1979

ABSTRACT

A simple two-country trading model was used to compare three methods of estimating export demand: OLS; TSLS; and TSLS applied to domestic ruves from which were derived excess demand. Estimator performance depends primarily on relative error variances around excess supply and demand. The third method was generally superior.

James K. Binkley is an assistant professor in the Department of Agricultural Economics at Purdue University. He holds a B. A. in Economics from Indiana University, an M. A. in Agricultural Economics from Washington State University, and a Ph.D. in Agricultural Economics from Virginia Polytechnic Institute. His major areas of interest are transportation and trade, econometrics, and resource economics.

Lance McKinzie is a graduate research assistant in the Department of Agricultural Economics at Purdue University. He completed his B. S. degree in Agricultural Economics at Purdue in May 1977 and is now a candidate for the M. S. degree. His major areas of interest are econometrics and international trade.

Paper presented at AAEA meetings, Pullman, July 29- Aug. 1, 1979.

A MONTE CARLO ANALYSIS OF THE ESTIMATION OF EXPORT DEMAND

by

James K. Binkley

and

Lance McKinzie

For nearly 30 years, since a seminal article by Guy Orcutt, a degree of controversy has existed concerning estimation of export demand functions. While there are several facets to this issue, much of the discussion has revolved around concern with the (now) familiar problem of bias associated with ordinary least squares estimation when variables are simultaneously determined. That such bias exists is beyond question; however, the consequences it brings to a given estimation situation are difficult to determine. Furthermore, all simultaneous estimation methods yield biased estimates, and the inconsistency of OLS may be of limited practical significance. In addition, a biased estimator may have otherwise desirable properties which justify its use, including ease of application. Undoubtedly this is the major reason for the prevalence of OLS in estimating export demands.

There are situations where estimating demand for exports via OLS is free from attack on methodological grounds. An example is the small country, price-taker case. In less clear-cut situations, researchers using OLS generally tend to acknowledge possible difficulties but justify their procedures on the basis that any biases introduced are likely to be "small." Of course this may well be the case. On the other hand, it may not.

As has been pointed out previously (Orcutt, Harberger) the existence of least squares bias in estimating export demand implies an underestimate of price elasticity. This generally acknowledged fact has generated attacks on those adopting the "elasticity pessimism" viewpoint (the pessimism referring to the fact that currency devaluation is not a viable tool for alleviating balance of payments difficulties), a viewpoint, it is claimed, which is based on erroneous parameter estimates obtained via OLS.

In noting the widely divergent views on the elasticity of demand for U.S. exports, Thompson has stated, "The debate revolves on basically an empirical question. The profession has generated few estimates of the relevant elasticities, and many of these are subject to criticism on methodological grounds." (p.2). Given the increasing importance of this elasticity for U.S. foreign and domestic agricultural policy, obtaining knowledge of the comparative properties of different methodologies represents an important area of inquiry.

This paper reports the results of a Monte Carlo analysis of the estimation of export demand. The primary purpose of the study was to compare the relative merits of three techniques for estimating the price coefficient in the excess demand curve under different circumstances. The simplest case was considered: a two country trading model in which the only estimation problem arises due to simultaneously determined variables. This can be taken as representative of the situation where a major exporter is facing the rest of the world. The estimation techniques examined and compared were: direct estimation of the export demand curve via OLS; direct estimation via two stage least squares (TSLS); and estimation of the appropriate domestic demand and supply curves by TSLS and then obtaining the estimated excess demand curve by subtraction. We call the latter method analytical least squares (ALS). Our results thus provide information concerning not only least squares bias but also any gains from incorporating more information (i.e., the nature of the domestic curves) into the estimation procedure.

Even in a relatively simple simultaneous model, analytical examination of the small sample behavior of OLS is difficult: analysis of the other two methods is less tractable. Meaningful comparison of these techniques through analytical methods is virtually impossible, and hence simulation is a useful method to examine the small sample behavior of these estimators, even though it is somewhat difficult to generalize from simulation results.

Description of the Experiments

The model used in the study consisted of eight equations: two sets (one each for the importer and exporter) of quantity-dependent supply and demand curves, and the following four identities:

(1) ES1 = QS1 - QD1 (exporter excess supply)

(2) ED2 = QD2 - QS2 (importer excess demand)

(3) P2 = P1 (arbitrage)

(4) ES1 = ED2 (market clearing condition)

QS and QD refer to domestic supply and demand quantities, respectively. These identities indicate that this is a free trade model (i.e., free of transport costs, tariffs, etc.). Thus equilibrium occurs where the prices in the two countries are equal and hence excess demand quantity equals excess supply quantity.

