# On the Measurement of Risks and Returns of Hedging with Options



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# On the Measurement of Risks and Returns of Hedging with Options

The newly traded options on agricultural futures offer an opportunity to create a countless number of risk and return profiles to agricultural producers and agribusinesses attempting to reduce price risk. The general purpose of this paper is to suggest a method which can be used to estimate the expected risks and returns of basic option-hedging strategies under different assumptions regarding the decision maker's price distributional expectations and risk preferences. Subsets of possible option strategies are then defined with a dominance criterion.

The paradigm under which risks and returns are measured was developed by Fishburn and by Holthausen. In the Fishburn-Holthausen model, return is a function of outcomes above a target level whereas risk is a function of outcomes below the target. Simulated in this study are cases in which a cattle feeder is choosing among what are assumed to be three mutually exclusive pricing strategies, including the purchase of a put, writing a call, or keeping an open (unhedged) position. The results are interpreted in two contexts. The first context emphasizes the market's expectation of a strategy's risk and return, offering objective measures which are available through the use of readily attainable input data. The second context is less objective in that risk preference parameters, price expectation, and price-variance expectation are defined to enable dominance analysis.

## I. Methodology

Option-hedging strategies are often defined in terms of effective price or return opportunities (e.g., Gammill and Stone; Kenyon (1984a); McKissick, Shumaker, and Williams; and many others). Quantification of the risks and returns of these strategies has been based on revenue streams generated under past conditions (e.g., Catlett and Boehlje; Hudson, Hauser, and Fortenbery; and Kenyon (1984b)) or on streams under various assumptions regarding price and/or production scenarios (e.g., Gardner; Ikerd; and Kenyon (1984c)). These types of analyses provide insight to the available strategies and to the sensitivity of the strategies' outcomes within select ranges of underlying factors. The focus of this paper is on measuring risks and returns of pricing techniques with a model which is more general in terms of price risk than those of the above studies and which relies on current market information to form the probabilistic characteristics needed. The method used to measure risk and return is described verbally and graphically in this section; the approach is described mathematically in the appendix.

Holthausen, in an extension of Fishburn's risk-return model, suggests that expected return to the decision maker should be based on outcomes above a target level, whereas expected risk should be based on outcomes below the target level. Holthausen then presents specific functions for risk and return based on this target-deviation concept which have an underlying utility function that is consistent with the von-Neumann-Morgenstern axioms for expected utility (see appendix for the risk, return, and utility functions). In this context, risk can be thought of as a weighted average of transformations of the absolute deviations from the target, given outcomes below the target. Likewise, return is a weighted average of transformations of the absolute deviations from the target, given outcomes above the target. Each deviation below the target is transformed by taking the deviation to the power $\alpha$; each return deviation is raised to the power $\beta$ ($\alpha>0$ and $\beta>0$). The values of $\alpha$ and $\beta$ reflect the decision maker's risk attitude. An individual is risk averse (seeking) over outcomes above the target t if $\beta$ is less (greater) than one, and the individual is risk averse (seeking) below t if $\alpha$ is greater (less) than one.[1]

Given the target, t, and the risk parameters, $\alpha$ and $\beta$, Holthausen argues that the $\alpha$-$\beta$-t model captures many of the features of empirically estimated utility functions. After converting various utility functions from other studies into $\alpha$-$\beta$-t terms, he finds that almost all of the functions have a point at which the shape changes markedly and that many combinations of risk-averse, risk-neutral, and risk-seeking behavior are displayed (p. 185). See Fishburn and Kochenberger for a similar analysis.

The $\alpha$-$\beta$-t model is used in this study under the assumptions that (a) a cattle feeder is choosing a pricing technique at current time $i_o$ for cattle which will be sold in five months at time I ($I-i_o=i$), (b) costs and production are certain and all net-revenue uncertainty is associated with the effective price received for the cattle, (c) options considered expire in five months, (d) the futures contract on which the options are offered has a current price of $60 per cwt., (e) the producer's target price is $60, and (f) the futures price follows a lognormal diffusion process.[2]

The estimation of expected risks and returns associated with simple option-hedging strategies under the above assumptions can be illustrated with the aid of Figures 1 and 2. Figure 1 is the expected lognormal density function of ending futures prices used to illustrate the put-purchase alternative. When the decision to hedge is made at time $i_o$, the hedger expects the ending futures price, $F_I$, to have the distribution shown. The hedger's expected return from buying a put is a function of those $F_I$ which are larger than the target plus the premium adjusted for time value; i.e., those $F_I > 60 + Pe^{ri}$, where P is the put premium and $e^{ri}$ is the appropriate discount factor. Since the hedger pays P at time $i_o$, $F_I$ must be greater than the target, t=60, by at least $Pe^{ri}$ to yield a price associated with return in the $\alpha$-$\beta$-t model. (We are assuming here that the basis (futures minus cash price) is zero and known. A non-zero basis expectation in itself can be reflected with a simple adjustment; however, in most cases, basis risk complicates the density function considerably.) The expected return, therefore, is found by taking each price under the unshaded area in Figure 1, subtracting $60+Pe^{ri}$ from this price, transforming this difference to reflect risk behavior; i.e., $(F_I - 60 - Pe^{ri})^{\beta}$, and then taking the weighted average of these transformed differences by using the probabilities associated with the $F_I > 60 + Pe^{ri}$.

Risk is defined with those $F_I$ which are less than $60+Pe^{ri}$ and is of two parts. When $F_I$ is less than the exercise price, X, then the hedger exercises the option, receiving an effective price of $X - Pe^{ri}$. This component of the risk is found by calculating the weighted average of the constant $(60 - X + Pe^{ri})^{\alpha}$ for $F_I$ corresponding to the lightly shaded area in Figure 1. When $X < F_I < 60 + Pe^{ri}$, the option is not exercised and the effective price is less than 60 by $60 - F_I + Pe^{ri}$. Thus the weighted average of the $(60 - F_I + Pe^{ri})^{\alpha}$ values for $F_I$ in the interval under the darkly shaded area is the second component of the put-purchase risk. Note that a darkly shaded area must exist for all strikes if the premium is at least as large as the option's "intrinsic value". Total risk is found by summing the two risk components associated with the light and dark areas. The expected risk and return described above, as well as those for the other pricing alternatives, are stated mathematically in the appendix.

