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Steady-State Solutions to Soil Salinity Optimization Problems

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This paper is concerned with the profit-maximizing choice of water quantities given the choice of crop and water quality but taking into account the dynamics of salt accumulation over time. The optimization model formulated in this paper is based on a detailed representation of the salt transport process in soils but assumes known or average rainfall levels. Optimal water quantities and soil salinities in steady state were calculated for navel oranges in two areas of California. Water applications sufficient to maintain maximum yields were found to be optimal in many cases.

Key words: navel oranges, optimal control, salinity, water.

A perennial problem in irrigated agriculture is soil salinity. All irrigation waters contain salts, and when these are allowed to accumulate in the soil, they result in reduced yields, with complete curtailment of crop production in extreme cases. Salts can be flushed out of the root zone by applying additional quantities of water at added expense to the farmer. Economic questions of interest concern the appropriate quantities of water to be applied per acre, choice of crops, and the appropriate mix of water from sources of differing qualities. Answers to these questions can provide guidance to individual irrigators as well as address various public policy issues and are of substantial concern in an era of water scarcity and increased salinization of water supplies in some areas.

This paper is concerned with the profit-maximizing choice of water quantities given the choice of crop and water quality but taking into account the dynamics of salt accumulation over time. Previous work by Matanga and Marino and Yaron and Olian has shown how this problem can be solved using dynamic programming under conditions of stochastic rainfall. The optimization model formulated in this paper is based on a detailed representation of the salt transport process in soils but assumes known or average rainfall levels. Because of nonlinearities and high dimensionality, the problem posed here cannot be solved using dynamic programming or other standard approaches. Rather than attempting to solve the full optimization problem, the paper considers only the steady-state solution. The steady-state solution, or optimal steady-state, refers to values for the state and control variables which, once achieved, will be maintained indefinitely under an optimal policy. Its value lies in the fact that the solutions to dynamic optimization problems typically converge to the optimal steady-state. Thus, knowledge of the optimal steady-state indicates the direction in which an optimal policy should be headed although not the rate. (An excellent discussion of optimal steady-states and their application to problems of natural resource management may be found in Burt and Cummings.) An algorithm for computing the optimal steady-state is developed and then applied to navel orange production in California.

The approach developed here can be used to formulate decision rules for water use at the farm level. The advantage is that detailed models of the underlying physical and biological processes can be incorporated into the analysis. The disadvantage is that the decision rules obtained are only approximately optimal in contrast to fully optimal rules obtained from dynamic programming or other approaches. While the specific topic of this paper is soil

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salinity, some conclusions of more general interest can also be noted. First, the results show that optimal steady-states sometimes can be obtained in situations where standard methods are infeasible. Second, dynamic optimization models in agricultural and natural resource economics are often formulated as highly aggregated, lumped-parameter models based on the average characteristics of the resource system under consideration. While the qualitative conclusions from such models are likely to be quite good, the analysis here suggests that the quantitative results may not always be accurate enough to form the basis for explicit policy recommendations.

Model

The optimization problem is to maximize

$$(1) \qquad \sum_{t=1}^{\infty} \alpha^t G(s_t,w_t)$$

subject to

$$(2) \qquad s_{i,t+1} = f_i(s_t,w_t) \qquad i = 1, \ldots, n$$

$$(3) \qquad w_t \geq ET,$$

where $s_{i,t}$ is the soil salinity in layer i ($i = 1, \ldots, n$) at the beginning of year t, s_t is the vector of soil salinities, w_t is the quantity of water applied during year t, ET is a parameter representing annual evapotranspiration from the root zone, α is the discount factor, and G is annual returns net of water costs. (Only perennial crops are considered here.) Expression (1) represents the present value of annual net returns, (2) is the equation of motion for soil salinity in each layer over time, and (3) guarantees that sufficient water is applied to meet crop needs at maximum yield. [Inequality (3) represents an implicit assumption that yield decrements are caused by soil salinity and not by moisture stressing.] Explicit functional forms for G and f_i are described in the following sections.