Parameters were selected such that country 1 was the exporter and country 2 the importer. However, trade in the other direction was not precluded, since this presented no problems in a free trade model.

Each domestic supply and demand quantity was expressed as a function of price and a single exogenous variable, domestic rainfall and income, respectively. The exogenous variables were pseudorandom uniform (0,20) deviates, the same set of data being used for all runs. The coefficient on each exogenous variable was 100 for every experiment. The other coefficients in the model varied and are discussed below. The error terms for the four behavioral equations were pseudorandom normal deviates with zero means and different variances, as described below. In all cases the errors across equations were uncorrelated.

Except for the random number generators, the computer program used in the experiments was completely self-contained, including having its own matrix inversion routine. The program was tested by comparing some of its results with SPSS and a Purdue-specific routine. Discrepancies for all estimates never exceeded one percent and were generally considerably less.

It is possible, at least heuristically, to determine a priori sources of (asymptotic) OLS biases in the export demand price coefficient in the above eight equation model. By eliminating the identities, the system can be reduced to two equations - the excess demand and supply schedule - in the two endogenous variables, quantity exported and price. This system involves the same four predetermined variables as before, which makes analytical derivation rather complicated. However, comparing this two equation model to yet simpler models which lend themselves to relatively easy analysis (see, for example, Rao and Miller, pp. 195-198) suggests that the bias in the price coefficient of the ED curve varies directly with the absolute value of the coefficient in ES and ED and with the ratio of the variance around the ED curve to the variance around the ES curve. The larger the random shifts in the excess supply curve relative to those of excess demand, the smaller will be the OLS bias. In the experiments reported here, attention is focused on these two sets of factors.

For each excess curve, we generated (by appropriate manipulation of domestic equations) three sets of slopes: one intersecting the abscissa at approximately a 45° angle (the "medium" slope case) and one each deviating from this curve in an upward ("steep" slope) and downward ("flat" slope) direction. For the excess supply curve, these were 333, 1000, 10000; for excess demand, -333, -1000, -10000. Using all combinations of slopes for the two curves generated nine cases to be examined.

For each of the nine cases, three types of experiments were run: (a) (relatively) stable demand - shifting supply; (b) shifting demand - (relatively) stable supply; and (c) both curves shifting. Each of these three was run under five levels of error variance. Table 1 lists the six levels of variance for the error terms in the four domestic equations for (a), (b), and (c) above.[1/] (Country one supply was given a larger error variance because it had higher levels of production.) The table also lists the fifteen average values for the R^2 from the eight reduced form equations for each combination of error structure and level over the experiments. This provides a reasonable measure of the relative amount of non-random behavior in the system in the various experiments.

For each parameter and error combination, 100 samples of size 20 were generated. For each sample, the excess demand curve was estimated using each of the three techniques.[2/] Given nine slope combinations, three error structures, and five levels of error variance, this entailed 135 sets of runs, each with 100 samples.

Results of the Experiments

Comparison of the performance of the three techniques will concentrate on four measures, all in reference to the price coefficient in the ED curve: the bias, the mean square error, the mean absolute deviation, and the ranking of the techniques in terms of the number of samples in which each generated the closest estimate to the true parameter.

Table 1. Variance of Structural Equation Error Terms, Five Levels Used in Experiments

Level	Exporter		Importer		Average Reduced Form R^2		
	Supply ('000)	Demand ('000)	Supply ('000)	Demand ('000)	Supply Shifting - Demand Stable	Demand Shifting - Supply Stable	Both Shifting
1	9	2	3	2	.91	.95	.99
2	90	20	30	20	.75	.85	.92
3	360	80	120	80	.57	.71	.74
4	900	200	300	200	.41	.54	.57
5	2250	500	750	500	.33	.42	.39
6	4500	1000	1500	1000			

Table 2 presents a summary of the results of the experiments. The entries in this table are listed by error structure, and indicate the number of 100-run experiments in which each of the three techniques "won" with respect to each of the four characteristics. It is seen that error structure played a significant role in determining the comparative performance of the techniques. Discussion of the results will thus be based on these structures.

