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AN EM-PM HYBRID POLICY MODEL FOR ANALYSIS OF AGRICULTURAL PRICE, PRODUCTION, AND RESOURCE USE

Policies and programs 1979

by

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INTRODUCTION

Historically, the U.S. Government has played an important role in national and regional agricultural production. At the national level, the Government uses production control programs, price supports, and loans to increase and stabilize farm commodity prices and also invests in research, development, and extension in promoting production supply. On the regional level, it has invested in various development projects such as the installation of irrigation to promote regional agricultural production, or to regulate regional land use to avoid permanent depletion of resources.

Implementing government programs usually has considerable differential effects over time on regional agricultural production patterns, or completion of individual regional projects frequently has a considerable impact on production in other regions. Quantitative evaluation of the impact of alternative national and regional programs on production, farm income, and food prices prior to policy enactment and implementation, can be useful to policy makers.

With advances in econometric modeling, operations research and computer increasing capacity and efficiency, numerous models have been developed and applied to analyze complex agricultural problems. Samples of econometric models are CED-CC, CHASE, CARD-RS, and DRI[1/] while programming models can be found in Heady and Srivastava (1975), and Schaller (1968).

[1/] CED-CC: Cross Commodity Forecasting System, developed at ESCS, USDA; CHASE: CHASE Agricultural Model; CARD-RS: Recursive Simulation Model developed at the Center for Agricultural and Rural Development, ISU; DRI: Data Resource, Inc. Agricultural Model.

Interest has been expressed in combining various types of models for policy analysis (Boss et al., 1977) to compliment the uses of each individual model. Some attempt has been made to develop hybrid models combining an econometric (positive) model with a programming (normative) model.[2/] Three hybrid models incorporating some different quantitative approaches are:

1. Quadratic Programming (Q.P.) Model (Takayama and Judge, 1964; Meister, Chen, and Heady, 1978).
   The model includes a set of econometric estimated demand and supply functions in a resource allocation programming model.

2. Input-Output and Linear Programming (IO-LP) Model (Penn et al., 1976)
   The model adds an IO model to a linear programming resource allocation model.

3. Recursive Programming (RP) Model (Day, 1961)
   A set of econometrically estimated flexibility restraints is added to a linear programming model.

This paper investigates the need and methods of combining an econometric model with a programming model, describes a specific hybrid model developed jointly by C.A.R.D. (Center for Agricultural and Rural Development) at Iowa State University and ESCS (Economics, Statistics, and Cooperatives Service) of USDA, and finally, presents some tests of the hybrid model.

[2/] Differences between positive and normative methodologies have been discussed in the literature; for examples, see Friedman (1953), Heady (1961), Kelso (1965), Quance and Tweeten (1971), and Shumway and Chang (1977). In short, the positive model concerns what was, is, or will be the consequence due to a change in the government program, while the normative model concerns what ought to or could be the consequence.

NEED FOR MODELS WITH SPACE AND TIME CHARACTERISTICS OF PRICE, PRODUCTION, AND RESOURCE USE

Because the government program has differential effects over time and space, there is a need for an analysis which incorporates these characteristics in detail. In general, four types of analysis of policies, resource use, agricultural structure, and environmental impacts can be identified. One is predictive analysis characterized by econometric or statistical models. These models best predict production and market behavior as expressed in time series or cross sectional data. One specific purpose is to estimate producer behavior; there are studies in which the main interest is not analyzing a producer's response, but in the future potentials of production and resource use in terms of resource availability and technology. A programming model frequently is used; there are studies to estimate the market impact should the production potential be realized in the absence of offsetting policies. The results are useful in suggesting needed policies to safeguard undesirable inequities or externalities. In these cases, an econometric model is integrated into a programming model to estimate market impact; there are also cases in which the main concern is to make an ex-ante, positive estimation of producer's response, market impact, etc., from a change of government environmental regulations and resource policies. In these cases a programming model will be used to complement an econometric model in providing policy variables that could not be specified in the econometric model, and in dealing with situations where unprecedented government programs are imposed. The hybrid model presented in this paper intends to accomplish this task. Emphasis is placed on combining a large-scale econometric model with a large-scale linear programming model.

SOME JUSTIFICATIONS

Economists frequently use an econometric model for positive estimation of the impact of policy implementation. Equilibrium of supply and demand is built into the model. Econometric estimation is proper only when the equilibrium solution falls inside the feasible production region defined on the left-hand side of the FR (feasible region as shown in Figure 1a) line which can be derived from a set of behavior production response equations, physical production restraints, and the production restraints due to implementation of a government program. Econometric estimation is invalid when the equilibrium solution is outside of the feasible region (Figure 1b). A policy change often shifts the relative position of the FR (and may also shift supply and demand). Since an econometric model does not explicitly specify the FR line it apparently is not known whether the solution is inside or outside of the region when it is used for ex-ante policy analysis. Thus, as shown in Figure 1b, an econometric model without knowledge of the location of the feasible region could underestimate the price and overestimate production. When demand, supply, and all the restraints that form the FR line are linear functions or nonlinear convex having some function properties, a QP model or a separate programming technique can be employed to obtain the correct estimates by maximizing net profit subject to restraints of supply and demand (and others). However,

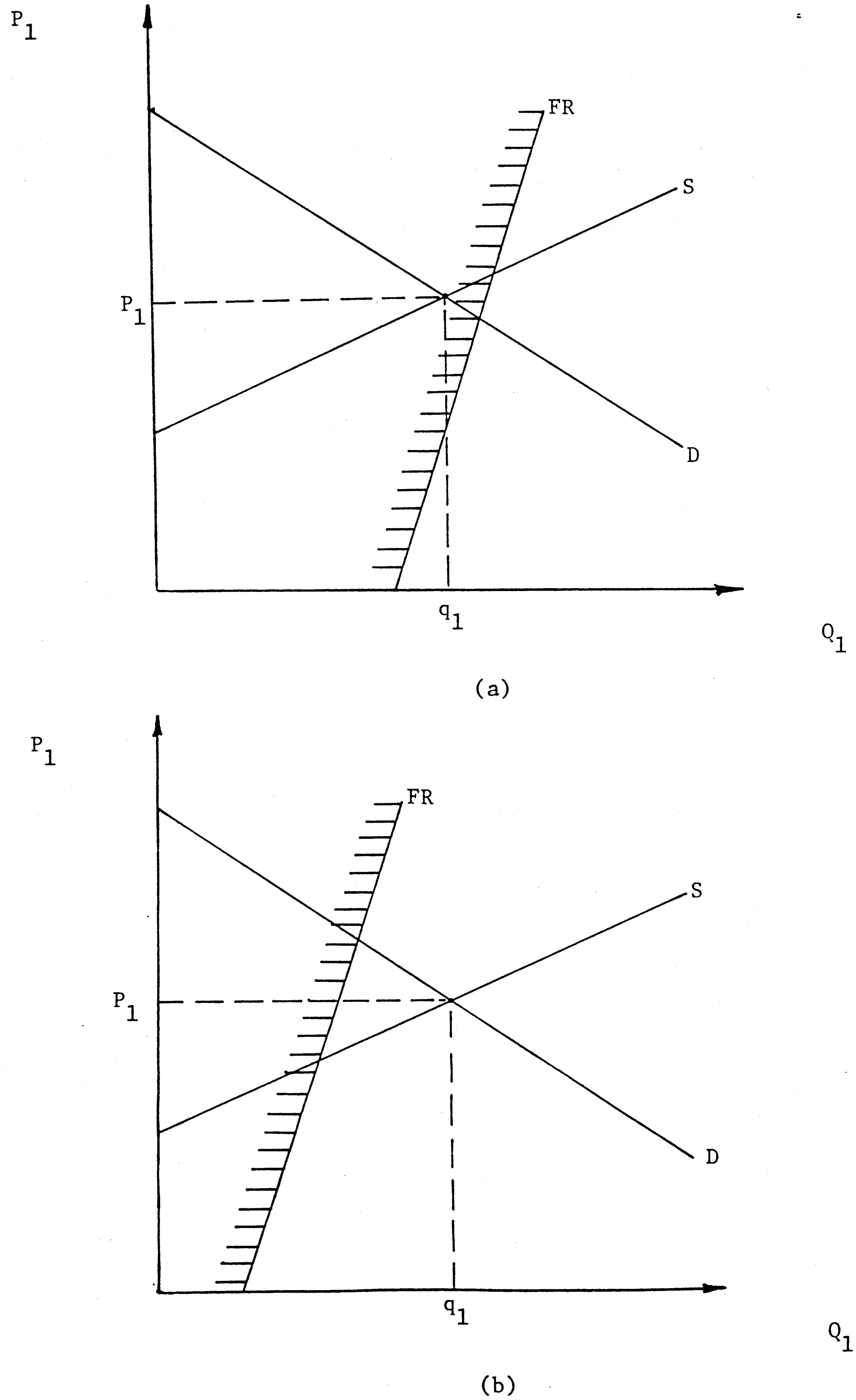

FIGURE 1. Demand and Supply (curves) of Crop 1 and its feasible production region on the left-hand side of the FR line.

in many cases, supply and the demand cannot be represented by the simple linear function and may be represented by a system of both linear and nonlinear functions. It also is possible for the FR line to be a nonlinear. Under any of these situations computational problems may arise. Under the above conditions, an alternative procedure is needed that consistently gives the appropriate estimates regardless of the location of the FR line relative to the demand and the supply lines. One approach is a hybrid model that utilizes features of both econometric and programming structure.

