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Elasticities in the Trade Liberalization Database

Walter H. Gardiner
Vernon O. Roningen
Karen Liu

Abstract: This paper describes the construction of elasticities of trade in the ZILIN database. It shows the behavior of each country in response to government policy. Various determinants of elasticities are examined.

Keywords: Elasticities, ZILIN, Trade Liberalization, Simulation, GATT, World Bank

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Page

- 8 Equations (5) and (6): Incorrect notation; " w " should be " δ ".
- 71 Equation (B.2): Incorrect notation; "]" should be " π " and "&" should be " Σ ".
Equation (B.3): Incorrect notation; "&" should be " Σ " and "m" should be " μ ".
Equations (B.4)-(B.6): Incorrect notation; " w " should be " δ " and "m" should be " μ ".
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Vernon O. Roningen, and Karen Liu, Agriculture and Trade Analysis Division,
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ABSTRACT

This report documents the elasticities that are available in the Trade Liberalization (TLIB) database, which are used by the Static World Policy Simulation (SWOPSIM) framework for generating TLIB models. The price elasticities of supply and demand are the key elements of models created from the TLIB database because these elasticities summarize the supply and demand behavior of each country to a price change as a result of a change in government policy. Various sources of elasticities have been used to compile a set of elasticities consistent with the data in the database.

Keywords: Elasticities, demand, supply, trade liberalization, database, models

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SUMMARY

Trade liberalization (TLIB) models are comparative static models generated by the Static World Policy Simulation (SWOPSIM) framework for use in policy analysis of agricultural markets. The database for TLIB models gives detailed coverage of 36 regions and 22 commodities that are important in world agricultural trade. The basic components of models are equations for supply, demand, net trade, and price, as well as a comprehensive set of policy measures. The elasticities are critical elements in TLIB models, since they summarize the supply and demand behavior of each country to a price change.

The elasticities in the TLIB database were obtained from various published and unpublished sources. The initial set of elasticities was taken from a survey of world agricultural trade models and various country/commodity studies. The elasticities were evaluated in terms of their magnitude and cross-price effects and tested for various properties, such as symmetry and homogeneity. Restrictions were imposed on the set of elasticities to assure model stability. The elasticities were then reviewed by country/commodity analysts in the Economic Research Service.

The elasticities used in TLIB models ultimately have to be evaluated by model performance, stability, and reasonableness of the results obtained from simulations with the model. TLIB models have been used in a wide range of applications in trade and policy research both inside and outside of ERS. TLIB models and the database are continually being refined to reflect new information on policies, prices, quantities, and elasticity estimates.

Elasticities in the Trade Liberalization Database

Walter H. Gardiner
Vernon O. Roningen
Karen Liu

INTRODUCTION

Models for analysis of trade liberalization exercises can be assembled from the Trade Liberalization (TLIB) database. TLIB models were developed as analytical tools for evaluating the effects of government policies on world agricultural markets. The database was initiated in anticipation of the Uruguay Round of multilateral trade negotiations that began in September 1986 under the auspices of the General Agreement on Tariffs and Trade (GATT). The task was to develop a modeling framework that would be as comprehensive as possible in terms of commodity and country coverage. The TLIB models were developed by the Economic Research Service (ERS), U.S. Department of Agriculture, using the Static World Policy Simulation (SWOPSIM) modeling framework (Roninen, 1986). The models are constructed in electronic spreadsheets and solved on microcomputers. These spreadsheets are designed to provide quick, yet extensive, analysis of a wide range of policy questions at a low cost to the user.

This report documents the elasticities that are available in the TLIB database. These elasticities are an integral part of the TLIB modeling activity in ERS because these elasticities reflect the response of supply and demand to price changes resulting from changes in policy variables.

COMMODITY AND COUNTRY COVERAGE IN THE TLIB DATABASE

The TLIB database consists of up to 22 commodities (table 1) and 36 countries or regions (table 2). Elasticities have been obtained for the major commodities in the regions listed. Each region does not contain the same commodity coverage because the commodity is either not produced or consumed, or the data were not available. Table 3 summarizes the commodity coverage for each region in the TLIB database.

TLIB MODEL STRUCTURE AND ELASTICITIES

TLIB models are comparative static models designed to compare alternative equilibrium states disregarding the process of adjustment. The basic components of the TLIB model structure consist of supply, demand, and net trade equations for each country and each commodity (Roninen, 1986). Simple standard equations for supply, demand, and net trade are specified as follows:

$$QS_i = CS_i * (1 + SSF_i) * (P_i^{**} Es_{ii}) * (P_j^{**} Es_{ij}) \quad (1)$$

$$QD_i = CD_i * (1 + DSF_i) * (P_i^{**} Ed_{ii}) * (P_j^{**} Ed_{ij}) \quad (2)$$

$$QT_i = QS_i - QD_i \quad (3)$$

where:

QS_i = the quantity supplied of commodity i
 QD_i = the quantity demanded of commodity i
 QT_i = the quantity traded of commodity i
 P_i = the price of the ith commodity
 P_j = the price of the jth commodity
 SSF_i = supply-shift factor of commodity i
 DSF_i = demand-shift factor of commodity i
 CS_i = constant term of supply equation
 CD_i = constant term of demand equation
 Es_{ij} = supply elasticity, for $i=j$ means own-price elasticity, $i=j$ the cross-price elasticity of i with respect to j
 Ed_{ij} = demand elasticity, for $i=j$ means own-price elasticity, $i=j$ the cross-price elasticity of i with respect to j
* = multiply
** = exponential power

Supply is defined as production; demand is defined as consumption plus stock changes; and trade is the difference between supply and demand. Supply and demand equations are functions of own- and cross-product prices. There is no technology variable in the supply equation and no income variable in the demand equation since the models are comparative static and used to evaluate the price impacts of policy changes, holding technology and income constant. Joint product relationships are used for dairy product supply and oilseed crushing demand, which are described in detail in another study (Haley).

The supply and demand equations are specified as nonlinear equations with constant elasticities over all price ranges. Constant elasticity functional

Table 1--Commodities in the TLIB database

Code	Livestock	Code	Crops	Code	Other
BF	Beef and veal	CN	Corn	SM	Soybean meal
PK	Pork	WH	Wheat	SO	Soybean oil
ML	Mutton and lamb	CG	Other coarse grains	OM	Other meals
PM	Poultry meat		Barley, millet,		Copra, cottonseed,
PE	Poultry eggs		mixed grains, oats,		linseed, palm
DM	Dairy milk		rye, sorghum		kernel, peanuts,
DB	Dairy butter	RI	Rice		rapeseed, safflower,
DC	Dairy cheese	SB	Soybeans		sesame, sunflower
DO	Dairy other	OS	Other oilseeds	OO	Other oils
	Condensed and		Copra, cottonseed,		Copra, cottonseed,
	evaporated milk,		flaxseed, palm kernels,		olive, palm,
	dry whole milk,		peanuts, rapeseed,		palm kernel, peanut,
	nonfat milk		safflower, sesame seed		rapeseed, safflower,
		CT	Cotton		sesame, sunflower
		SU	Sugar		
		TB	Tobacco		

Table 2--Regions in the TLIB Database

Code	Region/country
	Developed countries:
US	United States
CN	Canada
EC	European Community-10 (Denmark, France, West Germany, Greece, Ireland, Italy, Belgium/Luxembourg, Netherlands, United Kingdom)
SP	Spain
PT	Portugal
WE	Other Western Europe (Austria, Finland, Iceland, Malta, Norway, Sweden, Switzerland)
JP	Japan
AU	Australia
NZ	New Zealand
SF	South Africa
	Centrally planned economies:
EE	Eastern Europe (Albania, Bulgaria, Czechoslovakia, East Germany, Poland, Hungary, Romania, Yugoslavia)
SV	Soviet Union
CH	China
	Latin America:
MX	Mexico
CA	Central America and Caribbean (Belize, Costa Rica, El Salvador, Honduras, Guatemala, Nicaragua, Panama, Bahamas, Bermuda, Cuba, Dominican Republic, Haiti, Jamaica, Trinidad & Tobago, Barbados, Bonaire, Curacao, French West Indies, Guadeloupe, Martinique, Turks & Caicos, Cayman Islands, Aruba, British West Indies, Leeward-Windward Islands, St. Kitt, Netherlands Antilles, Antigua, Nevis, Montserrat, Guyana, French Guiana, Surinam)
BZ	Brazil
AR	Argentina
VE	Venezuela
LA	Other Latin America (Bolivia, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay)
	Sub-Saharan Africa and Middle East:
NG	Nigeria
AF	Other Sub-Saharan Africa (Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros Islands, Congo, Djibouti, Equatorial Guinea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Ivory Coast, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Reunion, Rwanda, Sao Tome/Principe, Seychelles, Sierra Leone, Somalia, Senegal, Sudan, Swaziland, Tanzania, Togo, Uganda, Zaire, Zambia, Zimbabwe)
EG	Egypt
MP	Middle East and North Africa Oil Producers (Syria, Iraq, Iran, Kuwait, Qatar, Saudi Arabia, United Arab Emirates, Oman, Bahrain, Algeria, Tunisia, Libya)
MO	Middle East and North Africa, Other (Turkey, Cyprus, Lebanon, Israel, Gaza, West Bank, Jordan, North Yemen, South Yemen, Morocco)
	Asia:
ND	India
OS	Other South Asia (Afghanistan, Bangladesh, Bhutan, Nepal, Pakistan, Sri Lanka)
DO	Indonesia
TH	Thailand
ML	Malaysia
PH	Philippines
SA	Other Southeast Asia (Brunei, Burma, Fiji)
SK	South Korea
TW	Taiwan
EA	Other East Asia (Hong Kong, Singapore)
OA	Other Asia (Cambodia, Laos, Mongolia, North Korea, Viet Nam)
	Other:
RW	Rest of the world

forms were used because they are easy to interpret and some types of model solution problems can be avoided (Thompson). This includes the problem of negative solution values for prices and quantities that sometimes occur during model simulation when linear equations are shifted away from their historical values. This is particularly true when simulating large changes in government policies, which, in turn, can cause large shifts in supply or demand.

The policy structure of the models is embedded in equations linking domestic and world prices. Policy measures used in the model are producer subsidy equivalents (PSE's) or consumer subsidy equivalents (CSE's), which are inserted as price wedges at the producer or consumer level. Price transmission elasticities are also used in TLIB models to characterize the degree to which domestic prices respond to changes in world prices. The relationship and the use of these elasticities and quantity and price data in TLIB models are explained in the report by Sullivan, Wainio, and Roningen.