Table 2. Number of Experiments in which each Technique was Superior, by Four Measures, by Error Structure

Relative Error Structure	Measure											
	MSE			Avg. % Deviation (Absolute Value)			Closeness Rank			Bias		
	OLS	TLS	ALS	OLS	TLS	ALS	OLS	TLS	ALS	OLS	TLS	ALS
Stable Demand - Shifting Supply	36	0	9	35	0	10	38	0	7	15	7	23
Stable Supply - Shifting Demand	0	0	45	0	1	44	0	0	45	0	34	11
Both Shifting	18	0	27	8	0	37	1	0	44	0	30	15

Case I: Excess Supply Shifting More than Excess Demand

This is the case in which OLS would be expected to do its best. With large random shifts in ES relative to ED, most points of equilibrium within the system will be close to the true excess demand curve, and thus fitting an OLS line through these points should yield a reasonably accurate estimate of demand. As Table 2 indicates, OLS did in fact perform well under this condition, completely dominating TSLS and being generally superior to ALS, which tended to do better than OLS only at lower variance levels. It was only with respect to bias that OLS displayed some relative weakness, being generally more biased than ALS and in some cases TSLS. However, none of the techniques generated highly biased estimates under this error structure, being generally less than +5%, except in the steep supply-flat demand case, in which the bias for all three approached +50% when variance levels were highest.

As random variance for ES increased (that for ED remaining the same), the performance of OLS improved both absolutely and relative to the other techniques. Of particular interest was the decline in the mean square error of OLS as the variance increased; the MSE of ALS tended to steadily rise, while that for TSLS generally became explosively large.

II. Excess Demand Shifting More than Excess Supply

While OLS appeared to be superior in the previous case, reversing the relative shifts in the two curves produced quite opposite effects. As Table 2 shows, in none of the 45 sets of experiments involved was OLS best by any of the four criteria. In some cases, OLS's performance was spectacularly bad. For example, in the steep supply-flat demand case, OLS displayed a positive bias ranging from 200 to 2,300 percent (leading to many positively sloped demand curves). In this particular case, the other two techniques were also seriously biased, although to a much less extent. For other slope combinations and for all but the lowest variance levels, the bias of OLS was at least +20% and generally much higher. In these cases, ALS and TSLS tended to display very little bias, with not a great deal of difference between them, although TSLS was generally less biased.

In terms of MSE, ALS completely dominated, having the smallest mean square error in each of the 45 experiments. The ranking was usually ALS, TSLS, OLS. At low levels of error variance, the differences were not great; higher levels produced rather significant differences, with OLS and TSLS tending to "take off" and ALS only steadily increasing. ALS was also clearly superior with respect to closeness of the estimated price coefficient to the true parameter. In each of the 45 sets of 100 samples, ALS was closest at least 60% of the time and in the majority of cases 90% or more. There was not a great deal of difference between OLS and TSLS with respect to the closeness ranking, although the latter appeared to be slightly better in this regard.

III. Both Curves Shifting

As might be expected, simultaneously increasing error variances in both curves produced less clear-cut results than in the former two cases. OLS always displayed the most bias, with the percentage bias varying directly with steepness of supply and flatness of demand. (The bias was as large as +900% in the steep supply-flat demand case; for most others at middle levels of error it was less than +50%). The TSLS estimates were usually the least biased, generally less than +30% at high levels of variance and virtually zero at mid and low levels. ALS was usually only slightly more biased than TSLS.

ALS was superior with respect to having the smallest mean absolute deviations, although any differences among the estimators were generally not unduly large. OLS tended to be worst of the three at low variance levels, though not markedly so, but improved its relative ranking at high levels of variance. At low to mid-level variances, all techniques generally produced mean absolute deviations significantly smaller than 50% of the magnitude of the true parameter. With respect to mean square errors, ALS was always smallest at low and mid levels of variance, with TSLS very close; both were always significantly smaller than OLS. However, at higher variance levels, OLS was generally very close to ALS and was usually smallest at the highest variance level, while the MSE for TSLS became much larger than that for the other two.

ALS clearly dominated with respect to the closeness ranking, being closest more times in every set of experiments (at least 60 percent of the time in most situations). In general there was very little difference between OLS and TSLS with respect to this measure except that at higher levels of error variance OLS tended to improve relative to the latter.

