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THE ROLE OF MARKET PRICE-WEIGHT RELATIONSHIPS IN OPTIMAL BEEF CATTLE MANAGEMENT MODELS

by

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Abstract

Optimal beef cattle diet and rate-of-gain analyses normally ignore the relationship between the animal's market price and its weight. Theoretical and empirical models are developed to show that optimal daily weight gain tends to be less than the feasible maximum when market price per pound falls rapidly with weight increases.

THE ROLE OF MARKET PRICE-WEIGHT RELATIONSHIPS IN OPTIMAL BEEF CATTLE MANAGEMENT MODELS

Approaches to the optimization of beef cattle management programs have changed significantly since they were first formulated in a linear programming context in the 1950's. Models were originally designed to identify feed rations which minimized the cost of providing minimum levels of energy and selected nutrients. Subsequent efforts sought to maximize overall farm or feedlot profits, including identification of optimal feeding weights, daily gains, and ration formulations [Kearl, Harris, and Fonnesbeck]. These have varied in attention to such details as number of head fed, variety of alternative feeds, seasonality factors, animal characteristics, and the manner in which energy requirements and appetite are modelled [Carlson; Harrison; Walker and Anderson].

An opportunity for significantly improving optimal beef feeding analysis was provided by development of the California net energy system, adopted by the National Research Council (NRC), which clarified the relationship between a feed ration's caloric density and the efficiency with which it could be converted into net energy for gain or maintenance. Brokken, et al., Wilson, and others subsequently cast the net energy system in an economic framework. These authors, notably Wilson, emphasized that over a wide range of concentrate and roughage prices, the system's net energy requirements favored *ad libitum* feeding of high-caloric-density rations, and thus maximization of daily weight gains. The purpose of the present paper is to show that a broader set of conclusions is reached when attention is paid to the relationship between cattle market price and cattle liveweight. Expressions for net revenue per day and feed cost per day are first defined, and implications of these formulations shown for optimal daily gain levels. Next, results of a maximum-profit farm feeding program are reported in which the importance of the formulations is illustrated.

Conceptual Issues

Marginal Net Revenue and Marginal Feed Cost

In a returns-per-day framework, the cattle feeder observes an animal's value at the beginning of a day and formulates the quantity and quality of its ration so as to maximize the difference between the animal's value at the end of the day and its beginning value plus feed cost. At any point in time, the market price of a steer or heifer is related, usually negatively, to its weight. It is here assumed for simplicity that this function is the same at the end as at the beginning of the day, and that it may be approximated by the linear form:[1/]

(1) $$P = a - bW,$$

where $a > 0$ and $b > 0$ are constants, P is market price in \$/lb, and W is market liveweight in lbs. Defining W_o and P_o as beginning weight and price, and W_e and P_e as ending weight and price, the difference or total net revenue (TNR) between the animal's value at the beginning and end of the day is

$$
\begin{aligned}
TNR &= W_eP_e - W_oP_o \\
&= W_e(a - bW_e) - W_o(a - bW_o). \\
&= a(W_e - W_o) - b(W_e^2 - W_o^2) \\
(2) \qquad &= a(W_e - W_o) - b[(W_e - W_o)^2 + 2W_eW_o - 2W_o^2] \\
&= a(W_e - W_o) - b(W_e - W_o)^2 - 2bW_eW_o + 2bW_o^2 \\
&= a(W_e - W_o) - b(W_e - W_o)^2 - 2bW_o(W_e - W_o) \\
&= (a - 2bW_o)(W_e - W_o) - b(W_e - W_o)^2.
\end{aligned}
$$

It is convenient now to define daily weight gain g as the difference between beginning and ending weight $(W_e - W_o)$ and to express (2) in the form

$$(2)' \qquad TNR = (a - 2bW_o)g - bg^2.$$

Net revenue at first rises, but at a continually decreasing rate, with increases in daily gain.2/ Dividing equation (2)' by g produces average net revenue (ANR), or the net increase in animal value per pound of gain added. Of greater economic importance is marginal net revenue (MNR), that is the addition to animal value caused by each additional pound added:

$$(3) \qquad MNR = dTNR/dg = (a - 2bW_o) - (2b)g.$$

Under the assumptions that a and b are positive, MNR is a negative and linear function of daily weight gain. The slope of this function (2b) varies directly with the slope of price-weight relationship (1) from which it is derived, whereas the intercept $(a - 2bW_o)$ increases with the intercept and decreases with the slope of the price-weight function.