Figure 2 helps to describe the risk and return expected when selling a call. In this case, the return is comprised of two parts and the risk is of one. Since the hedger receives premium C at time $i_o$, risk is a function of $F_I$ under the unshaded area of Figure 2 as each of these prices yields an effective price $(F_I + Ce^{ri})$ which is less than 60.[3] Hence, risk is the weighted average of the $(60 - F_I - Ce^{ri})^{\alpha}$ values, given $F_I < 60 - Ce^{ri}$. When $F_I > X$, the option is exercised and thus the effective price is $X + Ce^{ri}$, generating one return component which is the weighted average of the $(X + Ce^{ri} - 60)^{\beta}$ values under the lightly shaded area of Figure 2. The second return component is the weighted average of the $(F_I + Ce^{ri} - 60)^{\beta}$ values under the darkly shaded area. Again, a darkly shaded area must exist, given the call is valued at no less than its intrinsic value. Expected total return from selling a call is found by summing the two return components.

The expected risk and return estimates for the open or unhedged position are, respectively, the weighted average of $(60 - F_I)^{\alpha}$ values when $F_I < 60$ and the average $(F_I - 60)^{\beta}$ for $F_I > 60$.

To simulate the use of the three pricing alternatives, the density function of the ending futures, $L(F_I)$, and option premia are required. Derivation of the density relies on the assumption that the futures price follows a lognormal diffusion process, implying that the first difference in the natural logarithm of daily futures is normally distributed; i.e.:

$$(1)\quad \ln(F_d)-\ln(F_{d-1}) = \mu + \sigma Z,$$

where $F_d$ is the closing futures price at time d; $\mu$ is the mean logarithmic futures return per day; $\sigma$ is the standard deviation of the logarithmic return per day; and Z is the standard normal random variable. Given values for $\mu$, $\sigma$, and i, the expected lognormal distribution of $F_I$ at time $i_o$ can be found in a fashion analogous to the distribution projection underlying traditional option-pricing models.[4] The resultant lognormal distribution is used as the weighting function in the calculation of the weighted averages described above; i.e., equations A4 through A9 of the appendix are solved with the resultant lognormal densities. The solutions are found by numerical integration.

Considered in the next section, among other scenarios, are cases in which (a) the producer's price expectation is different than the current futures price and (b) the producer's variance expectation is different than that implied by the option premium. To derive reasonable estimates of the base-case variance and of the range in variance expectations, we considered variances observed during the first seven months of option trading on June live-cattle futures (November 1984-May 1985). Shown in Figure 3 are weekly averages of three types of variances--the 30-day historical variance, the volatility implied by Black's option pricing model, and the ex-post variance realized during the remainder of the option's life. Each variance is expressed in annualized standard deviation percentage units; i.e., the variance of the daily log-price changes is multiplied by 365 and the square root of this product is multiplied by 100. The most recent 30 trading days are used to calculate the historical variance of June live cattle prices. The implied volatilities are derived with the closing futures price, closing premium, and an annualized interest rate of eight percent.[5] Only those strikes which traded at least ten times during the day were considered to maintain market representativeness. The realized variance was calculated each day with the cattle price series which begins the next day and ends at option expiration. The latest price observation used is 14 days before expiration.

The historical variance and implied volatility can be thought of as two forecast estimates of the realized variance. The historical variance is a naive forecast insofar as the historical variance reflects the traders' expectation of future variance, whereas the implied volatility represents the market's forecast insofar as Black's model is correct. Note that during the first ten weeks of trading the historical variance was well below the market's average implied volatility, suggesting that the market did not believe that this historical variance represented future variance. However, historical variance and implied volatility rose together during the latter half of the period. For this particular contract and time period, the market consistently underestimated the ex-post realized variance. The average bias during the first 14 weeks was 2.5 units and the average bias during the last 14 weeks was 5.2 units.

Based on the general variance level and forecast bias observed during the first few months of cattle-option trading, the base case used for option-hedging simulation is defined with an annualized variance expectation of 16 while in the non-base cases, scenarios are considered in which the producer's expectation varies from the market's by four units. Although these estimates reflect recent variance levels, it is our opinion that the base-case level of 16 is relatively low given, for instance, that the average 40-day variance of the December live-cattle contract during 1973-1982 was about 26. Furthermore, past cattle price behavior suggests that the range used for expected volatility is reasonable if not conservative as the standard deviation of the variance is over nine.

The mean of the base case's price distribution is assumed to be the \$60 current futures price. In the base case, the producer's price expectation is also \$60 but in two other cases it is assumed that the producer expects the price to be either \$55 or \$65. Assuming that the expected mean for the ending futures price is the current price (i.e., $\mu = -\sigma^2/2$) then this \$5 deviation is well within the one standard deviation range of approximately \$52-69, calculated on the basis of the log-price return variances observed in the 1973-82 December futures cattle price series.

Option premia used for risk-return measurement are estimated by using an offshoot of Cox, Ross, and Rubinstein's binomial model with the log-normal approximating parameters suggested by Jarrow and Rudd (p. 188). This numerical technique was used instead of Black's futures option pricing model to avoid the boundary condition problem of Black's model (see Hauser and Neff).

The target-deviation weights ($\alpha$ and $\beta$) used were chosen to reflect various levels of risk averse and seeking attitudes. The simulations are not limited to risk-averse behavior because of the diverse behavior characteristics found by Holthausen within the $\alpha$-$\beta$-t context and supported by Young's (1979) survey of risk studies of Australian and American farmers in which "...approximately 50% of the sampled individuals manifested risk-preferring attitudes over at least some ranges when the measurement technique did not preclude this possibility."(p. 1067).

