form errors by stacking the successive columns of $(\tilde{\Pi} - \Pi)$,

$\tilde{G} = \text{diag}(\tilde{G}_1, \ldots\ldots, \tilde{G}_M)$, and $\tilde{G}_i$ is defined as $(\tilde{\Pi}_i, L_i)$, in which $\tilde{\Pi}_i$ is a submatrix of the estimated reduced form relating to the endogenous variables which appear on the right-hand side of the ith equation, and L_i is a selection matrix such that $XL_i = X_i$ which accounts for the predetermined variables in the ith equation,

$(\tilde{\beta} - \beta) = [(\tilde{\beta}_1 - \beta_1)' \ldots . (\tilde{\beta}_M - \beta_M)']'$ is a column vector with all errors of structural coefficients arranged in equation order, and

$(D' \otimes I_K)$ = an M K x M K matrix of the Kronecker product of the matrices $D' = [(1 - B)^{-1}]'$ and I_K.

Giving consistent estimate of Π as $\tilde{\Pi}$, the probability limit of $\tilde{G}_i$, say $\bar{G}_i$, is

$$\bar{G}_i = \text{plim } \tilde{G}_i = (\Pi_i, L_i), \qquad i = 1,2,.. M \tag{7}$$

Thus the asymptotic distribution of the restricted reduced form coefficients is given by

$$\sqrt{T} (\tilde{P} - P) \sim (D' \otimes I_k) \bar{G} \sqrt{T} (\tilde{\beta} - \beta) \tag{8}$$

Furthermore, giving the asymptotic covariance matrix of $\sqrt{T} (\tilde{\beta} - \beta)$, say Q, which is obtainable by various estimation techniques, the asymptotic covariance matrix of $\sqrt{T} (\tilde{P} - P)$, say W, is then obtained by

$$W = (D' \otimes I_K) \bar{G} Q \bar{G}' (D \otimes I_K) \tag{9}$$

By substituting the consistent estimates $\tilde{D}$ and $\tilde{G}$ for the unknown matrices $\bar{D}$ and G , the consistent estimate of W can be obtained straightforwardly from (9). However, a direct calculation based on this equation appears quite cumbersome in computation when the size of model is large. One effective strategy in programming is to partition some matrices of equation (9) into blocks. For a model with N stochastic equations, the estimated covariance matrix of structural estimates can be partitioned into N^2 blocks as $\begin{bmatrix} Q_{11} & \cdots & Q_{1N} \\ \cdots & \cdots & \cdots \\ Q_{N1} & \cdots & Q_{NN} \end{bmatrix}$, while the estimated covariance matrix of reduced form coefficients for M reduced form equations can be partitioned into M^2 blocks as $\begin{bmatrix} W_{11} & \cdots & W_{1M} \\ \cdots & \cdots & \cdots \\ W_{M1} & \cdots & W_{MM} \end{bmatrix}$.

Then a direct computational formula for equation (9) is obtained:

$$\begin{bmatrix} W_{11} & & W_{1M} \\ & \ddots & \\ W_{M1} & & W_{MM} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{N} \sum_{i=1}^{N} \tilde{d}_{i1} \tilde{d}_{j1} \tilde{G}_i Q_{ij} \tilde{G}_j' & \cdots\cdots & \sum_{j=1}^{N} \sum_{i=1}^{N} \tilde{d}_{i1} \tilde{d}_{jM} \tilde{G}_i Q_{ij} \tilde{G}_j' \\ & & \\ \sum_{j=1}^{N} \sum_{i=1}^{N} \tilde{d}_{iM} \tilde{d}_{j1} \tilde{G}_i Q_{ij} \tilde{G}_j' & \cdots\cdots & \sum_{j=1}^{N} \sum_{i=1}^{N} \tilde{d}_{iM} \tilde{d}_{jM} \tilde{G}_i Q_{ij} \tilde{G}_j' \end{bmatrix} \tag{10}$$

where $\tilde{d}_{ik}$ and $\tilde{d}_{jk}$ are (i,k) and (j,k) elements of $\tilde{D}$.

