ESTIMATION OF ECONOMIC AND ALLOCATIVE EFFICIENCIES RELATIVE TO STOCHASTIC FRONTIER PROFIT FUNCTION

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ABSTRACT

This paper presents a stochastic frontier profit function model. The (frontier) optimal demand functions for variable inputs are derived from this model, and used to calculate the allocative efficiency of individual variable inputs. These models are used to estimate the economic efficiency, optimal demand for variable inputs, and allocative efficiency of individual farms.

Key words: Stochastic Frontier Profit Function, Optimal Input Demand Functions, Economic Efficiency, Allocative Efficiency

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Since the release of the Farm Mangement data by the Indian government, probably the single issue most intensively investigated by Indian economists has been the relationship between farm size and farm output. Most of the major studies dealing with this topic completed before 1968 have been discussed by Bhagwati and Chakravarty, and those published before 1974 have been discussed by Bharadwaj. Such strong interest in this issue is quite natural, since a comparative analysis of the economics of small and large farms has important implications for policy issues like cooperative farming, land ceiling, and land redistribution. In brief, these studies invariably show that small farms produce higher output, and use more human labor and fertilizer per hectare. But these studies calculate only output input ratios, and hence do not measure economic efficiency and either of its components. A country like India, which has shortage of all factors of production except labor, obviously cannot afford to make an inefficient use of resources. It is, therefore, important to estimate the level of economic efficiency.

Lau and Yotopoulos, and Yotopoulos and Lau compared the relative economic efficiency of small and large farms in India, and concluded that small farms have higher economic efficiency compared to large farms. Sidhu (1974 b) did not find any significant difference in the economic efficiency of small and large wheat farms in the Indian Punjab. The method used by these authors, however, does not provide an actual numerical measure of economic efficiency. This paper estimates economic efficiency indices using the "composed error" model developed by Aigner, Lovell, and Schmidt.

The paper also estimates the optimum demand for variable inputs and uses this information to measure the allocative efficiency of each variable input.

Frontier Profit and Input Demand

Consider the following frontier production function,

$$(1) \qquad Q_i = Q^*_i (X_i; Z_i)\, e^{Q^o_i}, \qquad 0 \leq e^{Q^o_i} \leq 1$$

where Q_i is the i-th farm's actual output, X_i is a vector of variable inputs and Z_i is a vector of fixed inputs. The function $Q^* (X_i; Z_i)$ represents the maximum output which can be produced, given the set of inputs X_i and Z_i. It is the frontier production function. The farm-specific variable $e^{Q^o_i}$ is an index of neutral production technical efficiency. It is bounded between zero and one. Furthermore, assume that the farm, under given variable input prices and a given level of production efficiency $e^{Q^o_i}$, maximizes profit. The farm is then considered to be price inefficient in its input allocation if it fails to equate the marginal products of variable inputs with their normalized input prices (i.e., input price divided by output price). Assume that the marginal product of the j-th input is proportional to its normalized input price W_{ij} of the i-th farm,

$$(2) \qquad \frac{\partial Q_i}{\partial X_{ij}} = W_{ij}\, K^o_{ij}$$

The variable K^o_{ij} then measures the index of allocative (price) efficiency and it is input- and farm-specific.

The economic efficiency, which is a combination of technical and allocative (price) efficiency, of a farm is then reflected in the profit function derived from equations (1) and (2). The relation between the observed profit (π_i) and the frontier (efficient) profit (π_i^*), under given input prices and fixed inputs, is expressed as

$$(3) \quad \pi_i = \pi_i^* (W_i; Z_i)\, e^{\pi_i^o (Q_i^o, K_i^o)} \qquad 0 \leq e^{\pi_i^o (Q_i^o, K_i^o)} \leq 1$$

where W_i is the vector of input prices and K_i^o is the vector of allocative (price) efficiency index. The i-th farm's economic efficiency index $e^{\pi_i^o(Q_i^o, k_i^o)}$ is a function of production technical efficiency and allocative (price) efficiency.