This report documents the elasticities that are used in the TLIB database and, therefore, in models constructed from the database. The sources of elasticities, criteria for selecting the elasticities, and method for entering the elasticities in the models are discussed in the following sections.

Table 3--Commodity coverage for the TLIB regions 1/

^{1/} See table 1 for commodity codes and table 2 for country codes.

I = A set of elasticities is available for this commodity.

• ≡ No elasticities available for this commodity

SOURCES OF ELASTICITIES

The elasticities used in the TLIB database were based on elasticities from various published and unpublished sources and from researchers at the Economic Research Service (ERS). For developed countries, the principal sources of the elasticities were: the Ministerial Trade Mandate (MTM) model developed by the Organization for Economic Cooperation and Development (OECD), the Grain, Livestock, and Sugar (GLS) model developed by Tyers and Anderson, and the Grain, Oilseed, and Livestock (GOL) model developed by ERS. For developing countries, the principal sources of the elasticities were the GLS model, the World Food model developed by the Food and Agriculture Organization (FAO), the GOL model, and regional analysts at ERS. For centrally planned economies, elasticities were obtained primarily from the GLS model, the FAO World Food model, and from regional analysts at ERS. These comprehensive modeling efforts were supplemented with commodity studies from ERS, universities, and other research institutions and are listed in the References section.

A sample of the elasticities that were found in the literature for the supply of beef for selected regions is summarized in table 4. The first column contains the name of the country or region; the second column labeled BF (beef and veal) contains the own-price elasticity with respect to beef supply; columns PK (pork) through SM (soybean meal) contain the cross-price elasticities with respect to beef supply; and the last column indicates the sources of the elasticities. For example, the own-price elasticity of beef supply ranges from 0.30 to 0.72 for the United States, 0.24 to 0.61 for

Table 4--Price elasticities for beef supply, selected countries

Country	Commodity								Source
	BF	PK	ML	PM	DM	CN	CG	SM	
United States	0.69	--	--	--	0.01	-0.32*	--	--	OECD
	.72	-0.16	--	--	.02	--	-0.28	--	Tyers-Anderson
	.30	--	--	--	--	-.20	--	-0.05	USDA/ERS
Canada	.24	--	--	--	--	-.12*	--	--	OECD
	.61	-.18	--	--	.07	--	-.46	--	Tyers-Anderson
	.40	-.10	--	--	--	-.20	--	-.05	USDA/ERS
European Community	.62	--	--	--	.16	-.05*	.18	--	OECD
	1.02	-.48	--	--	.12	--	-.30	--	Tyers-Anderson
	.40	-.15	--	--	.15	-.20	--	-.10	USDA/ERS
Japan	.23	--	--	--	.68	-.44*	--	--	OECD
	.80	-.10	--	--	.04	-.16	--	--	Tyers-Anderson
	.50	-.10	0.20	-0.10	--	-.30	--	--	USDA/ERS
Australia	.34	--	-.06	--	.01	-.06*	.03	--	OECD
	.36	--	-.04	-.08	-.02	--	--	--	Tyers-Anderson
	.40	--	-.10	--	--	--	--	--	USDA/ERS

-- = Not applicable.

* = Elasticity with respect to feed cost of beef.

1/ See table 1 for commodity codes.

Canada, 0.40 to 1.02 for the European Community, 0.23 to 0.80 for Japan, and 0.34 to 0.40 for Australia.

Dairy-milk (DM) was the most frequently reported cross-price elasticity and was generally positive in value, indicating a complementary relationship with beef supply. Pork (PK) was the principal substitute for beef, except in Australia where mutton and lamb (ML) prevailed. Price elasticities for feeds (indicating the response of beef supply to a change in feed costs) were widely reported for most regions. Beef supply in Australia was the least responsive to changes in feed prices, while Japanese beef supply was the most responsive.

Commodity supply and demand elasticities that were found in the literature for various regions are summarized in Appendix A.

DETERMINING ELASTICITIES FOR THE TLIB DATABASE

TLIB models are designed to evaluate adjustments to policy changes that would occur over the medium term, which we define as 3-5 years. The OECD's MTM model is a medium-term adjustment model, while the Tyers and Anderson GLS model has both short- and long-term features. Most of the other studies contain elasticities that reflect a short-term response to price changes. These models were the basis for determining many of the elasticities used in the TLIB database. The rules for selecting these elasticities for the database were as follows:

- o If an elasticity from the MTM model and the GLS model were in agreement, then that elasticity was selected for use in the model.
- o If an elasticity from the two studies differed, their values were compared with other studies.
- o In general, the elasticities in the GLS model were greater than those in the MTM model and most other studies surveyed, so a value between the elasticities of the MTM and the GLS models was usually selected.
- o In some cases, important cross-price linkages were missing so other sources were consulted for these values.
- o The set of elasticities selected for each region was reviewed by country or commodity analysts and adjusted where appropriate.

After the elasticities were selected from the literature survey or obtained from country or commodity analysts, they were entered into separate microcomputer spreadsheets for each country or region. Table 5 shows a sample spreadsheet of the supply and demand elasticities for the countries in the TLIB database. The spreadsheets are divided into two blocks or matrices. The first block (SUPPLY) contains the supply elasticities, the second block (DEMAND) contains the demand elasticities. The first column of each spreadsheet contains the commodity codes for supply and demand. The first row contains the commodity codes for the associated price elasticities. Each row of the supply and demand blocks contains the price elasticities of the commodity listed in the first column.

The own-price elasticities are shown on the diagonal of the matrix and are all positive in value for supply and all negative for demand. The off-diagonal

Table 5--Supply and demand elasticities, United States

elements (those above or below the diagonal) are the cross-price elasticities that indicate the substitution or complementary relationships between the commodities. In the case of supply, a negative cross-price elasticity indicates the commodities are substitutes, while a positive sign indicates the commodities are complements. The opposite relationships hold for demand.

In the case of pork supply for the United States, the own-price elasticity is 1.0 and cross-price elasticities for beef and poultry meat are -0.02 and -0.01, respectively. U.S. pork supply also has cross-price elasticities with respect to corn (-0.25), other coarse grains (-0.05), and soybean meal (-0.07). On the demand side, pork has an own-price elasticity of -0.86 and cross-price elasticities of 0.08 for beef, and 0.03 for poultry meat.

After the elasticities were entered into spreadsheets, they were subjected to a number of tests to examine various properties from economic theory such as homogeneity and symmetry. Initially, supply and demand elasticities were entered only in the upper triangle of the matrix (cells on and above the diagonal), and the elasticities in the lower triangle (cells below the diagonal) were generated using the following equation:

$$E_{ij} = E_{ji} \frac{p_j q_j}{(p_i q_i)} \quad i, j = 1, \dots, n \quad (4)$$

where:

- E_{ij} = the cross-price elasticity of i with respect to j
- E_{ji} = the cross-price elasticity of j with respect to i
- p_i = the price of i
- p_j = the price of j
- q_i = the quantity of i
- q_j = the quantity of j

The equation is based on the "reciprocity relations" or symmetry conditions for output-supply and factor-demand functions:

$$\frac{wq_i}{wp_j} = \frac{wq_j}{wp_i} \quad i, j = 1, \dots, n \quad (5)$$

Equation (4) is simply equation (5) expressed in elasticities. Equation (5) is derived from the comparative statics of maximization models (Silberberg, 1978, pp. 112-114, 206, 270; Henderson and Quandt, 1980, p. 101). See Appendix C for the derivation of equation 5.

The reciprocity relations in terms of elasticities indicate the interdependencies of the cross-price elasticities for output-supply and factor-demand functions.

The symmetry conditions for consumer demand functions are represented by the Slutsky equations:

$$\frac{wq_i}{wp_j} + \frac{q_j wq_i}{wy} = \frac{wq_j}{wp_i} + \frac{q_i wq_j}{wy} \quad i, j = 1, \dots, n \quad (6)$$

where:

y = income

In terms of elasticities, the symmetry conditions can be rewritten:

$$E_{ij} = E_{ji} \frac{(p_j q_j)}{(p_i q_i)} + E_{jy} \frac{(p_j q_j)}{y} - E_{iy} \frac{(p_j q_j)}{y} \quad (7)$$

where:

E_{iy} and E_{jy} are the income elasticities of the i th and j th goods.

There are no income elasticities in the demand equations of the TLIB model because the model was designed to evaluate the price impacts of policy changes, holding income constant. Therefore, using equation (4) to calculate the cross-price elasticities in the lower triangle of the elasticity matrix for consumer demands (such as meat, eggs, and dairy products) may impart a bias by the amount that the income elasticities differ between any pair of goods. If the income elasticities are the same between any pair of goods, then the last two terms of equation (7) net out to zero and equation (7) is equivalent to equation (4). Equation (7) is appropriate for those goods that are used in final consumption such as meats, eggs, and dairy products. For those goods that are for intermediate consumption (grains, oilseeds, cotton, tobacco), equation (4) is the more appropriate formula.

The right column of each elasticity matrix, labeled "Elas. Row Sum," indicates the value obtained by summing across all elasticities in each row. This measure is used to check the homogeneity condition for each equation in the model. For output-supply and factor demands, the homogeneity condition in terms of elasticities yields:

$$E_{i1} + E_{i2} + \dots + E_{in} = 0 \quad i = 1, \dots, n \quad (8)$$

This simply states that the sum of all price elasticities is equal to zero. The output-supply and the factor-demand functions are homogeneous of degree zero in the prices (Silberberg, p. 272).

For consumer demands, the homogeneity condition can be expressed by

$$E_{i1} + E_{i2} + \dots + E_{in} + E_{iy} = 0 \quad i = 1, \dots, n \quad (9)$$

This says that the sum of all price elasticities and the income elasticity should be zero for consumer demand.

There are no price elasticities for inputs (such as fertilizer, machinery, and equipment) in the TLIB supply equations and no income elasticities in the demand equations since factor prices (except for feed) and income are exogenous to the model. Our interest lies in the commodity price adjustments from trade policy changes. Therefore, the sum of the price elasticities in each supply equation should be greater than zero (positive in value), while the sum of the price elasticities in each demand equation should generally be less than zero (negative in value).

