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MEASURING THE COVARIANCE MATRICES OF RESTRICTED REDUCED

FORM COEFFICIENTS WITH SMALL SIZE COMPUTERS

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The restricted reduced form equation system derived from a simultaneous structural model is the system used for forecasting and of immediate concern to policy decisionmakers and forecasters. For verifying the significance of restricted reduced form coefficients and for measuring the confidence of forecasts, the covariance matrices of these coefficients are required. However, the computation of the covariance matrix of restricted reduced form coefficients involves a heavy workload for a large size model, and the computer software packages for this computation are usually not available.

This paper formulates an operational procedure which can be easily implemented by applied econometricians for computing such a matrix. In particular, the procedure, which computes the covariance matrix in a sequential block by block fashion, is computationally efficient and suitable for programming in computers without larger core capacity. In fact, the limitation of core capacity is a critical factor to consider for programming with the increasing use of personal computers. Besides, as discussed later, the estimation procedure could be more accurate than that of the pioneer work developed by Goldberger, et al.(1961).

# I. The Procedures

A structural econometric model, containing M jointly dependent endogenous variables vector y, K predetermined variables vector x, and a corresponding vector of residuals u, can be expressed as

$$y' = y' B + x' C + u'$$
 (1)

where B and C are matrices of structural parameters. Given B and C as consistent estimates of B and C, the consistent estimates of the restricted reduced form coefficient matrix, say  $\tilde{\Pi}$ , can be derived by

$$\widetilde{\mathbf{H}} = \widetilde{\mathbf{C}} (\mathbf{I} - \widetilde{\mathbf{B}})^{-1}$$
(2)

The estimated reduced form coefficient matrix, which indicates the estimated effect of a change in predetermined variables on endogenous variables after taking account of the interdependence among the current endogenous variables, is of primary interest in many applications. Each derived reduced form coefficient in  $\widetilde{\Pi}$  is in general a nonlinear function of many structural coefficients in  $\widetilde{B}$  and  $\widetilde{C}$ . Now the problem is to obtain the asymptotic variances and covariances of the restricted reduced form coefficient estimates in terms of the asymptotic variances and covariances of the structural coefficient estimates.

The asymptotic covariance matrix of restricted reduced form coefficients was first derived by Goldberger, et al.(1961). Basically they used the first order Taylor expansion to approximate the error relationships between reduced form and structural form estimators as

$$\left(\begin{array}{c} \widetilde{\Pi} - \Pi \end{array}\right) = F \begin{bmatrix} \widetilde{B} - B \\ \widetilde{C} - C \end{bmatrix}$$

in which F is a matrix of first order partial derivatives of elements in  $\mathbb{I}$  with  $\sim$   $\sim$  respect to elements in B and C evaluated at their corresponding structural

(3)

parameters. By proper arrangement of the reduced form and the structural form errors in column vectors, the covariance matrix of reduced form coefficients is then derived.

The derivation of error relationships by Goldberger, et al.as shown in (3) is obviously not precise because of neglecting the terms contained in the higher powers of the expansion. Dhrymes(1973) proposed an alternative expression of the error relationships by a simple rearrangement of the components in the reduced form equation. Although his main concern was to derive and to compare the asymptotic efficiency of various reduced form estimators, his conceptual procedure in deriving the 'covariance matrix can be further transformed into an operational form as developed below. To facilitate practical measurements, the focus of this paper is on coding and computing the consistent estimates of the covariance matrix in an efficient way.

Defining  $D = (I - B)^{-1}$ , the exact error relationships can be expressed as  $(\widetilde{II} - II) = (\widetilde{II} - \widetilde{C}D + \widetilde{C}D - CD)$  $= [\widetilde{II}(I - B) - \widetilde{II}(I - \widetilde{B}) + \widetilde{C} - C]D$  $= (\widetilde{II}, I) \begin{bmatrix} \widetilde{B} - B \\ \widetilde{C} - C \end{bmatrix} D$  (4)

By rearranging the errors of reduced form and structural form coefficients in vectors and using the fact that  $vec(ABC) = (C' \otimes A) vec(B)$ , one can transform equation (4) into

$$\operatorname{vec}(\widetilde{\Pi} - \Pi) = [D' \otimes (\widetilde{\Pi}, I)] \operatorname{vec}\begin{bmatrix}\widetilde{B} - B\\\widetilde{C} - C\end{bmatrix}$$
$$= (D' \otimes I_{K})[I_{M} \otimes (\widetilde{\Pi}, I)] \operatorname{vec}\begin{bmatrix}\widetilde{B} - B\\\widetilde{C} - C\end{bmatrix}$$
(5)

By further rearranging of the errors of structural coefficients in equation order, while excluding zero entries in (B - B) and (C - C), equation (5) becomes

$$(\widetilde{P} - P) = (D' \otimes I_K) \widetilde{G} (\widetilde{\beta} - \beta),$$
 (6)

where

 $D' = [(1 - B)^{-1}]'$  and  $I_{K'}$ .