Given the dual relationship between the frontier production function $Q_i^* (X_i; Z_i)$ and the frontier (efficient) profit function $\pi_i^* (W_i; Z_i)$, the (frontier) optimum input demand (X_i^*) can be obtained as

$$(4) \quad X_i^* = - \frac{\partial \pi_i^*(W_i; Z_i)}{\partial W_i}$$

The efficiency of input utilization on a profit maximizing farm can then be defined as a ratio of or as a difference between the actual input used X_i and the optimum input demand X_i^*,

$$(5) \quad R_i^o = \frac{X_i}{X_i^*}, \quad D_i^o = X_i - X_i^*$$

Empirical estimation of the profit function (3) deserves special care since the index of economic efficiency $\pi_i^o (Q_i^o, K_i^o)$ is bounded between zero and one. Empirically, the discrepancy between the actual observed profit π_i and the frontier (efficient) profit π_i^* could be due to either (i) imperfect economic efficiency, i. e., $\pi_i^o (Q_i^o, K_i^o) < 1$, of a farm, or (ii) the effect of random shocks outside the farm's control and other statistical "noises". Let the random variable V_i, $(-\infty < V_i < \infty)$ stand for the random "noise" factor, multiplicative to the profit function (3). The empirical specification of the profit function, in logarithmic form, is given as

$$(6) \quad \ln \pi_i = \ln \pi_i^* (W_i; Z_i) + \quad \pi_i^o (Q_i^o, K_i^o) + V_i$$

Assume that the economic efficiency $\pi_i^o (Q_i^o, K_i^o)$ varies from farm to farm and is independent of the random error V_i. The regression equation (6) is then in fact a regression model with "composite error" $\varepsilon_i = e^{\pi_i^o(Q_i^o, K_i^o)}$ $+ V_i$. Assume that the random error $\ln E_i$ is normally distributed with zero mean and constant variance σ_v^2, i.e., $N(o, \sigma_v^2)$, and the distribution of the non-positive random component $\ln \pi_i^o (Q_i^o, K_i^o)$ is derived from a normal distribution, $N(0, \sigma_u^2)$, truncated from above at zero. The distribution of the composite error ε_i is derived by Aigner et al. The maximum likelihood estimation of the profit regression (6) can then be obtained by some numerical algorithm (such as Davidon-Fletcher-Powell method) as shown by Aigner et al. Alternatively, as shown by Huang, the estimation of equation (6) is more conveniently viewed as a model with the latent variable,

$$(7) \quad \ln \tilde{\pi}_i = \ln \pi_i^* (W_i; Z_i) + V_i$$

where $\tilde{\pi}_i$ represents the stochastic frontier (efficient) profit. The observed profit is then related to the stochastic frontier (efficient) profit as

$$(8) \quad \ln \pi_i = \ln \tilde{\pi}_i + \quad \pi_i^o (Q_i^o; K_i^o)$$

The advantage of the alternative formulation is that it allows the estimation of the stochastic frontier efficient profit regression and its economic efficiency index π_i^o (Q_i^o, K_i^o) via the expectation-maximization (EM) algorithm of the maximum likelihood method (Dempster, Laird, and Rubin). The algorithm is described in detail by Huang, and is not repeated here.

Data and Empirical Estimation

This study uses the stratified random sample data from 151 farms in Punjab and Haryana states in northwest India for the 1969-70 agricultural year to estimate the frontier (efficient) profit function π_i^* $(W_i; Z_i)$ through equation (6) and optimal input demand for labor, irrigation and fertilizer through equation (4). Variables and notations are defined as follow:

π = "profit" or value of farm output and by-products minus expenditure on hired labor, irrigation, and fertilizer.

X_1 = male-equivalent man-days of hired labor used on the farm.

X_2 = irrigation per farm in inch-hectares.

X_3 = kilograms of N, P, K nutrients applied to crops per farm.

Z_1 = male-equivalent man-days of family labor used.

Z_2 = land area cultivated in hectares.

Z_3 = annualized flow of capital services from farm machinery and equipment.

Z_4 = value in rupees of seeds and miscellaneous items per farm.

W_1 = daily wage rate in rupees paid to hired labor.

W_2 = price in rupees per inch-hectare irrigation.

W_3 = price in rupees per kilogram of N, P, K nutrients.

It is to be noted that although π is "profit" net of hired labor, irrigation and fertilizer cost only, multiple regressions implicit in our analysis serve to hold all other inputs, taken as fixed, statistically constant.

