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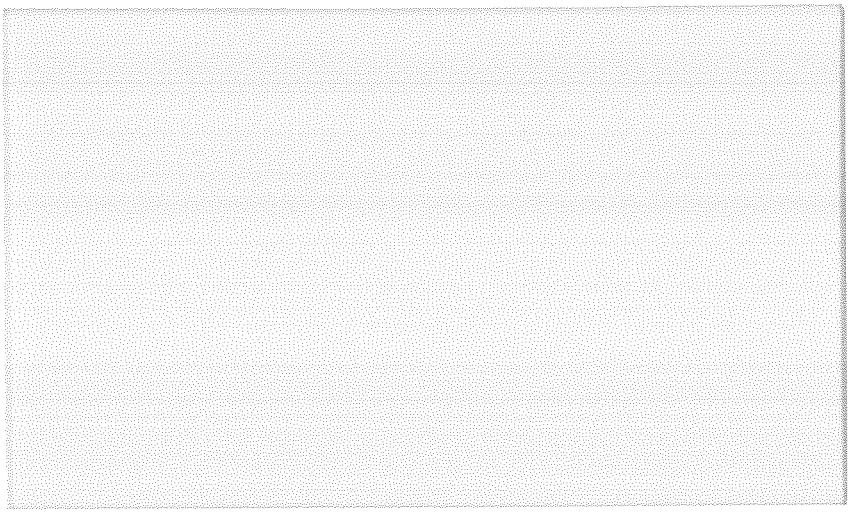

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"Exact" Welfare Measures in Agricultural Policy Analysis

By

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"Exact" Welfare Measures in Agricultural Policy Analysis

Abstract

A procedure for obtaining compensating and equivalent variations directly through integration of the Slutsky equation for a properly estimated demand system is developed. Unlike methods which integrate the ordinary demand using Roy's Identity to obtain the indirect utility function, this procedure is easily extended to analysis of multiple price changes.

"Exact" Welfare Measures in Agricultural Policy Analysis

Introduction

One of the important functions of agricultural and other applied policy analysis is to provide a sense of what different groups gain and lose, and by what amounts, under alternative policy regimes. Often the analyst is in possession of one or more equilibrium price-quantity points for the market being studied and estimates, from other studies, of various demand and supply elasticities (or coefficients). Under maintained hypotheses of linearity or constant elasticity, the welfare effects of different market interventions are traced.

This procedure is often fairly reasonable, particularly if the market parameters are borrowed from a study closely related to the problem at hand. Yet it is important to remember that errors from a number of sources can creep into empirical estimates generated thusly.

One avoidable source of error is the use of consumer's surplus as an approximation to the true measures of welfare change associated with price changes, the compensating and equivalent variations (CV and EV, respectively) defined by J.R. Hicks. Recent literature has shown that there is no need to use consumer's surplus as an approximation for welfare change, since the information needed for computation of CV or EV is available in the ordinary demand system which is estimated.

Hausman develops, for single price changes, closed-form solutions for the expenditure function implied by linear, constant-elasticity, and quadratic ordinary demands, and shows that even in situations where consumer's surplus approximates CV closely, it may give very inaccurate estimates of dead-weight

loss (DWL). Vartia offers several algorithms for calculating arbitrarily close approximations to the CV and EV of a (single or multiple) price change.

The line of reasoning developed here differs from that of Hausman and Vartia. The procedure to be developed here uses Shephard's Lemma, the fundamental theorem of calculus, and the Slutsky equation to obtain estimates of CV (and EV). So long as the ordinary demand system is estimated properly (i.e., consistent with the properties of and restrictions upon demand), recourse to the underlying utility function is unnecessary. This procedure is developed using less information about ordinary demand than Hausman's procedure, which assumes that the complete ordinary demand for the good whose price changes is known. Also, since it obtains compensated demands directly from the Slutsky equation, it can easily be extended to the analysis of multiple price changes, unlike the analysis of Hausman.