The target-deviation approach for describing risks and returns of option hedging was chosen for several reasons. First, as mentioned above, the underlying utility function is very flexible, allowing risk behavior to change at a particular inflection point. Holthausen provides empirical evidence supporting the need for this theoretical flexibility. Second, even without theoretical justification, the measurements are easy to interpret in that "risk" represents the average deviation below a price whereas "return" is an average deviation above a price. Whether these averages are viewed as risk and return or as simply an assessment of alternative portfolios, the information is useful to educators and hedgers by providing quantitative expectations across alternative hedging strategies. The third reason is that this measurement abstracts from the price floor and ceiling effects of put and call hedges, highlighting the point that call hedging can be used to create expected-deviation profiles that are similar to those which can be produced through put hedging. This aspect of option hedging is often ignored in educational materials while emphasis is usually placed on the fact that short hedging with puts results in a price floor without "downside-price risk" whereas call hedging creates a price ceiling with downside risk. If these characteristics are important, then they can be reflected in the target-deviation approach through the values of $\alpha$ and $\beta$. For instance, if the producer considers the price-floor effect of put hedging to be very beneficial, then this attitude can be reflected by increasing the value of $\alpha$; i.e., weighting deviations below the target heavily. However, because risk attitudes are difficult to define for either individuals or groups, the fourth reason for choosing this approach is that it is very amenable to displaying the market's expected price deviations. This is done by setting $\alpha = \beta = 1$ and by assuming that the option's implied volatility represents the market's variance forecast and that its price forecast is the current futures price, enabling one to define the market's forecast of the ending distribution. As a base case, these measurements reflect the market's expectation of the effective price deviations above and below the target in a probabilistic framework. Hence, the base estimates provide objective information to extension specialists, classroom instructors, exchange representatives, and others who wish to portray the risk-return profiles created by option hedging.[6]

## II. Results

Expected risk and return for each pricing alternative under ten scenarios are presented in Table 1. The base case (case 1) assumes $\alpha=\beta=1$, the producer's expected $F_I$ is the current price, and the producer's expected price variance is that of the market's and that used to price the options. Although option hedgers are well aware that options reduce risk and return, these estimates provide the market's judgement on the reductions.

The expected price deviation above as well as below 60 is 2.47 when not hedging in the base case. Risk can be reduced from 2.47 to 2.43 by using a put with a \$50 strike price, whereas the use of a \$70 put reduces risk to .37. This expected risk of 37 cents primarily reflects the difference between the \$60 current futures price and the effective price floor set by hedging with the put. The discounted premium ($Pe^{ri}$, where r=.08 and i=5/12) for the \$70 put is 10.39, meaning that a price floor is in effect at 60-.39. Since .39 is the risk associated with each outcome $0<F_I<70.39$

and since the probability of $F_I>70.39$ is very small, the expected risk of .37 is about equal to .39. Note that the undiscounted premium, P, is only 10.05 and thus most of the risk for the deep-in-the-money put hedge is due to interest payment or opportunity cost of the premium outlay. At the other extreme, the deep-out-of-the-money put with strike 50 has a premium of .085 and a risk of 2.43 which is comprised mostly of the average price deviation associated with $50<F_I+60+Pe^{ri}$.

When hedging with calls in the base case, risk can be reduced to .07 by selling the deep-in-the-money $50 call. This relatively small risk reflects the small probability that $F_I$ will be less than $60-Ce^{ri}$. The return from the $50 call hedge is analogous to the risk of the $70 put hedge in that it is comprised mostly of interest accrual. This is so because the return for each $F_I$ greater than 50 is $50+Ce^{ri}-60$ (.365 in this case) and therefore the expected return of .35 reflects the large probability of receiving the option's time value (.025) plus the interest on the 10.025 premium (.340).

Risk and return for the base case decrease (increase) when using puts (calls) as the strike increases, and, in general, the put (call) risk is slightly larger (smaller) than the respective return. The risk would equal the respective return had Black's model been used to estimate the premium as this model uses the same projected ending price distribution as that used in determining the risk and return. However, the general results and conclusions discussed below do not change when Black's estimate is substituted for our binomial estimate.

The focus of the analysis is on the identification of hedging strategies which are dominated under the $\alpha$-$\beta$-t dominance criterion given by

Holthausen (p. 183). A strategy, $S_1$, dominates another strategy, $S_2$, if the risk of $S_1$ is not greater than the risk of $S_2$, and the return of $S_1$ is not less than the return of $S_2$, and at least one of these inequalities is strict.[7]

In the base case, the put hedges using strikes 68 and 70 are dominated by two call-hedge alternatives. For example, the $70 put is dominated by the $50 call because the call risk (.07) is less than the put risk (.37) and the call return (.35) is greater than the put return (.18). The $70 put is also dominated by the $52 call and "almost" dominated by the $54 call. While it is recognized that, in theory, the "extent" of domination should not be evaluated (i.e., either an alternative is dominated or it is not dominated without cardinal measurement), from a practical viewpoint, it is difficult to argue that the risk (return) of .38 (.49) of the $54 call, for instance, is clearly preferrable to the $68 put's risk (return) of .44 (.31) given the small differences in the risks and returns. In addition, the risks related to early exercise and margin calls associated with call writing (see footnote 3) but not included in this analysis would tend to reduce the number of calls which dominate. It is interesting to observe, however, what happens to the number of alternatives dominated and the "extent" of domination when the decision maker's expectations or risk behavior change.

Domination results for each case are summarized in Table 2. Consider cases 1-5 for which $\alpha=\beta=1$. In case 1, where the producer's distributional expectations are those suggested by the market, the two deep-in-the-money puts are dominated because, conceptually, the interest charge on the premium outlay represents risk for put hedging but return for call hedging.