In summary, at the beginning of computation, the estimates of reduced form coefficients $\tilde{\Pi}$ and the matrix $\tilde{D}$ can be obtained from the structural coefficient estimates. Then we can calculate the matrix $\tilde{G}_i$ from the information in $\tilde{\Pi}$ and some selected matrix L_i as defined previously. Finally, the covariance matrix of restricted reduced form coefficients are computable from equation (10) by incorporating the variances and covariances of structural coefficients Q_{ij}. The measurement of each W_{km} for k, m = 1,2,.....,M in equation (10) uses the same information $\tilde{G}_i\ Q_{ij}\ \tilde{G}_j'$ for i,j = 1,2,....,N, but with different scale values of $\tilde{d}_{ik}$ and $\tilde{d}_{jm}$ depending on which W_{km} we want to estimate. It becomes apparent that, for defining and storing those common information sets in computers, it requires much less computing time and space. Furthermore, the estimated covariance matrix of reduced form coefficients corresponding to an endogenous variable can be obtained by simply referring to an appropriate diagonal block W_{ii}, $i = 1,2,\ldots,M$, and the square root of the diagonal elements of the block are estimated errors of reduced form coefficients. Obviously, the estimation procedure designated in a sequential block by block fashion is computational efficient and useful for programming in computers without larger core capacity.

II. An Illustration of Application

For illustration, the procedure developed in equation (10) is applied to compute the covariance matrices of the restricted reduced form coefficients of Klein´s model I of the U.S. economy. To have a basis of comparison with the results obtained by Goldberger, et al.(1961), prior information used in this study including the structural model estimators and their variances and covariances is compiled from that article.

The estimated reduced form coefficients along with two sets of alternative estimated standard errors (in parentheses) are presented in table 1. The figures in upper parentheses are obtained from Goldberger,et al., and those in lower parentheses are computed by the procedure outlined previously. Pairwise comparison of results from the two studies indicates that the estimated standard errors are quite different for those entries in the columns associated with variables P_{-1}, W_2, t, and the intercept. In view of symmetry in the estimated covariance matrix, table 2 presents only the lower triangle of the diagonal blocks at first and then the blocks in the lower off-diagonal triangle.

Table 1. Reduced form coefficients and estimated standard errors

	1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
C	42.8258	0.9473	0.6842	-0.1047	-0.1285	0.1788	0.1590	0.6636
	(8.6185)	(0.1878)	(0.0617)	(0.0373)	(0.2755)	(0.0236)	(0.0210)	(0.2368)
	(8.2901)	(0.1380)	(0.0777)	(0.0383)	(0.2834)	(0.0237)	(0.0305)	(0.2381)
I	25.8412	0.7358	-0.0291	-0.1820	-0.1759	-0.0076	-0.0068	0.1531
	(7.5674)	(0.1632)	(0.0396)	(0.0329)	(0.2384)	(0.0102)	(0.0091)	(0.2082)
	(6.4528)	(0.0809)	(0.0389)	(0.0318)	(0.2383)	(0.0102)	(0.0094)	(0.2081)
W_1	31.6353	0.8853	-0.1513	-0.1258	-0.1336	0.2218	0.1972	0.7973
	(7.2395)	(0.1614)	(0.0535)	(0.0312)	(0.2137)	(0.0159)	(0.0142)	(0.1975)
	(6.9857)	(0.1235)	(0.0458)	(0.0322)	(0.2109)	(0.0161)	(0.0382)	(0.1988)
P	37.0316	0.7978	-0.1935	-0.1609	-1.1708	-0.0506	-0.0450	1.0194
	(8.8331)	(0.1921)	(0.0448)	(0.0383)	(0.2748)	(0.0149)	(0.0132)	(0.2426)
	(7.9681)	(0.1065)	(0.0581)	(0.0377)	(0.2724)	(0.0148)	(0.0191)	(0.2424)
Y	68.6670	1.6831	0.6552	-0.2867	-1.3043	0.1712	0.1522	1.8167
	(15.3031)	(0.3324)	(0.0888)	(0.0663)	(0.4796)	(0.0287)	(0.0255)	(0.4205)
	(14.0657)	(0.2000)	(0.1009)	(0.0668)	(0.4828)	(0.0287)	(0.0312)	(0.4211)
K	25.8412	0.7358	-0.0291	0.8180	-0.1759	-0.0076	-0.0068	0.1531
	(7.5674)	(0.1632)	(0.0396)	(0.0329)	(0.2384)	(0.0102)	(0.0091)	(0.2082)
	(6.4528)	(0.0809)	(0.0389)	(0.0318)	(0.2383)	(0.0102)	(0.0094)	(0.2081)