A Cobb-Douglas production function is specified with hired labor (X_1), irrigation (X_2), and fertilizer (X_3) as variable inputs in production and other inputs, Z_1, Z_2, Z_3, and Z_4 are treated as fixed inputs. The i-th farm production function is given in equation (9).

$$(9) \quad Q_i = A \prod_{m=1}^{3} X_{im}^{\alpha_m} \prod_{j=1}^{4} Z_{ij}^{\alpha_j} e^{Q_i^o}$$

For the Cobb-Douglas production function, the correspondent profit function in logarithmic form is

$$(10) \quad \ln \pi_i = \ln B_o + \sum_{m=1}^{3} \beta_m \ln W_{im} + \sum_{j=1}^{4} \beta_j \ln Z_{ij} + u_i + v_i$$

where $U_i = \pi_i^o (Q_i^o, K_i^o)$ is the logarithm of the economic efficiency. The coefficients of the profit function are related to that of the production function as follows

$$(11) \quad \beta_m = \frac{-\alpha_m}{1-\alpha_m} \text{ and } \beta_j = \frac{\alpha_j}{1-\alpha_m}, \qquad m = 1,\text{------}3, \quad j = 1, 2,\text{---}, 4.$$

In empirical estimation, it is assumed that the production functions are different between large and small farms, and so are their production technical efficiency Q_i^o and the allocative (price) efficiency K_i^o. Thus two profit functions for large and small farms are estimated through the EM algorithm of the maximum likelihood method. The sample of 151 farms are divided into three groups with large farms defined as those with cultivated land greater than 8.0 hectares and the small farms are those with less than 6.0 hectares. The middle group of farms with cultivated land between 6.0 and 8.0 hectares is deleted from the sample estimation so that a more distinct comparison of economic efficiency and efficiency in input utilization between large and small farms can be achieved.

The results of the maximum likelihood estimation of the profit function (10) via the EM algorithm are tabulated in Table 1 along with the calculated coefficients of the production function from the relation (11). All estimated coefficients have the expected sign and are significantly different from zero in the one-sided test at the 5 percent level except for family labor whose coefficient in the case of small farms is not statistically different from zero. This is quite plausible and consistent with the labor surplus scenario in the development literature. Interestingly enough, the small farms show a higher return to scale than the large farms with both regressions indicating decreasing returns to scale. The latter findings are not, in general, at variance with the literature.

The results presented in Table 2 show that on an average the 78 small farms have slightly higher economic efficiency (77.94 percent) than that of the 48 large farms (75.56 percent). This result is in agreement with the conclusion of Yotopoulos and Lau that, in Indian agriculture, the small farms are relatively more efficient. But this result is in contrast to the finding of Sidhu that there is no significant difference in the efficiency of the small and large farms in the Indian Punjab. Economic efficiency was estimated for individual small and large farms. Frequency distributions of economic efficiency for small and large farms are presented in Table 3.

Table 1. Estimates of Frontier Profit and Production Functions

Variables in Logarithms	78 SMALL FARMS		48 LARGE FARMS	
	Profit Function	Production Function	Profit Function	Production Function
Constant	6.3681 (15.0600)	5.5290	6.1253 (26.6090)	4.8068
Land	.4585 (5.2108)	.3404	.1124 (3.9793)	.0713
Family Labor	.0192 (0.4336)	.0142	.1271 (8.2949)	.0806
Capital	.1830 (4.6440)	.1358	.1901 (11.6470)	.1206
Other Expenses	.2382 (3.9440)	.1768	.3407 (14.4430)	.2161
Hired Labor*	-.0919 (-1.4127)	.0682	-.0995 (-3.9433)	.0631
Irrigation*	-.0402 (-1.9705)	.0299	-.0460 (-4.1508)	.0292
Fertilizer*	-.2151 (-2.6247)	.1597	-.4307 (-8.8771)	.2733
σ_u^2	.10593		.14659	
σ_v^2	.03227		.00439	
λ	1.8118		5.7808	
Returns to scale		.9250		.8542

The numbers in the parentheses are the asymptotic t-values.

The estimates of elasticities in the production functions are calculated from equation (11).

*The unit prices of these inputs are used in the estimation of the stochastic frontier profit functions.

Table 2. Average Economic Efficiency and Allocative Efficiency (of Hired Labor, Irrigation, and Fertilizer) for Small and Large Farms.