For strikes less than 68, the interest-charge effect is less important and the puts are not dominated. When the producer's price expectation differs from the market's (cases 2 and 3), the set of put strikes dominated changes as price expectation changes, but none of the call options are dominated. At first glance, it is not intuitively appealing that put strikes are dominated when the producer expects a downward drift in price (case 2). However, note that the expected return of the open position in case 2 is only .66 and that the expected risk is 5.66, reflecting the small subjective probability that the producer expects $F_I$ to be larger than the $60 target. Thus, particularly for the low-strike cases in which the risks and returns of put hedging are quite sensitive to shifts in the expected distribution, puts are dominated because about the same return can be realized with call hedging but with considerably less risk. The significance of this result and other dominance results in cases where risks are very large should probably be tempered a bit from a practical standpoint because it would not take much risk aversion to cause hedging with futures to be the optimal hedging decision. In contrast to the low-price expectation scenario, the high-price expectation scenario (case 3) results in call domination because, in general, the returns are much higher given similar risk levels.

When the price variance expectation of the decision maker differs from the market's, puts tend to be dominated when the expectation deviates downward (case 4) and calls tend to be dominated when the producer expects a higher variance than the market (cases 5). Intuitively, this is appealing in that if the producer expects a smaller variance, ceteris paribus, then he believes that the premium is too large, encouraging call writing;

whereas, a large variance expectation relative to the market's expectation implies that he believes the option is underpriced and that put purchases are a "good deal". All put strikes are dominated in the low-variance case and all call strikes but one are dominated in the high-variance case. Furthermore, examination of Table 1 reveals that the "extent" of domination in these cases is very large. Based on these results, it is apparent that the producer's expectation of price volatility is an extremely important factor in determining how options will be used when price hedging. Thus, an important result of this analysis is in separating the price drift and variance effects and showing that the producer's implicit expectation of variance is as important if not more important than his expectation of price when choosing among option strategies.[8]

Cases 6-10 are scenarios in which the producer's $\alpha$ and $\beta$ are not one. For case 6, where $\alpha$ is .5, the producer's utility function (see appendix) is convex over outcomes below the target, implying a risk-seeking attitude. When $\alpha$ is 1.5 (case 7) the producer is risk averse over below-target outcomes. Another interpretation of these parameters can be placed in the context of the producer's attitude toward the price-floor effect of put hedging. For instance, a large $\alpha$ may reflect the producer's desire to have a minimum-price guarantee. In either the risk-behavior or price-floor contexts, the dominance results for cases 6 and 7 are intuitive. When $\alpha$ is 1.5 put hedges tend to dominate call hedges whereas, when $\alpha$ is .5, put hedges are dominated. For case 8 ($\beta$=.5) and case 9 ($\beta$=1.5), interpretation of the dominance results is more intuitive in a price-ceiling context than in the risk-behavior context. When $\beta$ is .5, relatively little weight is placed on above-target price deviations and thus the price-ceiling effect of call hedging does not cause calls to be dominated. When ß=1.5, calls are dominated by puts because a large exponential weight is placed on above-target price deviations. However, since a decreasing ß implies increasing risk aversion, it is not intuitive that puts should become less attractive relative to calls as risk aversion increases. This is not intuitive because risk aversion is usually described for an entire distribution (for instance, variance in E-V analysis) whereas ß applies to only part of the distribution. This result highlights an important feature of our risk and return measurements and, in this context, the importance of specifying whether a risk behavior description is for outcomes below or above the target. That is, the general contention that as the producer becomes more risk averse, puts become more attractive holds in the α-ß-t model for risk aversion below the target but, for outcomes above the target, increased risk aversion causes puts to be less attractive.

Case 10 (α=.4 and ß=.8) was simulated because this reflects Holthausen's rough transformation of Halter and Dean's utility function of net worth for grain farmers.[9] If, as in case 10, the farmer is a strong risk seeker for outcomes below the target (α=.4) but risk averse for outcomes above the target (ß=.8), then he will tend not to use puts.

In summary, these findings suggest that call hedging, relative to put hedging, is the preferred marketing alternative under many realistic and perhaps probable conditions regarding the producer's price expectation, variance expectation, and risk preference. Furthermore, call hedges dominate the no-hedge position in cases 2, 4, and 8.

Presented earlier was evidence that over a long period of time, the cases simulated may be atypical in that the base variance is relatively low. However, simulations of the same price and risk preference conditions under a larger variance (26) reveals that the general results presented in Table 2 still hold. While the risk and return levels are at higher levels, the only major difference in the results is that put strikes are less likely to be dominated when price expectation is greater than the market's (case 3) and that no puts are dominated when $\alpha$=1.5 (case 7). The fundamental reason why these differences exist is that the interest-charge effect plays a smaller role in determining put risks.

## III. Concluding Remarks

The general objective of this paper is to introduce a technique for estimating option-hedging strategies' risks and returns expected by the market in a probabilistic sense and by individual decision makers under various assumptions concerning distributional expectations. Given an individual's risk preferences, the option strategies can be evaluated in this risk-return framework through the $\alpha$-$\beta$-t model dominance criterion. Two points of the dominance results should be highlighted. First, under risk-neutrality, the decision maker's expected price drift affects the set of put strikes dominated but does not cause calls to be dominated. The individual's expected price variance, however, causes both puts and calls to be dominated under risk neutrality and often the "degree" of domination is large relative to the expected-price scenarios. Therefore, in terms of types of options dominated and the "degree" of domination, variance expectation is as important if not more important than price expectation. The second point is that care should be taken when describing the relationship between risk attitude and probable option strategies. As the producer becomes more risk averse over outcomes below the target, puts become more attractive. However, perhaps contrary to intuition, a risk-aversion increase for outcomes above the target causes puts to be less attractive.