Developing Exact Welfare Measures

Let $x_i = g_i(p,m)$ and $x_i^* = h_i(p,u)$ be the ordinary and compensated demands for good i, respectively, where p is a price vector, m is the consumer's income, and u represents utility level. At every initial equilibrium point in the analysis, (x_i^o, p_i^o), $x_i^o = g_i(p^o,m) = h_i(p^o,u^o) = x_i^{*o}$,

Consider a single price change p^o to p^1, where only the ith element in the price vector changes. The CV and EV of the price change are

$$CV = e(p^o,u^o) - e(p^1,u^o) = m - e(p^1,u^o)$$

$$EV = e(p^1,u^1) - e(p^o,u^1) = m - e(p^o,u^1)$$

where e(p,u) denotes the expenditure function derived from solution of the

dual problem min px s.t. u-u(x) = 0, and the superscripts denote initial and final levels of price and utility. The CV and EV are defined to be negative for a price rise and positive for a price decline.

By Shephard's Lemma, $h_i(p,u) = \partial e(p,u)/\partial p_i$. Also, by the fundamental theorem of integral calculus (see, e.g., Thomas, 1969, p. 172), if $F'(x) = f(x)$, then $F(b) - F(a) = \int_a^b f(x)dx$. Therefore, we can write

$$CV = e(p^o,u^o) - e(p^1,u^o) = \int_{p_i^1}^{p_i^o} h_i(p,u)dp_i,$$

since p_i is the only price which changes. Thus, analogous to the calculation of consumer's surplus, the CV (or EV) can be calculated as the area to the left of the compensated demand curve between the new and old prices. For convenience in exposition, we shall drop the subscript, with the understanding that we are integrating the (ordinary or compensated) demand curve for the good whose price changes.

The Linear Case

We consider first the linear case, since the development is somewhat more straightforward. As we move along the compensated demand for a good, its slope at each point (x*,p) can be obtained, using the Slutsky equation, from the income and price slopes of the (linear, in this case) ordinary demand which passes through the point (see, e.g., Deaton and Muellbauer). The Slutsky equation in partial derivative form proves useful in these derivations. Under the linear ordinary demand hypothesis, we can write

(1) $\partial x^*/\partial p = \alpha + \delta x^*$.

where $\partial x^*/\partial p$ is the slope of compensated demand, x* is quantity on the compensated demand and α and δ are coefficients for own price and income, respectively, for a good whose price change is to be evaluated.

Since as we move along the compensated demand, utility is constant, we can write (1) in total derivative form as

$$\frac{dx^*}{dp} = \alpha + \delta x^*$$

which is a simple ordinary differential equation with general solution

(2) $$x^* = -\frac{\alpha}{\delta} + k e^{\delta p}$$

To find the particular solution to (2), observe that $(x_o^*, p_o) = (x_o, p_o)$, so that

$$x_o = -\frac{\alpha}{\delta} + ke^{\delta p_o}$$

which requires that

$$k = \frac{x_o + \alpha/\delta}{\exp(\delta p_o)},$$

so that the equation for compensated demand passing through (x_o, p_o) is

(3) $$x^* = -\frac{\alpha}{\delta} + (x_o + \frac{\alpha}{\delta})\, e^{\delta(p-p_o)}.$$

Equation (3) is easily integrated to obtain the compensating variation of a price change from p_o to p, since

$$CV = -\int_{p_o}^{p} h(p,u)\,dp$$

$$= -\int_{p_o}^{p} -\frac{\alpha}{\delta} + (x_o + \frac{\alpha}{\delta})e^{-\delta p_o}\, e^{\delta p}\, dp$$

$$= \frac{\alpha}{\delta}(p - p_o) + \frac{1}{\delta}(x_o + \frac{\alpha}{\delta})(e^{\delta(p-p_o)} - 1).$$

It can be shown that for negative semidefiniteness in (1) to hold, the parameter restrictions are $-\alpha \geq \delta x_o$. These results are analogous to those obtained by Hausman.