	78 SMALL FARMS			48 LARGE FARMS		
Variable	Mean Effi	Observed	Frontier	Mean Effi	Observed	Frontier
Profit	0.77941	6937.3	8789.1	0.75561	13,401.00	17,974.00
Hired Labor	0.68520	106.65	149.80	0.81936	272.56	329.80
Irrigation	1.04650	2.7364	4.7492	0.64935	3.0680	7.5585
Fertilizer	0.30267	219.91	636.42	0.11262	333.333	2,633.70

Table 3. Frequency Distribution of Economic Efficiency for Small and Large Farms.

Efficiency Interval	Frequency	
	78 Small Farms (<6.00 Hects.)	48 Large Farms (>8.00 Hects.)
0.40 - 0.50	2 (2.56)	3 (6.25)
0.50 - 0.55	1 (1.28)	4 (8.33)
0.55 - 0.60	4 (5.13)	5 (10.42)
0.60 - 0.65	5 (6.41)	3 (6.25)
0.65 - 0.70	7 (8.97)	2 (4.17)
0.70 - 0.75	4 (5.13)	2 (4.17)
0.75 - 0.80	14 (17.95)	5 (10.42)
0.80 - 0.85	17 (21.80)	3 (6.25)
0.85 - 0.90	16 (20.51)	10 (20.83)
0.90 - 0.95	8 (10.26)	9 (18.75)
0.95 - 1.00		2 (4.17)
Average	0.7794	0.7556
Minimum	0.4860	0.4457
Maximum	0.9304	0.9680

The numbers in the parentheses are the percentages.

These results show that more individual small farms (70.52 percent) have economic efficiency above 0.75 as compared to individual large farms (60.42 percent). Furthermore, the small farm group is more (less) efficient in utilizing irrigation and fertilizer (hired labor). The small farms, on an average, have achieved absolute allocate efficiency in using irrigation (1.0465). The large farms as a group have underutilized all these variable inputs (irrigation, fertilizer, and hired labor), while the small farms have underutilized fertilizer and hired labor. The large farms are more efficient in using hired labor with average allocative efficnecy of 82 percent as compared to 69 percent for the small farms.

There is evidence that farm credit in general and purchase of tactors, tubewells, and to some extent even fertilizer have been subsidized in India (Johl; Rao; Sidhu 1974 a). In a situation of limited supply of credit, fertilizer and canal irrigation--subsidies (i.e. under pricing) can cause excess demand and hence crowding out of small farmers who lack both economic and political clout. Some small farmers may be unable to purchase proper amounts of variable inputs due to the lack of necessary funds and hence may fail to make optimal allocation of these inputs. There is further evidence that the extension workers tend to visit relatively large farmers (Sidhu 1974 b, p. 750). In light of this information, it is even more credible that small farms have achieved higher economic efficiency as well as allocative efficiency in using irrigation and fertilizer as compared to large farms.

Under the existing conditions relatively smaller farms have higher economic efficiency and are more efficient in utilizing irrigation and fertilizer as compared to large farms. The small farms also exhibit a higher return to scale than the large farms in northwest India. Therefore, one cannot argue for consolidation of small farms into cooperative farms on the basis of economies of scale/or lower level of economic efficiency. But on the other hand, the decreasing returns to scale, and lower economic and allocative efficiency on relatively large farms provide some evidence in favor of land ceiling, land redistribution, and/or progressive land tax policies. Furthermore, there does not seem to be any economic justification for credit and input subsidies which often benefit relatively larger farmers.

Summary and Conclusions

This paper estimates the stochastic frontier (efficient) profit functions and the optimum demand for variable inputs (hired labor, irrigation, and fertilizer), separately for small and large farms, using the 1969-70 farm level data from northwest India (i.e. Punjab and Haryana states), and compares the overall economic efficiency and the allocative efficiency of these two groups of farms. The results show that small farms have somewhat higher economic efficiency than that of large farms, although there is significant room for improving the economic efficiency on both types of farms. The small farms made almost optimal use of irrigation, and underutilized both hired labor and fertilizer, while large farms underutilized all three variable inputs. Small farms are more (less) efficient in utilizing fertilizer and irrigation (hired labor) as compared to large farms. Furthermore, small farms exhibit a higher return to scale than large farms. This latter result is in agreement with the previous results from India (Bardhan; Bhagwati and Chakavarty; Bharadwaj). In light of these results, one cannot advocate for consolidation of small farms into cooperative farms. On the other hand, this study provides some evidence in favor of land ceiling, land redistribution, and/or progressive land tax policies, but against subsidizing credit and farm inputs.

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