After the elasticities have been determined for each commodity and region, a batch program written in BASIC programming language takes this information along with price and quantity data for the base period and writes the model equations.

Appendix B contains tables of country/region elasticities in the TLIB database.

SENSITIVITY ANALYSIS

One way to evaluate the elasticities used in simulation models is to shock a particular component of a model, say a 10-percent drop in wheat production in the United States, and measure the subsequent adjustments to prices, production, consumption, and trade. Questions to examine might include:

- o Do these adjustments appear reasonable in terms of direction and magnitude for each commodity and each region?
- o What regions do the most/least adjusting to the shock?
- o Are the cross-commodity effects plausible?
- o Are the implied elasticities of export demand for the United States reasonable?

Other sensitivity test might include:

- o Simulating the model over a range of elasticity estimates for each country.
- o Increasing or decreasing all elasticities by a fixed percentage.
- o Using identical elasticities in each region.

To evaluate the elasticities in the TLIB database, individual country and regional models were first constructed and simulated to test for stability and reasonableness of results. In certain cases, elasticities that gave rise to stability and solution problems had to be modified. Next, a number of global models with various country/region combinations were built from the TLIB database and simulated under various policy environments. The results were examined in detail by country and commodity analysts. Results that were judged to be unreasonable led to further modification of elasticities. The final set of elasticities used in the TLIB database was examined thoroughly in terms of properties from economic theory and also in terms of its performance in the simulation of policy changes in a global commodity model.

CONCLUSIONS

In preparation for the Uruguay Round of GATT negotiations that began in September 1986, ERS constructed a database (TLIB) and a global modeling framework (SWOPSIM) to examine the implications for domestic and world markets of reducing assistance to agriculture. One of the earliest tasks was to assemble a set of price elasticities for a large number of commodities (22) and countries/regions (36). With only a few months to accomplish this task, ERS analysts did an indepth survey of global agricultural models and commodity market studies to obtain the most complete set of elasticities available. The elasticities were subjected to a battery of tests, scrutinized by country and commodity analysts, modified, retested, and reexamined. The present set of elasticities in the TLIB database has been used to construct commodity market models for numerous studies on the effects of trade liberalization on producers, consumers, and government budgets.

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APPENDIX A

**Commodity Supply and Demand Elasticities from the Literature for
Various TLIB Regions**

Appendix table 1-Price elasticities of supply for selected commodities, United States

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	.69 .72r .30					.01 .02d						-.32f -.28											OECD, Mar. 1986 Tyers and Anderson, Jan. 1986 Rojko, et al
PK													-.25f -.50										OECD Tyers and Anderson Rojko, et al
ML													-.21f										OECD
PM													-.22f										OECD Rojko, et al
PE																							OECD
DM	-.22 .12r					.55 .50d .40						-.19f -.12 -.30											OECD Tyers and Anderson Rojko, et al
DB							.35	-.80	-.04														OECD
DC							-.10	.29	-.05														OECD Rojko, et al
DO							-.10	-.59	1.31														OECD
WH										.50 .80 2.55 .56 .82		-.09 -.100 -.1.84 -.09 -.12										OECD Tyers and Anderson Rojko, et al Collins Shumway and Green, 1982	
CN																							OECD McKinzie, 1983 Shumway and Green
CG																							OECD Tyers and Anderson Rojko, et al
RI																							OECD Tyers and Anderson Rojko, et al
SB																							OECD Rojko, et al Shumway and Green, 1982 Williams, 1981 Collins
OS																							Gardiner, Oct. 1983
CT																							Gardiner, 1983
SU																							OECD Tyers and Anderson World Bank, 1981, p. III-3
TB																							

d = dairy products, f = feed costs, fb = feed costs-beef, n = nonruminant meat, r = ruminant meat.

Appendix table 2--Price elasticities of demand for selected commodities, United States

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source	
BF	-.96 -.90r -.70	.37 .30n .10		.03																			OECD, Mar. 1986, p. 9 Tyers and Anderson, Jan. 1986 Rojko, et al	
PK	.34 .30r .40	-.89 .80n -.80		.20																			OECD Tyers and Anderson Rojko, et al	
ML		.31	-.70																				OECD	
PM	.36 .30	.19 .20		-.74																			OECD Rojko, et al	
PE	.05			.07	-.15																		Huang, 1985 Gardiner, 1983	
DM						-.32 .30d .20																OECD Tyers and Anderson Rojko, et al		
DB							-.56 .70																OECD Rojko, et al	
DC								-.60 .50															OECD Rojko, et al	
DO									-.32															OECD
WH										-.06 .21 .20 .70												OECD Tyers and Anderson Rojko, et al Collins		
16											.12													
CN											.05													
CG	.29r 3.33h	.25n				.10d						.08 .07											OECD Tyers and Anderson Collins	
RI													.21 .50 .81											
SB														-.10 .10										OECD Tyers and Anderson Rojko, et al
SM	.07 .12	.27 .10																						Collins Griffith and Meilke, 1982 Gardiner, May 1986 Green and Hoskin, 1982
SO																								Rojko, et al Gardiner, July 1983 Griffith and Meilke, 1982 Green and Hoskin, 1982
OS																								Green and Hoskin Meyers and Hacklander, June 1979 Huysen, 1983 Gardiner, June 1983 Griffith and Meilke, 1982, p. 38
OM	.24		1.19		.16																			Gardiner, Oct. 1983
OO																								Gardiner, Jul. 1983
CT																								Gardiner, Jun. 1983
SU																								Gardiner, Oct. 1983
TB																								

d = dairy products, h = high-protein animal units, n = nonruminant meat, r = ruminant meat.

Appendix table 3--Price elasticities of supply for selected commodities, Canada

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	.24 .61r .40	-.18n -.10				.07d						-.12f -.46										OECD, Mar. 1986, p.4 Tyers and Anderson, Jan. 1986 Rojko, et al	
PK		.54 -.14r -.20										-.40f -.49										OECD Tyers and Anderson Rojko, et al	
ML										1.60			-1.23f									OECD Rojko, et al	
PM	-.10	-.20							.70			-.40											OECD Tyers and Anderson Rojko, et al
PE																							OECD Tyers and Anderson Rojko, et al
DM						.00 .50d .30						-.10 -.40										OECD Tyers and Anderson Rojko, et al	
DB							.46	-.84	-.10													OECD	
DC							-.49	1.10	-.10													OECD Rojko, et al	
DO								-.60	.60													OECD	
WH										.05	-.10	.20										OECD Tyers and Anderson Rojko, et al	
CN																							OECD
CG																							OECD Tyers and Anderson Rojko, et al
RI																							OECD
SB																							Griffith and Meilke, 1982
OS																							OECD Griffith and Meilke Rojko, et al
CT																							OECD
SU																							Tyers and Anderson
TB																							World Bank, 1981, p.III-3

d = dairy products, f = feed costs, n = nonruminants, r = ruminants.

Appendix table 4--Price elasticities of demand for selected commodities, Canada

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	-.85 -.40r -.60	.06 .11n .30	.01	.03																			OECD, Mar. 1986 Tyers and Anderson, Jan. 1986 Rojko, et al
PK	.11 .25r .40	-.96 .75n .70	.02	.03																			OECD Tyers and Anderson Rojko, et al
ML	.41 .42	.47 -1.87 -1.34		.01																			OECD Gardiner, 1983
PM	.18 .30	.10 .20		-.56																			OECD Rojko, et al
PE																							
DM																							OECD Tyers and Anderson Rojko, et al
DB																							OECD Rojko, et al
DC																							OECD Rojko, et al
DO																							OECD
WH																							OECD Tyers and Anderson
CN																							
I8																							
CG																							OECD Tyers and Anderson Rojko, et al
RI																							OECD Tyers and Anderson Rojko, et al
SB																							Griffith and Meilke, 1982, p.29
SM																							Rojko, et al Griffith and Meilke, p.43
SO																							Gardiner, June 1983 Griffith and Meilke
SO																							OECD Griffith and Meilke
OM																							Griffith and Meilke
OO																							Gardiner Griffith and Meilke, p. 38
CT																							
SU																							
TB																							OECD Tyers and Anderson

d = dairy products, n = nonruminant meat, r = ruminant meat.

Appendix table 5--Price elasticities of supply for selected commodities, European Community

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	.62 1.02r .40					.16 .12d .15						-.05f -.30			-.18fm -.10							OECD, Mar. 1986, p.8 Tyers and Anderson, Jan. 1986 Rojko, et al	
PK												-.47f -.75										OECD Tyers and Anderson Rojko, et al *	
ML												-.47f										OECD	
PM												-.53f										OECD Rojko, et al *	
PE																							
DM	.20 -.05					1.00 .51d .35						-.32f -.21			-.06fb -.20							OECD Tyers and Anderson Rojko, et al *	
DB							.29															OECD	
DC								.29														OECD Rojko, et al *	
DO									.15													OECD	
WH										.46 .90 .92												OECD Tyers and Anderson Rojko, et al *	
CN																							
CG																						OECD Tyers and Anderson Rojko, et al *	
RI																						OECD Tyers and Anderson Rojko, et al *	
SB																						OECD	
OS																						OECD Griffith and Meilke, 1982	
CT																						Gardiner, Oct. 1983	
SU																						OECD Tyers and Anderson World Bank, 1981, p.III-3	
TB																							

* = (2/3*EC6+1/3*EC3), d = dairy products, f = feed costs, fb = feed costs-beef, fm = feed costs-milk, n = nonruminant meat, r = ruminant meat.