Other reasons causing need for a hybrid model in policy analysis are: (1) For positive economic analyses, the econometric model is an appropriate tool. Its nature best allows it to simulate a real world or producer's responses as it provides a better short-run estimation with less modeling effort. However, it does not serve efficiently for ex-ante analyses relating to new policies which are unrelated to the past structure of agriculture and for which there is no historical data. It is not effective for interactions between the level of production activities[3/] and resource supplies, or in evaluating production potential beyond past production. (2) When unprecedented government policy relates to interaction between resource supplies concerned and production activities, incorporation of programming into the modeling structure becomes appropriate. (3) Traditionally, economists have considered a programming model to be normative have used it mainly for a

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[3/] Lee and Seaver (1973) demonstrated that a simultaneous equation approach can be used to investigate spatial equilibrium of the broiler market although considerable effort is required in finding the solution.

normative (what ought to or could be) purposes. However, there has been some success in using the programming model for positive estimation. An example is the incorporation of flexibility constraints into a recursive linear programming model (Day, 1961; Shaller and Dean, 1965) and specification of more realistic behavioral properties (Boussard and Petit, 1967) Considerable potential for using a programming model exists.

Ideally, a hybrid model might provide the analyst with the best features of both types of models, while eliminating certain conceptual problems associated with each. The hybrid should incorporate information on the spatial pattern of supply, resource use and the technical structure of production generated by the programming model, and utilize detailed information on market structure, processes, and prices provided by the econometric model. Consequently, for a given set of agricultural policies, the hybrid should simulate a dynamid sequence of events over space and through time. In this process, a consistent set of spatial and temporal real world economic performance indicators would be produced. The following section contains a discussion of six alternative approaches for linking econometric (E) and mathematical programming (P) models.

SIX METHODS OF COMBINING AN ECONOMETRIC AND A PROGRAMMING MODEL

The One-Way Communication Model (Figure 2)

The One-Way Communication Model is so named because the information flow is one way--from the econometric model to the programming

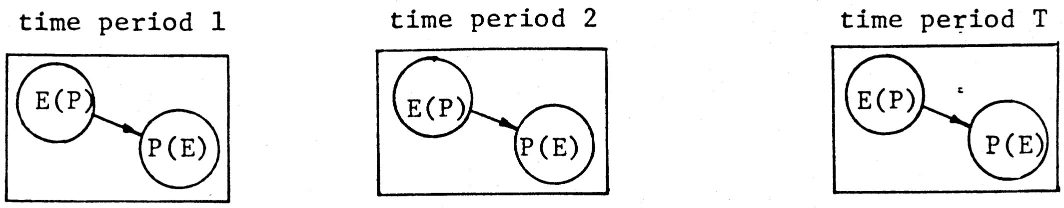


FIGURE 2. The one-way communications model.

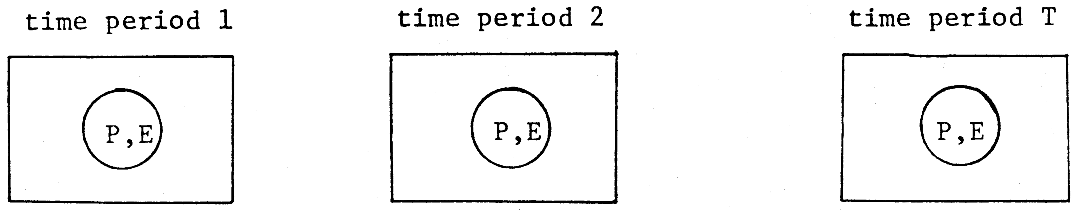


FIGURE 3. The simultaneous solution model.

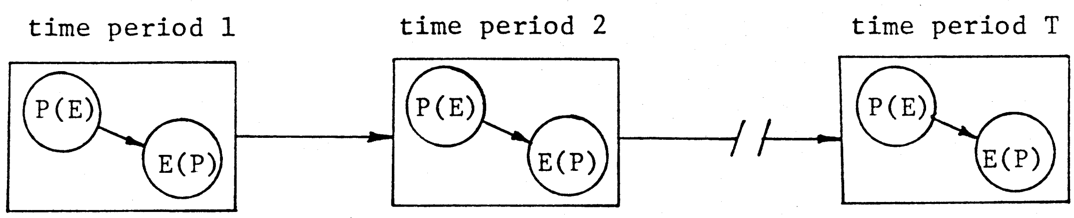


FIGURE 4. Recursive interactive programming model.

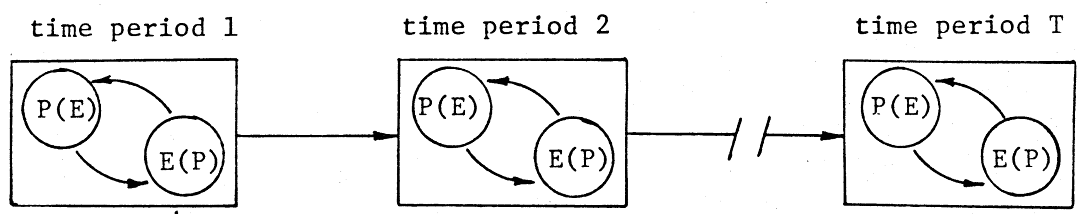
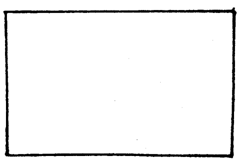
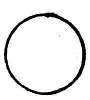


FIGURE 5. Recursive adaptive programming model.

 The solution of the hybrid model for time period t; t=1,2,...T

 The solution of a set of equations component of the hybrid model

P A set of equations obtained from a programming model

E A set of equations obtained from an econometric model

→ The transfer of information between components and/or models model (and visa versa). This hybrid is most easily characterized by a single-period and interregional programming model with fixed demands which are determined by a set of econometric equations. For example, a One-Way Communications model was used to analyze alternative future potentials for U.S. agriculture as defined for the National Water Assessment conducted under the auspices of the U.S. Water Resources Council (Meister, et al., 1975). At each point in time (i.e., the years--1985 and 2000) in this analysis, the quantities of agricultural products demanded (U.S. Water Resources Council, 1974 and 1975) were projected by the NIRAP econometric model (Quance, 1976). These demands were used as constraints in a linear interregional programming model (Meister and Nicol, 1975). The linear programming model was then used to project the least-cost (competitive equilibrium) spatial pattern of agricultural production and resource use subject to these minimum fixed demands. (An example of programming econometric model is a LP-ID model used by Sonka and Heady, 1973).

This One-Way Communication Model is currently available for this type of long range analysis. However, the ability of this model to simulate the short-run behavior of the agricultural sector is limited (not its origional purpose) by the fact that the model does not have a feedback from the programming to the econometric model within or between time periods. This model will encounter a problem of nonfeasilbe solutions when the econometrically estimated values of the linkage variables are outside of the feasible region defined by the restraints in the programming model. Proper adjustment of the econometric or programming model is required to obtain a feasible solution.

The Simultaneous Solution Model (Figure 3)

The Simultaneous Solution Model utilizes equations derived from an econometric model as identities (rather than inequality constraints) within the programming model. The conceptual appeal of this hybrid is that the solution to the model will simultaneously satisfy the assumptions of both parent models.

Penn, et al., (1976) used this approach to evaluate the short-run impacts of energy shortages on the U.S. economy. Their Simultaneous Solution Model incorporated Input-Output data developed by the U.S. Department of Commerce (1974) for eighty-five sectors into a linear programming model that contained two energy constraint equations.

