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SPECIFICATION OF THE PROFIT EQUATION AND EXPECTED PROFIT MAXIMIZATION

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Proper specification of the profit equation where profits are stochastic requires that special care be given to specifying the profit equation so that it is consistent with the characteristics of the profit maximization problem. It is shown here that the relationship of cost curves with the production function is not, in general, the same in the stochastic case as in elementary firm theory. It is also shown that although the "expected product price equals expected marginal cost" rule holds when expected marginal cost is interpreted as the expected marginal cost of planned production, the "expected input price equals expected marginal value product" rule for a planned input requires an adjustment for the increased cost of stochastic inputs.

Where profits are deterministic, economic theory provides two alternative specifications of the profit maximization problem. The primal problem for the single variable input-single product case is

(1) Max P = PqQ - PxX - TFC

where

(2) Q = F[X]

and

P = profit,
Pq = product price,
Q = production,
Px = variable input price,
X = quantity of variable input and
TFC = total fixed cost

which results in the following necessary conditions for maximizing profit:

(3) dP = PqF'dX - PxdX = 0.

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Dividing Eq. 3 by dX and rearranging terms results in the familiar condition that the marginal value product should be set equal to the input price:

(3a) $PqF' = Px$

A corresponding dual problem results from solving Eq. 2 for X.

(4) $X = G[Q]$

and maximizing profit after substituting for X in Eq. 1. The necessary condition for maximum profit derived from the dual problem is

(5) $dP = PqdQ - PxG'dQ = 0$

which when divided by dQ results in the familiar condition that the marginal cost is set equal to the product price:

(5a) $PxG' = Pq$

The fact that the primal and dual specification of the above profit maximization problem are two alternative specifications that provide the same necessary condition can be shown by totally differentiating Eqs. 2 and 4

(2a) $dQ = F'dX$

(4a) $dX = G'dQ$

and deriving Eq. 3 by substituting in Eq. 5 for dQ from Eq. 2a and for G'dQ from Eq. 4a.

If production is assumed stochastic and X is fixed, Eq. 1 can be restated as

(6) $\text{Max } E(P|X) = E(Pq)E(Q|X) - PxX - TFC$

and Eq. 2 restated as

(7) $E(Q|X) = F[X]$

where E is the expectations operator and the product price and production are assumed to be uncorrelated. [1/]

Eq. 3a is then also a necessary condition for maximization of expected profit where F' is interpreted as the expected marginal physical product of X in the production Q.

Expected profit can also be specified with the input level stochastic and output fixed where Eq. 1 is restated as

(8) $\text{Max } E(P|Q) = E(Pq)Q - PxE(X|Q) - TFC$

and Eq. 4 restated as

(9) $E(X|Q) = G[Q]$

Eq. 5a is then a necessary condition for the maximization of expected profit where PxG' is the expected marginal cost of producing Q. Just has suggested in the case of expected profit maximization "Assuming nonstochastic input prices, the relationship of cost curves with the production functions would be exactly as in elementary theory". However, the cost function $PxE(X|Q) = PxG[Q]$ in Eq. 8 can not be derived from the production function in Eq. 7 and in general, maximization of Eqs. 6 and 8 will not give the same result since they are different problems. Eq. 6 assumes that the input level and variable cost are fixed by the decision maker and output is stochastic with a marginal frequency distribution that is conditional upon the input level. To talk about the expected input level given the level of output is therefore meaningless in the context of the assumptions underlying Eq. 6. Similarly, it is not a matter of

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[1/] $E(YZ) = E(Y)E(Z) + COV(Y,Z)$ so that where the covariance of the product price and production is zero and $E(Pq|X) = E(Pq)$, expected revenues are $E(Pq)E(Q|X)$.+ Note that assuming zero covariance does not require mean independence or stochastic independence which are stronger conditions.

convenience that a production function is omitted from Eqs. 8 and 9, i.e. that output is not specified as a function of the input level. It has been assumed in Eqs. 8 and 9 that the expected input level depends upon the output selected and therefore a cost function exists, but not a production function, at least not in the usual sense. Consider, for example, machinery repair costs. If it is planned to grow 100 acres of a crop, there are a certain number of breakdown repair costs (repairs required after a breakdown) that can be expected given a particular preventive maintenance program. To ask how the number of acres grown depends upon breakdown repair costs is asking the question backwards assuming those repairs are always made that are required to carry out the plan. There is no doubt a relationship between the two variables i.e. higher breakdown repair costs would be expected with a larger acreage, ceteris paribus. However, a producer would not plan increased breakdown costs in an effort to increase acreage since breakdown costs are by assumption unplanned.

Clearly there are circumstances where profits depend upon both a cost function and a production function e.g. where expected profits are specified as:

$$E(P|Xp) = E(PqQ|Xp) - Px_p\, Xp - E(PxX|Xp) - TFC \tag{10}$$

where

E = expectations operator,
P = profit,
Xp = quantity of planned input,
Pq = product price,
Q = production,
Px_p = price of planned input,
Px = price of stochastic input,
X = quantity of stochastic input and
TFC = total fixed costs.