Giving consistent estimate of  $\Pi$  as  $\Pi$ , the probability limit of  $G_i$ , say  $G_i$ , is  $G_i = \text{plim } G_i = (\Pi_i, L_i)$ ,  $i = 1, 2, \dots$  M (7) Thus the asymptotic distribution of the restricted reduced form coefficients is given by

$$\sqrt{T}$$
 ( $\tilde{P} - P$ ) ~ (D'  $\otimes$  I<sub>k</sub>)  $\bar{G} \sqrt{T}$  ( $\tilde{\beta} - \beta$ ) (8)

Furthermore, giving the asymptotic covariance matrix of  $\sqrt{T}$  ( $\beta - \beta$ ), say Q, which is obtainable by various estimation techniques, the asymptotic covariance matrix of  $\sqrt{T}$  (P - P), say W, is then obtained by  $W = (D' \otimes I_K) \overline{G} Q \overline{G'} (D \otimes I_K)$  (9) By substituting the consistent estimates D and G for the unknown matrices D and G , the consistent estimate of W can be obtained straightforwardly from (9). However, a direct calculation based on this equation appears quite cumbersome in computation when the size of model is large. One effective strategy in programming is to partition some matrices of equation (9) into blocks. For a model with N stochastic equations, the estimated covariance matrix of structural estimates can be partitioned into  $N^2$  blocks as  $\lceil Q_{11} \cdots Q_{1N} \rceil$ ,

estimates can be partitioned into N<sup>2</sup> blocks as  $\begin{bmatrix} Q_{11} & \cdots & Q_{1N} \\ \vdots & \ddots & \ddots & \vdots \\ Q_{N1} & \cdots & Q_{NN} \end{bmatrix}$ , while the estimated covariance matrix of reduced form coefficients for M reduced form equations can be partitioned into M<sup>2</sup> blocks as  $\begin{bmatrix} W_{11} & \cdots & W_{1M} \\ \vdots & \cdots & \vdots \\ W_{M1} & \cdots & W_{MM} \end{bmatrix}$ .

Then a direct computational formula for equation (9) is obtained:

where  $d_{ik}$  and  $d_{jk}$  are (i,k) and (j,k) elements of D.

In summary, at the beginning of computation, the estimates of reduced form coefficients  $\Pi$  and the matrix D can be obtained from the structural coefficient estimates. Then we can calculate the matrix  $G_i$  from the information in  $\Pi$  and some selected matrix  $L_i$  as defined previously. Finally, the covariance matrix of restricted reduced form coefficients are computable from equation (10) by incorporating the variances and covariances of structural coefficients  $Q_{ij}$ . The measurement of each  $W_{km}$  for k, m = 1,2,...,M in equation (10) uses the same information  $\tilde{G}_i Q_{ij} \tilde{G}_j$  for i, j = 1,2,...,N, but with different scale values of  $\tilde{d}_{1k}$  and  $\tilde{d}_{jm}$  depending on which  $W_{km}$  we want to estimate. It becomes apparent that, for defining and storing those common information sets in computers, it requires much less computing time and space. Furthermore, the estimated covariance matrix of reduced form coefficients corresponding to an endogenous variable can be obtained by simply referring to an appropriate diagonal block  $W_{11}$ ,  $i = 1, 2, \ldots, M$ , and the square root of the diagonal elements of the block are estimated errors of reduced form coefficients. Obviously, the estimation procedure designated in a sequential block by block fashion is computational efficient and useful for programming in computers without larger core capacity.

## II. An Illustration of Application

For illustration, the procedure developed in equation (10) is applied to compute the covariance matrices of the restricted reduced form coefficients of Klein's model I of the U.S. economy. To have a basis of comparison with the results obtained by Goldberger, et al.(1961), prior information used in this study including the structural model estimators and their variances and covariances is compiled from that article.