Note: Two sets of figures in parentheses are estimated standard errors of the reduced form coefficients of Klein's model I; the upper figures are obtained from Goldberger, et al. (1961), and the lower figures are computed from this study. The names of variables are explained in the appendix.

Table 2 Covariance Matrix of Reduced Form Coefficients

A. Diagonal Blocks of cov ($\tilde{P}$ − P)

	1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
	Consumption (C): W_{11}							
1	68.7260							
P_{-1}	−0.0495	0.0190						
W_2	−0.4126	−0.0004	0.0060					
K_{-1}	−0.3051	−0.0003	0.0016	0.0015				
T	−1.7601	0.0061	0.0146	0.0083	0.0803			
$(W_1+T)_{-1}$	−0.0971	0.0007	0.0016	0.0004	0.0046	0.0006		
t	−0.0779	0.0009	0.0004	0.0005	0.0022	0.0001	0.0009	
G	1.5108	−0.0009	−0.0096	−0.0073	−0.0634	−0.0024	−0.0019	0.0567
	Investment (I): W_{22}							
1	41.6388							
P_{-1}	−0.0473	0.0065						
W_2	−0.0668	0.0005	0.0015					
K_{-1}	−0.2000	−0.0003	0.0003	0.0010				
T	−0.3692	0.0029	0.0092	0.0014	0.0568			
$(W_1+T)_{-1}$	−0.0179	0.0001	0.0004	0.0001	0.0024	0.0001		
t	−0.0152	0.0001	0.0004	0.0001	0.0022	0.0001	0.0001	
G	0.3143	−0.0026	−0.0080	−0.0011	−0.0496	−0.0021	−0.0019	0.0433
	Private wage bill (w_1): W_{33}							
1	48.8004							
P_{-1}	0.2357	0.0152						
W_2	−0.1904	−0.0011	0.0021					
K_{-1}	−0.2148	−0.0014	0.0008	0.0010				
T	−0.6657	0.0027	0.0071	0.0031	0.0445			
$(W_1+T)_{-1}$	−0.0064	0.0010	0.0004	0.0000	0.0021	0.0003		
t	−0.0170	0.0001	−0.0002	0.0003	0.0004	−0.0001	0.0015	
G	0.8126	0.0054	−0.0066	−0.0038	−0.0375	−0.0007	−0.0009	0.0395
	Profits (P): W_{44}							
1	63.4907							
P_{-1}	−0.0885	0.0113						
W_2	−0.2780	−0.0003	0.0034					
K_{-1}	−0.2903	−0.0004	0.0012	0.0014				
T	−1.2379	0.0014	0.0124	0.0056	0.0742			
$(W_1+T)_{-1}$	−0.0684	−0.0000	0.0008	0.0003	0.0031	0.0002		
t	−0.0614	−0.0002	0.0010	0.0002	0.0032	0.0002	0.0004	
G	1.0498	−0.0012	−0.0101	−0.0048	−0.0654	−0.0025	−0.0025	0.0587
	National income (Y): W_{55}							
1	197.8428							
P_{-1}	−0.1851	0.0409						
W_2	−0.8135	0.0000	0.0102					
K_{-1}	−0.9108	−0.0014	0.0035	0.0045				
T	−3.6208	0.0115	0.0378	0.0166	0.2331			
$(W_1+T)_{-1}$	−0.1938	0.0008	0.0027	0.0008	0.0105	0.0008		
t	−0.1788	0.0007	0.0011	0.0010	0.0066	0.0003	0.0010	
G	3.1134	−0.0060	−0.0297	−0.0145	−0.1992	−0.0076	−0.0057	0.1773