Constant Ordinary Demand Elasticity

We turn now to the case of ordinary demand elasticity, considering functions of the form

$$x = g(p,m) = \gamma p^{\alpha} m^{\delta}$$

For this class of functions, the Slutsky equation gives us, for every point on the compensated demand, the following relationship between the slope of the compensated demand at that point and the price and income slopes of the ordinary demand passing through that point:

$$(4) \qquad \frac{\partial x^*}{\partial p} = \alpha \frac{x^*}{p} + \delta \frac{x^{*2}}{m},$$

where both x* and m vary as p varies. The amount of income, m, required to keep utility constant as we move along compensated demand when price changes from p_o to p_1, can be written as

$$(5) \qquad m_1 = m_o \left(\frac{x_1^*}{x_1}\right)^{\frac{1}{\delta}} = m_o \left(\frac{x_1^*}{x_o}\right)^{\frac{1}{\delta}} \left(\frac{p_1}{p_o}\right)^{\frac{\alpha}{\delta}}$$

where $x_1 = \gamma p_1^{\alpha} m_o^{\delta}$, and $x_1^* = \gamma p_1^{\alpha} m_1^{\delta}$ are the values of ordinary and compensated demands at p_1, respectively, and $x_1 = x_o \left(\frac{p_1}{p_o}\right)^{\alpha}$.

Substituting (5) into (4), and writing in total differential form,

$$(6) \qquad \frac{dx^*}{dp} = \alpha \frac{x^*}{p} + \frac{\delta}{c} x^{*2-\frac{1}{\delta}} p_1^{\frac{\alpha}{\delta}},$$

where $c = x_o^{-\frac{1}{\delta}} p_o^{\frac{\alpha}{\delta}} m_o$ is a constant determined by the initial point through which the compensated demand passes.

The ordinary differential equation in (6) has the general solution

$$(7) \qquad x^* = \left[kc \frac{1+\alpha}{1-\delta}\right]^{\frac{\delta}{\delta-1}} w^{-\frac{(\alpha+\delta)}{\delta-1}}, \quad \delta \neq 1$$

where w = p+a, and k and a are arbitrary constants determined by the values of compensated demand and compensated demand slope at the initial point.

It is immediately apparent from (7) that certain parameter restrictions upon ordinary demand follow from the negative semidefiniteness of the Slutsky matrix, which requires that $\frac{dx^*}{dp} \leq 0$. Sufficient conditions for this are:

(a) $\frac{\alpha + \delta}{\delta - 1} \geq 0$: for $\delta > 1$, $\delta \geq -\alpha$;

for $\delta < 1$, $\delta \leq -\alpha$;

(b) $\frac{1 + \alpha}{1 - \delta} \geq 0$, $k > 0$: for $\delta > 1$, $-\alpha \geq 1$;

for $\delta < 1$, $-\alpha \leq 1$.

Restrictions (a) and (b) imply that when $\delta \neq 1$, either $1 < -\alpha \leq \delta$ or $\delta \leq -\alpha < 1$, will assure that the ordinary demand represents utility maximizing behavior. While these conditions are not strictly necessary, there are only a few special cases in which they would not hold and the ordinary demand would still represent utility maximization.

Making use of the fact that $x_o^* = x_o$, and that $w \frac{dx^*}{dp} = -\left(\frac{\alpha + \delta}{\delta - 1}\right) x^*$, we can substitute the value of compensated demand and the slope of compensated demand at the initial price into (7) and, upon solving for k and a, obtain

$$(8) \quad x^* = x_o \left[\frac{v}{p - p_o + v}\right]^z = x_o v^z \left[p - (p_o - v)\right]^{-z}$$

where $z = \frac{\alpha + \delta}{\delta - 1}$, and $v = -\left(\frac{p_o}{\delta - 1}\right) z$.

Thus, compensated demand, corresponding to a constant elasticity ordinary demand function and intersecting ordinary demand at the initial point $(x_o^*, p_o) = (x_o, p_o)$, can be easily calculated from (8) as a function of ordinary demand parameters and the initial price-quantity point.