Quadratic programming models also belong to the simultaneous solution category. The QP incorporates demand and supply equations into a linear programming model (Takayama and Judge, 1964) and has been applied extensively to study the problem of U.S. agricultural production (e.g., Plessner, 1965; Meister, Chen, and Heady, 1978).

Some problems will arise in future applications of a Simultaneous Solution Model under any one of the following three conditions:

1. Where the feasibility region defined by the equations derived from the positive model is smaller than the computational errors inherent in the linear programming software package.

2. Where a static equilibrium solution is imposed on a dynamic disequilibrium system (see Baumol, 1951).

3. Where nonlinear equations derived from the econometric component result in prohibitive computational costs when cast within a mathematical programming framework.

A Simultaneous Solution Model constructed using equations, for example, from the CED-CC Forecasting System and the NWA programming model would contain thousands of equations and tens of thousands of variables. A Simultaneous Solution Model of this size would be conputationally infeasible, and/or prohibitively costly (particularly if bounding procedures are used).

Recursive Interactive Programming (RIP) Models (Figure 4)

Characteristic of RIP Models

The basic features which characterize Recursive Interactive Programming (RIP) models are presented below:

1. The hybrid model develops a unique description of the agricultural economy for each stage in a sequence of time periods (t=1, 2,...T).

2. The hybrid model consists of at least one programming and at least one econometric component.

3. Within each stage the individual components are solved once in a prespecified sequence. The former component (the programming model or the econometric model in Figure 4) is solved before the latter component (the econometric model or the programming model in Figure 4) is run.

4. For the purpose of this discussion, each component has three categories of variables. These categories are not necessarily mutually exclusive. These are described below.

   (a) Exogenous variables are not determined within either component. The "explanation" for their behavior resides outside the component. When the equations within a component are solved these variables are taken as given.

   (b) Endogenous variables are those whose values are "explained" or determined by the operation of the component model.

   (c) Linkage variables are the exogenous variables in one component whose values are determined by the operations of the preceding component.

5. The solution procedure for a hybrid model begins by running the former component for the first stage. The input data are the values of the former component's endogenous variables;

The solution vector produced by this run contains the values of the former component's endogenous variables; this is used to determine the values of the linkage variables is the latter component. Subsequently, the latter component is solved. The solution of the latter component in this stage is used to determine the starting values of the linkage variables in the former component of the next stage.

6. Using the recursive relationship identified in "5", above, the hybrid model moves forward stage by stage.

Two examples of RIP model are described:

First: The national economic model (Shaller, 1968 and Sharples and Shaller, 1968) developed by Farm Production Economic Division. Economic Research Service is one example of the RIP model. It consists of about 90 profit-maximization linear programming submodels. At year t, these submodels estimate planned acreage which is used to estimate the planned production as well as quantity of production input used. These estimates then are fed into a national econometric model to calculate equilibrium prices and then expected prices. These prices, estimated costs, production, and input uses are fed into each programming submodel for estimating the production for year t+1.

Second: Baum (1978) has built and empirically tested a national recursive interactive programming model for U.S. agriculture. Baum utilized the crop sector of the abridged version of the National Water Assessment linear programming model with a revised econometric simulation model based on one developed by Ray and Heady (1974). Within each stage of his analysis, the profit maximizing linear programming model was run first to estimate national crop acreage and production. (In the first example, 90 profit maximization submodels are used.) The values of these linkage variables were then passed to the simulation model. The simulation model was subsequently run to estimate the values of market sector variables for the same stage in time. The output of the simulation component was used to revise the coefficients in the linear programming model in the following stage to estimate the net return coefficients in the objective function, the values of the activity flexibility restraints, i.e., the upper and lower limits on crop acreage response by region, and the values of the transformation or input-output coefficients, optimal nitrogen fertilizer rates and crop yields in the linear programming model.

The RIP models have many advantages over those described earlier:

They allow for a two way flow of communication--one way within in each stage, and the other way between stages. This is a higher degree of interaction between components than is achieved by the One-Way Communications Model;

They present less of a computational problem than the Simultaneous Solution Models do, because the feasibility set is not restricted to equality solutions of the econometric model;

Finally, they dynamically simulate a sequence of events over space and through time in a nonsimultaneous, or cobweb solution framework.

In contrast to a purely econometric model based on part time series data, the RIP allows the evaluation of potential supply capacities for the future. When it is needed for this purpose, it is an appropriate tool.

The RIP approach also has limitations:

An RIP hybrid begun by running with LP model tends to overestimate total production and underestimate prices when the interest is in positive predictions, because the linear programming component produces an economically efficient use of resources. This overestimates production as input to econometric models, therefore, it will underestimate the prices.

> The RIP hybrid begun by running with Econometric model may encounter the infeasible solution problem as described in the one-way communication hybrid model. The econometric component may give an estimated production that exceeds the capacity of regional productions.
>
> If either of the components has been specified incorrectly, the recursive nature of the model may result in a propagation of errors over time, between stages.

The first problem can be ameliorated by introducing psuedo behavioral constraints into the programming component; Shaller's and Baum's procedure of adjusting upper and lower bounds on regional acreage limitations in response to the price impacts produced by the econometric component is an appropriate methodology. Additional research is needed to improve the accuracy of regionally specific acreage (or production) response equations. The second and third problems can be addressed in part by incorporating a two way flow of communications between components within each stage of the analysis. This concept of a corrective adjustment within stage feedback mechanism is similar to a self-adaptive control system (D'Azzo and Houpis, 1966); it is defined as a model which has the capability of changing values of linkage variables through an internal process of estimation evaluation, and adjustment according to a pre-setup rule. It forms the basis for the Recursive Adaptive Programming (RAP) models which are described in the following discussion.

Recursive Adaptive Programming (RAP) Model (Figure 5)

The RAP model is constructed from the RIP model by including a feedback structure in each stage. A RAP model is constructed by using an econometric model as the <u>former component</u> and a linear programming model as the <u>latter component</u>. This basic structure is used to construct the CARD-NRED hybrid model in latter chapter. The econometric model is used as the former component based on the following reasoning. For evaluating the short-run impacts of agricultural policies, it is natural to relate the econometric model as the principal component in the hybrid model and to use the linear programming model to act in the following subordinate and complementary role:

1. For each commodity, the LP contains an accounting row that measures the deviation between aggregate production as forecast by the econometric component and the aggregate contained in the LP solution. Large penalty costs have been assigned to the deviational variables in the profit maximizing objective function in order to force the LP solution to come as close as possible to the econometric solution.

   If all of the deviational (production) variables in the LP solution vector are equal to zero, the solutions produced by the two components are assumed to be consistent. In this case, the LP has validated the results of the econometric component, and the RAP model begins the computations for the next stage in time.

2. However, if any of the deviational variables in the LP solution vector are not equal to zero, then the production forecast by the econometric component is outside the production possibilities region defined by the feasibility constraints in the LP component. In this case, the pre-setup adaptive feedback mechanism is invoked. Within this stage, the production variables programmed in the econometric component become linkage variables from the programming component to the econometric component; they are set equal to the LP solution values. The econometric component is resolved producing a new set of prices. Then the RAP model goes forward to the next stage of analysis.

Alternative Feedback Adjustment Procedures for RAP Models

The key problem in building the RAP hybrid model is to find the best pre-setup procedure to adjust the production (or acreage) when the equilibrium solution is outside the feasible region.

Harrison (1976) suggested an iteration procedure to find the equilibrium prices in a hybrid model. Covergence of shadow prices from linear programming is used as the criterion. This procedure is appropriate when both the econometric and the programming components in the hybrid model have a simple model structure and are inexpensive to run. This is not the case for integrating large-scale models. Four potential procedures are examined.

Figure 6 is used to illustrate four procedures that can be used to adjust the econometrically estimated equilibrium solution $A(q_1, q_2)$ which is outside of the feasible region in a two crop model.

(a) Shortest Distance Approach (SD) This approach will give the final solution indicated by B in Figure 6. AB is the shortest distance as measured by the sum of squares of the quantity of the two crops to be adjusted from A to the feasible region. (b) Independent Adjustment Approach (IA) This approach gives the solution indicated by C. This approach adjusts only the crop production which is larger than the feasible range (OE). The approach, however, does not adjust the crop production which is less than the feasible range (OH). (c) Maximization Approach (MA) This approach ignores the equilibrium solution generated by the econometric model. The approach obtains the adjusted value simply running the programming component. One of the likely solutions is indicated by D in Figure 6. (d) Minimum Absolute Distance Approach (MAD) This approach gives the solution which has the least adjustment of absolute values of the production. The solution may be either C or B, depending on the slope of DE as shown in Figure 6.

