A DYNAMIC ANALYSIS OF U.S. AGRICULTURAL PRODUCTION

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A large body of production economics literature concerns studies of output supply and input demand systems based on the assumption of instantaneous adjustment to prevailing prices by firms (Christensen, Jorgensen, and Lau; Fuss and McFadden). Quantities of variable inputs are assumed to adjust instantaneously to their desired levels. This static theory does not allow sluggish adjustments that characterize such inputs as land and capital. Adjustment cost theory has been utilized to accommodate quasi-fixed factors. Recently considerable interest has focused on incorporating dynamic elements in the analysis of input demand. Developments in adjustment cost theory allow a consistent dynamic theoretical framework for estimating input demand and output supply relationships (Lucas; Mortensen; Epstein; Vasavada). Generally dynamic input demand and output supply functions are estimated for a single output technology.

Highly volatile agricultural output prices in the United States have resulted in a need for diversification of farm enterprises. As a consequence, production of more than one output indicates that agricultural production should be studied in a framework of multiple output technologies. Input demand and output supply have been investigated using a dual approach in a static framework for multi-output technologies (Shumway; Just, Zilberman, and Hochman). The objective of this paper is to apply dynamic adjustment theory to multi-output technologies to empirically estimate input demand and output supply equations for aggregate U.S. agriculture.

Theoretical Model

The agricultural firm is assumed to maximize the discounted present value of an infinite stream of future profits. The value function of the firm can be written

$$(1)\ J(P,W,Q,K_0) = \max_{Y,L,I > 0} \int_0^\infty e^{-rt}[\ (P,W,K,I) - Q^T K]\,dt$$

$$\text{s.t.}\quad K = I - \delta K$$
$$K(0) = K_0$$

where

J = value function of the firm,

π = profit function solved by

$$\max_{Y,L} \pi = P^T Y - W^T L \text{ s.t. } F(Y,L,K,I) = 0,$$

Y = vector of outputs,

K = vector of quasi-fixed inputs,

L = vector of variable inputs,

I = vector of gross investment on quasi-fixed inputs,

P = price vector of Y,

W = price vector of L,

Q = price vector of K,

δ = diagonal matrix with positive depreciation rates of quasi-fixed inputs on the diagonal,

r = the discount rate, and

t = time.

$\pi(P,W,K,I)$ is assumed to be convex in P, W, K, and I and twice differentiable.

Assuming a constant real discount rate and the above conditions are satisfied, the value function J satisfies the Hamilton-Jacobi equation

$$(2)\ rJ(P,W,Q,K) = \max_{Y,L,I > 0} [\ (P,W,K,I) - Q^T L + J_K(P,W,Q,K)(I - \delta K)]$$

where the subscripts represent derivatives.

Optimal input demand and output supply equations are obtained by applying Hotelling's Lemma to (2). Differentiation with respect to W, Q, and P yields respectively

$$(3)\ L^* = -rJ_W + J_{WK}K^*$$

$$(4)\ \dot{K}^* = J_{QK}^{-1}(rJ_Q + K)$$

$$(5)\ Y^* = rJ_P - J_{PK}\dot{K}^*$$

The above derivations assume J is homogeneous of degree 1 in P, W, and Q. Normalization of prices relaxes this requirement of homogeneity. If all prices are normalized by a certain output price, then the respective output supply equation for that particular output becomes

$$(6)\ Y_n^* = r[J - J_P P - J_W W - J_Q Q] + [P^T J_{PK} + W^T J_{WK} + \bar{Q}^T J_{QK} - J_K \dot{K}^*]$$

The remaining output equations take the form of (5).

Empirical Model

For purposes of illustration, assume U.S. agriculture can be characterized by three outputs, Y_1, Y_2, and Y_3, two variable inputs, L_1 and L_2, and one quasi-fixed input, K. Furthermore assume all prices are normalized with respect to Y_1. Therefore consider the following value function

$$(7)\ J(P,W,Q,K) = A_0 + [a_1^T\ a_2^T\ a_3^T\ a_4^T] \begin{bmatrix} P \\ W \\ Q \\ K \end{bmatrix} +$$

$$+ 1/2\ [P^T W^T Q^T K^T] \begin{bmatrix} A & E & F & G \\ E^T & B & H & S \\ F^T & H^T & C & M^{-1} \\ G^T & S^T & M^{-1T} & D \end{bmatrix} \begin{bmatrix} P \\ W \\ Q \\ K \end{bmatrix}$$

where P is a 2 x 1 vector, W is a 2 x 1 vector, Q is a 1 x 1 scalar, and K is a 1 x 1 scalar. Also A, B, C, and D are symmetric.