Regardless of risk attitude of individuals, the model can be used to ascertain the market's expectation of risk and return by setting $\alpha=\beta=1$ and by estimating the market's expectation of the ending futures price distribution. This distribution can be generated under two plausible assumptions. First, the market expects no drift in price. In theory, risk premium arguments based on backwardation or contango arguments might be given to dispute this assumption but, given the weak empirical evidence of risk premiums and the difficulty of measuring them, this issue does not seem to be of much practical importance. The second assumption is that one can determine the market's expectation of the variance of the price diffusion process by calculating the implied volatility from option premiums. Given the market's drift and variance expectations, the model's risk and return measurements can be interpreted as the market's expected deviation below and above the target, respectively, for different pricing strategies. As opposed to more conventional measurements in, say, an EV framework, these risk and return concepts may be more meaningful to the decision maker.

It should be emphasized that the purpose of this paper is to introduce the measurement technique and that there are many variations of the empirical procedure used which analysts may want to consider. In our analysis, the individual's expectations of variance was based on the general level and forecast performance of implied volatilities observed during the first seven months of cattle option trading. More detailed analysis of implied volatilities over a longer period of time is needed to determine the reliability of the market's variance forecast and the factors such as season, price level, contract, time to maturity, etc. which affect this forecast. Given the importance of the producer's variance forecast relative to the market's in the above assessment of option hedging strategies, this area of research may be most beneficial to educators and extension specialists when evaluating alternatives.

A second area of study might involve the simulation of additional pricing scenarios. Of interest here are the effects of varying time lengths and targets, using combinations of futures and options, incorporation of non-zero basis expectations and basis risk, consideration of option hedges for commodity purchases, and many others.

The last area of research suggested regards the decision maker's subjective expectations. Do farmers, agribusinesses, and other hedgers agree with the market's distributional expectations? Once these subjective probabilities are elicited, how do the risk and return expectations vary from the market's expectations under various risk attitudes? These types of questions are interesting but very difficult to address empirically. Furthermore, the results would represent a cross section of decision makers which may not be very useful when advising or making individual decisions. Because of this, we believe that the most needed research at this time should involve the quantification of "objective" risk and return expectations and which provides useful benchmarks for individual decision makers.

Footnotes

[1] The relationships here are strict if k in equation A3 of the appendix is one. If $k \neq 1$, these risk behavior relationships with $\alpha$ and $\beta$ may not hold for outcomes spanning t (see Holthausen for the behavior characteristics around t).

[2] Lognormality is assumed because most option-pricing models are based on its diffusion process. Recently, Sarassoro completed an in-depth analysis in which live-cattle log-price changes of all contracts traded during 1973-1982 were examined for independence and normality. His results provide fairly strong support, particularly for recent years, for the lognormality assumption.

[3] Because the producer does not make the exercise decision when selling calls, it is possible that the option could be exercised before expiration. However, less than one percent of the trades in the cattle and soybean option markets during the first six months have been exercises. Few options are exercised because, if exercise is desired, it is often more attractive to simply offset the option and earn the return through the premium change. This analysis also ignores the margin call possibility when writing calls.

[4] In Black and Scholes' model, the implicit projection of the distribution is done under the condition that $\mu+\sigma^2/2$ is equal to the risk-free interest rate, whereas Black's futures option model sets $\mu+\sigma^2/2$ to zero. The distributions projected in this paper are under various levels of $\mu+\sigma^2/2$, depending on the assumed drift and variance expectation of the hedger.

Footnotes Cont'd.

[5] Option data used are those published by the Chicago Mercantile Exchange in "IOM Futures Daily Information Bulletin."

[6] In this sense, the model used is not based on subjective probabilities and thus is open to criticism by advocates of the "personal probability" approach. We, however, agree with Young (1980, p. 2) in that if "...the explicit objective of research is to provide information or recommendations that will help decision makers improve their business and production decisions, I see little theoretical value, and less practical feasibility, in insisting that extensionists or researchers use elicited subjective risk assessments in place of computed historical risk measures."

[7] Holthausen and Fishburn show that this criterion produces an efficient set which is a subset of the relevant stochastic-dominance criterion's efficient set.

[8] An example of the importance of volatility expectation in "real-world" situations is given by Avery in a recent Futures article (p. 72) in which he describes Jerry Ostry's program for replacing soybean futures with options when hedging soybean crush spreads. For instance, the choice of put purchases and/or call sales to replace short futures depends on the hedger's volatility expectation.

[9] His transformation of their orchard farmer's utility function results in $\alpha=\beta=1$.

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## Appendix

The specific functions for risk (RK) and return (RT) suggested by Holthausen are:

(A1) $RK = \int_{-\infty}^{t}(t-y)^{\alpha}F(y)dy$ and

(A2) $RT = \int_{t}^{\infty}(y-t)^{\beta}F(y)dy,$

where t is a target level for outcomes, below which outcome y is associated with risk and above which outcome y is associated with return; $\alpha$ and $\beta$ are risk preference parameters; and F(y) is a probability density function. Underlying this model is a utility function U(y) that is consistent with the von-Neumann-Morgenstern axioms for expected utility.

(A3) $$U(y) = \begin{cases} (y-t)^{\beta} \;\forall\; y \geq t \\ -\,k(t-y)^{\alpha} \;\forall\; y \leq t \end{cases}$$

For a particular strike, the following equations describe the expected risk and return of the three pricing techniques considered in this analysis. E[RTP] and E[RKP] are expected return and risk, respectively, of purchasing a put. E[RTC] and E[RKC] are return and risk of selling a call. E[RTO] and E[RKO] are return and risk of keeping an open or naked position.