Appendix table 6--Price elasticities of demand for selected commodities, European Community

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	-.76 -.60r -.68 -.41	.30 .25n .27 .25	.01	.04																		OECD, Mar. 1986, p. 7 Tyers and Anderson, Jan. 1986 Rojko, et al Herlihy	
PK	.26 .25r .42	-.79 -.90n -.80 -.31	.03																			OECD Tyers and Anderson Rojko, et al Herlihy	
ML	.22	.10	-1.19	.10																		OECD	
PM	.11 .36	.09 .42 .14	.08	-.60 -.91 -.61																	OECD Rojko, et al Herlihy		
PE				-.12																		Herlihy	
DM	.02r	.02n																				OECD Tyers and Anderson Rojko, et al	
DB																						OECD Rojko, et al Herlihy	
DC																						OECD Rojko, et al Herlihy	
DO																						OECD	
WH	.02	.05																				OECD Tyers and Anderson Rojko, et al Herlihy	
CN																							
CG	.15r .50	-.45n .50																				OECD Tyers and Anderson Rojko, et al Herlihy	
RI																						OECD Tyers and Anderson Rojko, et al Herlihy	
SB																						Huyser, p.171 Gardiner, May 1986	
SM	1.40 .19																					Rojko, et al Gardiner, May 1983 Gardiner, May 1986 Griffith and Meilke, 1982, p.43	
SO																						Gardiner, May 1983 Griffith and Meilke, p. 38	
OS																						Gardiner, Sep. 1983 Griffith and Meilke	
OM																						Gardiner, Sep. 1983 Gardiner Griffith and Meilke	
OO																						Gardiner, June 1983 Griffith and Meilke	
CT																						Gardiner, Sep. 1983	
SU																						OECD Tyers and Anderson	
TB																							

d = dairy products, n = nonruminant meat, r = ruminant meat.

Appendix table 7--Price elasticities of supply for selected commodities, Spain

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO.	CT	SU	TB	Source
BF	1.60 .70r .40 1.53					.12 .04d .15						-.76f -.30			-.06fm -.10							OECD, Mar. 1986, p.20 Tyers and Anderson, Jan. 1986 Rojko, et al Barcelo, 1985	
PK		.29 .09r .20 1.69										-.30 -.30		-.20f -.75								OECD Tyers and Anderson Rojko, et al Barcelo	
ML			.11											-.07f								OECD	
PM				.35 2.39										-.22f								OECD Barcelo	
PE																							
DM	.12					1.32 1.92								-.15f			-.06fb					OECD Barcelo	
DB							.46	-1.00	-.06													OECD	
DC								-.37	.99	-.06												OECD	
DO									-.37	-1.00	1.87											OECD	
WH												.82 .90 1.42			-.58 -.51							OECD Tyers and Anderson Barcelo	
CN													1.09									Barcelo	
21																							
CG																						OECD Tyers and Anderson Barcelo	
RI																						OECD Tyers and Anderson Rojko, et al	
SB																						OECD Rojko, et al	
OS																						OECD Barcelo	
CT																						Gardiner, Oct. 1983 Barcelo	
SU																						OECD Tyers and Anderson Barcelo	
TB																							

d = dairy products, f = feed costs, fb = feed costs-beef, fm = feed costs-milk, n = nonruminant meat, r = ruminant meat.

*Barcelo's estimates are long run.

Appendix table 8--Price elasticities of supply for selected commodities, Portugal

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	1.60 .70r .40	.29 .09r .20				.12 .04d .15						.76f .30										OECD, Mar. 1986, p.20 Tyers and Anderson, Jan. 1986 Rojko, et al	
PK													.20f .75										OECD Tyers and Anderson Rojko, et al
ML			.11										.07f										OECD
PM				.35									.22f										OECD
PE																							
DM	.12					.32							.15f										OECD
DB							.46	-1.00	-.06													OECD	
DC								.37	.99	-.06												OECD	
DO									.37	-1.00	1.87											OECD	
WH												.82 .91											OECD Tyers and Anderson
CN																							
CG													.31 .30										OECD Tyers and Anderson
22	RI													.57 .40 .30								OECD Tyers and Anderson Rojko, et al	
SB														.80 .20									OECD Rojko, et al
OS																							OECD
CT																							Gardiner, Oct. 1983
SU																							OECD Tyers and Anderson
TB																							

d = dairy products, f = feed costs, fb = feed costs-beef, fm = feed costs-milk, n = nonruminant meat, r = ruminant meat.

Appendix table 9--Price elasticities of demand for selected commodities, Spain and Portugal

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	-.61 -.90r -.17																					OECD, Mar. 1986, p. 19 Tyers and Anderson, Jan. 1986 Barcelo, 1985	
PK																						OECD Rojko, et al Tyers and Anderson Barcelo	
PM																						OECD Rojko, et al Barcelo	
ML																						OECD	
PE																						OECD	
DM																						OECD Tyers and Anderson Rojko, et al Barcelo	
DB																						OECD Rojko, et al	
DC																						OECD Rojko, et al	
DO																						OECD	
WH																						OECD Tyers and Anderson Rojko, et al Barcelo	
23																							
CN																							
CG																						OECD Tyers and Anderson Rojko, et al	
RI																						OECD Tyers and Anderson Rojko, et al Barcelo	
SB																						OECD, p. 19 Huyser, 1983, p. 178	
SM																						Huyser	
SO																							
OS																						Gardiner Barcelo	
OM																							
OO																							
CT																						Gardiner, Sep. 1983 Barcelo	
SU																						OECD Tyers and Anderson Barcelo	
TB																							

d = dairy products, n = nonruminant meat, r = ruminant meat.

Appendix table 10--Price elasticities of supply for selected commodities, Other Western Europe

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	.63 .69r .40	-.21 .48n -.15				.36 .12d .15					-.97f .30			-.10fm .10								OECD, Mar. 1986, pp. 12, 18 * Tyers and Anderson, Jan. 1986 Rojko, et al	
PK		.45 .99n .20									-.30	-.17f .75										OECD Tyers and Anderson Rojko, et al	
ML			.15									-.09f											OECD
PM	-.20	-.20				.55 .60						-.19f											OECD Rojko, et al
PE																							
DM						.28 .30						-.35	-.09f										OECD Rojko, et al
DB							.26	-.39	-.05														OECD
DC								-.92	1.62	-.05													OECD
DO									-.92	-.39	1.94												OECD
WH											.76 .90 .50		-.38 .84 .25									OECD Tyers and Anderson Rojko, et al	
CN																							
CG												-.11 .14 .19	.34 .91 .49									OECD Tyers and Anderson Rojko, et al	
RI																							
SB																							
OS																							OECD
CT																							Gardiner, Oct. 1983
SU																							OECD Tyers and Anderson World Bank, 1981, p.III-3
TB																							

* = (.1 * p. 12 + .9 * p. 18), d = dairy products, f = feed costs, fm = feed costs-milk, fp = feed costs-poultry, n = nonruminant meat, r = ruminant meat.

Appendix table 11--Price elasticities of demand for selected commodities, Other Western Europe

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	-.81 -.70r	.03 .32n																				OECD, Apr. 1986 *	
PK	.02 .20	-.50 .70								.20												Tyers and Anderson, Jan. 1986	
ML	.01	.01	-.47	.01																		OECD	
PM	.10	.20		-.41 .80																		OECD	
PE																						OECD	
DM																						OECD	
DB																						Tyers and Anderson	
DC																						GOL	
DO																						OECD	
WH																						OECD	
25 CN																						Tyers and Anderson	
CG																						GOL, p. 100	
RI																						OECD	
SB																						Tyers and Anderson	
SM		1.00 .32																				GOL	
SO																						Gardiner, May 1983	
OS																						Gardiner, June 1983	
OM																						OECD	
OO																						Gardiner	
CT																						Gardiner	
SU																						OECD	
TB																						Tyers and Anderson	

* = (.10*p. 11 + .9*p. 17), n = nonruminant, r = ruminant.

Appendix table 12--Price elasticities of supply for selected commodities, Japan

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	.23 .80r .50	-.10n -.10		.20	-.10		.68 .04d					-.44f .16										OECD, Mar. 1986, p.16 Tyers and Anderson, Jan. 1986 Rojko, et al	
PK		1.50 -.06r .70										-.44f .03										OECD Tyers and Anderson Rojko, et al	
ML																							
PM																						OECD Rojko, et al	
PE																							
DM	.05 -.09r																					OECD Tyers and Anderson Rojko, et al	
DB																						OECD	
DC																						OECD	
DO																						OECD	
WH																						OECD Tyers and Anderson Rojko, et al	
CN																							
CG																						OECD Tyers and Anderson Rojko, et al	
RI																						OECD Tyers and Anderson Rojko, et al	
SB																						OECD Rojko, et al	
OS																						Gardiner, Oct. 1983 Griffith and Meilke, 1982	
CT																						Gardiner	
SU																						OECD Tyers and Anderson	
TB																							

d = dairy products, f = feed costs, n = nonruminant meat, r = ruminant meat.

Appendix table 13--Price elasticities of demand for selected commodities, Japan

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	-1.00 -1.20r	.12 .40n																					OECD, Mar. 1986, p.15 Tyers and Anderson, Jan. 1986
PK	.50 .20	-1.00 -.90																					OECD Rojko, et al
ML																							OECD
PM	.04 .50	-.01 .17																					OECD Rojko, et al
PE																							OECD
DM																							OECD Tyers and Anderson Rojko, et al
DB																							OECD Rojko, et al
DC																							OECD Rojko, et al
DO																							OECD
WH																							OECD Tyers and Anderson Rojko, et al
CN																							OECD
CG																							OECD Tyers and Anderson Rojko, et al
27																							OECD Tyers and Anderson Rojko, et al
RI																							OECD Tyers and Anderson Rojko, et al
SB																							OECD Huysen, p. 184 Griffith and Meilke, 1982, p.29
SM	1.20 .08																						Rojko, et al Gardiner, July 1983 Huysen Griffith and Meilke, p.43
SO																							Gardiner, June 1983 Griffith and Meilke
OS																							Gardiner, June 1983 Griffith and Meilke
OM																							Gardiner, June 1983 Griffith and Meilke
OO																							Gardiner, June 1983 Griffith and Meilke
CT																							OECD
SU	.15	.45																					OECD Tyers and Anderson
TB																							

n = nonruminant meat, r = ruminant meat.

Appendix table 14--Price elasticities of supply for selected commodities. Australia

d = dairy products, f = feed costs, n = nonruminant meat, r = ruminant meat.

Appendix table 15--Price elasticities of demand for selected commodities, Australia

	Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
	BF	-.92 -.63r	-.01 .33n		.27	-.01																		OECD, Mar. 1986, p.5 Tyers and Anderson, Jan. 1986
	PK	.43 .20r	-1.25 -.40n	.24	.17																			OECD Rojko, et al. Tyers and Anderson
	ML	.78	.24	-1.43	.31																			OECD
	PM	.15	.05	.08	-.76																			OECD
	PE																							
	DM																							OECD Tyers and Anderson Rojko, et al.
	DB																							OECD Rojko, et al.
	DC																							OECD Rojko, et al.
	DO																							OECD
	WH																							OECD Tyers and Anderson Rojko, et al.
29	CN																							
	CG																							
	RI																							
	SB																							
	SM																							
	SO																							
	OS																							
	OM	.74	.97																					
	OO																							
	CT																							
	SU																							
	TB																							

d = dairy products, n = nonruminant meat, r = ruminant meat.