IV. Other Considerations

To assess the inferential accuracy of the estimators, confidence intervals based on the t statistic and coefficient standard errors as generally calculated were constructed, although of course these are not strictly applicable. Except in the variable supply-stable demand case, the performance of OLS in this regard can only be termed abysmal. In several experiments, out of 100 95% confidence intervals based on the OLS estimate of the price coefficient and the appropriate tabular t value, not a single one contained the true parameter. However, with excess demand relatively stable, generally 95 or more confidence intervals bracketed the true coefficient. TSLS and ALS were much more consistently correct regarding inferences; generally the 95% confidence intervals contained the true parameter at least 90% and often more than 95% of the time. Only in those cases (such as the stable supply-variable demand case with large variances) where these techniques performed quite poorly (although better than OLS) did this percentage drop as low as 75%. This was true even though there was a consistent tendency for the usual formulas for the variances of the estimates to overstate this variance (as compared with the sample variance, i.e., $\sum_{i}(\hat{\beta}_i-\bar{\hat{\beta}})^2/n$). Evidently use of the t distribution induces a counterbalancing bias.

The performance of the three techniques in estimating the intercept of the excess demand equation was very similar to their behavior in estimating the price coefficient. For example, OLS performed well in the variable supply-stable demand case but quite poorly when the relative variations were reversed. The over-all performance of the techniques in estimating the parameters of the exogenous variables was better than was true of the price coefficient or the intercept, a not unexpected result. Even when the estimates of the latter were quite poor, those for the former were at least reasonably accurate. In most cases they were quite accurate. In general, ALS performed best, particularly with respect to mean square errors. This is not surprising given the fact that ALS estimated these coefficients directly in the domestic curves.

Conclusions

As has been seen, there are cases in which OLS performs quite well in estimating the parameters of export demand functions. In particular, if the variability in excess supply is large relative to excess demand, OLS appears to be the best technique of the three examined. If the relative random shifts in the two curves is reversed, then the performance of OLS is exceedingly poor. If there is little difference in the relative variation, then OLS is definately inferior to ALS and generally inferior to TSLS, except possibly in systems with a great deal of non-systematic behavior.

The performance of all three techniques depended upon the slopes of the true curves. While the magnitude of the bias of OLS (and the others) varied with the absolute value of the slopes (in accordance with *a priori* expectations), the *percentage* bias was greater the flatter the demand curve. Indeed, the absolute performance of each estimator was worse the steeper the supply and the flatter the demand. All three estimators tended to do their best and their worst under similar sets of slope coefficients; the error levels and structures caused the relative differences among them.

Given the estimation situation, then, what factors should govern choice of estimation technique? Since the performance of the estimators relative to each other was for the most part insensitive to slope parameters, possible values of these are probably not of overriding concern. The major consideration is clearly the relative variance around the true curves.

In early domestic demand studies for agricultural commodities, use of OLS was rationalized on the basis that supply was more variable than demand. That this situation should have led to reasonably sound estimates has been borne out in this study. Indeed, under this condition, OLS appears to be the preferred technique. However, in international trade, it is difficult to argue that supply is more variable than demand.

Consider the case of estimating the export demand for feed grains from the U.S. The unexplained variation in supply is determined by unexplained variation in domestic supply and demand; that for demand by variation in supply and demand for all importers. It would seem that, in a case such as this, the variation in excess demand would, if anything, exceed the variation in supply.

It thus seems likely that cases in which OLS performs well (other than the price-taker case) may not be common in estimating excess demand functions. This suggests choosing a consistent estimation technique. Of the two examined here, ALS was clearly superior. Although usually displaying somewhat more bias than TSLS, ALS generated estimates which, both in a linear and quadratic sense, tended to be closer to the true parameter in individual samples. In fact, one of the more interesting results of this study was the mediocrity displayed by TSLS in estimating export demand. Since the same data are required to implement both ALS and TSLS, ALS is only slightly more bothersome to use (requiring estimation of more individual equations). In cases where the choice is between the two, our results indicate that ALS is the better selection.

FOOTNOTES

1/ Six levels were required because the experiments with both curves shifting began at the lowest variance level and then increased this through four levels for both curves. In the other experiments, the variance for one curve remained at the lowest level and that for the other increased through five levels of variance, beginning with the second lowest.

2/ For TSLS and ALS, the first stage of the estimation involved regressing the endogenous variables on all four predetermined variables. The OLS estimation only used the predetermined variables from the importer, since those from the exporter do not appear in the ED curve.

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