(A4) $$E[RTP] = \int_{60+Pe^{ri}}^{\infty} (F_I - Pe^{ri} - 60)^{\beta}\, L\,(F_I)d(F_I)$$

(A5) $$E[RKP] = \int_{0}^{X}(60-X+Pe^{ri})^{\alpha}\, L\,(F_I)dF_I + \int_{X}^{60+Pe^{ri}}(60-F_I+Pe^{ri})^{\alpha}\, L\,(F_I)dF_I.$$

(A6) $$E[RTC] = \int_{60-Ce^{ri}}^{X}(F_I + Ce^{ri}-60)^{\beta}\, L\,(F_I)dF_I + \int_{X}^{\infty}(X+Ce^{ri}-60)^{\beta}L(F_I)dF_I,$$ and

(A7) $$E[RKC] = \int_{0}^{60-Ce^{ri}}(60-F_I-Ce^{ri})^{\alpha}L(F_I)dF_I$$

(A8) $$E[RTO] = \int_{60}^{\infty}(F_I-60)^{\beta}L(F_I)dF_I$$

(A9) $$E[RKO] = \int_{0}^{60}(60-F_I)^{\alpha}L(F_I)dF_I$$

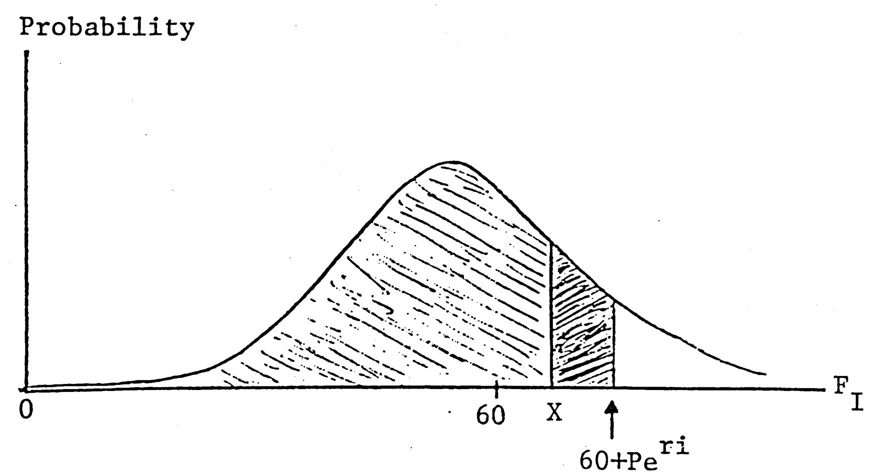


Figure 1. Risk and Return of Put Purchase.

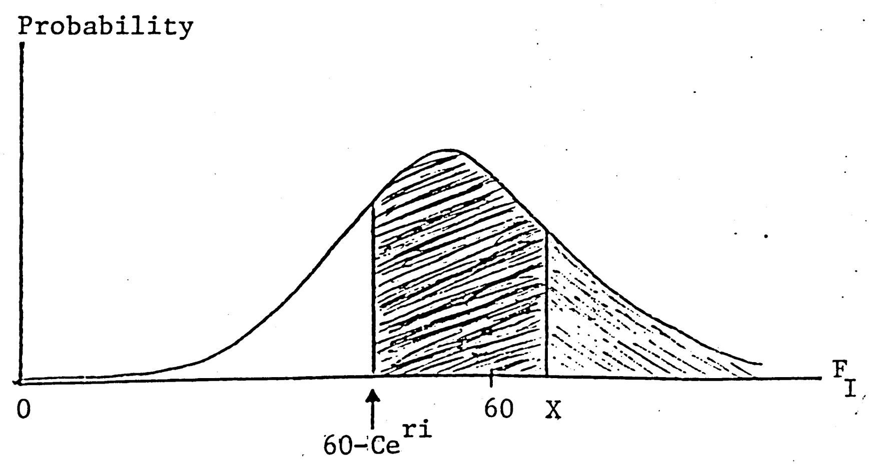


Figure 2. Risk and Return of Call Sale.