Table 2. (continued)

B. Off-Diagonal Blocks of Cov($\tilde{P}$ − P)

		1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
		Consumption (C): W_{21}							
Investment (I)	1	43.7389	-0.0094	-0.2104	-0.1967	-0.8010	-0.0460	-0.0562	0.6976
	P_{-1}	-0.0789	0.0072	-0.0002	-0.0002	0.0012	-0.0000	-0.0003	-0.0012
	W_2	-0.1237	0.0002	0.0013	0.0006	0.0082	0.0003	0.0000	-0.0072
	K_{-1}	-0.2091	-0.0006	0.0010	0.0010	0.0037	0.0002	0.0003	-0.0033
	T	-0.6905	0.0013	0.0059	0.0033	0.0480	0.0014	0.0003	-0.0444
	$(W_1+T)-1$	-0.0328	0.0000	0.0003	0.0001	0.0021	0.0001	0.0000	-0.0019
	t	-0.0295	0.0000	0.0003	0.0001	0.0019	0.0001	-0.0000	-0.0017
	G	0.5906	-0.0013	-0.0049	-0.0028	-0.0418	-0.0012	-0.0002	0.0387
		Consumption (C): W_{31}							
Private Wage bill (W_1)	1	51.9613	0.1800	-0.2613	-0.2207	-0.9456	-0.0333	-0.0387	0.9797
	P_{-1}	0.0005	0.0158	-0.0001	-0.0004	0.0075	0.0010	0.0003	-0.0003
	W_2	-0.2471	-0.0010	0.0032	0.0010	0.0091	0.0007	0.0002	-0.0075
	K_{-1}	-0.2354	-0.0011	0.0011	0.0011	0.0045	0.0001	0.0003	-0.0047
	T	-1.0859	0.0024	0.0090	0.0051	0.0555	0.0025	0.0011	-0.0474
	$(W_1+T)_{-1}$	-0.0512	0.0007	0.0009	0.0002	0.0034	0.0004	0.0001	-0.0018
	t	-0.0351	0.0010	-0.0005	0.0003	0.0007	-0.0002	0.0011	-0.0013
	G	0.9837	0.0036	-0.0062	-0.0047	-0.0416	-0.0008	-0.0009	0.0425
		Investment (I): W_{32}							
Private wage bill (W_1)	1	39.6149	-0.0639	-0.0862	-0.1911	-0.4950	-0.0240	-0.0210	0.4175
	$P-1$	0.0225	0.0062	0.0002	-0.0007	0.0011	0.0000	0.0000	-0.0012
	W_2	-0.1314	0.0001	0.0013	0.0006	0.0068	0.0003	0.0003	-0.0058
	K_{-1}	-0.1822	-0.0003	0.0004	0.0009	0.0022	0.0001	0.0001	-0.0018
	T	-0.5222	0.0018	0.0076	0.0023	0.0461	0.0020	0.0018	-0.0402
	$(W_1+T)_{-1}$	-0.0232	0.0001	0.0003	0.0001	0.0016	0.0001	0.0001	-0.0014
	t	-0.0396	-0.0003	-0.0000	0.0003	0.0002	-0.0000	-0.0001	-0.0002
	G	0.4953	-0.0015	-0.0068	-0.0022	-0.0421	-0.0018	-0.0016	0.0366
		Consumption (C): W_{41}							
Profits (P)	1	60.5036	-0.2390	-0.3617	-0.2751	-1.6156	-0.1098	-0.0954	1.2288
	$P-1$	-0.1290	0.0104	-0.0005	-0.0001	-0.0002	-0.0004	0.0003	-0.0018
	W_2	-0.2893	0.0007	0.0042	0.0012	0.0136	0.0012	0.0002	-0.0093
	K_{-1}	-0.2788	0.0002	0.0015	0.0013	0.0075	0.0005	0.0005	-0.0058
	T	-1.3647	0.0049	0.0115	0.0065	0.0728	0.0035	0.0014	-0.0604
	$(W_1+T)_{-1}$	-0.0787	-0.0000	0.0011	0.0003	0.0033	0.0003	0.0001	-0.0025
	t	-0.0723	-0.0001	0.0013	0.0003	0.0035	0.0004	-0.0002	-0.0023
	G	1.1177	-0.0059	-0.0083	-0.0055	-0.0637	-0.0028	-0.0012	0.0528
		Investment (I): W_{42}							
Profits (P)	1	45.7628	-0.0623	-0.1043	-0.2179	-0.5646	-0.0266	-0.0238	0.4874
	P_{-1}	-0.0792	0.0076	0.0004	-0.0002	0.0031	0.0001	0.0001	-0.0027
	W_2	-0.1458	0.0002	0.0016	0.0006	0.0083	0.0004	0.0004	-0.0071
	K_{-1}	-0.2145	-0.0003	0.0004	0.0011	0.0025	0.0001	0.0001	-0.0021
	T	-0.6480	0.0024	0.0097	0.0028	0.0586	0.0025	0.0023	-0.0512
	$(W_1+T)_{-1}$	-0.0407	0.0000	0.0004	0.0002	0.0022	0.0001	0.0001	-0.0019
	t	-0.0318	0.0001	0.0004	0.0001	0.0022	0.0001	0.0001	-0.0019
	G	0.5167	-0.0023	-0.0085	-0.0022	-0.0519	-0.0022	-0.0020	0.0454