Eq. 10 is a general specification of expected profits conditional upon the level of a single planned input where the expected level of a single stochastic input is assumed to be conditional upon the level of planned input. This equation could be further generalized where Q, Pq, Xp, Px_p, X and Px are all vectors representing a multifactor-multiproduct case. It is assumed that expected profits are being evaluated at a time when the quantity and price of the planned input can be determined with certainty. Where production and the product price are assumed to be uncorrelated and the level of the stochastic input, X, and its price, Px, are assumed to be uncorrelated and $E(Px|Xp)=E(Px)$, expected profits can be represented as:

$$(10a) \quad E(P|Xp)=E(Pq)E(Q|Xp)-Px_pXp-E(Px)E(X|Xp).$$

Letting

$$(11) \quad E(Q|Xp) = F[Xp]$$

and

$$(12) \quad E(X|Xp) = H[Xp]$$

the first order condition for maximum expected profits in Eq. 10a is

$$(13) \quad E(Pq)F' - E(Px)H' = Px_p.$$

The first order condition in Eq. 13 not only deviates from the "marginal value product equals the input price" rule in Eq. 3a because of the introduction of stochastic production, but it also deviates from the usual statement of the "expected marginal value product equals the expected input price" rule. Actually, the left hand side of Eq. 13 is the expected marginal value product of Xp where the expected marginal value product is calculated as net of the expected "indirect" costs, $E(Px)H'$, associated with a marginal increase in Xp.

If stochastic output, Q, and the level of the stochastic input, X, are related through a production function such as

(14) $X = G[Q]$[2/]

substituting for X in Eq. 10a results in expected profits specified as a function of the expectation of a function of Q i.e. as a function of $E(X|Xp)=E(G[Q]|Xp)$. If $G[Q]$ is linear, expected profits will depend upon $E(Q|Xp)$. However, if $G[Q]$ is nonlinear, expected profits will depend upon higher moments of Q. For example, for $G[Q] = \alpha Q+\beta Q^2$ it follows that $E(G[Q]) = \alpha E(Q)+\beta E(Q^2) = \alpha E(Q)+\beta V(Q)+\beta(E(Q))^2$ where $V(Q)$ is the variance of Q. In other words, expected profits depend upon the variance of production and an expected profit maximizer (risk neutral) will respond to changes in the variance of production as does the risk averse utility maximizer as shown by Just. If all producers are affected by the same general weather conditions, the product price realized and production level achieved by individual producers would be expected to be correlated as well and the expected profits would further depend upon higher moments of Q.[3/]

An alternative approach to specifying expected profits is to view plans in terms of a planned production level, Qp, where

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[2/] The relationship between X and Q is assumed here to be deterministic for convenience. This relationship could be stochastic also.

[3/] See footnote 1 for the expectation of a product.

(15) $E(P|Qp)=E(PqQ|Qp)-TVCp[Qp]-E(TVC[Q,Qp])-TFC$

where

$TVC_p[Qp]$ = total planned variable costs

and

$TVC[Q,Qp]$ = total variable costs that are associated with actual production.

Again assuming the product price and production are uncorrelated, expected profits are

(15a) $E(P|Qp)=E(Pq)E(Q|Qp)-TVCp[Qp]-E(TVC[Q,Qp])-TFC$

and assuming $E(Q|Qp)=Qp$, the first order condition for maximizing expected profits is

(16) $E(Pq) = TVC'p + \partial E(TVC[Q,Qp])/\partial Qp.$

The usual first order condition results where the expected product price is set equal to the expected marginal cost of (planned) production.[4/]

Again where total variable costs are a nonlinear function of output, Q, the first order conditions for expected profit maximization will depend upon higher moments of Q e.g. expected profit maximizers will be sensitive to changes in the variability of production as shown by Just.

It was argued above that in expected profit maximization the relationship of cost curves with the production function would not be exactly as in elementary theory. This observation applies in particular to the relationships involving stochastic variables. However, a production function and corresponding cost function do exist for planned production and planned inputs and in that context Eq. 10 is the stochastic equivalent of the primal problem and Eq. 15 is the stochastic equivalent of the dual problem and maximizing Eq. 10 with respect to the level of the planned input and maximizing Eq. 15 with respect to planned

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4/ Just interprets the expected marginal cost as $E(\partial TVC[Q,Qp]/\partial Q)$ and concludes that since in general $E(\partial TVC[Q,Qp]/\partial Q \neq \partial E(TVC[Q,Qp])/\partial Qp$ the rule is not to equate the expected product price and the expected marginal cost. The above inequality is correct but the right hand side of the inquality is the expected marginal cost of planned output while the left hand side is the expectation of the marginal cost of actual output.

output will result in the same expected profit maximizing factor - product mix and therefore the same level of maximum expected profit.

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References Cited

Just, Richard E., "Risk Aversion Under Profit Maximization", *Amer. J. Agr. Econ.*, 57 (1975): 347-52.