Simulation

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A Review of Monte Carlo Applications in Agricultural Research

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Leonard E. Burman
University of Minnesota

INTRODUCTION

Monte Carlo Programming (MCP) involves random sampling from the feasible set of some farm planning or investment analysis model, the feasible set being defined in much the same way as in the more traditional linear programming (LP) problem. The primary advantages cited by those who used MCP were: it allowed the inclusion of an arbitrary number of objective functions; integer level constraints were easy to specify; and the computer algorithm was very easy to program using available machines. Among its disadvantages were the high cost of using the procedure and the lack of ability to find the optimal solution in most cases. The cost factor, combined with the improvement in competing analytic methods such as mixed integer programming (MIP) and quadratic programming (QP), ultimately extinguished much of the enthusiasm for the traditional kind of MCP and there has been no significant new published research of that type since 1969.

In this paper, I review the basic MCP methodology used in farm planning and investment evaluation models in the late 1960's. Key features of this early research are discussed along with a brief survey of more modern extensions of the basic techniques. Finally, the advantages and disadvantages of MCP as a research tool are considered.

BASIC METHODOLOGY

Although a number of articles appeared in 1966-1967 espousing basically similar methods, Stryg (15,16) was apparently the first to apply Monte Carlo methods to farm planning problems. In a paper which immediately followed, Lindgren and Carlsson (11) developed a "seeking" procedure for efficiently zeroing in on solutions which were near optimal in terms of the objective function(2). At about the same time, Thompson (17), Dent and Thompson (6), and Donaldson and Webster (7,8) produced slightly modified versions of Stryg's or Lindgren's models and applied these to various farm planning problems. For the most part, these papers were simply expository of the basic methodology and did not treat problems which were especially important in their own right.

Since the predominant analytic tool used for farm planning problems in the 60's was linear programming, the Monte Carlo programming problem was invariantly set up in a similar manner (this was also practical since the results were often compared to LP results for similar problems (8,15,16)). The objective function and constraints were usually presented in a tableau, although the solution algorithm operated on an equation-by-equation basis and involved no matrix operations. The algebraic form of the Monte Carlo method is presented here, following the terminology of Carlsson, Hovmark and Lindgren (3).

The objective function(s) is defined as a function of the n activities:

$$Z(x_1, x_2, \ldots, x_j, \ldots, x_n)$$

where x_j is the level of the jth activity. Z may be of almost any form (smoothness and convexity in the range of the feasible set are desirable, but not essential, features). x_j may be integer or real, subject to:

$$x_j^{min} \leq x_j \leq x_j^{max} \quad \text{or } x_j = 0$$

k activities are assumed to be independent random variables. The remaining n-k are dependent on one or more of the k independent activities according to the equation:

$$x_{k+h} = \sum_{j=1}^{k} D_{hj} x_j \quad (h=1,2, \ldots, n-k)$$

Constraints are formulated as linear functions of the independent activities.[1/] The form of the constraints varies from the LP in that a fixed and a variable resource cost is specified for each activity relative to each constraint:

$$\sum_{j=1}^{k} R_{ij} \leq b_i \quad \text{where:} \quad R_{ij} = \begin{cases} q_{ij} + a_{ij}x_j & \text{if } x_j \neq 0 \\ 0 & \text{if } x_j = 0 \end{cases}$$

$$a_{ij} \geq 0, \; b_i \geq 0 \text{ and } q_{ij} \geq -a_{ij}x_j^{min}$$

Finally, a constraint may be imposed on the number of activities which may enter a solution.

These constraints define a feasible region which will most likely look much different from that of the standard LP problem. The constraints on range, integer constraints, the resource cost or efficiency component (q_{ij}; which may or may not enter the solution, thus varying the net resource constraint level) and the limits on number of activities virtually assure that the feasible set is non-convex.