Table 2. (continued)

B. Off-Diagonal Blocks of Cov($\tilde{P}$ − P)

		1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
				Private wage bill (W_1): W_{43}					
Profits (P)	1	42.7757	−0.2127	−0.1880	−0.2027	−0.9423	−0.0681	−0.0577	0.6664
	P_{-1}	−0.1196	0.0067	0.0002	0.0001	0.0015	−0.0002	0.0005	−0.0033
	W_2	−0.1571	0.0012	0.0024	0.0007	0.0095	0.0008	−0.0004	−0.0064
	K_{-1}	−0.2030	0.0003	0.0008	0.0010	0.0043	0.0003	0.0003	−0.0031
	T	−0.7749	0.0058	0.0088	0.0036	0.0572	0.0029	0.0005	−0.0461
	$(W_1+T)_{-1}$	−0.0510	0.0000	0.0007	0.0002	0.0024	0.0002	−0.0001	−0.0019
	t	−0.0427	0.0002	0.0007	0.0001	0.0025	0.0002	−0.0004	−0.0016
	G	0.5846	−0.0069	−0.0067	−0.0028	−0.0501	−0.0025	−0.0007	0.0395
				Consumption (C): W_{51}					
National Income (Y)	1	112.4650	−0.0589	−0.6230	−0.5017	−2.5612	−0.1431	−0.1341	2.2085
	P_{-1}	−0.1284	0.0262	−0.0006	−0.0005	0.0073	0.0007	0.0006	−0.0021
	W_2	−0.5363	−0.0002	0.0074	0.0022	0.0227	0.0019	0.0004	−0.0169
	K_{-1}	−0.5141	−0.0009	0.0026	0.0025	0.0120	0.0006	0.0008	−0.0106
	T	−2.4506	0.0073	0.0205	0.0116	0.1283	0.0060	0.0025	−0.1078
	$(W_1+T)_{-1}$	−0.1299	0.0007	0.0020	0.0005	0.0067	0.0006	0.0001	−0.0043
	t	−0.1074	0.0009	0.0007	0.0006	0.0042	0.0002	0.0009	−0.0036
	G	2.1014	−0.0022	−0.0145	−0.0101	−0.1052	−0.0036	−0.0021	0.0954
				Investment (I): W_{52}					
National Income (Y)	1	85.3777	−0.1262	−0.1905	−0.4091	−1.0596	−0.0506	−0.0447	0.9049
	P_{-1}	−0.0567	0.0137	0.0007	−0.0009	0.0042	0.0001	0.0001	−0.0039
	W_2	−0.2772	0.0003	0.0028	0.0012	0.0151	0.0007	0.0007	−0.0129