Appendix table 16--Price elasticities of supply for selected commodities, New Zealand

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	1.05 .36r .40	.35 -.04n -.10										-.27f -.02		.09fm								OECD, Mar. 1986, p.14 Tyers and Anderson, Jan. 1986 Rojko, et al	
PK		1.00 -.10r -.10											-.48f -.06									OECD Tyers and Anderson Rojko, et al	
ML	-.03		.21							-.15				-.09f								OECD	
PM				2.00										-.86f								OECD	
PE																							
DM	.04 -.18r					.60 :.40 .20d								-.09f -.20								OECD Rojko, et al Tyers and Anderson	
DB							.34															OECD	
DC								-.84						1.66								OECD Rojko, et al	
DO									-.84						1.77							OECD	
WH																						OECD Tyers and Anderson Rojko, et al	
30 CN																							
CG																						OECD Tyers & Anderson Rojko, et al	
RI																							
SB																							
OS																							
CT																							
SU																							
TB																							

d = dairy products, f = feed costs, fm = feed costs-mutton, n = nonruminant meat, r = ruminant meat.

Appendix table 17--Price elasticities of demand for selected commodities, New Zealand

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	-.56 -.63r	.20 .05n	.26																				OECD, Mar. 1986, p.13 Tyers and Anderson, Jan. 1986
PK	.36 .20	-.55 .40	.06																				OECD Rojko, et al
ML	.29		-.54																				OECD
PM			-.50																				OECD
PE																							
DM																							OECD Tyers and Anderson Rojko, et al
DB																							OECD Rojko, et al
DC																							OECD Rojko, et al
DO																							OECD
WH																							OECD Tyers and Anderson Rojko, et al
31																							
CN																							
CG																							
RI																							
SB																							
SM																							
SO																							
OS																							
OM																							
OO																							
CT																							
SU																							
TB																							

d = dairy products, n = nonruminant meat, r = ruminant meat.

Appendix table 18--Price elasticities of supply for selected commodities, South Africa

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source
BF	.72r	-.08n																					Tyers and Anderson, Jan. 1986
PK	-.15r	.90n																					Tyers and Anderson
ML			.20																				OECD, Mar. 1986, p. 22 Gardiner, Sept. 1983
PM				-.47	.47																		Gardiner
PE						.50																	Gardiner Gardiner
DM							.80d																Tyers and Anderson
DB								.33															Tyers and Anderson
DC									1.15														Tyers and Anderson
DO									1.30														OECD
WH										.60													Tyers and Anderson Rojko, et al
CN											.20												Gardiner, Sept. 1983
CG												.60											Tyers and Anderson Rojko, et al
RI													.17										Tyers and Anderson
SB																							Rojko, et al
OS																							
CT																							Gardiner, Oct. 1983
SU																							Tyers and Anderson World Bank, 1981, p.III-3
TB																							

d = dairy products, n = nonruminant meat, r = ruminant meat.

Appendix table 19--Price elasticities of demand for selected commodities, South Africa

Code	BF	PK	ML	PM	PE	DM	DB	DC	DO	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Source	
BF	-1.16r	.20n																					Tyers and Anderson, Jan. 1986, p.	
PK	.40r	-1.2n																					Tyers and Anderson	
ML			-.37	.37																			Gardiner, Sept. 1983	
PM				.56	-.56																		Gardiner	
PE					-.34																		Gardiner	
DM						-.80d																	Tyers and Anderson	
DB							-.50																OECD, Mar. 1986, p.22	
DC								-.50															OECD	
DO									-.55														OECD	
WH										-.15	.05												Tyers and Anderson Rojko, et al	
CN																								
CG											.05	-.30											Tyers and Anderson Rojko, et al	
RI												.05	.10	-.50									Tyers and Anderson Rojko, et al	
33																								OECD
SB																								Gardiner, May 1983
SM																								Gardiner, June 1983
SO																								
OS																								
OM																								
OO																								
CT																								
SU																								
TB																								

d = dairy products, n = nonruminant meat, r = ruminant meat.

APPENDIX B

Supply and Demand Elasticities Spreadsheets in TLIB Database

Appendix table 20--Supply and demand elasticities, United States

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.65	.01									.09	.02											.54	
PK	.02	1.00									.25	.05											.59	
ML			.80	.01							.38	.08											.34	
PM				.02							.11	.02											.37	
PE											.08	.01											.40	
DM	.03										.06	.01											.44	
DB																							.05	
DC																							.05	
DP																							.05	
WH																							.31	
CN																							.31	
CG																							.33	
RI																							.40	
SB																							.36	
SM																							.36	
SO																							.05	
OS																							.05	
OM																							.12	
OO																							.41	
CT																							.19	
SU																							.25	
TB																							.19	
Demand:																								.62
BF	.70	.05																					.74	
PK	.08	-.86																					.33	
ML		.37	-.70																				.42	
PM	.09	.06																					.35	
PE																							.02	
DM																							.63	
DB																							.60	
DC																							.65	
DP																							.10	
WH																							.09	
CN																							.10	
CG																							.05	
RI																							.17	
SB																							.22	
SM																							.09	
SO																							.28	
OS																							.36	
OM																							.20	
OO																							.08	
CT																							.24	
SU																							.20	
TB																							.20	

Appendix table 21--Supply and demand elasticities, Canada

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	.50	-.03																					
PK	-.03	1.50																					
ML			.50																				
PM			-.16																				
PE				.70																			
DM					.50																		
DB						.45																	
DC							.17																
DP								.34															
WH									.46														
CN										.34													
CG											.35												
RI												.75											
SB													.10										
SM														.35									
SO															.30								
OS																.12							
OM																		.85					
OO																			.97				
CT																				.30			
SU																					.10		
TB																						.30	
																							.20
Demand:																							
BF	-.80	.06																					
PK	.08	-.86																					
ML			.19	-1.00																			
PM	.09	.07			-.67																		
PE						.30																	
DM							.18																
DB								.02															
DC									.13														
DP									.02														
WH										.50													
CN											.20												
CG												.02											
RI													.06										
SB														.21									
SM															.09								
SO																.22							
OS																	.25						
OM																		.40					
OO																			.25				
CT																				.40			
SU																					.20		
TB																							.20

Appendix table 22--Supply and demand elasticities, European Community

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																							.60	
BF	-.55	-.07																					.71	
PK	-.05	.90																					.52	
ML			.70	-.01																			.50	
PM					.04	-.08																	.56	
PE							.80																.66	
DM								.02	.75														.05	
DB										.65													.05	
DC											.11	.23	-.30	.23									.05	
DP												.23	-.12	.53	-.12								.05	
WH													.11	.22	-.29	.22								.05
CN														.50	-.05	.16								.24
CG															.21	-.60	.15							.24
RI																.31	-.07	.55						.35
SB																		.35						.30
SM																			.40					.05
SO																				.37	.30	.12		.05
OS																								.05
OM																								.05
OO																								.14
CT																								.11
SU																								.15
TB																								.20
Demand:																								.48
BF	-.70	.20	.02																					.63
PK	.12	-.80	.02																					.51
ML	.04	.25	-.90	.10																			.64	
PM	.06	.15	.04	-.90																			.20	
PE																								.02
DM																								.42
DB																								.38
DC																								.34
DP																								.13
WH																								.08
CN																								.12
CG																								.23
RI																								.05
SB																								.14
SM																								.19
SO																								.05
OS																								.26
OM																								.50
OO																								.52
CT																								.50
SU																								.50
TB																								.50

Appendix table 23--Supply and demand elasticities, Spain

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum		
Supply:																									
BF	.65	-.09																							
PK	-.03	.81																						.59	
ML			.65		-.03																			.55	
PM				.10																				.46	
PE					.70	-.04																		.23	
DM						-.02	.70																	.57	
DB							.60																		.60
DC								.15	.45	-.70	.45													.05	
DP									.27	-.14	.60	-.14												.05	
WH										.13	.40	-.62	.40											.05	
CN											.80		-.05	.38										.37	
CG												.08	.70	-.30										.32	
RI													.28	-.13	.70									.28	
SB														.48											.48
SM															.60										.30
SO																.09	.10	.04	-.30					.05	
OS																	.09	.10	.04					.05	
OM																			.45					.45	
OO																				.10	.02	.13		.05	
CT																								.50	
SU																								.50	
TB																								.20	
Demand:																									
BF	-.75	.30	.02	.10																					
PK	.10	-.60	.01	.10																					.33
ML	.07	.10	-.70	.10																					.39
PM	.11	.34	.03	-.80																					.43
PE																									.32
DM																									.20
DB																									.08
DC																									.30
DP																									.40
WH																									.20
CN																									.13
CG																									.10
RI																									.14
SB																									.18
SM																									.05
SO																									.28
OS																									.15
OM																									.05
OO																									.18
CT																									.59
SU																									.48
TB																									.21
																									.20

Appendix table 24--Supply and demand elasticities, Portugal

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.60	.10									.02												.57	
PK	-.05	.80									-.08	.02											.52	
ML			.70								-.05	.01											.53	
PM			-.13								-.01	.08	.02										.30	
PE											-.05	.01											.50	
DM												.03												.60
DB																								.05
DC																								.05
DP																								.37
WH																								.28
CN																								.20
CG																								.48
RI																								.60
SB																								.05
SM																								.05
SO																								.45
OS																								.05
OM																								.50
OO																								.50
CT																								.40
SU																								.20
TB																								.20
Demand:																								.40
BF	-.75	.20	.05	.10																				.38
PK	.10	-.60	.02	.10																				.17
ML	.23	.19	-.70	.10																				.29
PM	.14	.28	.03	-.73																				.20
PE																								.05
DM																								.35
DB																								.40
DC																								.20
DP																								.17
WH																								.11
CN																								.17
CG																								.26
RI																								.05
SB																								.29
SM																								.20
SO																								.05
OS																								.18
OM																								.53
OO																								.58
CT																								.30
SU																								.50
TB																								.50

