MARKET EQUILIBRIUM WITH RANDOM PRODUCTION

by

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This paper investigates competitive equilibrium when production is subject to random deviations from intended output. We find that the most commonly used specification of equilibrium price and output -- the intersection of the demand function and the intended supply function -- is incorrect. This result has possibly important implications for some areas of current interest, including the literature on gains from price stabilization and the empirical modeling of supply response to risk.

Determination of Equilibrium

Consider the following simple model for a single product. The demand function is

$$X_t = D(P_t) \quad (1)$$

and production is subject to a random error,

$$X_t = \phi(X_t^*, u_t), \quad (2)$$

Where X_t and X_t^* are actual and intended output, and u_t is a random error term. The standard specifications of ϕ are linear ($X_t = X_t^* + u_t$) or multiplicative, ($X_t = X_t^* u_t$), but our argument is not sensitive to the functional form. The error term is observed only after production decisions have been made, and it is assumed that it is too late to change intended production after the random event (e.g., drought) has occurred. Thus, this year's supply is perfectly inelastic in response to current market price, even though next year's intended production is responsive to next year's expected profit prospects.

What is the meaning of "equilibrium" in the context of this model? In any particular year there will exist a value for X_t^* and u_t that results in a particular value, X_t^o. This X_t^o will clear the market at a price determined by the demand function P_t^o. This market-clearing price is the short-run equilibrium price, and is just the same as standard theory would give us for equilibrium with perfectly inelastic supply at X_t^o. What is different about instability is the difficulty of specifying longer-run equilibrium. To make sense of this concept at all we must refer to mean values over a series of seasons, i.e., the statistical expectation of X_t and P_t.

The standard method of specifying equilibrium for such a model is to represent intended output X_t^* as a function of expected price P_t^* (see, Turnovsky, p. 713):

(3) $$X_t^* = S(P_t^*).$$

Equilibrium is then said to be attained when producer expectations of price equal the mean market price, i.e., $P_t^* = \overline{P}$. By taking the expectations of equations (1) to (3) we find equilibrium as the intersection of the intended supply function (3) with demand.

While this is a plausible specification of equilibrium, it is incorrect. It is, in general, inconsistent with long run competitive equilibrium.

The fundamental long run equilibrium condition under competition is that expected profits be zero (assuming risk-neutrality), i.e., expected average revenue equals average costs:

(4) $$\frac{E(X_t P_t)}{E(X_t)} = AC_t.$$

In our usual static models, equation (4) reduces to the equilibrium condition that price equals average cost. However, under random production mean revenue per unit output is not equal to mean price because

$$\frac{E(X_t P_t)}{E(X_t)} \neq \frac{E(X_t)E(P_t)}{E(X_t)}\ .$$

The nature of the inequality can be investigated using Jensen's inequality (Rao, 1973), which states that

$$E[f(\varepsilon)] \gtreqless f[E(\varepsilon)] \quad \text{when} \quad \frac{\partial^2 f}{\partial \varepsilon^2} \gtreqless 0 ,$$

where ε is a random variable, and f represents the functional form. It means that $E[f(\varepsilon)]$ is greater, equal, or less than $f[E(\varepsilon)]$ when f is respectively a convex, linear or concave function of ε. In the case considered, the random variable is output X and the function is the total revenue function $X \cdot P(X)$. We have

$$(5) \qquad \frac{\partial^2[X \cdot P(X)]}{\partial X^2} = 2\frac{\partial P(X)}{\partial X} + \frac{\partial^2 P(X)}{\partial X^2} \cdot X$$

Consider two special cases. First, for linear demand,

$$P = a + bX \qquad (a > 0,\ b < 0),$$

which implies that

$$\frac{\partial^2(X \cdot P)}{\partial X^2} = 2b < 0.$$

Therefore, for linear demand, it is always true that

$$\frac{E(X \cdot P)}{E(X)} < E(P).$$

For a demand function with constant elasticity η it turns out that equation (5) is negative for $-\infty < \eta < -1$ and positive for $\eta > -1$. Therefore, from Jensen's inequality,

$$\frac{E(XP)}{E(X)} < P[E(X)] \quad \text{if the elasticity of demand is greater than 1, and}$$

$$\frac{E(XP)}{E(X)} > P[E(X)] \quad \text{if the elasticity of demand is between zero and 1.}$$

In general, mean revenue per unit is not equal to expected price. A special case when they are equal is a unit elastic demand curve, when revenue is constant.