The estimated reduced form coefficients along with two sets of alternative estimated standard errors (in parentheses) are presented in table 1. The figures in upper parentheses are obtained from Goldberger, et al., and those in lower parentheses are computed by the procedure outlined previously. Pairwise comparison of results from the two studies indicates that the estimated standard errors are quite different for those entries in the columns associated with variables  $P_{-1}$ ,  $W_2$ , t, and the intercept. In view of symmetry in the estimated covariance matrix, table 2 presents only the lower triangle of the diagonal blocks at first and then the blocks in the lower off-diagonal triangle.

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				and the second					
	:	1	P_1	W2	K_1	T	$(W_1+T)_{-1}$	t	G
С	:	42.8258 (8.6185) (8.2901)	0.9473 (0.1878) (0.1380)	0.6842 (0.0617) (0.0777)	-0.1047 (0.0373) (0.0383)	-0.1285 (0.2755) (0.2834)	0.1788 (0.0236) (0.0237)	0.1590 (0.0210) (0.0305)	0.6636 (0.2368) (0.2381)
I	:	25.8412 (7.5674) (6.4528)	0.7358 (0.1632) (0.0809)	-0.0291 (0.0396) (0.0389)	-0.1820 (0.0329) (0.0318)	-0.1759 (0.2384) (0.2383)	-0.0076 (0.0102) (0.0102)	-0.0068 (0.0091) (0.0094)	0.1531 (0.2082) (0.2081)
w <sub>1</sub>	:	31.6353 (7.2395) (6.9857)	0.8853 (0.1614) (0.1235)	-0.1513 (0.0535) (0.0458)	-0.1258 (0.0312) (0.0322)	-0.1336 (0.2137) (0.2109)	0.2218 (0.0159) (0.0161)	0.1972 (0.0142) (0.0382)	0.7973 (0.1975) (0.1988)
Ρ	:	37.0316 (8.8331) (7.9681)	0.7978 (0.1921) (0.1065)	-0.1935 (0.0448) (0.0581)	-0.1609 (0.0383) (0.0377)	-1.1708 (0.2748) (0.2724)	-0.0506 (0.0149) (0.0148)	-0.0450 (0.0132) (0.0191)	1.0194 (0.2426) (0.2424)
Y	•	68.6670 (15.3031) (14.0657)	1.6831 (0.3324) (0.2000)	0.6552 (0.0888) (0.1009)	-0.2867 (0.0663) (0.0668)	-1.3043 (0.4796) (0.4828)	0.1712 (0.0287) (0.0287)	0.1522 (0.0255) (0.0312)	1.8167 (0.4205) (0.4211)
K	::	25.8412 (7.5674) (6.4528)	0.7358 (0.1632) (0.0809)	-0.0291 (0.0396) (0.0389)	0.8180 (0.0329) (0.0318)	-0.1759 (0.2384) (0.2383)	-0.0076 (0.0102) (0.0102)	-0.0068 (0.0091) (0.0094)	0.1531 (0.2082) (0.2081)

Table 1. Reduced form coefficients and estimated standard errors

Note: Two sets of figures in parentheses are estimated standard errors of the reduced form coefficients of Klein's model I; the upper figures are obtained from Goldberger, et al. (1961), and the lower figures are computed from this study. The names of variables are explained in the appendix.

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Covariance Matrix of Reduced Form Coefficients . Table 2 . ~