Let $J(s_t)$ be the present value of annual net returns in all future periods under optimal operation, given that soil salinity at the beginning of t is s_t. From dynamic programming arguments (Bertsekas), J must satisfy the following functional equation:

$$(4) \quad J(s_t) = \max_{w_t \geq ET} G(s_t,w_t) + \alpha_J[f(s_t,w_t)],$$

where $f(s_t,w_t)$ denotes the vector $[f_1(s_t,w_t), \ldots, f_n(s_t,w_t)]$. Let w^* and s^* represent the optimal steady-state water quantity and soil salinities, respectively. Then $w^* > ET$, since otherwise salt builds up in the soil indefinitely, and so inequality (3) holds strictly. The optimality of w^* implies that it must solve the maximization problem on the right-hand side of (4). Differentiating the right-hand side of (4) yields the first-order condition to be solved by w^*:

$$(5) \quad \frac{\partial G(s^*,w^*)}{\partial w} + \alpha \sum_{i=1}^{n} \frac{\partial J[f(s^*,w^*)]}{\partial s_i} \frac{\partial f_i(s^*,w^*)}{\partial w} = 0.$$

Differentiating both sides of (4) with respect to s_i, applying (5), and rearranging yields

$$(6) \quad \frac{\partial J(s^*)}{\partial s_i} = \frac{\partial G(s^*,w^*)}{\partial s_i} + \alpha \sum_{j=1}^{n} \frac{\partial J[f(s^*,w^*)]}{\partial s_j} \frac{\partial f_j(s^*,w^*)}{\partial s_i}.$$

In addition w^* and s_i^*, $i = 1, \ldots, n$, must satisfy

$$(7) \qquad s_i^* = f_i(s^*,w^*).$$

Equations (5), (6), and (7) determine the optimal steady-state solution to the general soil salinity model (1)–(3). These equations represent a system of $2n + 1$ equations in $2n + 1$ variables (w^*, s_i^*, $\partial J(s^*/\partial s_i)$). The system is likely to be quite difficult to solve in general when n is large, G and f_i are nonlinear, and if $\partial G/\partial w$, $\partial f_i/\partial w$, $\partial f_i/\partial s_j$ are discontinuous. The next section discusses the special case where the f_i are specified using relations developed by Bresler. In this case a simple procedure can be devised for solving (5), (6), and (7) when n is large.

Salt Balance Relations and Algorithm

Bresler developed a model for predicting salt concentrations in the root zone after irrigation as a function of salt concentrations before irrigation and the quantity and quality (salinity) of irrigation water. This model can be used to specify the aggregate salt balance relations (2) for a given intrayear irrigation scheduling policy.

Assume that there are T irrigations within the year, and let $w_t^\tau = h^\tau(w_t)$ be the quantity of water applied during irrigation τ as a function of total water use during the year. The functions h^τ are assumed to be given and satisfy $\sum_{\tau=1}^{T} h^\tau(w) = w$ for all possible values of w. Salt concentrations in layer i after irrigation τ in year t are denoted by s_{it}^τ and determined by

$$(8)\quad s_{it}^\tau = s_{it} \qquad \tau = 0$$

$$s_{it}^\tau = \left[w_t^\tau q - \sum_{k=1}^{i-1} (s_{kt}^\tau - s_{kt}^{\tau-1})V_k + \left(V_i - \frac{D_{it}^\tau}{2}\right) s_{it}^{\tau-1}\right] \Bigg/ \left[V_i - \frac{D_{it}^\tau}{2} + w_t^\tau - \sum_{k=1}^{i} ET_k^\tau\right], \qquad \tau = 1, \ldots, T$$

where $D_{it}^\tau = \text{Min}\left(w_t^\tau - \sum_{k=1}^{i} ET_k^\tau, 2V_i\right)$, q is the salt concentration of the irrigation water (a parameter), V_i is the depth of water in soil layer i at saturation, and ET_i^τ is soil moisture deficit (evapotranspiration) from soil layer i before irrigation τ. (Salt concentrations of the soil solution are defined on the basis of a saturated paste extract. Also assume that $w^\tau \geq \sum_{i=1}^{n} ET_i^\tau$ for all τ.) According to Bresler's model, salt concentrations at the end of the year are equal to salt concentrations after the last irrigation; and so the equations of motion (2) are defined implicitly by (8) once the intrayear irrigation scheduling policy has been specified.