The relationship of daily feed cost to daily gain may be derived by utilizing the NRC gain and maintenance net energy functions together with a specified ration and appropriate feed prices. In the NRC functions, a steer's or heifer's daily requirement of net energy for maintenance (NE_m) is a linear function of its metabolic weight, and its daily requirement of net energy for gain (NE_g) is a function of both metabolic weight and daily weight gain.3/ The price of a unit of maintenance net energy (P_{nem}) or gain net energy (P_{neg}) may be determined by dividing each feed price per unit dry matter by its NE_m or NE_g concentration per unit dry matter, multiplying by the proportion of ration NE_m or NE_g accounted for by each feed, and summing these products. Total daily feed cost is then

$$(4) \qquad TFC = (NE_m)(P_{nem}) + (NE_g)(P_{neg}).$$

Expressing on a lb-weight basis the NRC requirement functions for a steer, and substituting into (4),

$$(4)' \quad TFC = .077(W/2.2)^{.75}(P_{nem}) + (.02396g + .00141g^2)(W/2.2)^{.75}(P_{neg}).$$

For any weight $W = (W_e + W_o)/2$, the corresponding average feed cost (AFC), or cost per lb of gain, is found by dividing (4)' by g. The marginal feed cost function (MFC), that is the feed cost of an additional pound of gain, is

$$(5) \qquad MFC = dTFC/dg = (.02396 + .00282g)(W/2.2)^{.75}(P_{neg}),$$

a positive and linear function of daily weight gain.

Optimal Daily Weight Gain

Returns per day over feed and cattle purchase cost are now maximized by equating falling marginal net revenue (3) with rising marginal feed cost (5) and solving for the maximum-return rate of daily gain g*:

$$(6) \qquad g^* = \frac{a - 2bW_o - (.02396)(W/2.2)^{.75}(P_{neg})}{2b + (.00282)(W/2.2)^{.75}(P_{neg})} .$$

Equation (6) defines a wide range of optimal daily gains, depending upon levels of a, b, W, and P_{neg}. Only some of these optimal levels are feasible in the sense of being consistent with the steer's ability to consume dry matter, an ability related to the steer's weight and to the caloric density of the ration. Let g_m be the absolute maximum daily gain achievable by a steer at a given weight. Then if $g^* > g_m$, a corner solution prevails at g_m. If $g^* < g_m$, the marginal conditions are fulfilled and returns are optimized by operating at less than the maximum daily gain level. Less-than-maximum daily gains may be achieved by feeding less than ad libitum or by feeding a high roughage diet.

The responsiveness of optimal daily gains g* to the cattle price-weight structure is characterized by differentiating (6) with respect to price-weight intercept a and slope b:

$$(7) \qquad dg^*/da = \frac{1}{2b + .00282(W/2.2)^{.75}P_{neg}}$$

$$(8) \quad dg^*/db = \frac{-2W_0[2b+.00282(W/2.2)^{.75}P_{neg}] - 2[a-2bW_0-.02396(W/2.2)^{.75}P_{neg}]}{[2b+.00282(W/2.2)^{.75}P_{neg}]^2} .$$

Since all terms in the denominator of (7) are positive, equation (7) itself is positive, meaning that decreases in the cattle price-weight intercept reduce optimal daily weight gains. The reduction is explained by the fact that a downward shift of the price-weight intercept in (1) also reduces the intercept of marginal net revenue function (3).

Under realistic feed price structures, term $[a-2bW_0-.02396(W/2.2)^{.75}P_{neg}]$ in equation (8) is positive, so that the entire right-hand-side of (8) is negative.[4/] Thus increases in the rate at which cattle prices decline with increases in weight are associated with a decline in the optimal daily gain. The decline is caused by a downward shift of the intercept <u>and</u> an increase in the negative slope of the marginal net revenue function as the negative price-weight slope increases. Hence, given a particular set of feed prices, there exists some negative cattle price slope b above which optimal gains g* fall below absolute maximum feasible daily gain g_m. This situation is depicted in Figure 1, where g_1^* refers to an infeasible optimal daily gain given price-weight slope b_1, and g_2^* a feasible optimal daily gain given slope b_2 $(|b_2| > |b_1|)$.