The solution process simply involves randomly sampling from the so-defined feasible set. The activities to enter the solution are chosen randomly from among the independent activities. A random value (rounded for integers) within the predetermined limits (x_j^{min} to x_j^{max}) is assigned to the selected activity and each of the resource levels (constraints) is decreased by the resource use of that activity. If any of the constraints are violated, the activity level is reduced until either all resource levels are non-negative or the minimum activity level is passed

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1/ Several authors suggest that non-linear constraints might be used, but none appeared in the surveyed literature.

($x_j < x_j^{min}$). If the latter occurs, the activity level is set equal to zero. Another activity is randomly selected and the process is repeated until the prespecified number of activities have entered the solution or until all of the activities have been tried.

A second stage process moves the solution from the interior to the boundary of the feasible set (if this is desirable). Each of the activity levels is increased (in the order in which they were selected) until at least one constraint is reached for each. Finally the value of the objective function(s) is calculated.

The solution process is repeated a specified number of times to produce a characteristic sample of the feasible set. The number of iterations, usually 1,000 or more, is determined based on an estimate of how large the sample need be to get the desired number of solutions which are near optimal with respect to the various objective functions with a given probability.

If some activities are preferred to others, a weighting scheme is used to increase the probability that the preferred activities will enter the solution. Since earlier-selected activites tend to have higher values, the preferred activities will be larger on average with weighting than without. If,a priori, certain activities are known to be more important in the objective functions, appropriate weighting will produce more solutions near optimum levels.

Carlsson, et al. (3) suggest a "multi-step seeking process" which takes a systematic approach to calculating weights combined with "interval reduction" which results in a much higher percentage of solutions in the neighborhood of the optimum. A set of solutions is produced using the basic procedure outlined above without prior weights. The solutions with objective function values above certain minimum levels are selected. If certain activities never occur in these solutions, they are removed from further consideration. The range limits of the included activites are reset to the observed range in the sample ("interval reduction"). Weights are calculated for these variables based on their relative frequencies in the acceptable solutions. Using the more precise intervals and the calculated weights, a new set of solutions is computed using MCP.

Other improvements were made in the Monte Carlo methodology, although these were fairly straightforward extensions of the basic techniques. Most notably, Dent and Thompson (6), in their analysis of feed ration problems, made three modifications. In the first part of the solution process, violations of constraint minima are ignored. In most cases, after the specified number of activities entered the solution, there was no problem in achieving feasible results in the second step when the activity levels were increased.[2/] Another extension was the inclusion of step-sizes for activity levels (greater than unity). This was desirable because many of the feeds could only be bought in multi-unit packages (i.e., bushels rather than pounds). Finally, following the method suggested earlier by Thompson (17), certain activities were constrained to be mutually exclusive so that some of the solutions would contain each of the mutually exclusive activities, but none would have more than one of them. All of the above modifications testify to the simplicity of modifying the computer program to build in any conceivable type of boolean or linear constraint.

2/ It is likely that earlier writers did not address this issue simply because their problems did not include minimum constraints.

Several techniques were used to process the MCP output into a meaningful form for analysis. Most popular among these is the estimation of efficiency lines which show the trade-off between various objectives. An efficiency line is a chart of optimum levels of one objective for given levels of a second objective. If the objectives are maximum profit and minimum variance, for instance, this is equivalent to a sample estimate of the E-V frontier. The advantage of MCP is that efficiency lines may be derived for all possible pairs of objectives with no additional computation.

SOME APPLICATIONS

An example of how these techniques might be applied is given by Dent and Byrne (5) in their investment planning paper. They compare efficiency lines for mutually exclusive investments, where minimized capital investment for given levels of net present value are the objectives. In the case of land improvement for sheep versus cattle, they find that cattle produces higher returns for low levels of investment, but sheep are more profitable at higher levels. If capital is tight in the near-term, but further investment is planned later, the less profitable investment in the short-run, sheep, would be the best choice for the long-run. When more capital becomes available, this would go to further land improvement for sheep, producing higher total profits overall. Dent and Bryne suggest that this type of analysis would be applicable for any investments which are additive over time.