An important point to notice is that this model has a recursive structure. Salinity in each layer depends only on salinities in soil layers above it. Given s_t and w_t, salinities throughout the year can be computed sequentially starting with the first layer and irrigation. The recursive structure also implies that $\partial f_i(s_t,w_t)/\partial s_j = 0$ for $j > i$.

These facts can be used to design an efficient iterative scheme for solving (5)–(7). First, choose some initial value for w which is sufficient to meet or exceed moisture deficits at each irrigation. For a fixed value of w, (8) is linear in the salinities. Steady-state salinities can then be readily computed for each layer starting with the first. (In the steady-state, beginning and ending salinities will be identical. Salinities within the year will not necessarily be equal to beginning and ending salinities but will be identical to salinities measured at the same time during succeeding years once the steady-state is achieved. The system is in steady-state in the sense that the time path of salinities throughout the year remains the same from one year to the next.) Given the steady-state salinities, the derivatives of f and G with respect to initial salinities can then be evaluated.

The next step is to solve (6) for $\partial J/\partial s_i$ as follows. Since $\partial f_j/\partial s_i = 0$ for $j < i$, solve first for $\partial J/\partial s_n$, then for $\partial J/\partial s_{n-1}$, and so on until the entire subsystem (6) is solved. Next, evaluate $\partial f_i/\partial w$ and $\partial G/\partial w$, and finally calculate the left-hand side of the first-order condition (5). If this is sufficiently close to zero, then the optimal steady-state has been reached. If the left-hand side of (5) is positive (negative), then increase (decrease) w and repeat the procedure.

Computational experience with this algorithm has been good. A scan over plausible values indicated a single solution. As w increased, the left-hand side of (5) steadily decreased (assuming average salinity within the range between maximum yield and zero yield). Convergence was obtained within arbitrarily small limits rapidly and cheaply and for many soil layers.

Data and Model Specification

The model is applied to navel orange production in two regions of California. Oranges are considered to be a relatively sensitive crop with respect to soil salinity (Maas and Hoffman). Annual net revenues are defined by

$$G(s_t, w_t) = pg(s_t, w_t) - c(q)w_t,$$

where p is the output price, $g(s_t, w_t)$ is the salinity response function, and $c(q)$ is the per-unit cost of water with salt concentration q. A simple average of salt concentrations in each layer at the beginning of the year and after each irrigation is used to estimate crop yields. Following Maas and Hoffman, maximum yield is achieved for average root zone salinity levels up to a salinity level s^l. Yield declines linearly after this level until average root zone salinity reaches an upper limit s^u, after which yield is zero. Soil layers are assumed to be of equal depth and composition. Therefore, V_i is calculated as V/n, where V is

Table 1. Water and Crop Data, Navel Orange Production, California

Parameter[a]	Description	Kern County	Coachella Valley
P	Output price	\$311.50/ton	\$332.50/ton
s^l	Salinity at maximum yield	17.5 meq/l[b]	17.5 meq/l
s^u	Salinity at zero yield	96 meq/l	96 meq/l
Y^{max}	Maximum yield	5.37 tons/acre	5.75 tons/acre
V	Water content in root zone at saturation	.75 feet	.75 feet
q	Salt concentration of irrigation water	7.1 meq/l	14.0 meq/l
i	Annual interest rate	.05	.05

[a] P and Y^{max} are taken from Agricultural Commissioner's Reports and are computed as an average of 1979 and 1980; s^l and s^u are taken from Maas and Hoffman; V is soil moisture content at saturation times root zone depth; root zone depth is taken from the California Department of Water Resources; soil moisture content at saturation is taken from a local soil and water specialist; and q is taken from Rhoades.

[b] Meq/l is milliequivalents per liter.

water content in the root zone at saturation. Crop data and salt concentrations of irrigation waters are listed in table 1.