Assuming consistent aggregation of investment across firms, then $J_{KK} = D = 0$ (Blackorby and Schworm). The input demand and output supply functions are derived from the value function (7) utilizing equations (3)-(6) to yield

(8) $L^* = -r[a_2 + E^T P + BW + HQ] - S[rK - \dot{K}^*]$

(9) $\dot{K}^* = M[r(a_3 + F^T P + H^T W + C^T Q)] + [ru + M]K$

(10) $Y_1^* = rA_0 - r[.5P^T AP + P^T EW + P^T FQ + .5WBW + W^T HQ + W^T SK + .5Q^T CQ] + a^T[rK - \dot{K}^*]$

(11) $Y_{n-1}^* = r[a^T + P^T A + W^T E + Q^T F^T] + G[rK - \dot{K}^*]$

where u is the identity matrix of appropriate dimensions and n is the number of outputs.

The investment demand equations in (9) can be written in the form of a multivariate flexible accelerator with constant adjustment coefficients (Nadiri and Rosen).

(12) $\dot{K}^* = N(K - \overline{K})$

where N = (ru + D) is an adjustment matrix and $\overline{K}$ is the vector of steady state stocks. K is determined by solving the system with $\dot{K}^* = 0$ to yield

(13) $\overline{K} = (ru + M)^{-1} M[r(a^T + P^T F + W^T H + Q^T C)]$.

Data

Index numbers for agricultural output and input prices and quantities for the period 1948-79 were obtained from Ball. Divisia price and quantity indexes (Diewert) were then computed for three outputs, two variable inputs, and one fixed input. The three outputs were (1) field crops, (2) livestock and dairy, and (3) fruits, nuts, and vegetables. Variable inputs include (1) labor and (2) intermediate materials while the fixed input included capital. Labor represents hired labor, self-employed, and unpaid family labor. Intermediate materials represent feed, seed, purchased livestock, chemical fertilizer, lime, pesticides, petroleum fuels, natural gas, and electricity. Capital price and quantity indexes consist of farm produced durables, producers' durable equipment, and land. A real discount rate of 0.1 is assumed.

Results

Input demand and output supply equations (8) through (11) were estimated using iterated nonlinear three stage least squares (N3SLS). The iterated N3SLS estimator yields consistent and asymptotically efficient estimates since it is asymptotically equivalent to full-information maximum likelihood (Hausman). The estimated parameters of the dynamic system are presented in Table 1. This model is characterized as a short-run dynamic model because $\dot{K}^*$ is not restricted to a steady state value of zero.

Nine of the twenty-seven short-run parameter estimates are significant at the 1% level; in addition, four estimates are significant at the 10% level. The own price derivatives of both variable and quasi-fixed factors are significant at the 1% level, while those for outputs are not significant. Convexity of the value function in prices occurs if J_{WW},

Table 1. Short-Run Parameter Estimates for the Dynamic Multiple Output Model[a]