More recent research has attacked more concrete problems with widely varying implementations of the Monte Carlo method. Cassidy, Rodgers and McCarthy (4) used an MCP-type approach to simulate risky outcomes. The notion of the feasible set was replaced by a joint probability space. For each activity, the minimum, maximum and most likely outcome were determined subjectively based on questioning of the farm planner. These three parameters are sufficient to specify a triangular distribtuion. The sampling procedure simply involved generating random values for each activity based on its derived probability density function. Results of the simulation were presented to the planner in the form of a sample cumulative density function for each of the objectives.

For analyzing a particular investment, Cassidy's method provides a different kind of information than earlier MCP studies did. The planner can judge the probability of acheiving each of his various objectives. Cassidy points out that this approach can also indicate the value of better information (reduced variance) in terms of ultimate objectives. The elements of the system which contribute most to reducing risk can be identified as targets for further research.[3/]

A limitation to the gross sampling approach to Monte Carlo programming is that a large number of solutions must be generated to produce meaningful information. Anderson (1) suggests analyzing alternatives based on conditions of third degree stochastic dominance to select the "risk-efficient" set of farm plans. In applying this to Hazell's (10) four-enterprise vegetable farm problem, only 48 plans needed to be generated to find 20 in the risk efficient set.

Using the same approach, he showed how "risk-efficient Monte Carlo programming" (REMP) may be used to evaluate farm policy. REMP was applied

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3/ Etherington (9) uses a similar approach to simulate price and yield variability over time in a rubber tree replacement model.

to two taxing alternatives in Australia. Since the characteristics of the risk-efficient sets generated in each case were essentially identical, Anderson concluded that taxation policy considerations should focus on the stabilization effects and "... need not be complicated by attempts to account for price effects caused by policy-induced changes in aggregate levels of production." (1, p. 104)

Anderson suggests that REMP is a good alternative to conventional risk-programming approaches when risk is non-normal (he uses beta distributions in the examples) or when utility is not quadratic. Compared to the MCP approach of presenting the farm planner with a large set of alternative plans with probable outcomes and letting him evaluate the risk subjectively, REMP requires the elicitation of subjective probabilities and some assumptions about the nature of the farmer's utility function.

COMPARISON WITH OTHER METHODS

Donaldson and Webster (8), in their 1967 farm planning study, elucidate some of the major features of farm enterprises and constraints which could not suitably be modelled using LP (and which could be using MCP). First among these is "lumpiness of resource flows." Inputs such as labor and machinery cannot be bought in small quantities or in fractional quantities. MCP deals with this problem by specifying the range and step-size for activities which use these resources.

Some capital items, such as buildings for livestock and special cropping equipment command an economic rent whether or not they are used. These "short-run resource fixities" of available farm inputs result in a preselection of certain activities. MCP can treat these activities by using weighting to increase the probability of their inclusion.

Other factors in the production process which are better handled by MCP are non-linear production functions and economies of scale. Since the objective function in MCP may take almost any form, the first problem is trivial. For the latter, the specification of the constraints allows for large start-up costs (via the q_{ij} term) with average variable costs decreasing with higher activity levels.

Aside from constraints on physical relationships, MCP offers several direct advantages over LP for the farm manager. Although he may wish to consider many alternatives, he often prefers to limit the total number of enterprises engaged in and this can't be accomplished using standard LP.

A disadvantage of linear programming is that it can only consider solutions which lie at the vertex of the boundary of the feasible set. Renborg (14) has shown that in farm planning problems there is likely to be a fairly wide range of near-optimal solutions and these cannot be adequately represented using LP. If the farmer's preferences can be reduced to a single objective function then this is obviously not a problem. If there are only two objectives, such as minimum risk for each level of income, and if the risks associated with each activity may be quantified, then a sample of plans giving various income levels with minimum risk may be generated by minimizing risk with income levels as a constraint varied parametrically. With crop rotation as a secondary objective, Powell and Hardaker (13) concluded that this was a superior approach in one of the few cases where MCP was compared to LP methods and lost.