Six different irrigation scheduling policies are considered, depending on the time at which irrigation occurs, the distribution of water uptake by the plant from the root zone, and the quantity of water applied at each irrigation. Four of the policies assume that irrigation occurs whenever the soil moisture deficit reaches a specified level determined as annual evapotranspiration divided by the number of irrigations. Two of the policies assume that irrigation occurs at equal time intervals during the year, in which case soil moisture deficits at irrigation differ from one irrigation to the next. Two different water uptake distributions are used. The first is a uniform water uptake distribution, and the second is an exponential water uptake distribution. (These distributions are used to calculate moisture deficits in individual layers at the time of irrigation.) The irrigation scheduling policies also differ in terms of the quantity of water applied at each irrigation. Two strategies for determining water quantities at irrigation time are used. The first assumes equal leaching fractions at every irrigation. [Leaching fractions are defined as $(W - ET)/W$.] This implies equal water quantities when irrigation occurs at equal moisture deficits but unequal water quantities when irrigation occurs at equal time intervals. The second strategy assumes that all leaching occurs at the beginning of the year (first irrigation), with water quantities at succeeding irrigations just sufficient to cover moisture deficits. In every case, twelve irrigations during the year are assumed. The specific policies used are given in table 3. Evapotranspiration data used to calculate moisture deficits at the time of irrigation are listed in table 2.

Results

The results are reported in table 3. The first two columns detail the irrigation scheduling policies used and the number of soil layers. The third column shows average annual salinities in the steady-state for a specified quantity and quality of irrigation water (assuming other conditions as in the Coachella Valley). Columns four through seven show optimal steady-state levels of water applications and average annual soil salinities for two water prices in each region. Water prices of \$25.00 per acre foot and \$75.00 per acre foot are within the current range of water prices in

Table 2. Monthly Evapotranspiration, Navel Oranges, California

Month	Kern County[a]	Coachella Valley[b]
	(inches)	
January	.9	1.3
February	1.7	1.7
March	1.9	3.2
April	2.7	4.2
May	3.9	5.4
June	4.5	6.8
July	4.7	6.5
August	4.0	5.8
September	2.9	4.7
October	2.0	3.4
November	1.5	1.9
December	.7	1.2
Total	31.4	46.1

[a] California Department of Water Resources.

[b] ET data for the Coachella Valley is not available. Values reported here are for the Imperial Valley.

Table 3. Optimal Steady-State Levels of Annual Water Quantities and Average Annual Soil Salinities, Navel Oranges

Irrigation Scheduling Policy	Number of Layers	Average Soil Salinity (meq/l) w = 5.0 a-f/acre q = 14 meq/l	Optimal Steady-State Water Quantities (a-f/acre/year) (Average Steady-State Soil Salinities (meq/1))			
			Kern County		Coachella Valley	
			$25/a-f	$75/a-f	$8/a-f	$35/a-f
A1	1	60.43	4.40 (17.5)	3.55 (27.0)	15.19 (18.74)	7.70 (27.90)
	4	33.38	3.35 (17.5)	3.35 (17.5)	12.57 (17.5)	7.88 (20.96)
	10	29.13	3.12 (17.5)	3.12 (17.5)	11.23 (17.5)	7.46 (20.43)
	20	27.85	3.03 (17.5)	3.03 (17.5)	10.78 (17.5)	7.29 (20.28)
B1	20	27.85	3.03 (17.5)	3.03 (17.5)	10.79 (17.5)	7.29 (20.28)
C1	20	36.08	3.48 (17.5)	3.48 (17.5)	13.83 (17.52)	8.31 (21.33)
A2	20	54.73	3.27 (23.23)	2.93 (26.06)	5.68 (52.95)	4.72 (56.12)
B2	20	57.42	3.22 (24.47)	2.92 (26.98)	5.42 (56.35)	4.64 (59.11)
C2	20	61.79	3.82 (24.33)	3.27 (27.86)	7.10 (54.15)	5.49 (58.41)

Notes: Optimal steady-state levels of average soil salinities are in parentheses. All soil salinities are measured as the concentration of a saturated paste extract. Irrigation scheduling policies are as follows: A is irrigation at equal moisture deficits, uniform water uptake distribution; B is irrigation at equal time intervals, uniform water uptake distribution; C is irrigation at equal moisture deficits, exponential water uptake distribution; 1 is equal leaching fractions for water applied; 2 is all leaching during first irrigation.

Kern County. The current price of water in the Coachella Valley is roughly $8.00 per acre foot. A price of $35.00 per acre foot represents an estimate of the value of this water in alternate uses (Howitt, Mann, and Vaux).