	K_{-1}	−0.3967	−0.0006	0.0008	0.0020	0.0046	0.0002	0.0002	−0.0040
	T	−1.1702	0.0042	0.0173	0.0050	0.1048	0.0045	0.0041	−0.0914
	$(W_1+T)_{-1}$	−0.0639	0.0001	0.0007	0.0003	0.0038	0.0002	0.0002	−0.0033
	t	−0.0714	−0.0002	0.0004	0.0004	0.0024	0.0001	0.0001	−0.0021
	G	1.0119	−0.0038	−0.0152	−0.0044	−0.0940	−0.0040	−0.0036	0.0820
				Private wage bill (W_1): W_{53}					
National Income (Y)	1	91.5762	0.0230	−0.3784	−0.4175	−1.6080	−0.0744	−0.0747	1.4790
	P_{-1}	0.1161	0.0219	−0.0009	−0.0014	0.0043	0.0008	0.0007	0.0021
	W_2	−0.3475	0.0001	0.0045	0.0015	0.0166	0.0012	−0.0006	−0.0130
	K_{-1}	−0.4178	−0.0011	0.0016	0.0020	0.0074	0.0003	0.0006	−0.0069
	T	−1.4406	0.0085	0.0159	0.0067	0.1017	0.0050	0.0009	−0.0837
	$(W_1+T)_{-1}$	−0.0573	0.0010	0.0010	0.0002	0.0045	0.0004	−0.0002	−0.0026
	t	−0.0597	0.0003	0.0005	0.0004	0.0029	0.0001	0.0010	−0.0025
	G	1.3972	−0.0015	−0.0133	−0.0066	−0.0876	−0.0032	−0.0015	0.0791
				Profits (P): W_{54}					
National Income (Y)	1	106.2665	−0.2081	−0.4351	−0.4933	−2.0128	−0.1193	−0.1041	1.6344
	P_{-1}	−0.3012	0.0180	0.0009	−0.0001	0.0072	0.0000	0.0000	−0.0081
	W_2	−0.4600	−0.0001	0.0057	0.0020	0.0212	0.0015	0.0017	−0.0168
	K_{-1}	−0.4930	−0.0003	0.0018	0.0024	0.0092	0.0005	0.0004	−0.0076
	T	−2.1802	0.0030	0.0219	0.0099	0.1314	0.0055	0.0057	−0.1155
	$(W_1+T)_{-1}$	−0.1364	−0.0002	0.0016	0.0006	0.0060	0.0004	0.0005	−0.0050
	t	−0.1191	0.0004	0.0006	0.0006	0.0037	0.0002	−0.0001	−0.0032
	G	1.7162	−0.0045	−0.0165	−0.0080	−0.1116	−0.0044	−0.0042	0.0983