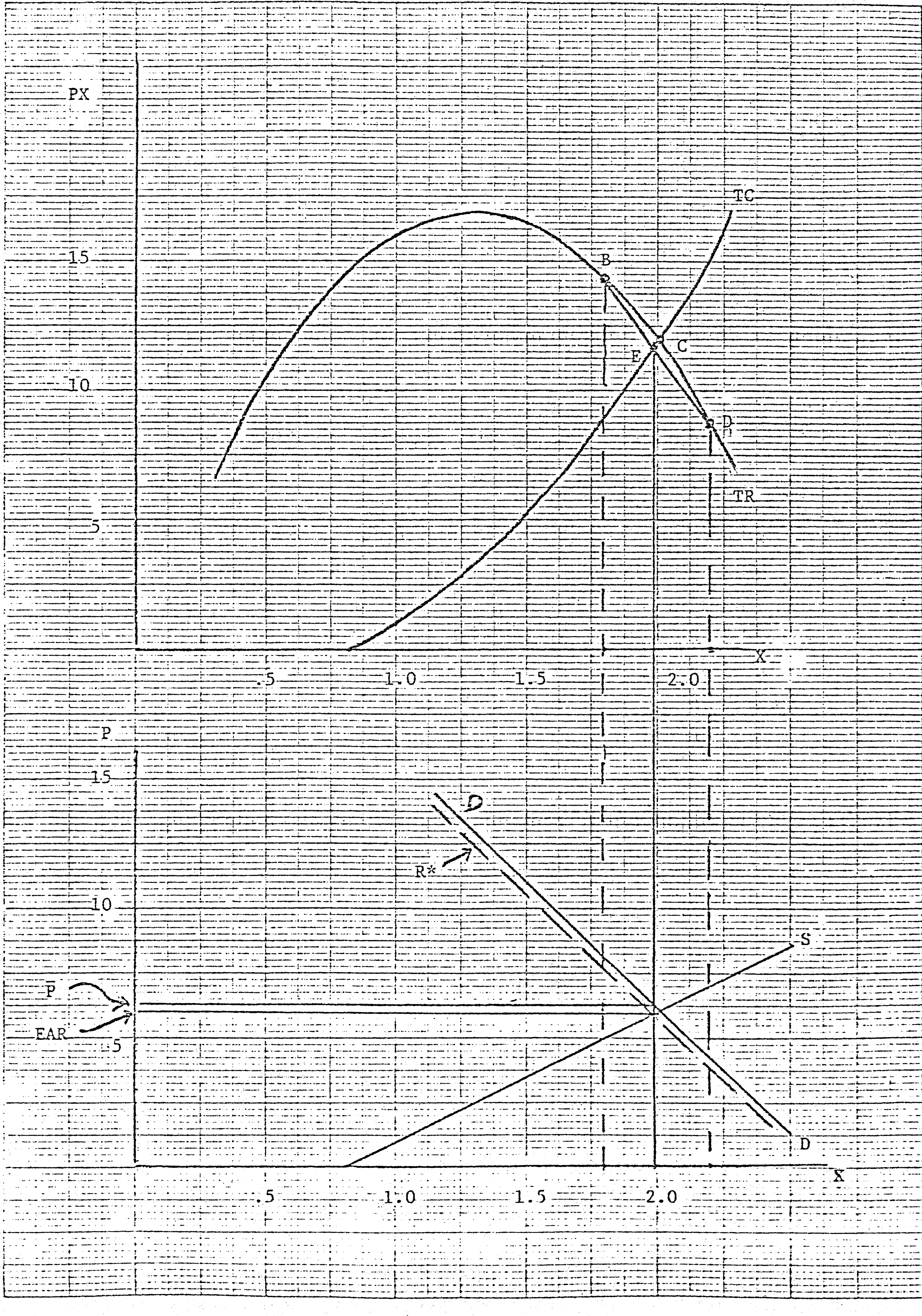
The appropriate specification of equilibrium can be depicted diagramatically with the use of total revenue and total cost functions. Figure 1 shows total revenue and the corresponding demand function for a case of linear demand. It shows total cost and the corresponding intended supply function for a linear version of equation (3). The error structure for equation (2) is additive with u_t taking the value $\pm$ .2 with equal probability.