Appendix table 25--Supply and demand elasticities, Other Western Europe

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	.57	.07																					
PK	-.04	.80																					.62
ML			-.80	-.04																			.61
PM			-.01		.75	-.02																	.32
PE							.75																.46
DM								.06															.60
DB									.60														.61
DC										.22	.26	-.25	.26										.05
DP											.30	-.10	.55	-.10									.05
WH											.36	.40	-.38	.40									.05
CN																							.05
CG																							.35
RI																							.25
SB																							.23
SM																							.20
SO																							.45
OS																							.05
OM																							.30
OO																							.05
CT																							.05
SU																							.20
TB																							.45
Demand:																							
BF	-.70	.18																					.52
PK	.08	-.60																					.50
ML			-.47	.01																			.46
PM			.22	-.65																			.43
PE						.35																	.35
DM							.16	.02	.09	.03													.02
DB								.45															.45
DC									.48														.48
DP										.40													.40
WH											.35	.02	.15	.01									.17
CN												.05	-.75	.40									.25
CG													.07	.08	-.45								.28
RI														.08									.36
SB																							.05
SM																							.44
SO																							.25
OS																							.05
OM																							.45
OO																							.45
CT																							.45
SU																							.20
TB																							.50

Appendix table 26--Supply and demand elasticities, Japan

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.40	-.10				-.06				.27				-.05	-.02				-.01				.43	
PK	-.03	.83				-.06								-.07	-.03				-.04				.58	
ML			.45																				.45	
PM	-.05	-.14				1.27	-.03								-.16	-.08			-.08				.71	
PE															-.06	-.03			-.03				.66	
DM	.15														-.01				-.01				.52	
DB																							.05	
DC																							.05	
DP																							.34	
WH																							.30	
CN																							.15	
CG																							.49	
RI																							.42	
SB																							.05	
SM																							.05	
SO																							.84	
OS																							.05	
OM																							.05	
OO																								
CT																								
SU																								
TB																								
Demand:																								.64
BF	-1.00	.26																						
PK	.10	-.95																						
ML			-.35																					.35
PM	.10	.20																						.80
PE																								.30
DM																								.09
DB																								.54
DC																								.68
DP																								.63
WH																								.17
CN																								.25
CG																								.07
RI																								.19
SB																								.06
SM																								.15
SO																								.08
OS																								.05
OM																								.20
OO																								.19
CT																								.30
SU																								.54
TB																								.50

Appendix table 27--Supply and demand elasticities, Australia

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	.70	-.03	-.10	-.01																			
PK	-.15	.80																					.50
ML	.26		.70																				.53
PM	-.08			.80	-.02																		.37
PE					-.02	.60																	.64
DM	-.09					.50																	.49
DB							.30	.65	.95	.65												.37	
DC							.33	-.44	1.26	-.44												.05	
DP								.24	.54	-.79	.54												.05
WH										-.10													.05
CN											.90												.05
CG												.04											.60
RI												.71											.44
SB													.11										.11
SM														.60									.49
SO															.38								.20
OS																.38							.05
OM																	.13						.05
OO																		.60					.12
CT																			.70				.05
SU																				.25			.05
TB																					.50		.50
Demand:																							
BF	-.78	-.10	.15																				
PK	-.22	-1.02	.18	.15																			.53
ML	.28	.15	-1.20	.17																			.47
PM		.24	.32	-.80																			.59
PE					.25																		.25
DM						.17		.04	.09	.04													.25
DB							.45																
DC								.40															.45
DP									.45														.40
WH										.24													.45
CN											.35												.20
CG												.01											.20
RI													.36										.34
SB														.45									.45
SM															.42								.05
SO																.60							.25
OS																	.93						.23
OM																		.53					.06
OO																			.16				.27
CT																			.44				.58
SU																				.65			.20
TB																							.25
																							.50

Appendix table 28--Supply and demand elasticities, New Zealand

Appendix table 29--Supply and demand elasticities, South Africa

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	.50																						
PK		.75																					
ML			.50																				
PM				-.16																			
PE					-.04																		
DM						.09																	
DB							.45																
DC								.24															
DP									.27														
WH										.21													
CN											.45												
CG												.26											
RI													.04										
SB														.16									
SM															.10								
SO																.50							
OS																	.13		.33				
OM																		.35					
OO																			.20				
CT																				.37			
SU																					.30		
TB																						.49	
Demand:																							
BF																							
PK																							
ML																							
PM																							
PE																							
DM																							
DB																							
DC																							
DP																							
WH																							
CN																							
CG																							
RI																							
SB																							
SM																							
SO																							
OS																							
OM																							
OO																							
CT																							
SU																							
TB																							

Appendix table 30--Supply and demand elasticities, Eastern Europe

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum		
Supply:																									
BF	.30	-.05			-.03		.10				.02	.01											.28		
PK	-.02	.45									-.03	-.04	-.03										.34		
ML			.35								-.03	-.04	-.03										.24		
PM	-.06			.70							-.03	-.11	-.09										.32		
PE					.35						-.02	-.03	-.02										.27		
DM	.04					.30						-.01												.30	
DB							.19	.25	.26	.25													.05		
DC							-.13	-.24	.65	-.24													.05		
DP							-.06	.12	-.12	.12													.17		
WH											.25	-.02	-.05											.15	
CN											-.03	-.25	-.07											.16	
CG											-.08	-.08	.35											.23	
RI												-.07		.30											.22
SB													.18		.45										.05
SM														.36	.30	.11									.05
SO														-.03	-.03										.23
OS																									.05
OM																									.05
OO																									.15
CT																									.20
SU																									.20
TB																									.20
Demand:																									
BF	-.20	.06																							.14
PK	-.02	-.50																							.46
ML			-.28	.02																				.28	
PM	.15		-.25																					.10	
PE					.10																			.10	
DM																									.04
DB																									.15
DC																									.16
DP																									.40
WH																									.21
CN																									.16
CG																									.13
RI																									.15
SB																									.05
SM																									.25
SO																									.30
OS																									.05
OM																									.18
OO																									.13
CT																									.15
SU																									.30
TB																									.35

Appendix table 31--Supply and demand elasticities, Soviet Union

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.25	-.03			-.01																			
PK	-.12	.40																						
ML			.30																					
PM	-.07			.50	-.01																			
PE						.25																		
DM	.08						.20																	
DB								.03	-.05	-.02	-.05													
DC									.50	-.12	.80	-.12												
DP										-1.00	.66	-.26	.66											
WH											.23	-.02	-.04											
CN												.10	.38	-.05	-.05									
CG													.03	.23										
RI														.08	-.16									
SB															.45									
SM																.16								
SO																	.20	.18	.07					
OS																		.20	.18	.07				
OM																			.15					
OO																				.52	.18	.39		
CT																					.52	.18	.39	
SU																					.15			
TB																						.16		
Demand:																								
BF	-.19	.02				.02																		
PK	.05	-.18																						
ML			.15																					
PM	.09			.25																				
PE					.15																			
DM						.09																		
DB							.15																	
DC								.15																
DP									.15															
WH										.25	-.03	.03												
CN											.12	-.35	.05											
CG											.04	.01	-.25											
RI																								
SB																								
SM																								
SO																								
OS																								
OM																								
OO																								
CT																								
SU																								
TB																								

Appendix table 32--Supply and demand elasticities, China

Code	BF	PK	ML	PM	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																						
BF	.21	-.04	-.01																			.25
PK		.50																				.45
ML		-.03	.25																			.18
PM		-.07		.49																		.29
DM	.14																					.41
DB																						.05
DC																						.05
DP																						.05
WH																						.10
CN																						.14
CG																						.14
RI																						.09
SB																						.05
SM																						.05
SO																						.08
OS																						.05
OM																						.05
OO																						.10
CT																						.15
SU																						.15
TB																						.15
Demand:																						
BF	-.80	.35		.10																		.35
PK		-.40																				.39
ML			-.30																			.30
PM		.07	.12	-.60																		.41
DM																						.05
DB																						.30
DC																						.35
DP																						.30
WH																						.06
CN																						.11
CG																						.16
RI																						.11
SB																						.11
SM																						.11
SO																						.11
OS																						.16
OM																						.18
OO																						.24
CT																						.10
SU																						.46
TB																						.05

Appendix table 33--Supply and demand elasticities, Mexico

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.30	.05																						
PK	-.06	.55																						
ML			.44																					
PM				.70																				
PE					.70																			
DM						.05																		
DB							.35																	
DC								.04																
DP									.05															
WH										.01														
CN											.01													
CG												.02												
RI													.03											
SB														.05										
SM														.05										
SO															.05									
OS																.05								
OM																	.05							
OO																		.05						
CT																			.05					
SU																				.05				
TB																					.05			
Demand:																								
BF	-1.16	.42	.01	.07																				
PK	.49	-1.00		.04																				
ML	.16		-.60																					
PM	.27	.13		-1.10																				
PE																								
DM																								
DB																								
DC																								
DP																								
WH																								
CN																								
CG																								
RI																								
SB																								
SM																								
SO																								
OS																								
OM																								
OO																								
CT																								
SU																								
TB																								

Appendix table 34--Supply and demand elasticities, Central America & Caribbean

Code	BF	PK	ML	PM	PE	DM	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																				
BF	.40	-.04			-.01				.02			.01							.32	
PK	-.16	.60							-.02			-.08							.27	
ML			.40						-.13			-.07							.15	
PM	-.04			.50	-.01				-.03			-.12							.19	
PE					-.02	.50			-.06			-.04							.33	
DM	.04					.35			-.04			-.02							.30	
WH							.47		-.05			-.05							.12	
CN								.22	-.01			-.04							.12	
CG								-.04	.28			-.05							.17	
RI									-.03			.58							.44	
SB										.45									.45	
SM											.09								.05	
SO											-.09								.05	
OS												.10							.44	
OM													.04						.05	
OO														.45					.45	
CT															.42				.05	
SU															.08				.05	
TB															.39				.05	
Demand:																				
BF	-.80	.20																	.50	
PK	-.63	-1.10																	.36	
ML			-.50																.50	
PM	.36	.11																	.43	
PE		.03																	.47	
DM																			.18	
WH																			.15	
CN																			.09	
CG																			.26	
RI																			.52	
SB																			.05	
SM																			.44	
SO																			.70	
OS																			.05	
OM																			.38	
OO																			.93	
CT																			.50	
SU																			.30	
TB																			.20	