Ignoring for the moment the runs with less than twenty layers and assuming equal leaching fractions at each irrigation (policies A1, B1, C1), we see that generally enough water is applied to maintain maximum yields. This holds for both prices in Kern County and the low water price in the Coachella Valley. Yield reductions of 3%–5% are suffered in the Coachella Valley at water prices of $35.00 per acre foot, depending on the scheduling policy being considered. In theory, profit maximization and yield maximization occur at different input levels. These results show that in some situations this difference may be negligible. More substantial yield reductions are incurred for those scheduling policies with all leaching at the beginning of the year (A2, B2, C2). Yield losses at the optimal steady-state range from a low of 7% (Kern County, $25.00 per acre foot) to a high of 53% (Coachella Valley, $35.00 per acre foot). Using the data in table 1, the loss in profits from policies with all leaching at the beginning of the year is substantial compared to the policies where leaching is spread throughout the year. These results suggest that all leaching at the beginning of the year is an inferior policy for the situations considered here.

Economic intuition suggests that water quantities will decline as water prices increase. This proposition is generally supported by the results in table 3. In the Coachella Valley an increase in water prices from $8.00 per acre foot to $35.00 per acre foot results in reduction of optimal steady-state levels of annual water quantities ranging from 14% to 40%. Maximum yields are maintained for policies A1, B1, and C1 in Kern County so optimal water quantities are not reduced with an increase in price in these cases. Reductions of 9%–14% are obtained for policies A2, B2, and C2 in Kern County. Another factor determining profit-maximizing water quantities is the salt concentration of the water. At a water price of $25.00 per acre foot, leaching fractions in Kern County range from 9%–25% depending on the scheduling policy, while leaching fractions in the Coachella Valley at $35.00 per acre foot range from 17%–54%. While the results are not directly comparable because of differences in crop prices and yields, they suggest that increased salt loads imply significant increases in water quantities.

Table 3 shows the effects of dividing the root zone into different numbers of layers for one scheduling policy (A1). The movement of salt and water in the root zone is a continuous process, and so the more layers considered the better the approximation. Predicted average salinity for a water quantity of 5 acre feet per acre per year is 60 milliequivalents per liter (meq/l) with one layer and 28 meq/l with twenty layers. Thus an error of over 100% can be made by assuming a single layer. The differences in optimal steady-state levels of water quantities and soil salinities are smaller but still substantial. Depending on the area and water price, optimal water quantities with one layer are 6%–45% greater than optimal water quantities with twenty layers, while optimal steady-state levels of soil salinities with one layer differ by as much as 54% from those with twenty layers. These results suggest that some minimum level of disaggregation in the salinity problem is necessary if accurate solutions are to be obtained.

Concluding Remarks

This paper has illustrated a procedure for determining the steady-state solution to dynamic optimization problems relating to soil salinity. The procedure is based on a specific physical model of the soil salinity process (Bresler), but it can be applied to other models in which the movement of water and salt is primarily downward—a reasonable assumption in many situations. The solutions can be used in several ways to guide decision making at the farm level. Optimal steady-state water quantities can be applied in each year, with salinities eventually reaching the associated steady-state level. Another option is to leach down immediately to the optimal steady-state level and then maintain salinities at that level using the steady-state water quantity. A third option is to construct an approximately optimal decision rule using the steady-state values of the first derivative of the optimal value function as estimates of the marginal user cost of soil salinities. Which alternative is best can be evaluated using simulation.

Optimal water quantities and soil salinities in the steady-state were calculated for navel oranges in two areas of California. Water applications sufficient to maintain maximum yields were found to be optimal for both prices in the region with good quality water and for the current water price in the region with low water quality. (This result assumes uniform water applications in the field. It is not likely to hold under conditions of nonuniformity.) As a result of various legal and institutional barriers to water transfers, water often remains trapped in low-valued agricultural uses. When the price of water in the Coachella Valley is increased from its current level to an estimate of its value in alternate uses, water quantities decline by 17%–40% depending on the irrigation scheduling policy. Profits are substantially lower for the policies with all leaching at one time, suggesting that, for the situations considered here, leaching at several different times throughout the year is desirable. Finally, the solutions vary by as much as 50% depending on the level of aggregation used in specifying the soil salinity process. The implication is that including detailed descriptions of underlying biological and physical processes in economic analyses of agriculture and natural resources may have substantial payoffs.

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