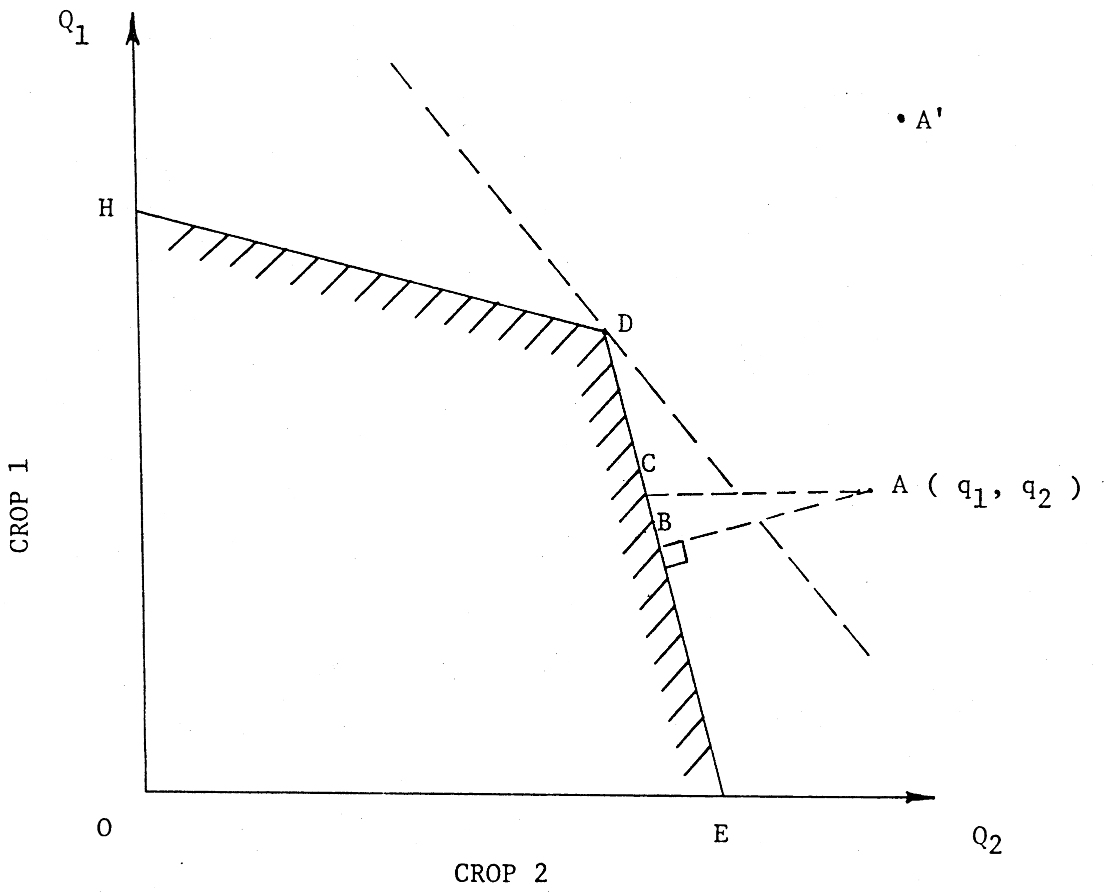


Figure 6. Points C, B and D are candidates for adjusted values from infeasible solution A (q_1 , q_2)

Each approach has its own appeal: It should be noted that it is possible that the SD, MA, and MAD approaches may obtain the same adjusted values when the econometric estimated value is at A' in Figure 6. The SD has the least squares of quantity of production to be adjusted; the IA has only to adjust the crop which is outside of the feasible region; the NA approach gives the adjusted value which maximizes net return from the production; the MAD has at least absolute value in adjustment. Furthermore, the MAD gives a solution either the same as the solution of the SD or the solution of the IA. Because of this characteristic, the MAD approach is used in building the RAP hybrid model.

An Experiment To Investigate Characteristics of Hybrid Models

An experiment is conducted to show solution results from various methods of linkage. Although the experiment use a specific simple model, the test results give several fundamental characteristics resulting from each method of linkage. The experiment uses a simple two crop econometric model which is formulated as:

$P_1 = 250 - 0.003\ q_1$ (P_1 and q_1 are price and quantity of crop 1

$C_1 = -1125 + 0.250\ q_1$ (C_1 is cost of production of crop 2)

$P_2 = 400 - 0.080\ q_2$ (P_2 and q_2 are quantity of crop 2)

$C_2 = -1143 + 1.4\ q_2$ (C_2 is cost of production of crop 2)

The programming model in the experiment uses only land resource constraint and is expressed as:

	Abundant Land Resource L=150 (q_1, q_2)	Tight Land Resource L=120 (q_1, q_2)	Note
TRUE (surplus) MODEL	(5434, 1042)	(4844, 925)	
ECONOMETRIC EM_t	(5434, 1042)	(5434, 1042)	Correct solution for L=150, Invalid solution for L=120
ONE-WAY $EM_t \rightarrow LP_t$	(5434, 1042)	None	Correct solution for L=150, Nonfeasible solution for L=120
ONE-WAY $LP_t \rightarrow EM_t$; $MRP_1 > MRP_2$ $MPR_2 > MRP_1$	(7500, 0) (0, 6000)	(6000, 0) (0, 4800)	Incorrect solution for both L=150 and L=120
RIP $\rightarrow EM_t \rightarrow LP_t \rightarrow$	(5434, 1042)	None	Correct solution for L=150, Nonfeasible solution for L=120
RIP $\rightarrow LP_t \rightarrow EM_t \rightarrow$; $MRP_1 > MRP_2$ $MRP_2 > MRP_1$	(7500, 0) (0, 6000)	(6000, 0) (0, 4800)	Incorrect solution for both L=150 and L=120
SIMULTANEOUS $EM_t + LP_t$	(2691, 521)	(2691, 521)	Incorrect solution for both L=150 and L=120
RAP $\rightarrow EM_t \rightarrow LP_t \rightarrow EM_{t+1}$; $MRP_1 > MRP_2$ $MRP_2 > MRP_1$	(5434, 1042)	(4697, 1042) (5434, 453)	Correct solution for L=150, Incorrect solution for L=120

Maximize

$$f(P_1, P_2, C_1, C_2, q_1, q_2)$$

Subject to:

$$\frac{q_1}{50} + \frac{q_2}{40} \leq L$$

$$P_1 > C_1; \; P_2 > C_2$$

Table 1 shows quantities of Crop 1 and Crop 2 produced in each case. The true model uses maximization of consumer's and producer's surplus as the objective function. This model has maximum net return among all the cases considered. All other cases (except the EM model) use maximization of net profit as their objective function.

Several conclusions can be drawn from these results.

1. In the abundant land resource case, only the model beginning with the EM model gives the correct solution. This implies that any hybrid model beginning with the LP model will give a wrong solution. To avoid this shortcoming; production flexibility constraints are frequently added to the LP model.

2. In the tight land resource case, none of the model yields the correct solution. The one-way and RIP models begun by running the EM model, give a nonfeasible solution, while models begun by running the LP model give results with extreme values. The Simultaneous model underestimates the production. The solution from the RAP model, although incorrect, is relatively close to the correct solution. Especially so when the Marginal Revenue Product (MRP) of land can be estimated accurately. The RAP solution is (4697, 1042) as compared with the true solution of (4844, 925).

3. The poor performance of the simultaneous model (with maximization of net profit as its objective function and with demand and supply constraints) in estimating production should not be overlooked. This model does not give the maximum value in the objective function and use of the model will underestimate production.

The true model, Table 1, is also a simultaneous model but with maximization of the surplus as its objective function. A correct formulation of objective function to reflect behavior of production response is the key to success in using the simultaneous model.

4. As mentioned in (1), any hybrid model begun by running a programming model will give poor results in simulating production response. In practice, using a PM as a predictive model is very difficult. Unlike an EM the PM does not have the capability of a search method such as the least-square-error method to fit the model to the observed time series data. The PM approach depends on the trial and error method and a high level of expert knowledge about the whole production system in finding a proper model to simulate time series data. This approach is inefficient in achieving the accuracy that is readily obtained, with less effort, by an EM. At this stage of research, any hybrid model intended for use in the prediction should use the EM as its main structure.