	Estimate	T-Ratio
Intercept terms:		
A_0(FLD)	10.4556	7.10**
A_{11}(LVST)	9.2560	3.57**
A_{12}(FNV)	9.1537	10.27**
A_{21}(LABOR)	-5.5742	-2.84**
A_{22}(INTM)	-18.5027	-6.80**
A_3(CAP)	-3.3831	-0.24
Outputs:		
A_{11}(LVST)	1.9469	1.02
A_{22}(FNV)	0.5660	0.11
Inputs:		
B_{11}(LABOR)	18.4226	5.41**
B_{22}(INTM)	18.2140	4.91**
C(CAP)	-3.2081	-2.79**
Cross Effects:		
Outputs:		
A_{12}(LVST/FNV)	-0.3485	-0.58
Inputs:		
B_{12}(LABOR/INTM)	-16.0353	-10.74**
Output-Input:		
A_4(FLD/CAP1)	0.7755	2.31*
E_{11}(LVST/LABOR)	-7.5146	-2.86*
E_{12}(LVST/INTM)	2.8107	1.64
E_{21}(FNV/LABOR)	0.0581	0.13
E_{22}(FNV/INTM)	0.6711	0.64
F_1(LVST/CAP)	1.8200	1.29
F_2(FNV/CAP)	0.2777	1.26
G_1(LVST/CAP1)	-1.1300	-1.81*
G_2(FNV/CAP1)	0.0102	0.13
H_1(LABOR/CAP)	0.7818	0.60
H_2(INTM/CAP)	1.2970	1.93*
S_1(LABOR/CAP1)	0.4833	0.80
S_2(INTM/CAP1)	-0.1741	-0.69
M (ADJCOMP)	-0.0984	-0.47

[a]FLD for field crops, LVST for livestock and dairy, FNV for fruits, nuts, and vegetables, LABOR for labor, INTM for intermediate materials, CAP for change in capital, CAP1 for capital minus change in capital, and ADJCOMP for a component in the adjustment factor.

*Significant at the 10% level.

**Significant at the 1% level.

J_{PP}, and J_{QQ} are non-negative. Since $J_{QQ} = -M \cdot C = -0.3157$, convexity of the value function cannot be accepted.

Hypotheses can be tested concerning the fixity of the factors. When all factors are perfectly variable, the adjustment matrix N in equation (12) must equal -u. Since the system contains only one quasi-fixed input, N is a scalar. Therefore if capital is perfectly variable, N = -1. This hypothesis was rejected at the 1% significance level, indicating that the sluggish adjustment of capital characterizes U.S. agriculture. The adjustment coefficient is estimated to be -0.1984 for this system.

In order to obtain long-run relationships, equations (10) through (13) were re-estimated assuming $\dot{K}^* = 0$. The long-run input demand equation for capital becomes equation (13). Long-run parameter estimates are reported in Table 2. Ten of the long-run estimates are significant at the 1% level, while three estimates are significant at the 10% level. A_{11}, the own price derivative of livestock and dairy attains a significance level of 1% in the long run.

Short-run and long-run price elasticities are calculated from the regression results presented in Tables 1 and 2, respectively. Short-run elasticities are presented in Table 3. Short-run own price elasticities for livestock and dairy and fruits, nuts, and vegetables are positive and highly inelastic. Own price elasticities for variable inputs, labor and intermediate materials, are negatively inelastic and elastic, respectively. A surprising result was the positive short-run own price elasticity for capital. The cross output-input price elasticities perhaps reveal information concerning the capital input. An increase in the price of either intermediate materials or capital results in an increase in

Table 2. Long-Run Parameter Estimates for the Dynamic Multiple Output Model[a]

	Estimate	T-Ratio
Intercept terms:		
A_0(FLD)	2.3451	0.44
A_{11}(LVST)	-10.8381	-1.46
A_{12}(FNV)	18.6348	4.85**
A_{21}(LABOR)	-28.0790	-5.53**
A_{22}(INTM)	-6.8480	-1.26
A_3(CAP)	0.9508	0.78
Outputs:		
A_{11}(LVST)	11.6654	4.27**
A_{22}(FNV)	-2.8387	-1.54
Inputs:		
B_{11}(LABOR)	17.1169	5.21**
B_{22}(INTM)	9.8626	2.79**
C(CAP)	-0.0435	-0.51
Cross Effects:		
Outputs:		
A_{12}(LVST/FNV)	-0.4514	-0.31
Inputs:		
B_{12}(LABOR/INTM)	-10.6816	-5.40**
Output-Input:		
A_4(FLD/CAP)	7.2428	1.36
E_{11}(LVST/LABOR)	-15.0064	-4.82**
E_{12}(LVST/INTM)	6.0786	3.86**
E_{21}(FNV/LABOR)	3.9956	2.77
E_{22}(FNV/INTM)	-2.4656	-1.40
F_1(LVST/CAP)	0.2420	0.64
F_2(FNV/CAP)	-0.2890	-0.68
G_1(LVST/CAP)	16.6485	1.90*
G_2(FNV/CAP)	-6.3659	-2.52*
H_i(LABOR/CAP)	0.1659	0.62
H_2(INTM/CAP)	-0.3462	-0.68
S_1(LABOR/CAP)	25.2515	4.67**
S_2(INTM/CAP)	-6.7050	-1.86*
M (ADJCOMP)	-0.0984	-0.77