		•	A. Diag	onal Bloc	ks of cov	(P - P)		
	1	P_1	W2	K_1	T	$(W_1+T)_{-1}$	t	G
		Consum	ption (C):	W,,				· · · · · · · · · · · ·
1	68,7260			11			•	
P_1	-0.0495	0.0190						
W2	-0,4126	-0.0004	0.0060					•
K_1	-0.3051	-0.0003	0.0016	0.0015				
T	-1.7601	0.0061	0.0146	0.0083	0.0803			
$(W_1+T)_{-1}$	-0.0971	0.0007	0.0016	0.0004	0.0046	0.0006		
t	-0_0779	0.0009	0.0004	0,0005	0.0022	0.0001	0.0009	
G	1.5108	-0.0009	-0.0096	-0.0073	-0.0634	-0.0024	-0.0019	0,0567
		·				· · · · · ·		
		Investm	ent (I):	W22				
	41.0388	· · ·						
P-1	-0.0473	0.0055						
_w2	-0.0550	0.0005	0.0015					
<u>^-1</u>	-0.2000	-0.0003	0,0003	0.0010				
11 J.T.)	-0.0170	0.0029	0.0092	0.0014	0.0568	•		
w1+1)-1	-0.0177	0.0001	0.0004	0.0001	0.0024	0,0001		
E C	0 71 17		0.0004	0.0001	0.0022	0.0001	0,0001	
6	0.3143	-0.0020	-0.0000	-0,0011	=0.0496	-0.0021	-0.0019	0.0433
		Private	wage bill	(w1): W3	3			
1	48.8000				-			. ·
	0 2157	0:0153						
	-0 1004	-0 0011	0 0031					
<sup>2</sup>	-0 2148	-0.0011	0.0021	0 1010				
- 4 T	-0 6657	0 0027	0.0071	0.0010				
(W.+T) .	-0.0004	0.0010	6 0000	0.0031	0.0445			
····	-0.0170	0_0001	-0.0002	0.0000	0.0021	0.0003		
G	0.8126	0-0054	-0.0002	-0.00038		-0.0001	0,0015	
					-0-0313	-0.0007	-0.0009	0.0345
		Profits	(P): W.					
1	63,4907							•
Р,	-0.0885	0-0113					•	
	-0.2780	-0.0003	0.0034					
K_1	-0.2903	-0.0004	0.0012	0.0014				
T	-1.2379	0.0014	0.0124	0.0056	0 0742	•		
;+T) ,	-0_0684	-0.0000	¥.0008	0.0003	0 0031	0 0003		
1 E	-0.0614	-0.0002	0.0010	5000.0	0.0032	0 0002	0 0000	
G	1.0498	-0.0012	-0.0101	-0.0048	-0.0654	-0.0025	-0.0004	
						-0.0053	-0.0025	0.0307
		National	income (Y	)• W				
1	197.8428		(1	~~ ~55				
P_1	-0,1851	0.0400						
W2	-0.8135	0.0000	0.0102					
K_1	-0.9108	-0.0014	0.0035	0.0045	•			
T	-3.6208	0.0115	v.0378	0.0105	0.2331			
(+T)_1	-0,1938	0.0008	0.0027	0.0008	0.0105	0.0008		
⁺ ¯ŧ	=0,1788	0.0007	0.0011	0.0010	0_0066	0.0003	0 0010	
G	3,1134	-0.0060	-0.0297	-0.0145	-0-1992	+0_007A	-0.0010	A 1777
-	1		-				-0.002/	v.1//3

Table 2. (continued)