MATHEMATICAL FORMULATION TO ILLUSTRATE ESSENCE OF HYBRID MODELS

An econometric component and a programming component are used to illustrate the essence of constructing the hybrid model. Assuming an econometric component consisted of N equations in the hybrid model is expressed as

$$Y_{nt} = \sum_i a_i Y_{it} + \sum_j b_j Y_{jt-1} + \sum_k c_k Z_{kt} + e_{nt} \qquad (1)$$

for $n=1,2,\ldots,N$

Where Y_{nt} and Z_{kt} denote endogenous and exogenous variables respectively; a_i, b_j, and c_k are coefficients; e_{nt} is an error term. The first I (I<N) endogenous variables are linking variables to a programming component which is expressed as

$$\text{Maximize } [\sum_{ij}\sum(P_{ijt} - C_{ijt})X_{ijt} - \alpha_1(\sum_i(V_i^+ + V_i^-)) - \alpha_2(\sum_{ij}\sum(W_{ij}^+ + W_{ij}^-))] \qquad (2)$$

Subject to: (1) National Production Balance Restraints

$$\sum_j X_{ijt} + V_i^+ - V_i^- = Y_{it} \qquad (3)$$

for $i = 1,2,\ldots,I$

(2) Regional Production Response Balance Restraints

$$X_{ijt} + W_{ij}^+ - W_{ij}^- = \beta_{ijt} X_{ijt-2} \qquad (4)$$

for $i = 1,2,\ldots,I$
$j = 1,2,\ldots,J$

(3) Production Resource Restraints

$$\sum_i\sum_j V_{ijl} X_{ijt} \leq R_{lt} \qquad (5)$$

for $l = 1,2,\ldots,L$

Where: P_{ijt} = Farm Price for Crop i in producing Region j in time period t

C_{ijt} = Cost of Production for Crop i in producing Region j in time period t

X_{ijt} = Quantity of Production of Crop i in Region j in time period t

α_1 and α_2 = Two arbitrary large constant values satisfying the following conditions: $\alpha_1, \alpha_2 > (P_{ijt} - C_{ijt})$ for all i,j, and t

V_i^+, V_i^- = Positive or Negative Deviation from econometric estimated production of Crop i ($V_i^+, V_i^- \geq 0$)

W_{ij}^+, W_{ij}^- = Positive or Negative Deviation from econometric estimated production of Crop i in Region j ($W_{ij}^+, W_{ij}^- \geq 0$)

V_{ijl} = Technological coefficients for using resource l by Crop i in Region j

R_{lt} = Maximum amount of resource l available in time t

β_{ijt} = Coefficient used to predict the products X_{ijt} from X_{ijt-1}. The value of β_{ijt} is estimated from a regression equation which has independent variables such as expected price and other variables.

The objective function (2) is to maximize production net returns and minimize the absolute deviation between the programming solution and values of national and regional econometric estimates. The formulation of minimizing the absolute deviation is described in Sposito (1975). Another form of the objective function, e.g., minimizing production costs and the deviation formulated, or minimization of the deviation can also be formulated.

Properly assigning values for V_i^+, V_i^-, W_{ij}^+, and W_{ij}^- the model ((1) to (5)) can be transformed into a national, regional, and simultaneous national-regional hybrid model. The model becomes a one-way (N) national hybrid model by setting V_i^+ and V_i^- equal to zero. The model is a national model because the production in the solution from the programming component is set to the value estimated by the econometric component. The N model usually is used to analyze regional production response to meet a national target quantity of the production. Similarly, the model can become a one-way regional (R) hybrid model by setting W_{ij}^+ and W_{ij}^- equal to zero. The model is a regional one because the national production is determined by summing all the regional production. This R model is useful to investigate possible inpact from regional production expansion. When this R model is structured in such a way that the sum of all the regional production is used in the next time period (stage) the model becomes a two-way communication or RIP model as mentioned earlier.

The model also can be transformed into simultaneous national and regional hybrid model when V_i^+, V_i^-, W_{ij}^+ and W_{ij}^- are not set equal to zero, or ranges are used to replace these variables. One variation of the NR simultaneous model is to employ a pair of flexibility restraints to give a range for each crop production in each region. The model is a simultaneous one, because the final solution production in the programming component is jointly determined by national and regional estimates. This is the basic structure of the RAP model explained earlier. At each time period, the RAP model checks whether ΣX_{ijt} is equal to Y_{it}. If not, (either V_i^+ or V_i^- is not equal to zero) the value of $\sum_j X_{ijt}$ replaces the value of Y_{it} and is fed back to the econometric model to adjust the values of all endogenous variables before the RAP model starts the simulation for next time period (stage t+1).

If the first term (the net profit) in the equation (2) is set at zero, this RAP model is equivalent to a restricted statistical model which is fitted with least-absolute-value to a series of production data generated by the econometric component (1).

MODEL VALIDATIONS

Description of the Test Model

The RAP test model consists of two components: an econometric model represented by the CED Cross Commodity model and a programming model represented by the CARD-NRED LP model. The CED model includes livestock and crop sectors (Teigen, 1977). It has 127 exogenous variables and 164 endogenous variables represented by 164 regression and identity equations.[4/] These equations are divided into ten groups:

[4/] One endogenous variable, soybean oil price, is exogen ized. Therefore only 163 endogenous variables are in the test model.

retail demand, retail product supply relations in the dairy sector, farm demand for the livestock sector, capital stocks, livestock supply, crop demand product stocks, planted acreage relations, supply and utilization identity, and index definitions. The crop sector includes corn, sorghum, barley, oats, wheat, and soybeans. The CED-CC model can be expressed as:

$$Y_{it} = a_{it} + \sum_{n=1}^{164} b_{1n} b_{1n} Y_{nt} + \sum_{n=1}^{164} (b_{2n} Y_{nt-1} + b_{3n} Y_{nt-2})$$

$$+ \sum_{m=1}^{127} (b_{4m} Z_{mt}) + e_{it} \qquad (1)'$$

$$i=1,\ldots,164$$

Where Y_{nt} and Z_{mt} denote endogenous and exogenous variables, respectively; b_{1n}, b_{2n}, b_{3n}, and b_{4m} are coefficients, while e_{it} is an error term. The CARD-NRED model is a reduced version of the CARD-NWA model (see An Overview of Data Processing Activities in CARD-NRED LP model by Huang, Weisz, and Alt, 1978). To reduce the cost of the test, the programming component of the model has only one land class and uses only land as the resource restraint. The programming component can be expressed as:

Maximize

$$\sum_{i=1}^{6} \sum_{j=1}^{105} \left[\sum_{k=1}^{k_j} (XD_{ijkt} + XI_{ijkt}) P_{ijt} - \sum_{k=1}^{k_j} XD_{ijkt} CD_{ijkt} - \sum_{k=1}^{k_j} XI_{ijkt} CI_{ijkt} \right]$$

$$- \alpha_1 \sum_{i=1}^{6} (V_i^{+} + V_i^{-}) \qquad (2)'$$

Subject to

National production balance restraints

$$\sum_{j=1}^{105} \sum_{k=1}^{K_j} (XD_{ijkt} + XI_{ijkt}) + V_i^{+} - V_i^{-} = Q_{it}$$

$i=1,\ldots,6$; k_j varies from region to region (3)

Regional production response restraints

$$\sum_{k=1}^{k_j} (XD_{ijkt} + XI_{ijkt}) \leq [\bar{\beta}_{ijt}] [\sum_{k=1}^{k_j} (XD_{ijkt-1} + XI_{ijkt-1})] \quad (4)$$

$$\sum_{k=1}^{k_j} (XD_{ijkt} + XI_{ijkt}) \geq [\underline{\beta}_{ijt}] [\sum_{k=1}^{k_j} (XD_{ijkt-1} + XI_{ijkt-1})]$$

Land restraints 5/

$$\sum_{i=1}^{13} \sum_{k=1}^{k_j} VD_{ijkt}\, XD_{ijkt} \leq LD_{jt}$$

$$\sum_{i=1}^{13} \sum_{k=1}^{k_j} VI_{ijkt}\, XI_{ijkt} \leq LI_{jt}$$

$j=1,\ldots,105$ (5)

Where XD_{ijkt} (or XI_{ijkt}) is defined as the quantity of production of crop i using rotation and tillage practice k on dry (or irrigated) land in producing area j in time period t. CD_{ijkt} or CI_{ijkt} is the cost of producing one unit of XD_{ijkt} or XI_{ijkt}, respectively. VD_{ijkt} or VI_{ijkt} is acres of land use to produce one unit of XD_{ijkt} or XI_{ijkt}, respectively. LD_{it} or LI_{it} is total dry or irrigation land available in producing area j in time period t. P_{ijt} is Farm Price for Crop i

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5/ i=7 to 13 refers to corn silage, nonlegume hay, legume hay, cotton, summer fallow, and sugar beets.

in producing region j in time period t. $\bar{\beta}_{ijt}$ and $\underline{\beta}_{ijt}$ are respectively the maximum and minimum proportionate increase or decrease of production of crop i in PA j from year t-1 to year t; the price elasticities are used to determine their values.