This model is identical in form to that of Turnovsky. Turnovsky's approach to finding the equilibrium identifies the point at which expected producer price equals mean market price, i.e., the intersection of S and D in figure 1. However expected total revenue lies on the midpoint of a chord connecting $\overline{X} \pm .2$ on the total revenue function. The sequence of midpoints as $\overline{X}$ changes gives expected total revenue as a function of $\overline{X}$. Dividing by $\overline{X}$ yields R_t^*, expected average revenue, which is plotted as a dotted curve beneath the demand function. The intersection of this curve with intended supply is the long run equilibrium point. Equilibrium is represented in the total curves as point E, where expected total revenue equals total cost.

Thus we find that equilibrium output under random production is less and mean price higher than under deterministic production with the same cost and demand structure. And the more variable production is, the greater the reduction in equilibrium production. The basic economic reason is the shape of the total revenue function, which yields lower mean revenues from variable production. (It has nothing to do with risk aversion, which is absent from this model.)

To give concreteness to these results consider how the equilibrium is calculated for the linear model illustrated in figure 1. First, if we supposed the equilibrium to be at the intersection of intended supply and the demand function,

Figure 1. Equilibrium as expected total revenue equal to total costs



the frequency distribution of outcomes, and the resulting mean values, are as follows;

frequency	u_t	X_t	P_t	$(X \cdot P_t)$	Cost
.5	-.2	1.8	8	14.4	12.0
.5	+.2	2.2	4	8.8	12.0
mean	0	2.0	6	11.6	12.0

While mean price is the same as the market-clearing price if u_t were zero, mean total revenue is less than in the certainty case. Thus, expected losses exist, and we do not have a long run competitive equilibrium. To find the stochastic long run equilibrium, note that equation (4) can be multiplied by $E(X_t)$ to obtain expected revenue equal to expected total cost. The latter is nonstochastic since producers aim to produce $\overline{X}$ each year. The condition that total cost equals total revenue is

$$(6) \qquad S^{-1}(\overline{X})\overline{X} = \sum_t D^{-1}(X_t)X_t Pr(u_t)$$

where S^{-1} and D^{-1} are the inverse supply and demand functions. Equation (6) is the basic relationship which allows us to solve for equilibrium $\overline{X}$ in the stochastic environment. In the case shown in figure 1, we have $D^{-1}(X_t) = 26 - 10P_t$, $S^{-1}(\overline{X}) = -4 + 5\overline{X}$, $u_t = -.2$ or $+.2$, each with probability .5, and that $X_t = \overline{X} + u_t$ so that,

$$(-4 + .5\overline{X})\overline{X} = .5(26 = 10(\overline{X} - .2))(\overline{X} - .2) + .5(26 - 10(\overline{X} + .2))(\overline{X} + .2).$$

This reduces to the quadratic equation:

$$15\overline{X}^2 - 30\overline{X} + .4 = 0,$$

for which the root that satisfies both the intended supply and the demand equation is $\overline{X} = 1.987$. (Note that if $U_t = 0$, the nonstochastic case, then the quadratic equation becomes

$$15\overline{X}^2 - 30\overline{X} = 0,$$

i.e., $\overline{X} = 2$. This is the graphical certainty-equivalent equilibrium in figure 1.)

To confirm these values as equilibria:

frequency	u_t	X_t	P_t	$(X \cdot P)_t$	cost
.5	-.2	1.787	8.13	14.53	11.78
.5	+.2	2.187	4.13	9.03	11.78
mean	0	1.987	6.13	11.78	11.78

Implications

The preceding discussion indicates that the modeling of competitive equilibrium under uncertainty but with risk neutrality is a more complex task than might at first be supposed. If risk aversion by producers were introduced, the problem would become more complicated still. In order to examine the problems that can arise from an insufficiently careful treatment of market equilibrium under uncertainty, consider three areas of recent literature: (1) studies of the producer and consumer gains from price stabilization, (2) the result of Hazell and Scandizzo that competitive equilibrium is suboptimal in certain stochastic models, and (3) empirical estimation of supply functions, particularly producers' response to risk.