III. Concluding Remarks

This study has formulated an operational procedure for computing the covariance matrix of restricted reduced form coefficients, which are essential estimates for various applications of an econometric model. The procedure is illustrated by applying it to the Klein's model I of the U.S. economy, where some prior information is obtained from Goldberger, et al.(1961). This set of prior information, which is an approximation of the variances and covariances from a structural model estimated by two-stage least squares, may not be the best choice (McCarthy,1972). However, the purpose of this study is to demonstrate the procedure for obtaining the covariance matrix of restricted reduced form coefficients but not the properties of this particular set of prior information per se. Assuming that the information is obtained from some proper structural estimates, this study applies this classical example so that the results are comparable to those estimates from an alternative procedure of Goldberger, et al. Even though the size of model in this example is rather small, the two procedures yield substantially different results. As mentioned before, since the covariance matrix obtained in this study is derived from a set of exact estimated error relationships, the results thus obtained should be more accurate.

Appendix. Basic Prior Information Used in this Study

The following prior information used for computing the asymptotic covariance matrix of reduced form coefficients is obtained from Goldberger, et al. (1961) Klein's model I consists of three stochastic equations and three identities; the structure estimates were computed by Klock and Mennes (1960) using the two-stage least squares:

$$C = \underset{(1.3197)}{16.5548} + \underset{(0.1179)}{0.0173}\,P + \underset{(0.1072)}{0.2162}\,P_{-1} + \underset{(0.0402)}{0.8102}\,(W_1 + W_2)$$

$$I = \underset{(7.5225)}{20.2782} + \underset{(0.1728)}{0.1502}\,P + \underset{(0.1624)}{0.6159}\,P_{-1} - \underset{(0.0360)}{0.1578}\,K_{-1}$$

$$W_1 = \underset{(1.1471)}{1.5003} + \underset{(0.0356)}{0.4389}\,(Y + T - W_2) + \underset{(0.0388)}{0.1467}\,(P + W_1 + T)_{-1} + \underset{(0.0291)}{0.1304}\,t$$

$$Y + T = C + I + G$$

$$Y = P + W_1 + W_2$$

$$K = K_{-1} + I$$

where variables are C (consumption), P (profits), W_1 (private wage bill), W_2 (government wage bill), I (net investment), K (end-of-year capital stock), Y (national income), T (business taxes), t (time measured from 1931 as origin), G (government expenditures). The standard errors are given in parentheses.

Appendix Table Covariance Matrix of Structural Coefficients

Equation		(1) Consumption equation				(2) Investment equation				(3) Private wage bill equation			
		1	P	P_{-1}	W_1+W_2	1	P	P_{-1}	K_{-1}	1	$Y+T-W_2$	$(P+W_1+T)_{-1}$	t
(1)	1	1.7416											
	P	-0.0153	0.0139										
	P_{-1}	0.0047	-0.0096	0.0115									
	W_1+W_2	-0.0327	-0.0015	-0.0005	0.0016								
(2)	1	2.1462	0.0589	0.0207	-0.0834	56.5893							
	P	-0.0459	0.0041	-0.0047	0.0013	-0.9199	0.0298						
	P_{-1}	0.0177	-0.0032	0.0052	-0.0012	0.7735	-0.0256	0.0264					
	K_{-1}	-0.0082	-0.0004	-0.0001	0.0004	-0.2676	0.0042	-0.0039	0.0013				
(3)	1	-0.6106	-0.0005	0.0327	0.0016	0.3607	-0.0017	-0.0152	-0.0004	1.3159			
	$Y+T-W_2$	0.0092	-0.0018	0.0012	0.0001	0.0131	0.0006	-0.0003	-0.0001	-0.0067	0.0013		
	$(P+W_1+T)_{-1}$	0.0007	0.0019	-0.0018	-0.0001	-0.0197	-0.0006	0.0006	0.0001	-0.0154	-0.0012	0.0015	
	t	0.0028	0.0007	0.0005	-0.0005	-0.0313	0.0005	-0.0007	0.0002	0.0156	-0.0003	0.0001	0.0008

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