Three sets of endogenous variables are selected as linkage variables to transfer information from the econometric component to the programming component. These three sets (expressed as Y_{it} in econometric component) are regional crop price P_{ijt}, cost of production CD_{ijkt} (and CI_{ijkt}), and national aggregate crop production Q_{it}. At time period t the values of P_{ijt} and CD_{ijkt} are used to revise the coefficient in the objective function; the values of P_{ijt} are used in the regional production response restraints; the value of Q_{it} is used as the value of the right-hand side of the national aggregate production balance restraints.

The final production $[\sum_{j=1}^{105} \sum_{k=1}^{k_j} (XD_{ijkt} + CI_{ijkt})]$, determined by the program component, is used as the linkage variable to transfer information from the programming component to the econometric component. When the final production (denoted as Q_{it}^*) differs from Q_{it}, Q_{it}^* will be considered a better estimated value of the actual production and will then be used in the econometric component to adjust the values of other endogenous variables in the component. The adjusted values subsequently are used for determining the linkage variables in time period t+1. When Q_{it}^* is equal to Q_{it}, no adjustment is performed.

Test methods

Two test methods were used to evaluate the performance of the hybrid model in estimating agricultural production, prices, and levels of other agricultural activities. These two methods are: (1) static simulation and (2) dynamic simulation. Each method is applied to the hybrid model and to the CED-CC model, respectively. Estimated values from these two types of models are compared with actual observations. The difference between the first and second method is: In the first method, for each time period, actual observed data are used for all predetermined variables (includes lagged and exogenous variables), while in the second method, the lagged endogenous variables are estimated recursively, and used as input in the next time period. The first method attempts to conduct "ex-post" analysis. Result from this method provide information indicating how well the model can perform when error from input data is removed or kept at a minimum. Results from the second method provide information indicating how well the model can be used for multi-period simulation; for example, how seriously the error accumulated in previous time periods will affect the performance of the model in later time periods. This information is extremely important if the model is designed for making ex-ante analysis of more than one time period.

The years 1969 and 1972 were arbitrarily selected for the test (or ex-post test) of the hybrid model.[6/] Years 1969 to 1973 and 1972

[6/] In conducting an expost analysis, it is necessary to use actual values for all predetermined variables as input data. Although this requirement poses no difficulty in the econometric component, it does pose difficulty in the programming component. The LP component uses extensively synthesized data that do not have observed values. Furthermore, the ex-post analysis also requires forecast values which should be outside the sampling period in which all the regression coefficients in the model were estimated. Therefore, it is an approximation of ex-post analysis.

through 1976 were selected for the dynamic test. However, only the results from years 1969 to 1973 are presented.

Data

The regression coefficients of the econometric component (CED-CC model) were established in 1977 by using historical data from years 1950 to 1977. Endogenous and exogenous data from 1960 to 1977 were also updated.

The data set in the programming component (CARD-NRED LP model) was derived from 1975 LP data base residing in CARD. Initial data (1968 and 1971) were derived from this data base. The production cost is adjusted according to cost indices for production, interest, taxes, and wage rates (Agricultural Statistics, 1976). Projected production costs were adjusted by a constant rate from test period 1969 and 1972. Stoecker's (1974) yield function was used to estimate yield for 1969 and 1971. Constant yield was assumed during the test period. The derived regional to national price ratio (1972-74) was assumed unchanged. The values of elasticities are from Richardson and Day (1975).

Results

Each year's simulation of the econometric component determines 164 values for endogenous variables including livestock and crop production, utilization, and marketing activities. The programming component gives spatial distributions of thousands of crop production activities and land use patterns in 105 producing areas. Because of space limitations only key portions of results are presented.

Table 2. Ex-post simulation results

Crop	Actual	Estimated	Error (%)
	1969 National Production		
Corn	4687	4487	.27
Soybeans	1133	1116	1.50
Oats	965	959	.62
Wheat	1442	1453	.76
	1969 Iowa Production		
Corn	1,012,563	1,001,146	1.13
Soybeans	179,850	182,530	1.49
Oats	93,840	108,720	13.69
Wheat	1,320	1,755	32.95
	1972 National Production		
Corn	5570	5444	0.24
Soybeans	1270	1312	3.31
Oats	690	784	13.62
Wheat	1546	1601	3.56
	1972 Iowa Production		
Corn	1,212,200	1,154,493	4.76
Soybeans	217,800	215,161	2.92
Oats	70,000	81,362	16.23
Wheat	1,238	1,360	9.90

Static simulation test results of national and Iowa production of corn, soybeans, oats, and wheat are shown in Table 2. In general, the model performed well in estimation of corn and soybeans at both national and state levels. At the national level, both cases show less than a 5 percent error in estimation. For 1969, on the state level, the error was less than 2 percent and was less than 4 percent in the 1972 simulation. Oats and wheat are minor crops in Iowa. The model performed poorly in estimating production at the state level as well as the national level (more than 13 percent error). Although the model performed well (less than 4 percent error) in estimating wheat production on the national level, it did poorly on the state level estimations (32.9 percent error in 1969 and 9.9 percent error in 1972). These simulation results indicate that the hybrid model does well estimating major crop production at either state or national levels while performing poorly in estimation of minor crops. However, this poor performance can be improved significantly if a more accurate regional crop production response is available and implemented into the LP component of the hybrid model.

Dynamic simulation test results from the first simulation run indicate most of the national crop production generated by the econometric component was adjusted by the programming component.[7/] This causes a significant descrepancy in estimates of the national crop production and prices between the hybrid model and the CED-CC model. The following information can be drawn from figures 7 through 10: (a) The hybrid model using the regional restraints fail to give a better