To obtain the compensating variation, (8) can be integrated over the price interval (p_o, p). Alternatively, we make note of the relationship between expenditure and demand given in (5):

$$CV = m_1 - m_o = m_o \left(\frac{x_1^*}{x_o}\right)^{1/\delta} \left(\frac{p_1}{p_o}\right)^{-\alpha/\delta} - m_o$$

$$= m_o \left[\left(\frac{v}{p - p_o + v}\right)^{z/\delta} \left(\frac{p_1}{p_o}\right)^{-\alpha/\delta} - 1\right].$$

Recall that the development thus far has been for $\delta \neq 1$. When $\delta = 1$, the differential equation in (6) simplifies, so that the compensated demand can be written

$$x^* = x_o (p_1/p_o)^{\alpha} \exp\left[\left(p^{\alpha+1} - p_o^{\alpha+1}\right)/(\alpha+1)c\right], \quad \delta = 1, \alpha \neq -1$$

$$x^* = x_o (p/p_o)^{-1+\frac{1}{c}}, \quad \delta = 1, \alpha = -1 \tag{9}$$

Note that the compensated demand in (9) is a special case where compensated elasticity is constant, which holds for Cobb-Douglas utility functions.

A point should be made regarding the price domain of the compensated demand in (8). Recall that as p varies, the function $x^*(w) = x^*(p - p_o + z)$ satisfies the differential equation in (6), in addition to which $x^*(w_o) = x(p_o)$. Thus, as p varies from p_o to p_1, w varies from w_o to w_1, where $w_1 = \left(\frac{p_1}{p_o}\right) w_o$.

Multiple Price Changes

The advantage of deriving compensated demands directly from the Slutsky equation, rather than by integrating through Roy's Identity to the indirect utility function, and inverting to obtain the expenditure function, is its use in analyzing multiple price changes. While Hausman was able to integrate to

obtain quasi-expenditure functions in the many good, single price change case, he was unable to extend his analysis to the multiple price change case.

The procedure developed in this paper can easily be extended to anlysis of multiple price changes. While a full development cannot be presented here, the extension can be obtained from the author. Consider a system of n ordinary demands that are properly estimated (i.e., consistent with the properties of utility maximization-homogeneity, adding up, Cournot and Engel aggregation, and negative semi-definite Slutsky matrix). Suppose, for illustration, that the prices of goods 1 and 2 change. Shephard's Lemma and the fundamental theorem of calculus allow us to evaluate the change in the expenditure function as the sum of two line integrals of compensated demand.

$$CV(p_1^o \to p_1^1,\ p_2^o \to p_2^1) = e(p_1^o,\ p_2^o,\ p,\ u^o) - e\ (p_1^1,\ p_2^1,\ p,\ u^o)$$

$$= e(p_1^o,\ p_2^o,\ p,\ u^o) - e(p_1^1, p_2^o,\ p,\ u^o) + e(p_1^1, p_2^o,\ p,\ u^o) - e(p_1^1,\ p_2^1,\ p,\ u^o)$$

$$(10) \qquad = -\int_{p_1^o}^{p_1^1} h_1(p_1, p_2^o,\ p,\ u^o)dp_1 - \int_{p_2^o}^{p_2^1} h_2(p_1^1, p_2, p, u^o)dp_2,$$

where p denotes the other prices being held constant. According to (10), the multiple price change can be evaluated sequentially. The amount of compensation required to keep utility constant as the first price changes (given by the first line integral) is allowed to determine new levels of demand for all commodities, then the second line integral is evaluated as p_2 changes. The compensating variation is the sum of the partial compensations determined by each line integral . The procedure can be used to evaluate k price changes in the same way.

Summary and Conclusions

We have developed a procedure for obtaining exact measures of welfare change resulting from a single price change, from the ordinary demand for that good. This procedure uses the Slutsky equation, Shephard's Lemma, and the fundamental theorem of integral calculus. It was applied to linear and constant elasticity demand specifications, and we showed that exact measures of compensating variation (CV) can be obtained from knowledge of income and price slopes and an initial equilibrium point. Thus, the information requirements of this procedure are less than those of, e.g., Hausman, who assumed that the complete demand equation was known. We also showed that this procedure is easily extended to analysis of multiple price changes, unlike Hausman's results.

This procedure has implications for policy analysis. Heretofore calculations of consumer's surplus have been used in policy analysis under a maintained hypothesis of linearity or constant elasticity of the ordinary demand function. Using essentially the same information and the same maintained hypotheses, calculations of CV are possible. The use of this procedure should eliminate one source of error in policy analysis.

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