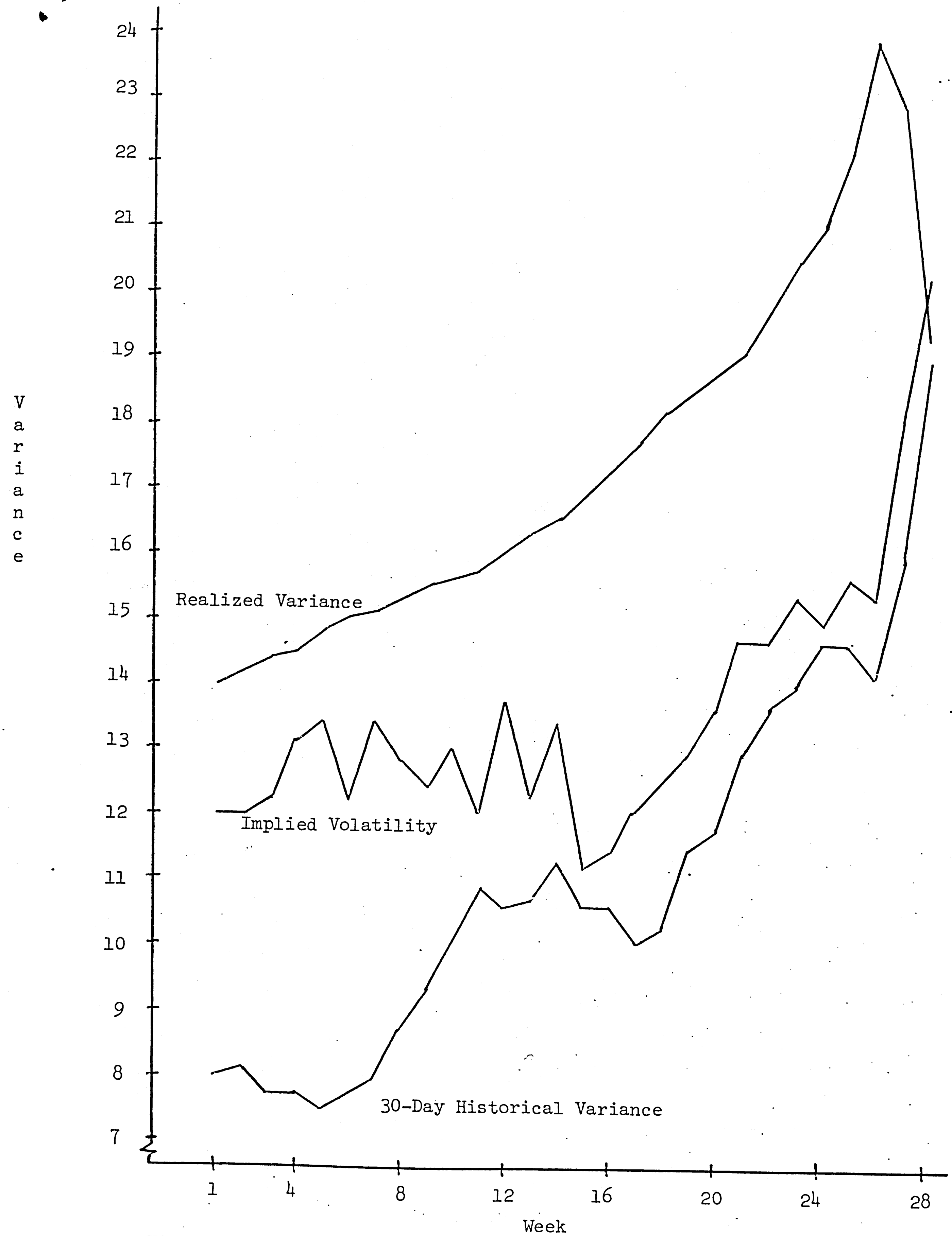


Figure 3. Annualized Implied Volatility, Historical Variance, and Realized Variance for Live Cattle.

Table 1. Risk and Returns in Dollars per cwt. by Risk Preference, Expected Price and Variance, Option Type, and Strike.