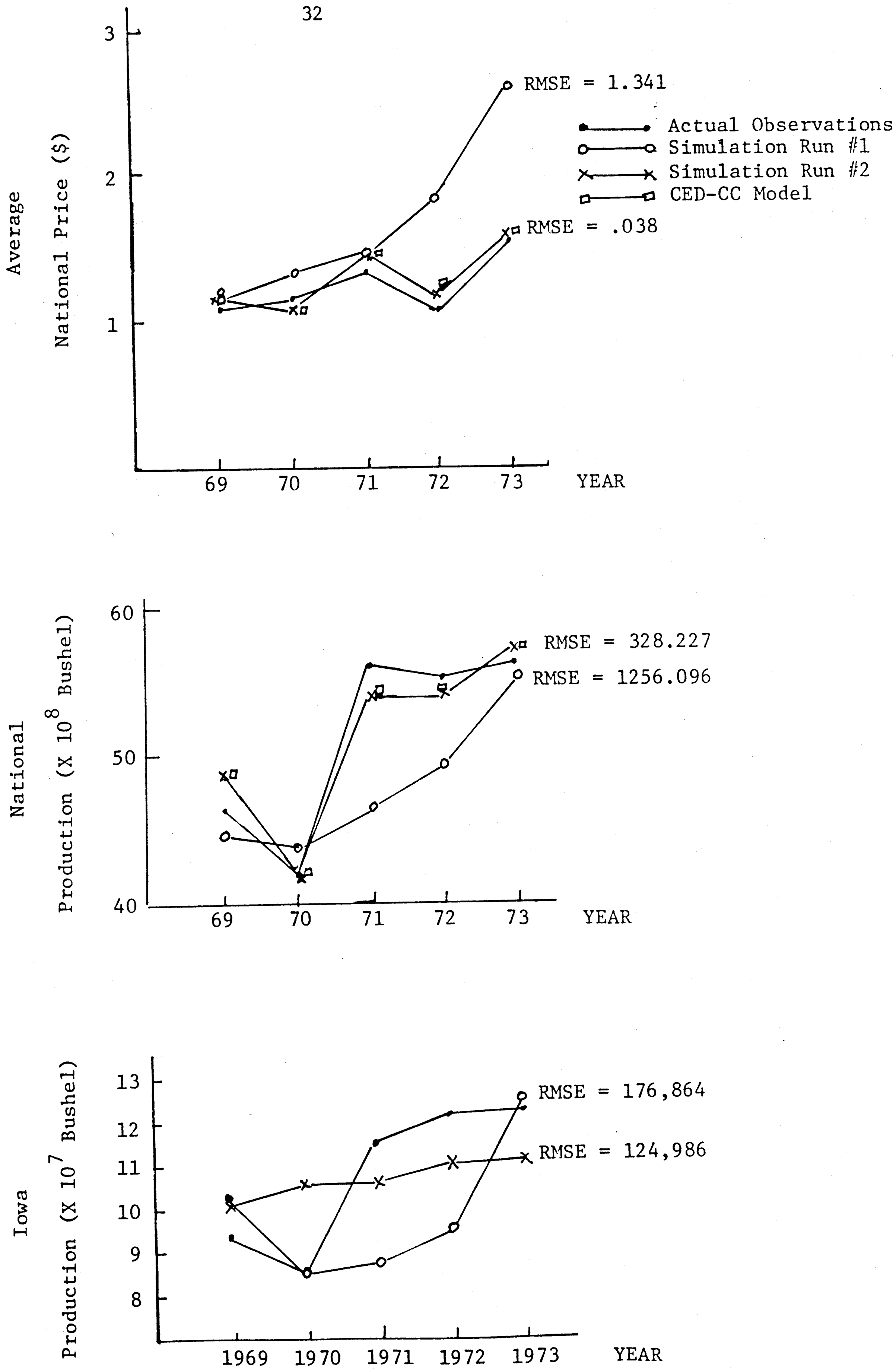

Figure 7. Corn; comparison of performance of the hybrid and CED-CC models to actual observations

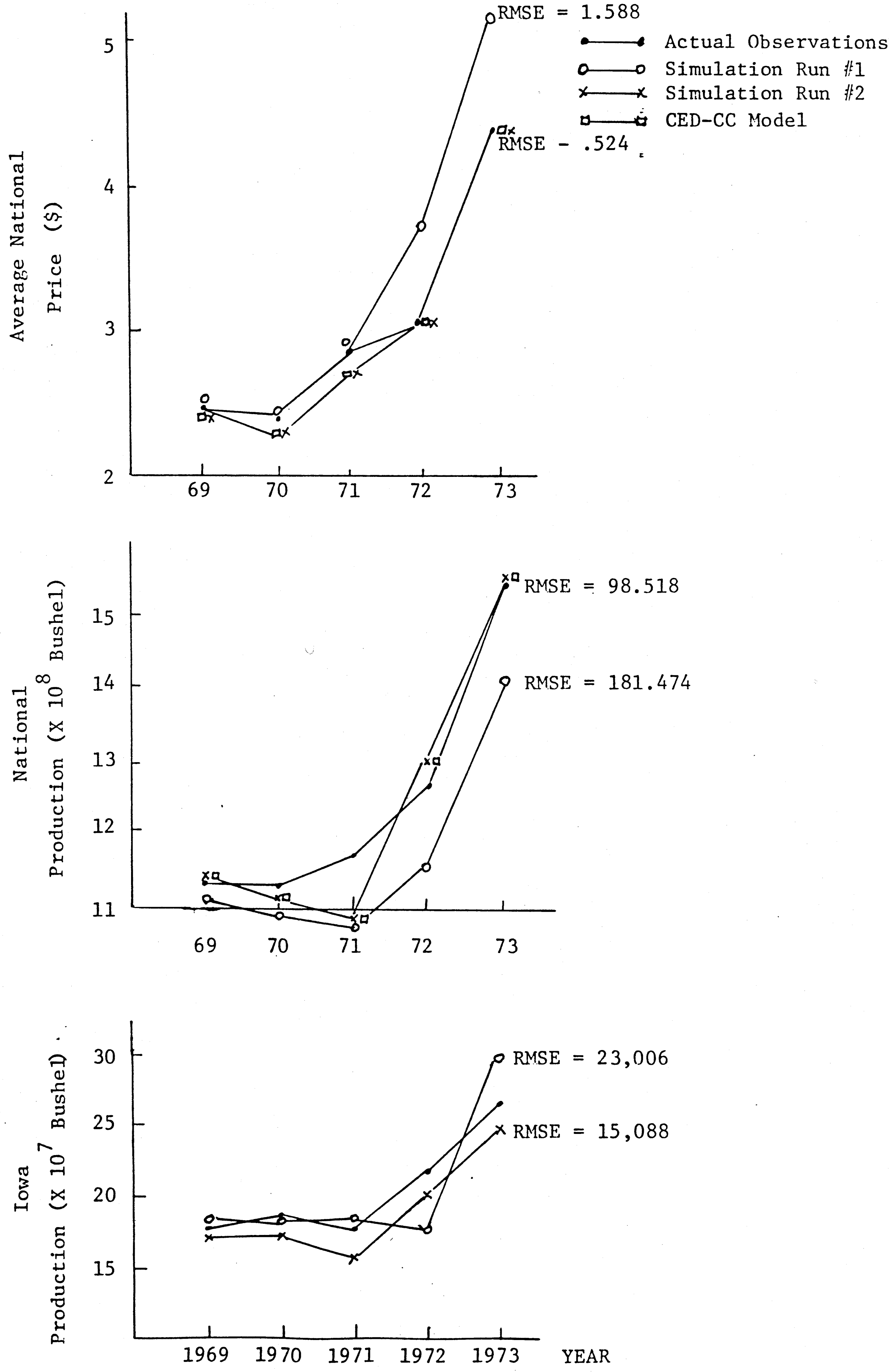

Figure 8. Soybeans: comparison of performance of the hybrid and CED-CC models to actual observations

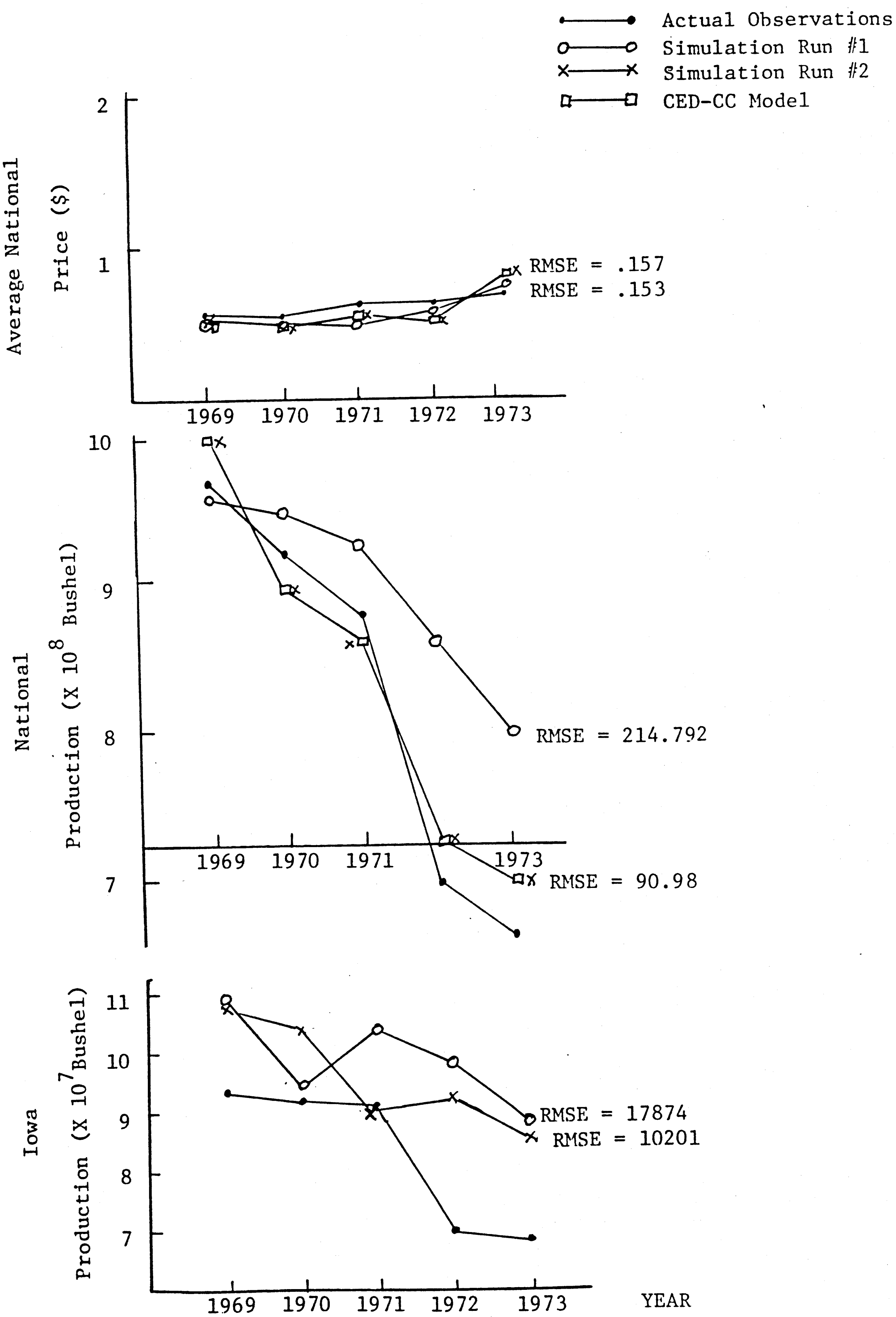

Figure 9. Oats; comparison of performance of the hybrid and CED-CC models to actual observations