B. Off-Diagonal Blocks of  $Cov(\tilde{P} - P)$ 

	1	P_1	W <sub>2</sub>	К_1	T	(W1+T)-1	t	G
	Consumption (C): W <sub>21</sub>							
$ \begin{array}{c} 1 \\ P_{-1} \\ \overline{w}_{2} \\ K_{-1} \\ T \\ T \end{array} $	43.7389 -0.0789 -0.1237 -0.2091 -0.0905	-0.0094 0.0072 0.0002 -0.0006 0.0013	-0.2104 -0.0002 0.0013 0.0010 0.0059	-0.1967 -0.0002 0.0006 0.0010 0.0033	-0.8010 0.0012 0.0082 0.0037 0.0480	-0.0460 -0.0000 0.0003 0.0002 0.0014	-0.0562 -0.0003 0.0000 0.0003 0.0003	0.6976 -0.0012 -0.0072 -0.0033 -0.0444
2 (w <sub>1</sub> +1)-1 2 t 4 G	-0.0295 0.5906	0.0000	0.0003 0.0003 −0.0049	0.0001	0.0021 0.0019 -0.0418	0.0001 0.0001 -0.0012	-0.0000 -0.0000 -0.0000	-0.0019 -0.0017 0.0387
•		Consu	aption (C)	: W <sub>31</sub>	•			
Mage bil Mage bil Mage Vil Mage Vil	51,9613 0,0005 -0,2471 -0,2354	0.1800 0.0158 -0.0010 -0.0011	-0.2613 -0.0001 0.0032 0.0011	-0.2267 -0.0004 0.0010 0.0011	-0.9456 0.0075 0.0091 0.0045	-0.0333 0.0010 0.0007 0.0001	-0.0387 0.0003 0.0002 0.0003	0.9797 -0.0003 -0.0075 -0.0047
T 1 رمان 1 رمان 2 روم 2 رم 2 رم 2 رم 2 رم 2 رم 2 رم 2 رمم 2 رم 2 رمم 2 رم 2 ر	-1.0859 -0.0512 -0.0351 0.9837	0.0024 0.0007 0.0010 0.0036	0.0090 0.0009 -0.0005 -0.0062	0.0051 0.0002 0.0003 -0.0047	0.0555 0.0034 0.0007 -0.0416	0.0025 0.0004 -0.0002 -0.0008	0.0011 0.0001 0.0011 -0.0009	-0.0474 -0.0018 -0.0013 0.0425
1		Invest	ment (I):	W32	•			
(I 1 P-1 W2 W2	39,6149 0,0225 =0,1314 =0,1822	-0.0639 0.0062 0.0001	-0.0862 0.0002 0.0013	-0.1911 -0.0007 0.3006	-0.4950 0.0011 0.0068	-0.0240 0.0000 0.0003	-0.0210 0.000 0.0003	0.4175 -0.0012 -0.0058
T W1+T)-1 T	-0.5222 -0.0232 -0.0396 0.4953	0.0018 0.0001 -0.0003	0.0076 0.0003 -0.0000	0.0023 0.0001 0.0003	0.0461 0.0016 0.0002	6,0020 0,0001 -0,0000	0.0018 0.0001 -0.0001	-0.0018 -0.0014 -0.0014 -0.0002
				-0.0022	-0.0-21	-0.0010	-0.0018	0.0300
1	60 5036	Consum	ption (C):	W41				4 3398
$ \begin{array}{c} P-1 \\ W_2 \\ \hline K_1 \end{array} $	-0.1290 -0.2893 -0.2788	0.0104 0.0007 0.0002	-0.0005 0.0042 0.0015	-0.0001 0.0012 0.0013	-0.0002 0.0136 0.0075	-0.0004 0.0012 0.0005	0.0003	1,2208 -0,0018 -0,0093 -0,0058
T ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲ ۲	-1.3647 -0.0787 -0.0723 1.1177	0.0049 -0.0000 -0.0001 -0.0059	0.0115 0.0011 0.0013 -0.0083	0.0065 0.0003 0.0003 -0.0055	0.0728 0.0033 0.0035 -0.0637	0.0035 0.0003 0.0004 -0.0028	0.0014 0.0001 -0.0002 -0.0012	=0.0604 =0.0025 =0.0023 0.0528
		Invest	ment (I):	W42		•		
1 P-1 W2 K_1 T (W1+L)-1 t G	45.7628 -0.0792 -0.1458 -0.2145 -0.6480 -0.0407 -0.0318 0.5167	-0.0623 0.0076 0.0002 -0.0003 0.0024 0.0000 0.0001 -0.0023	-0.1043 0.0004 0.0016 0.0004 0.0097 0.0004 0.0004 0.0085	-0.2179 -0.0002 0.0006 0.0011 0.0028 0.0022 0.0001 -0.0022	-0.5646 0.0031 0.0063 0.0025 0.0566 0.0022 0.0022 -0.0519	-0.0266 0.0001 0.0004 0.0001 0.0025 0.0001 0.0001 -0.0022	-0.0238 0.0001 0.0004 0.0001 0.0023 0.0001 0.0001 -0.0020	$\begin{array}{c} 0.4874 \\ -0.0027 \\ -0.0071 \\ -0.0021 \\ -0.0512 \\ -0.0019 \\ -0.0019 \\ 0.0454 \end{array}$

Table 2. (continued) B. Off-Diagonal Blocks of Cov(P - P)