Gains from price stabilization. Waugh and Oi couched their original papers on this subject in terms of the behavior of microeconomic units in response to exogenous price fluctuations. However, since Massell's work of ten years ago the focus has switched to the study of market equilibria under stochastic supply and demand functions. The characterization of market equilibrium in this literature, without exception as far as we can determine, has been to replace producers' and consumers' expected prices by mean prices. It is then said that the economic agents' expectations are borne out, so there is no reason for them to alter their behavior and we have the stochastic analog to competitive equilibria in the standard nonstochastic supply-demand treatment. While this paper does not consider stochastic demand or producer response to current price as Oi and Massell do, it was shown above that in the stochastic production model as used by Turnovsky -- the model that seems most appropriate for agricultural crops -- this characterization of "equilibrium" is unsatisfactory. The comparisons of unstabilized and stabilized situations are not comparisons of alternative equilibria. Thus, when Turnovsky (p. 714) finds that stabilization increases producers' surplus under rational expectations, what he measures is in fact the gain in mean total revenue. This gain is correctly characterized as producers' surplus only when the intended supply function is perfectly inelastic. When producers can expand intended production in response to expected profit opportunities, there will be an increase in mean production, and consequently a decline in mean price. Therefore, some of the producer gains will be dissipated and Turnovsky's formula for producer gains (p.713) will not hold.

Welfare under competition. Hazell and Scandizzo find that under a particular specification of equations like (1) to (3) the mean output generated by competitive market equilibrium is nonoptimal in that it does not maximize the expected sum of producers' plus consumers' surplus. As in the preceding discussion, their analysis does not involve risk aversion or risk preference. They derive an "optimal distortion price" (pp. 645-46) which does result in the maximization of expected producers' plus consumers' surplus.

Hazell and Scandizzo use a nonstochastic linear demand function and a linear intended supply function, as in figure 1, but they introduce the error multiplicatively. They then calculate equilibrium price by the same method that we have been criticizing as used in the Turnovsky paper. It is this "equilibrium" that they find suboptimal. However, applying equation (6) to their model it can be shown that the full equilibrium value of R_t^* is in fact their "optimal" price. Thus, the problem that Hazell and Scandizzo are exploring is not one of suboptimality of the competitive equilibrium but rather the inability of our usual techniques to specify equilibrium propoerly when production is stochastic.

Empirical specification of supply functions. Suppose production is stochastic as in equations (1) to (3) but we no longer have stationarity, i.e., the functions D and S shift from time to time. In this case we would trace out a series of values of $E(P_t)$ and intended output X_t^* as conditions changed. $E(P_t)$ would still not be equal to R_t^*. Therefore, it is strictly speaking inappropriate to regress, say, acreage (as a proxy for intended output) on expected price in supply analysis. In practice, however, when demand shifts the resulting change in expected price would typically be associated with an almost proportional change in expected average revenue. Thus, expected price may well be a good proxy for R_t^* in equation (3).

A more serious empirical problem arises in the investigation of supply response to risk. With a linear demand function, increased variability in production reduces expected average revenue for a given mean price. Therefore, an increase in variability will be observed in association with a reduction in intended production in a regression which also holds price constant. We are not surprised to observe such a negative supply response to variability, but those who have estimated these responses tend to attribute them to risk aversion. The same effect is found in this paper as strictly a matter of reduced expected returns as production variability increases, and has nothing to do with subjective preference concerning risk. This is not to say that observed acreage responses to variability in output (or in price variability which results from output variability) are not in fact risk responses. They may well be. The point is only that a significant regression coefficient on variability is not a powerful test for risk response. The appropriate specification to analyze risk response would be to include expected average revenue rather than expected price in the regression equation.

Conclusion

The term "equilibrium" is ambiguous as applied to a stochastic environment. Random errors cause outcomes almost always to deviate from expectations so that one may never observe a combination of price and output that would correspond to static equilibrium in a deterministic model. It is true that there is a market-clearing price in each period which may be called an equilibrium price, but this is a strictly transitory concept of equilibrium. There is no reason for producers or consumers to base future plans on these random prices. Instead, economic actors are most plausibly considered as basing their plans on expected prices as inferred from observation of a series of market prices and outlook information. In this context, equilibrium is most usefully thought of as a state of affairs in which the actions of all economic agents are mutually consistent with one another. For producers, this means that the series of market outcomes does not lead them either to expand or contract intended production. For consumers, it means that the frequency distribution of price is consistent with mean desired consumption equal to mean production.

This paper has argued that the appropriate long run equilibrium condition for competitive producers is that expected average revenue (EAR) equals average cost. This criterion applies to both deterministic and stochastic models. In deterministic models, however, EAR equals price: $E(P_tX_t)/(E(X_t) = P_t$. The approach which specifies equilibrium in terms of mean price paid by consumers lying on the intended supply curve implies that expected profits or losses exist, depending on the shape of the total revenue function. This result has serious implications for the price stabilization literature, which attempts to compare stabilized equilibria with producer and consumer well-being in "equilibria" under instability.

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