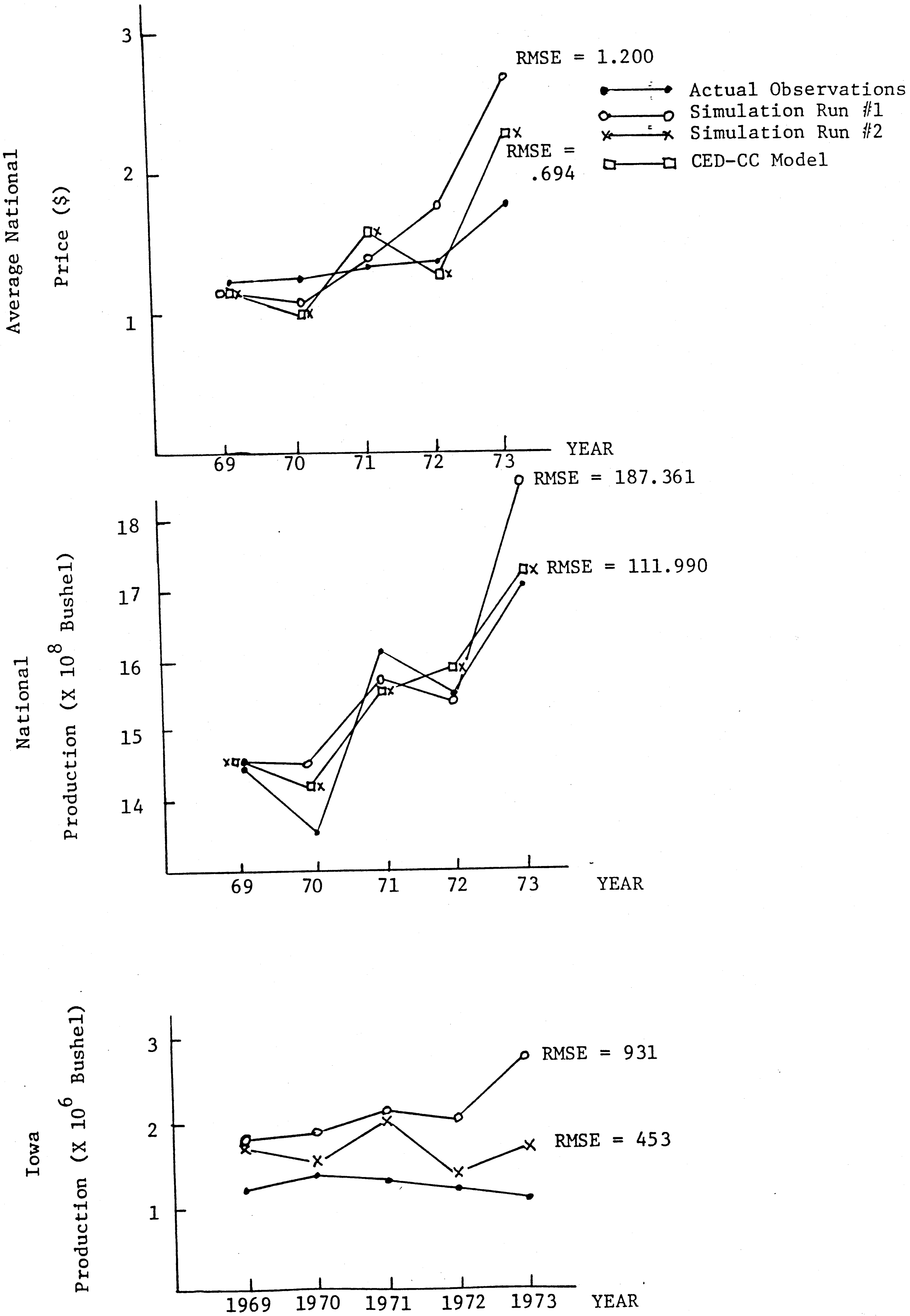

Figure 10. Wheat; comparison of performance of the hybrid and CED-CC models to actual observations

estimation on aggregate national production and price as compared with the estimates generated by using the CED-CC model alone. The failure is due to the fact that restraint constructed by using price elasticities is not aduquate to represent the regional response. (b) The

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7/ Crop production adjustments by the LP are indicated by "*" in table below:

Simulation year	Corn	Sorghum	Oats	Barley	Wheat	Soybeans
1969	*	*	*	*		*
1970	*	*	*	*		
1971	*	*	*			*
1972	*		*		*	
1973	*		*	*		

adjustment mechanism in the hybrid model assumes that national aggregated production can be better estimated by summing the individually regional production, than estimated by using national aggregated data as done by the CED-CC econometric model. This assumption can be true only if a set of accurate regional response functions can be formulated. In order to improve the performance of the hybrid model considerable effort is needed to develop regional restraints. (c) The time recursive structure, as used by the hybrid model, will accumulate error and pass it on to the next time period. This was found in the results (i.e., estimates of the corn and soybean prices in the figures). To reduce this error perhaps the regional restraints should be formulated as a function of the endogenous variable in the econometric component rather than by depending heavily on the previous year's production, as formulated in (4). From these findings it is suggested that whenever accurate regional response restraints are not available, probably the one-way communication may perform better between time periods than any model with a recursive structure.

In the second simulation run, the regional restraints (4)' were not included; instead, four regression equations representing corn, soybeans, oats, and wheat production responses[8/] were used as regional restraints for Iowa. These equations were used to generate the RHS values of the regional restraints for Iowa. As expected, the hybrid model gave the same estimation of the national production as the estimates generated by the CED-CC model. Meanwhile, a significant improvement on simulation of the Iowa crop production is achieved (as judged by the values of RMSE). This outcome demonstrates that if a better econometrically estimated regional response function is used, the hybrid model can give a better estimate of national production as well as regional production and price.

[8/] The four regression equations are:

(1) $$Y_t^c = \underset{(59289)}{746326} - \underset{(177184)}{13119.56}(P_c^t - P_{t-1}^c) + \underset{(7373)}{35297T} - \underset{(59485)}{81412}\, P_{t-1}^c$$

$$R^2 = 0.747$$

(2) $$Y_t^s = \underset{(14329)}{53528} + \underset{(0.35)}{5.27}\, A_t^s$$

$$R^2 = 0.9345$$

(3) $$Y_t^o = \underset{(13694)}{-27826} + \underset{(0.55)}{5.6}\, A_t^o$$

$$R^2 = 0.8658$$

(4) $$Y_t^w = 1286.5 - 509(P_t^w - P_{t-1}{}^w) + \underset{(183)}{390.94}\, P_{t-1}{}^w$$

$$R^2 = .4432$$

Where crop production Y_t^c, Y_t^s, Y_t^o, and Y_t^w = crop production of the four crops: corn soybeans, oats, and wheat. P_t^c, P_t^w = national prices of corn and wheat. A_t^s A_t^o = planted acres of soybeans and oats. Values for these variables are generated from the econometric components in the hybrid model.

CONCLUSION

Need for a policy model with space and time characteristics of price, production, and resource use leads to development of hybrid models combining econometric and programming components and to investigating methods of linking econometric with programming components resulting in a recursive adaptive (RAP) hybrid model. The model uses a programming component to validate the estimates by the econometric component and adjusts the estimates when they are outside the production feasible region. The static simulation tests of the model show the model performing well in estimating corn and soybean production at both national and regional (state of Iowa) levels, but show inconsistencies in estimating production of oats and wheat. The dynamic simulation tests show both national and regional (state of Iowa) estimates follow the general movement of the observations, but has cumulative error. By relaxing the bounds of regional restraints, the model could be used as a national model. Considerable effort in improving the regional restraints is required in order to use the model for regional or national and regional simultaneous positive policy analyses.

SUMMARY

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