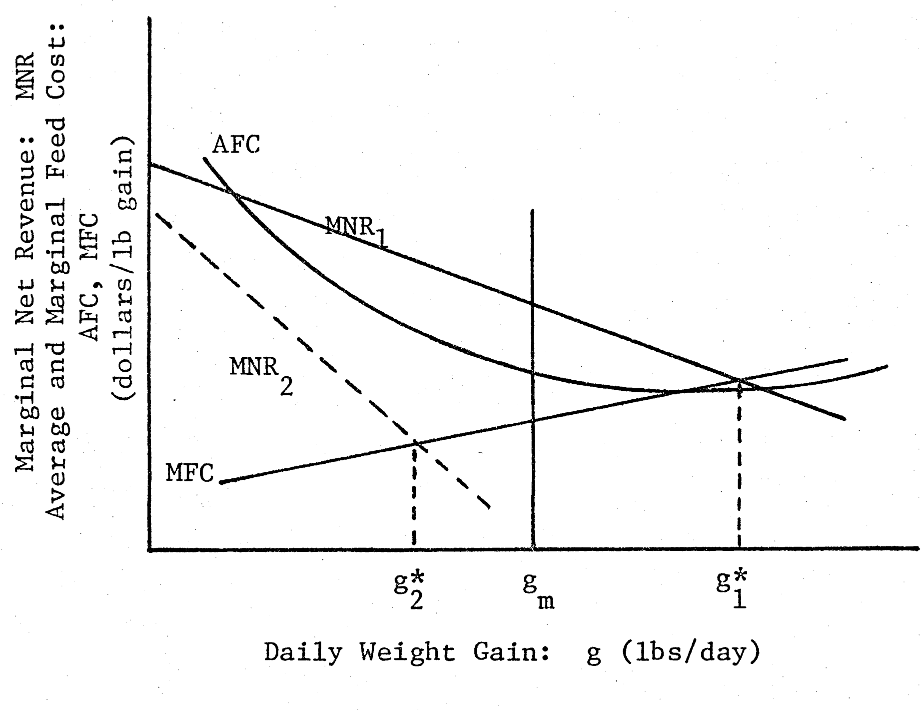

Figure 1. Illustration of daily weight gain optimization when cattle prices are negatively related to weight.

A similar derivative of equation (6) with respect to feed ration price P_{neg} may also be shown negative, suggesting that increases in ration prices dampen optimal gain levels by shifting upward both the slope and intercept of the marginal feed cost function.

Empirical Issues

The question naturally arises whether optimal daily gain g* would fall below maximum feasible gain g_m under historically realistic cattle price-weight functions and realistic sets of feed prices. To derive empirical estimates of the relationships outlined above, a programming model was developed of a mixed crop and beef feeding operation.

Model Structure

The model spanned a one-and-a half year time horizon in which a farm operator was considered to make cattle feeding, purchase, and sale, and crop production, purchase, and sale decisions at the beginning and end of each quarter. Potential crop production activities included corn grain and stover, corn silage, alfalfa pasture and hay, orchard grass-ladino clover pasture, and fescue pasture and hay. Crops could be produced in the first two quarters of the program (spring and summer) and were available for subsequent cattle feeding or sale by way of storage activities. The model selected fertilization rates endogenously and could produce crops on a variety of soil classifications.

Steers were available for purchase, or subsequent sale, at 100-lb weight increments between 500 and 1000 lbs, and heifers at 75-lb increments between 450 and 900 lbs. At each such weight, the operator had the option to feed at maintenance level (zero weight gain), a lower level of gain (1.1 lbs/day for steers, .83 lb/day for heifers), or a higher level of gain (2.2 lbs/day for steers, 1.67 lbs/day for heifers). Net energy requirements for maintenance and gain at each weight and gain level, and energy concentrations of feeds, were taken from the National Research Council (see footnote 3). Minimum protein requirements were those reported by Carlson, and dry matter appetite relations were as specified in Nino and Hughes. The program's objective function maximized returns to the farm operator's own land, labor, and capital. The operator could purchase additional capital or labor if profitable.

Model Results

In the baseline model run, feed and cattle market prices in each season were the 1968-1977 averages for that season in the Appalachian area of Virginia, as inflated to 1977 dollars. An average cattle price-weight function for each season and sex was calculated by separately averaging the intercepts and slopes for each season and sex over the ten-year period. For the sake of brevity, only solution results involving fall steers are reported here.[5/] The baseline solution called for purchasing 583 500-lb steers on October 1 and feeding them a daily ration consisting of 44% corn silage, 28% corn grain, 21% corn stover, and 7% orchardgrass-clover (dry matter basis) to gain the maximum 2.2 lbs per day. They were sold on January 1 as 700-lb yearlings. The steers' sale price ($0.534/lb) was essentially the same as their purchase price ($0.539/lb) since the January 1 price-weight function had the same slope as, but higher intercept than, the October 1 price-weight function.