[a]FLD for field crops, LVST for livestock and dairy, FNV for fruits, nuts, and vegetables, LABOR for labor, INTM for intermediate materials, CAP for capital, and ADJCOMP for a component in the adjustment factor.

*Significant at the 10% level.

**Significant at the 1% level.

Table 3. Short-Run and Long-Run Elasticities for the Dynamic Multiple Output Model

Output Supply and Input Demand Equations	Livestock and Dairy	Fruits, Nuts, and Vegetables	Labor	Intermediate Materials	Change in Capital	Capital
			Short-Run Price Elasticities			
Livestock and Dairy	0.1939	-0.0483	-0.5423	0.3147	0.0976	-
Fruits, Nuts, and Vegetables	-0.0303	0.0068	0.0037	0.0655	0.1299	-
Labor	0.4148	-0.0045	-0.7367	0.9951	-0.0232	-
Intermediate Materials	-0.3240	-0.1077	1.3393	-2.3602	-0.0805	-
Change in Capital	-0.0186	-0.0040	-0.0058	-0.0149	1.0251	-
			Long-Run Price Elasticities			
Livestock and Dairy	1.1619	-0.0626	-1.0830	0.6806	-	0.0130
Fruits, Nuts, and Vegetables	-0.0392	-0.3433	0.2514	-0.2407	-	-0.0135
Labor	0.8284	-0.3072	-0.6847	0.6629	-	-0.0049
Intermediate Materials	-0.7007	0.3958	0.8921	-1.2780	-	0.0215
Capital	0.3579	-0.5952	0.1778	-0.5756	-	-0.0347

output of livestock and dairy and fruits, nuts, and vegetables. Accordingly, as the prices of these outputs are increased, use of intermediate factors and captal decreases suggesting the two factors may be inferior. The positive cross-price elasticities for labor and intermediate materials suggest the two are gross substitutes, while the negative cross-price elasticities of demand for labor and capital in addition to intermediate materials and capital indicate that labor and intermediate materials are complementary to capital .

Long-run elasticities are also reported in Table 3 and differ somewhat from their short-run counterparts. The own price elasticity of livestock and dairy remains positive and becomes inelastic, while the own price elasticity for fruits, nuts, and vegetables remains inelastic but becomes negative. An interesting result is the own price elasticity of capital becomes negative and inelastic in the long run. As in the short run, long-run cross price elasticities for labor and intermediate factors indicate substitutability between the two factors. The positive cross price elasticity for labor and capital suggests the two factors are gross substitutes in the long run. The negative cross price elasticity between intermediate factors and capital in the long run can perhaps be rationalized by concluding that the two inputs are complements.

Conclusions

This study applied dynamic duality theory to multiple output technologies to estimate aggregate input demand and output supply equations. The empirical results suggest that capital can be characterized as quasi-fixed indicating it is slow to adjust to changes in prices which implies an advantage of the present dynamic approach over static analyses. The differences observed in the supply responses of the various categories of outputs indicates an advantage of the present multiple-output, dynamic approach over earlier dynamic analyses that considered only a single category of aggregate output.

Some of the empirical highlights of the study are as follows. Comparing short-run and long-run own price elasticities indicated that livestock and dairy supplies became more elastic through time, as expected, but the opposite was the case for fruits, nuts, and vegetables. The demands for capital became more elastic through time, but the demands for labor and intermediate materials became more inelastic through time. Labor and intermediate factors were found to be gross substitutes in both the short and long run. The cross price relationships implied that intermediate materials exhibit a complementary relationship with capital in both the short and long run, while labor complements capital in the short run and serves as a substitute for capital in the long run.

This paper has demonstrated the application of the framework of multiple output technologies. The results should be viewed as tentative. However, the paper provides a foundation for future research in the area.

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