A concensus among the authors who adopted MCP is that farmers have many objectives and that these objectives cannot easily be quantified. Dent and Byrne (5) represent this position fairly well:

> It has become clear ... that the objectives of farmers, particularly their long term objectives, cannot be reduced to a single criterion. The list of factors which influence the selection of farm plan will include the stability of the plan in a changing economic and physical environment, the ease with which the business can be expanded (or in some cases contracted), the need for new capital investment, the position on the farmer's concept of social scale which a particular way of farming permits, as well as short term profitability and maintenance of family income ... Therefore, the alternative to an optimizing approach is to offer the farmer a number of feasible plans. These plans should be similar in terms of ... the primary criterion ... In addition, values of a number of other criteria may be calculated ... The farmer may then make a choice from the range of plans offered with some knowledge about the financial and physical implications of each plan. (pp. 104-105)

An obvious shortcoming of MCP is that it will virtually never report the true optimum. Candler, Cartwright and Penn (2), in their critique of Thompson's farm planning model, caution that simulation methods may fall far short of the true optimum. Using Thompson's example, they were able to incorporate the same kinds of constraints in a MIP model. The MIP solution for the objective function (gross margin) was 15% higher than the best MCP solution. Candler summarizes: "... if the solution space is fairly flat, then we may be almost indifferent between any of the top 1 percent of solutions ... The question is: How often is the solution space an n-dimensional pancake and how often an n-dimensional orange?" (p. 238) Although the probability of obtaining near-optimal solutions may be increased by using heuristic algorithms such as weighting or Carlsson's seeking procedure, these bear the risk of excluding the optimum entirely if misspecified: "... raising the expected value of plans selected does not necessarily also raise the expected value of the best plan selected." (2, p. 238)

Thompson (18), in a rebuttal, argued that MCP's main advantage lies in its handling of multiple objectives. The estimation of efficiency lines, for instance, is comparatively simple using MCP, whereas the MIP efficiency frontier, while technically more precise, involves much greater difficulty and cost.

Thompson contends that whether or not you hit upon the optimal value of the objective function is irrelevant:

> "The Monte Carlo algorithm can handle large numbers of objectives with little extra effort, so that its role becomes less that of an optimum seeker and more a portrayer of the significant relationships present in the system under study. The emphasis on an optimum (a concept which is largely illusory in agricultural systems) is then replaced by emphasis on a 'road map' from which a farmer may judge the consequences of different courses of action." (p. 241)

If the relevance of an optimum in agricultural research is an arguable point, the cost of MCP is not. It is easy to see that as the number of activities increases the size of the feasible set tends to increase exponentially. Thus, the size of the sample must be very large for more than trivial models. Where reported, the number of iterations ranged from 1,000 to 4,000 in the early MCP studies. All of the problems were very small by LP standards. Aside from the computing cost, the sophisticated multivariate analysis requires that many or all of the sample plans be stored for analysis and this adds significantly to total cost. The result is that MCP is impractical for large problems. Candler, et al. suggest that the cost of achieving the added information which MCP provides compared to deterministic alternatives may often excede the value of the information.

CONCLUSION

MCP is a useful tool for agricultural production economics, although its promise as a routine instrument for farm planning was probably overstated by early authors. Due to its flexibility, it offers a means of dealing with problems which are too mathematically complex to be treated with deterministic methods. Application of MCP to simulating the outcomes of stochastic processes, although somewhat far afield from the initial research, seems a particularly appropriate area for future research. Where analytic alternatives are available, however, they are usually the more cost-effective alternative. Some research in developing methods like Anderson's REMP, which help to identify near-optimal solutions as they are generated and thus reduce costs, might vastly increase the applicability of MCP.

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