Subsequent to the baseline solution, corn purchase and sale prices were parametrically increased and decreased in $0.10/bu increments from the 1968-1977 mean prices. This was followed by similar parametric alteration of corn grain and corn silage yields, thereby affecting their costs of production per unit weight. When corn prices were varied from $1.80 to $3.20/bu, unfertilized corn grain yields from 1166 to 3498 lbs DM per acre (soil class II), and unfertilized corn silage yields from 3678 to 11,034 lbs DM per acre (soil class II), profits shifted drastically but little variation occurred in optimal cattle production activities. Ration caloric density changed only slightly (from 2.62 to 2.70 Mcals ME/kg DM), optimal daily gain levels remained at their maximum, and no shifts occurred in purchase or sale weights.

There was a more dramatic reaction to changes in cattle price-weight functions. In order to render these results comparable to the above theoretical exposition, the steer price-weight function for January 1 was first adjusted to equal that for October 1 (P = .6349 - .000191W). The negative slopes of this function were then simultaneously increased (decreased algebraically) for both periods in intervals of .00001, that is in intervals of $0.001 per lb per 100-lb weight increase, over the range .000191 to .000301. As the negative slope increased from .000191 to .000281, operator profits fell and fewer steers were purchased for feeding, but they continued to gain 2.2 lbs per day. When the slope reached .000291, the optimal gain fell to 1.1 lbs per day. Reflecting the increased relative importance of maintenance net energy at this low gain level, corn grain dropped from the optimal ration and was replaced with corn silage and stover, fed *ad libitum*.

Given average feed prices and crop yields, therefore, optimal daily steer gains in the present model drop below their feasible maximum when market steer prices decrease more than $0.029 per lb per 100-lb weight increase. To give some idea of the frequency of such occurence, slopes steeper than this (1977 dollar basis) have characterized Virginia fall Choice feeder steer sales during 10 of the past 20 years. Among all steer grades as a group, slopes have exceeded this during 4 of the past 20 years. Because the price-weight functions utilized here are linear approximations, and because the model solutions are derived from a certain feed cost, price, and yield structure, the particular solution values reported cannot of course be taken as a general guideline for farm beef feeders.

Summary and Conclusions

The present analysis further qualifies the conclusion reached by several researchers that beef cattle feeding profits are highest when daily weight gains are at their maximum. Despite wide variations in concentrate and roughage prices or costs, operator returns in a fall feeding situation were usually maximized by feeding at the maximum daily gain level. However, this only held true under a restricted range of cattle market price-weight relationships. In general, maximum daily weight gains are optimal when cattle market prices respond weakly to cattle sale or purchase weight. When cattle prices fall rapidly with increases in weight, marginal net feeding revenue may intersect marginal feed cost at a less-than-maximum daily weight gain. Price-weight relationships of such an order have occurred frequently during the past two cattle cycles.

Footnotes

1/ Market prices of beef cattle typically decline, at a continually decreasing rate, as weight rises. Constant rates of decline (linear relationships) are also often encountered and serve as a suitable approximation for analytic work.

2/ In some cases, weight increases are associated with increases in grade. This effect may be incorporated by including a grade term in the intercept of (2)', or by including an expected grade change in the slope calculation.

3/ Where NE_m and NE_g are expressed in magacalories and W in kilograms, the NRC requirement functions for steers are $NE_m = .077W^{.75}$; $NE_g = W^{.75}(.05272g + .00684g^2)$.

4/ During 1968-1977, intercept a averaged \$0.6349 and slope b averaged \$0.00019 (1977 \$ basis). Even when corn prices are near \$3.50/bushel DM, P_{neg} is in the order of \$0.092/MCal NE_g. Using these values, $[a - 2bW_o - .02396(W/2.2)^{.75}P_{neg}] = .2014$ for a 700-lb animal.

5/ Virginia price-weight relationships were first estimated in linear form for each season and sex during the period 1968-77. Variables other than weight, such as breed, grade, and lotsize, were included in the regressions to avoid confounding weight effects with the latter effects. Steer functions used in the baseline solution were: for fall, P = .6349 - .000191W; and for winter, P = .6679 - .000191W. Corn prices averaged \$2.59/bushel, as fed basis, 1977 dollars.

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