Appendix table 35--Supply and demand elasticities, Brazil

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.50	-.02																						
PK	-.04	.60																						
ML			.35																					
PM				.65																				
PE					.50																			
DM						.43																		
DB							.35																	
DC								.40																
DP									.35															
WH										.38														
CN											.06													
CG												.02												
RI													.18											
SB														.65										
SM															.02									
SO																.06								
OS																								
OM																								
OO																								
CT																								
SU																								
TB																								
Demand:																								
BF	-.70	.17																						
PK	.37	-.95																						
ML			-.60																					
PM	.50	.23																						
PE																								
DM																								
DB																								
DC																								
DP																								
WH																								
CN																								
CG																								
RI																								
SB																								
SM																								
SO																								
OS																								
OM																								
OO																								
CT																								
SU																								
TB																								

Appendix table 36--Supply and demand elasticities, Argentina

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																							.46	
BF	.50																						.59	
PK		.70																					.39	
ML			.42																				.34	
PM				.45	-.01																		.38	
PE						.45																	.05	
DM	-.10						.55																.05	
DB							-.25	.35	-.40	.35													.05	
DC							-.28	-.04	.40	-.04													.05	
DP								.78	.96	-1.10	.96												.45	
WH												.60	-.10	-.05										.36
CN												-.13	.66	-.07										.34
CG												-.17	-.19	.80	-.01									.49
RI												-.09	-.07	-.05	.80									.43
SB												.03	-.03	-.01										.05
SM																							.05	
SO																							.05	
OS																							.05	
OM																							.23	
OO																							.46	
CT																							.30	
SU																							.30	
TB																								
Demand:																								.56
BF	-.59		.01	.02																				.49
PK		-.69		.20																				.45
ML	.35		-.80																					.35
PM	.26	.24		-.85																				.50
PE																								.02
DM																								.30
DB																								.66
DC																								.85
DP																								.17
WH																								.19
CN																								.13
CG																								.20
RI																								.05
SB																								.75
SM																								.63
SO																								.05
OS																								.18
OM																								.87
OO																								.20
CT																								.52
SU																								.20
TB																								

Appendix table 37--Supply and demand elasticities, Venezuela

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	.55	.01																					
PK	-.02	.55																					
ML			.50																				
PM				.50	.01																		
PE					-.01	.45																	
DM						.04																	
DB							.40																
DC								.20															
DP									.50														
WH										.75													
CN											.50												
CG												.10											
RI													.05										
SB														.04									
SM															.02								
SO																.05							
OS																	.05						
OM																		.42					
OO																			.70				
CT																				.20			
SU																					.55		
TB																							.50
Demand:																							
BF	-.88	.20																					
PK	.40	-1.10																					
ML	.26		.60																				
PM	.34	.25																					
PE																							
DM																							
DB																							
DC																							
DP																							
WH																							
CN																							
CG																							
RI																							
SB																							
SM																							
SO																							
OS																							
OM																							
OO																							
CT																							
SU																							
TB																							

Appendix table 38--Supply and demand elasticities, Other Latin America

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum	
Supply:																								
BF	.45																							.45
PK		.50																						.45
ML			.55																					.52
PM				.55																				.48
PE					.01																			.45
DM	.03					.50																		.45
DB							.45																	.05
DC								.03																.05
DP									.05															.05
WH										.47														.13
CN											.32													.14
CG												.32												.14
RI													.38											.21
SB														.10										.40
SM															.05									.35
SO																.05								.37
OS																	.05							.37
OM																		.05						.30
OO																			.25					.30
CT																				.30				.26
SU																					.30			.20
TB																								.20
Demand:																								.56
BF	-.62		.03			.03																		.55
PK	.15		-.90			.20																		.45
ML				-.45																				.54
PM	.13		.18			-.85																		.50
PE							.50																	.07
DM								.10																.10
DB									.80															.58
DC										.02														.81
DP										.07														.14
WH											.88													.15
CN												.30												.17
CG													.10											.24
RI														.13										.05
SB															.24									.69
SM																.28								.58
SO																	.07							.05
OS																		.39						.39
OM																			.29					.77
OO																				.20				.20
CT																					.20			.20
SU																								.20
TB																								.20

Appendix table 39--Supply and demand elasticities, Nigeria

Demand:

54

WH	.93	.02	.10	.02	.79
CN	.01	-.30	.01		.28
CG	.02		.20	.01	.17
RI	.01		.03	-.51	.46
SB				-.13	.09
SM				-.20	-.18
SO				-.20	-.15
OS					.05
OM					.18
OO					.20
CT					.20
SU					.10
TB					.20

Appendix table 40--Supply and demand elasticities, Other Sub-Saharan Africa

Demand:

Appendix table 41--Supply and demand elasticities, Egypt

Code	BF	ML	PM	PE	DM	DB	DC	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																					
BF	.35																				.31
ML		.35																			.22
PM	-.10		.80	-.02																	.13
PE			-.01	.35																	.12
DM					.35																.28
DB						.35															.05
DC							.05														.05
WH								.30													.26
CN									.20												.18
CG										.02											.23
RI										.07	.11										.18
SB											.45										.16
SM												.02									.05
SO													.05								.05
OS													.12								.19
OM														.20							.05
OO															.59						.05
CT																.30					.15
SU																	.34				.09
TB																		.18			.10
Demand:																					
BF	-.40		.02	.05																	-.33
ML		.17			.60	.30															-.13
PM			.25			.18	.60														-.17
PE								.50													-.50
DM									.10												.04
DB										.30											.30
DC											.06										.30
WH												.20									.17
CN													.40								.37
CG														.01							.28
RI														.02							.25
SB															.05						.05
SM																.42					.18
SO																	.26				.20
OS																		.11			.07
OM																			.05		.07
OO																			.14		.05
CT																			.20		.20
SU																				.40	.40
TB																				.10	.10

Appendix table 42--Supply and demand elasticities, Middle East & North Africa Oil Producers

Code	BF	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																						
BF	.21																					.26
ML		.50																				.31
PM			.60																			.24
PE				.50																		.28
DM	.06				.40																	.31
DB					-.04	.07	-.05	.07														.05
DC					-.25	-.15	.60	-.15														.05
DP					-.70	.58	-.42	.58														.30
WH									.30													.30
CN										.50												.24
CG											.25											.15
RI												.15										.20
SB													.20									.05
SM														.09								.05
SO															.10							.15
OS																.04						.05
OM																	.15					.05
OO																		.42				.45
CT																			.07			.40
SU																				.45		.10
TB																						.10
Demand:																						.28
BF																						.22
ML																						.35
PM																						.30
PE																						.04
DM																						.30
DB																						.30
DC																						.30
DP																						.10
WH																						.23
CN																						.22
CG																						.20
RI																						.05
SB																						.23
SM																						.06
SO																						.05
OS																						.20
OM																						.13
OO																						.30
CT																						.10
SU																						.10
TB																						.10

Appendix table 43--Supply and demand elasticities, Middle East & North Africa - Others

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	.21																						
PK		.20																					
ML			.50																				
PM				.50																			
PE					.40																		
DM						.60																	
DB							.25																
DC								.40															
DP									.25														
WH										.60													
CN											.25												
CG												.50											
RI													.30										
SB														.20									
SM															.30								
SO																.13							
OS																	.05						
OM																		.15					
OO																			.75				
CT																				.75			
SU																					.62		
TB																							
Demand:																							
BF	-.28																						
PK		-.20																					
ML			.02																				
PM				.02																			
PE					.04																		
DM						.30																	
DB							.09																
DC								.20															
DP									.20														
WH										.02													
CN											.11												
CG												.40											
RI													.10										
SB														.03									
SM															.05								
SO																.01							
OS																	.03						
OM																		.15					
OO																			.75				
CT																				.18			
SU																					.62		
TB																							

Appendix table 44--Supply and demand elasticities, India

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							20
BF	.20																						.10
PK		.30																					.33
ML			.35																				.33
PM				.40																			.35
PE					.40																		.29
DM						.30																	.05
DB							.03																.05
DC								.34															.05
DP									.69														.05
WH										.38													.05
CN											.45												.22
CG												.09											.21
RI													.60										.21
SB														.10									.21
SM															.05								.21
SO																.15							.21
OS																	.35						.21
OM																		.38					.21
OO																			.15				.21
CT																				.28			.21
SU																					.68		.21
TB																						.50	.24
																							.19
Demand:																							.66
BF	-.20																						.66
PK		-.20																					.60
ML			-.50																				.75
PM				-.30																			.55
PE																							.84
DM																							.20
DB																							.49
DC																							.50
DP																							.48
WH																							.44
CN																							.74
CG																							.50
RI																							.58
SB																							.05
SM																							.13
SO																							.23
OS																							.06
OM																							.18
OO																							.36
CT																							.65
SU																							.60
TB																							.80

Appendix table 45--Supply and demand elasticities, Other South Asia

Code	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:														
WH	.40	.01	-.01	-.03										.28
CN	-.09	.46		-.05										.28
CG	-.22		.50	-.04										.20
RI	-.01			.40										.18
SB					.20									.34
SM						.53	.30	.28						.19
SO							.53	.30	.28					.05
OS								.40						.05
OM		-.10	-.02											.17
OO				-.09										.05
CT									.63	.30	.38	-.02		.05
SU										.63	.30	.38		.38
TB											.50			.29
												.45		.34
Demand:														
WH	-.35	.03	.01	.20										.15
CN	.31	-.60		.20										.24
CG	.26		-.60	.20										.28
RI	.11	.01		-.50										.41
SB					.65	.31	.29							.05
SM														.07
SO														.97
OS		-.25	-.12	-.05	-.46									.07
OM														.10
OO														.59
CT														.54
SU														.53
TB														.64

Appendix table 46--Supply and demand elasticities, Indonesia

Code	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:														
WH														.20
CN														.21
CG														.29
RI														.23
SB														.05
SM														.05
SO														.37
OS														.05
OM														.05
OO														.25
CT														.45
SU														.45
TB														.45
Demand:														
WH														.20
CN														.28
CG														.30
RI														.49
SB														.77
SM														.90
SO														.05
OS														.51
OM														.90
OO														.70
CT														.33
SU														.50
TB														.50