	1	P-1	W <sub>2</sub>	K_1	T	(W <sub>1</sub> +T)_1	t	G
		Priva	ate wage b	111 (W1):	W/3		· · · · · · · · · · · · · · · · · · ·	
, ,	42 7757	-0 2127	-0 1880	-0 2027	-0 9423	-0 0681	-0 0577	0 6664
. P.	-0 1196	0.0007	0 10002	0.0001	0.0015	=0.0002	- 0.0005	-0.0033
	-0 1571	0 0012	0 0024	0.0007	0.0095	0.0008	-0.0004	-0.0064
E x 7	-0.2030	0.0003	0.0008	0.0010	0.0043	0.0003	0.0003	-0.0031
	-0.7749	0.0058	0.0068	0.0036	0.0572	0.0029	0.0005	-0.0401
+(N+T)	-0.0510	0.0000	0.0007	5000.0	0.0024	0.0002	-0.0001	=0.0019
	-0.0427	5000.0	0.0007	0.0001	0.0025	5000.0	-0.0004	-0.0016
L C	0.5846	-0.0069	-0.0067	8500.0-	-0.0501	-0.0025	-0.0007	0.0395
	1	Const	umption (C	): W <sub>51</sub>			4	
1(	112,4650	-0.0589	-0.6530	-0,5017	-2.5612	-0.1431	-0.1341	2.2085
· Ĕ · P-1	1-0.1284	2020.0	-0.0706	-0,0005	0.0073	0.0007	0.0006	1500.0-
. ຍິ W2	-0.5363	-0.0005	0.0074	0.0022	0.0227	0.0019	0.0004	-0.0169
F K	-0.5141	-0.0009	0.0026	0.0025	0.0120	0.0706	0.0008	-0.0100
	-2.4500	0.00/3	0.0205	0.0116	0.1283	0.0050	0.0025	-0.10/0
Ē(₩1+T)-1	-0,1299	0.0007	0.0020	0,0005	0.0067	0.0005	0.0001	-0,0045
T . T	-0.10/4	0.0004	0.0007	-0.0006	0.0042		0.0009	-U UUSD
Ra	2.1014	-0.0022	-0.0145	-0.0101	-0-1032	•0•0038	-0.0021	0.0934
		Tove	stment (T	)• W				
1	05. 3 - 77			52	1 0504			6 0000
	05.3///	-0.1202	-0,1905	-0.4091	-1-0340	•U•U5U8	-0.0447	-0 -0019
		0.0137	0.0007		0.0042	0.0001	0.0001	
			0.0020	0.0015	0.0151	0.0007	0.0007	
<b>E </b>		-0-0008	0 0173	0.0020	0.0040	0.0002	0.0002	-0 0914
	-1 0439	0.0042	0.0175	0.0003	0.1048	0.0043	0 00041	-0.0033
		-0.0002	0 0004	0 0004	0.0024	0.0001	0.0001	=0.0021
4	1 0119	=0.00002	-0.0152	-0.0044	=0_0940	-0.0040	-0.0036	0.0820
Nat								••••
		Priv	ate wage 1	bill (W <sub>1</sub> ):	W <sub>53</sub>			
	191,5762	0.0730	-0.3784	-0,4175	-1.6080	-0,0744	-0.0747	1.4790
ES P	0 1161	0.0219	-0.0009	-0,0014	0.0043	0.0008	0.0007	ü.0021
DO W	-0.3475	0.0001	0.0045	0.0015	0.0166	0.0012	-0.0006	-0.0130
E K	1-0.4178	-0.0011	Ú.0016	0.0050	0.0074	0.0003	0.0006	-0.0069
:	i <b>−1</b> _4406	0.0085	0,0159	0.0057	0.1017	0.0050	0.0009	=0.ú837
₩(W1+T)_	1-0.0573	0.0010	0.0010	0.0005	0.0045	0.0004	-0.0005	-0.0026
0 1 -	Ė=0.0597	0.0003	0.0005	0.0004	0.0029	0.0001	0.0010	-0.0025
at	q 1.3972	=0.0015	-0,0133	-0.0006	-0.0876	-0.0032	-0.0015	Ů <b>.</b> 0791
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~						••••••		
		Prof	its (P):	W54				
: 월유 1	(106,2665	i =0.2081	-0.4351	-0,4933	-2.0128	-0.1193	-0,1041	1,6344
8 P-1	1-0.3012	2 0.0100	0.0009	-0.0001	0.0072	0.0000	0.0000	-0,0081
5 W2	-0.4600	0.0001	0.0057	0.0020	0.0212	0.0015	J.0017	′ <b>=</b> û,0168
. K_1	-0.4930	-0.0003	0.0018	0.0024	0.0092	0.0005	0.0004	-0.0076
T E	-2.180	0.0030	0.0219	0.0099	0.1314	0,0055	0.0057	-0.1155
ೆ ್ಷ (₩1+T)-	1 -0.1304	-0-0005	<b>U</b> .0016	0.0006	0.0060	0.0004	0.0005	-0,0050
at	t-0,1191	0.0004	0.0006	0.0006	6.0037	0.0005	-0.0001	-0.0032
- <b>Z</b>	q 1.716	-0.0045	-0,0165	•0.0080	-0.1116	-0.0044	-0.0042	0.0983