| Case | α | β | $E[F_T]$ | $E[\sigma]$[a] | Option | | Strike 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | Open Position Risk | Open Position Return |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 60 | 16 | Put | Risk | 2.43 | 2.37 | 2.25 | 2.06 | 1.80 | 1.48 | 1.14 | 0.84 | 0.59 | 0.44 | 0.37 | 2.47 | 2.47 |
| | | | | | | Return | 2.43 | 2.37 | 2.25 | 2.07 | 1.80 | 1.47 | 1.12 | 0.79 | 0.52 | 0.31 | 0.18 | | |
| | | | | | Call | Risk | 0.07 | 0.18 | 0.38 | 0.67 | 1.01 | 1.37 | 1.70 | 1.96 | 2.16 | 2.29 | 2.37 | | |
| | | | | | | Return | 0.35 | 0.36 | 0.49 | 0.74 | 1.06 | 1.40 | 1.72 | 1.97 | 2.16 | 2.29 | 2.37 | | |
| 2 | 1 | 1 | 55 | 16 | Put | Risk | 5.21 | 4.82 | 4.27 | 3.59 | 2.86 | 2.14 | 1.51 | 1.02 | 0.68 | 0.48 | 0.39 | 5.66 | 0.66 |
| | | | | | | Return | 0.65 | 0.62 | 0.58 | 0.51 | 0.42 | 0.32 | 0.22 | 0.13 | 0.08 | 0.04 | 0.02 | | |
| | | | | | Call | Risk | 0.46 | 0.90 | 1.53 | 2.28 | 3.06 | 3.78 | 4.38 | 4.83 | 5.16 | 5.37 | 5.50 | | |
| | | | | | | Return | 0.30 | 0.28 | 0.33 | 0.43 | 0.54 | 0.62 | 0.67 | 0.69 | 0.69 | 0.68 | 0.68 | | |
| 3 | 1 | 1 | 65 | 16 | Put | Risk | 0.82 | 0.83 | 0.84 | 0.83 | 0.81 | 0.75 | 0.65 | 0.53 | 0.42 | 0.34 | 0.31 | 0.81 | 5.81 |
| | | | | | | Return | 5.74 | 5.64 | 5.46 | 5.14 | 4.67 | 4.07 | 3.37 | 2.64 | 1.95 | 1.35 | 0.88 | | |
| | | | | | Call | Risk | 0.01 | 0.03 | 0.07 | 0.14 | 0.24 | 0.36 | 0.49 | 0.59 | 0.67 | 0.73 | 0.76 | | |
| | | | | | | Return | 0.36 | 0.39 | 0.56 | 0.90 | 1.42 | 2.06 | 2.76 | 3.45 | 4.07 | 4.60 | 5.00 | | |
| 4 | 1 | 1 | 60 | 12 | Put | Risk | 1.89 | 1.91 | 1.93 | 1.89 | 1.77 | 1.53 | 1.22 | 0.90 | 0.64 | 0.46 | 0.39 | 1.85 | 1.85 |
| | | | | | | Return | 1.81 | 1.75 | 1.64 | 1.45 | 1.20 | 0.90 | 0.61 | 0.37 | 0.19 | 0.09 | 0.04 | | |
| | | | | | Call | Risk | 0.01 | 0.04 | 0.13 | 0.30 | 0.54 | 0.83 | 1.12 | 1.36 | 1.55 | 1.67 | 1.75 | | |
| | | | | | | Return | 0.36 | 0.38 | 0.54 | 0.81 | 1.16 | 1.48 | 1.72 | 1.87 | 1.92 | 1.92 | 1.91 | | |
| 5 | 1 | 1 | 60 | 20 | Put | Risk | 2.88 | 2.72 | 2.48 | 2.18 | 1.83 | 1.46 | 1.10 | 0.79 | 0.56 | 0.42 | 0.35 | 3.09 | 3.09 |
| | | | | | | Return | 3.05 | 2.99 | 2.87 | 2.68 | 2.41 | 2.07 | 1.68 | 1.30 | 0.95 | 0.66 | 0.44 | | |
| | | | | | Call | Risk | 0.22 | 0.43 | 0.73 | 1.11 | 1.53 | 1.94 | 2.29 | 2.57 | 2.77 | 2.90 | 2.98 | | |
| | | | | | | Return | 0.33 | 0.34 | 0.46 | 0.69 | 1.00 | 1.35 | 1.70 | 2.03 | 2.31 | 2.54 | 2.71 | | |
| 6 | .5 | 1 | 60 | 16 | Put | Risk | 1.05 | 1.06 | 1.06 | 1.05 | 1.03 | 0.98 | 0.91 | 0.81 | 0.71 | 0.63 | 0.59 | 1.05 | 2.47 |
| | | | | | | Return | 2.43 | 2.37 | 2.25 | 2.07 | 1.80 | 1.47 | 1.12 | 0.79 | 0.52 | 0.31 | 0.18 | | |
| | | | | | Call | Risk | 0.05 | 0.11 | 0.21 | 0.35 | 0.50 | 0.65 | 0.77 | 0.87 | 0.94 | 0.99 | 1.02 | | |
| | | | | | | Return | 0.35 | 0.36 | 0.49 | 0.74 | 1.06 | 1.40 | 1.72 | 1.97 | 2.16 | 2.29 | 2.37 | | |
| 7 | 1.5 | 1 | 60 | 16 | Put | Risk | 6.08 | 5.70 | 5.08 | 4.23 | 3.25 | 2.28 | 1.45 | 0.87 | 0.49 | 0.31 | 0.23 | 6.40 | 2.47 |
| | | | | | | Return | 2.43 | 2.37 | 2.25 | 2.07 | 1.80 | 1.47 | 1.12 | 0.79 | 0.52 | 0.31 | 0.18 | | |
| | | | | | Call | Risk | 0.13 | 0.35 | 0.78 | 1.44 | 2.30 | 3.24 | 4.14 | 4.89 | 5.46 | 5.84 | 6.08 | | |
| | | | | | | Return | 0.35 | 0.36 | 0.49 | 0.74 | 1.06 | 1.40 | 1.72 | 1.97 | 2.16 | 2.29 | 2.37 | | |
| 8 | 1 | .5 | 60 | 16 | Put | Risk | 2.43 | 2.37 | 2.25 | 2.06 | 1.80 | 1.48 | 1.14 | 0.84 | 0.59 | 0.44 | 0.37 | 2.47 | 1.00 |
| | | | | | | Return | 0.98 | 0.96 | 0.92 | 0.85 | 0.75 | 0.63 | 0.50 | 0.36 | 0.24 | 0.15 | 0.09 | | |
| | | | | | Call | Risk | 0.07 | 0.18 | 0.38 | 0.67 | 1.01 | 1.37 | 1.70 | 1.96 | 2.16 | 2.29 | 2.37 | | |
| | | | | | | Return | 0.58 | 0.58 | 0.65 | 0.76 | 0.86 | 0.93 | 0.97 | 0.99 | 1.00 | 1.00 | 1.00 | | |
| 9 | 1 | 1.5 | 60 | 16 | Put | Risk | 2.43 | 2.37 | 2.25 | 2.06 | 1.80 | 1.48 | 1.14 | 0.84 | 0.59 | 0.44 | 0.37 | 2.47 | 6.89 |
| | | | | | | Return | 6.76 | 6.57 | 6.22 | 5.64 | 4.84 | 3.89 | 2.90 | 1.99 | 1.27 | 0.74 | 0.41 | | |
| | | | | | Call | Risk | 0.07 | 0.18 | 0.38 | 0.67 | 1.01 | 1.37 | 1.70 | 1.96 | 2.16 | 2.29 | 2.37 | | |
| | | | | | | Return | 0.21 | 0.23 | 0.38 | 0.72 | 1.33 | 2.15 | 3.12 | 4.09 | 4.94 | 5.62 | 6.11 | | |
| 10 | .4 | .8 | 60 | 16 | Put | Risk | 0.90 | 0.91 | 0.92 | 0.92 | 0.92 | 0.91 | 0.87 | 0.81 | 0.74 | 0.68 | 0.65 | 0.90 | 1.69 |
| | | | | | | Return | 1.66 | 1.62 | 1.55 | 1.43 | 1.25 | 1.03 | 0.80 | 0.57 | 0.38 | 0.23 | 0.13 | | |
| | | | | | Call | Risk | 0.04 | 0.10 | 0.19 | 0.31 | 0.44 | 0.56 | 0.67 | 0.75 | 0.81 | 0.85 | 0.87 | | |
| | | | | | | Return | 0.43 | 0.44 | 0.55 | 0.74 | 0.97 | 1.19 | 1.36 | 1.49 | 1.57 | 1.63 | 1.66 | | |

[a] Annualized standard deviation percentage.

Table 2. Summary of Calls, Puts, and Open Positions Domianted.

| Case | $\alpha$ | $\beta$ | $E[F_I]$ | $E[\sigma]$[a] | Call Strikes Dominated[b] | Put Strikes Dominated[b] | Call Strikes Dominating Open Position[c] |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 60 | 16 | --- | 68–70 | --- |
| 2 | 1 | 1 | 55 | 16 | --- | 50–68 | 62–70 |
| 3 | 1 | 1 | 65 | 16 | --- | 58–70 | --- |
| 4 | 1 | 1 | 60 | 12 | --- | 50–70 | 64–70 |
| 5 | 1 | 1 | 60 | 20 | 52–70 | --- | --- |
| 6 | .5 | 1 | 60 | 16 | --- | 52–70 | --- |
| 7 | 1.5 | 1 | 60 | 16 | 54–70 | 68–70 | --- |
| 8 | 1 | .5 | 60 | 16 | 70 | 50–70 | 66–70 |
| 9 | 1 | 1.5 | 60 | 16 | 54–70 | --- | --- |
| 10 | .4 | .8 | 60 | 16 | --- | 52–70 | --- |

[a] Annualized standard deviation percentage.

[b] Ranges of strikes dominated are inclusive. Listed are those call strikes which are dominated by call strikes, and put strikes dominated by call strikes.

[c] The open or no-hedge position is never dominated by put hedges; ranges are inclusive.