#### III. Concluding Remarks

This study has formulated an operational procedure for computing the covariance matrix of restricted reduced form coefficients, which are essential estimates for various applications of an econometric model. The procedure is illustrated by applying it to the Klein's model I of the U.S. economy, where some prior information is obtained from Goldberger, et al. (1961). This set of prior information, which is an approximation of the variances and covariances from a structural model estimated by two-stage least squares, may not be the best choice (McCarthy, 1972). However, the purpose of this study is to demonstrate the procedure for obtaining the covariance matrix of restricted reduced form coefficients but not the properties of this particular set of prior information Assuming that the information is obtained from some proper structural per se. estimates, this study applies this classical example so that the results are comparable to those estimates from an alternative procedure of Goldberger, et al. Even though the size of model in this example is rather small, the two procedures yield substantially different results. As mentioned before, since the covariance matrix obtained in this study is derived from a set of exact estimated error relationships, the results thus obtained should be more accurate.

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### Appendix. Basic Prior Information Used in this Study

The following prior information used for computing the asymptotic covariance matrix of reduced form coefficients is obtained from Goldberger, et al.(1961) Klein's model I consists of three stochastic equations and three identities; the structure estimates were computed by Klock and Mennes (1960) using the two-stage least squares:

- $C = 16.5548 + 0.0173 P + 0.2162 P_{-1} + 0.8102 (W_1 + W_2)$ (1.3197) (0.1179) (0.1072) (0.0402)
- $I = 20.2782 + 0.1502 P + 0.6159 P_{-1} 0.1578 K_{-1}$ (7.5225) (0.1728) (0.1624) (0.0360)

 $W_1 = 1.5003 + 0.4389 (Y + T - W_2) + 0.1467 (P + W_1 + T)_{-1} + 0.1304 t$ (1.1471) (0.0356) (0.0388) (0.0291)

$$Y + T = C + I + G$$

 $Y = P + W_1 + W_2$ 

$$K = K_{-1} + I$$

where variables are C (consumption), P (profits), W<sub>1</sub> (private wage bill), W<sub>2</sub> (government wage bill), I (net investment), K (end-of-year capital stock), Y (national income), T (business taxes), t (time measured from 1931 as origin), G (government expenditures). The standard errors are given in parentheses. Appendix Table

# Covariance Matrix of Structural Coefficients

Equation	(1) Co 1	onsumption P	equation P_1	W <sub>1</sub> +W <sub>2</sub>	(2) In 1	ivestment o P	equation <sup>P</sup> -1	к <sub>-1</sub>	(3) Private wage bill equation $1   Y+T-W_2   (P+W_1+T)_{-1}   t$
(1) 1 P = 1 $W_1 + W_2$ (2) 1 P = 1 $K_{-1}$	1.7416 -0.0153 0.0047 -0.0327 2.1462 -0.0459 0.0177 -U.0082	0.0139 -0.0096 -0.0015 0.0589 0.0041 -0.0032 -0.0004	0.0115 -0.0005 0.0207 -0.0047 0.0052 -0.0001	0,0016 -0,0834 0,0013 -0,0012 0,0004	56.5873 -0.9199 0.7735 -0.2676	0.0298 -0.0256 0.042	v.0264 -0,0039	U.0013	
$(3) 1Y+T-W_2(P+W_1+T)_{-1}t$	-0.6106 0.0092 0.0007 0.028	-0.0005 -0.0018 0.0019 0.0007	0.0327 0.0012 -0.0018 0.0005	0.0016 0.0001 -0.0001 -0.0005	0,3607 0.0131 -0.0197 -0,0313	-0.0017 0.0006 -0.0006 0.0005	-0.0152 -0.0003 0.0006 -0.0007	-0.0004 -0.0001 0.0001 0.0002	$\begin{array}{r} 1.3159 \\ -0.0067 & 0.0013 \\ -0.0154 & -0.0012 & 0.0015 \\ 0.0156 & -0.0003 & 0.0001 & 0.0008 \end{array}$

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