Appendix table 47--Supply and demand elasticities, Thailand

Appendix table 48--Supply and demand elasticities, Malaysia

Code	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:														
WH		.10												.10
CN			.25											.15
CG				.25										.25
RI					.50									.49
SB						.35								.20
SM							.09							.05
SO								.10						.05
OS									.04					.05
OM										.15				.35
OO											.35			.05
CT												.04		.05
SU													.30	.30
TB														.49
Demand:														
WH														.13
CN														.11
CG														.30
RI														.26
SB														.06
SM														.69
SO														.39
OS														.05
OM														.63
OO														.60
CT														.35
SU														.64
TB														.40

Appendix table 49--Supply and demand elasticities, Philippines

Appendix table 50--Supply and demand elasticities, Other Southeast Asia

Code	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:														
WH														.20
CN														.23
CG														.38
RI														.27
SB														.20
SM														.05
SO														.05
OS														.21
OM														.05
OO														.25
CT														.50
SU														.40
TB														.40
Demand:														
WH														.15
CN														.11
CG														.45
RI														.29
SB														.27
SM														.53
SO														.35
OS														.05
OM														.48
OO														.44
CT														.50
SU														.50
TB														.59

Appendix table 51--Supply and demand elasticities, South Korea

Appendix table 52--Supply and demand elasticities, Taiwan

Code	BF	PK	ML	PM	PE	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																			
BF	.40	-.10																.30	
PK	.65																	.54	
ML		.50																.50	
PM			.50															.31	
PE				.50														.38	
WH																			
CN																			
CG																			
RI																			
SB																			
SM																			
SO																			
OS																			
OM																			
OO																			
CT																			
SU																			
TB																			
Demand:																			
BF	-1.24	-.50																.74	
PK	.03	-.65																.54	
ML			-.59															.59	
PM			.34	-.60														.26	
PE					-.50													.50	
WH																			
CN																			
CG																			
RI																			
SB																			
SM																			
SO																			
OS																			
OM																			
OO																			
CT																			
SU																			
TB																			

Appendix table 53--Supply and demand elasticities, Other East Asia

Code	BF	PK	ML	PM	PE	WH	CN	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																		
BF	.30						.07	0										.23
PK		.35					.03	0										.30
ML			.30				.07	0										.23
PM				.40				.02	.06	0								.29
PE					.30				.06	0								.20
WH																		
CN							.50	.02		.05								.33
RI							.03	.50		.15								.32
SB									.30									.20
SM										.36	.30	.11						.15
SO										.36	.30	.11						.05
OS										.10								.05
OM																		.25
OO																		.05
CT																		.05
SU																		.25
TB																		.40
Demand:																		
BF	-.30																	-.30
PK		-.30																-.30
ML			-.30															-.30
PM				-.73														-.73
PE					-.30													-.30
WH																		
CN							-.27	.20		.40		.01						.66
RI							.20	.50				.05						.25
SB									.16		-.25							-.09
SM									.02	.08		-.27	.14	.05				.08
SO																		.50
OS																		.30
OM																		.13
OO																		.84
CT																		.68
SU																		.60
TB																		.60
																		.49
																		.49

Appendix table 54--Supply and demand elasticities, Other Asia

Demand:

69

Appendix table 55--Supply and demand elasticities, Rest of the World

Code	BF	PK	ML	PM	PE	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
Supply:																							
BF	-.40	-.02																					.42
PK	-.02	.40																					.38
ML			.40																				.38
PM				.30																			.40
PE					.40																		.20
DM						.50																	.32
DB							.06																.19
DC								.05															.05
DP									.05														.05
WH										.02													.05
CN											.30												.05
CG												.05											.16
RI													.01										.16
SB														.02									.16
SM															.02								.15
SO																.02							.15
OS																	.02						.15
OM																		.25					.05
OO																			.04				.05
CT																				.04			.05
SU																					.15		.15
TB																						.37	.20
Demand:																							
BF	-.70	.01																					.59
PK	.20	-.70																					.35
ML			-.45																				.35
PM	.32	.02		.28																			.19
PE																							.24
DM																							.05
DB																							.69
DC																							.37
DP																							.30
WH																							.23
CN																							.29
CG																							.35
RI																							.35
SB																							.35
SM																							.08
SO																							.80
OS																							.79
OM																							.05
OO																							.11
CT																							.66
SU																							.13
TB																							.13

APPENDIX C

Reciprocity Relationships from Maximization Models

Given a firm with n outputs and s inputs. The production function in implicit form can be written:

$$F(q_1, \dots, q_n, x_1, \dots, x_s) = 0 \quad (B.1)$$

where (B.1) is an increasing function of the q 's (the outputs) and a decreasing function of the x 's (the inputs). The profit function for a multi-output, multi-input firm can be written:

$$J = p_i q_i - r_k x_k \quad (B.2)$$

where

- p_i = the price of output i
- q_i = the quantity of output i
- r_k = the price of input k
- x_k = the quantity of input k

The objective of the firm is maximize profits subject to the technical constraints given by the production function:

$$\text{maximize } J = p_i q_i - r_k x_k + m F(q_1, \dots, x_s) \quad (B.3)$$

The first order conditions for profit maximization yield:

$$\frac{\underline{wJ}}{wq_i} = p_i + m F_i = 0 \quad i = 1, \dots, n \quad (B.4)$$

$$\frac{\underline{wJ}}{wx_k} = -r_k + m F_{s+k} = 0 \quad k = 1, \dots, s \quad (B.5)$$

$$\frac{\underline{wJ}}{wm} = F(q_1, \dots, x_s) = 0 \quad (B.6)$$

where $F_i (i = 1, \dots, n + s = m)$ is the partial derivative of (B.1) with respect to the i th argument.

The second order conditions yield the comparative-static relations known as the "reciprocity relations" or the substitution effects:

$$\frac{\underline{wq_i}}{wp_j} = \frac{\underline{wq_j}}{wp_i} \quad i, j = 1, \dots, n \quad (B.7)$$

$$\frac{\underline{wx_k}}{wr_j} = \frac{\underline{wx_j}}{wr_k} \quad k, j = 1, \dots, s \quad (B.8)$$

$$\frac{\underline{wq_i}}{wr_k} = \frac{-\underline{wx_k}}{wp_i} \quad i = 1, \dots, n \quad (B.9)$$

These partial derivatives indicate the rate of change of quantity with respect to a price change. The partial derivatives are symmetric and indicate that the substitution effects for the firm are symmetric. The reciprocity relations can be converted into elasticities to reveal the interdependencies of the cross-elasticities:

$$e_{ij} = e_{ji} \frac{(p_j q_j)}{(p_i q_i)} \quad (B.10)$$

$$e_{kj} = e_{jk} \frac{(r_j x_j)}{(r_k x_k)} \quad (B.11)$$

$$e_{ik} = e_{ki} \frac{(r_k x_k)}{(p_i q_i)} \quad (B.12)$$

The above analysis can be applied to consumer demand.

If there are n commodities, the consumer's utility function can be expressed by:

$$U = f(q_1, q_2, \dots, q_n) \quad (B.13)$$

and the budget constraint is

$$y - \sum p_i q_i = 0 \quad (B.14)$$

The consumer's objective is to maximize (B.13) subject to (B.14), that is the consumer desires to maximize the Lagrange function

$$L = f(q_1, q_2, \dots, q_n) + m(y - \sum p_i q_i) \quad (B.15)$$

The first order conditions for utility maximization are:

$$\frac{\partial L}{\partial q_i} = f_i - m p_i = 0 \quad i = 1, \dots, n \quad (B.16)$$

$$\frac{\partial L}{\partial m} = y - \sum p_i q_i = 0 \quad i = 1, \dots, n \quad (B.17)$$

The second order conditions yield the following symmetry conditions:

$$\frac{wq_i}{wp_j} + \frac{q_j wq_i}{wy} = \frac{wq_j}{wp_i} + \frac{q_i wq_j}{wy} \quad i, j = 1, \dots, n \quad (B.18)$$

In terms of elasticities, the symmetry conditions can be rewritten:

$$e_{ij} = e_{ji} \frac{(p_j q_j)}{y} + B_j \frac{(p_j q_j)}{y} - B_i \frac{(p_j q_j)}{y} \quad (B.19)$$

where B_i and B_j are the income elasticities of the i th and j th goods, respectively.

These partial derivatives measure the rate of change of quantity with respect to a price change. The partial derivatives are symmetric and indicate that the substitution effects for two items are symmetric. The reciprocity relations can be converted into elasticities to reveal the interdependence of the cross-elasticities:

$$\epsilon_{ij} = \epsilon_{ji} \frac{p_{ij} q_{ji}}{(p_{ii} q_{ii})}$$

$$\epsilon_{ik} = \epsilon_{ki} \frac{p_{ik} q_{ki}}{(p_{kk} q_{kk})}$$

$$\epsilon_{ik} = \epsilon_{ki} \frac{p_{ik} q_{ki}}{(p_{kk} q_{kk})}$$

The above analysis can be applied to consumer demand.

If there are n commodities, the consumer's utility function can be represented by:

$$U = f(q_1, q_2, \dots, q_n)$$

and the budget constraint is

$$Y = p_1 q_1 + p_2 q_2 + \dots + p_n q_n$$

The consumer's objective is to maximize (B.12), subject to (B.13), that is the consumer desires to maximize the Lagrange function

$$L = U(q_1, q_2, \dots, q_n) - \lambda(Y - p_1 q_1 - \dots - p_n q_n)$$

The first order conditions for utility maximization are

$$\frac{\partial L}{\partial q_i} = \frac{\partial U}{\partial q_i} - \lambda p_i = 0$$

$$\frac{\partial L}{\partial q_j} = \frac{\partial U}{\partial q_j} - \lambda p_j = 0$$

The second order conditions yield the following symmetry conditions:

$$\begin{aligned} \frac{\partial^2 L}{\partial q_i \partial q_j} &= \frac{\partial^2 U}{\partial q_i \partial q_j} - \lambda(p_i p_j) = 0 \\ \frac{\partial^2 L}{\partial q_j \partial q_i} &= \frac{\partial^2 U}{\partial q_j \partial q_i} - \lambda(p_j p_i) = 0 \end{aligned} \quad (B.18)$$

In terms of elasticities, the symmetry conditions can be rewritten:

$$\epsilon_{ij} = \epsilon_{ji} \frac{p_{ij} q_{ji}}{(p_{ii} q_{ii})} + \epsilon_{ik} \frac{p_{ik} q_{ki}}{(p_{ii} q_{ii})} - \lambda \frac{p_i p_j}{(p_{ii} q_{ii})}$$

where ϵ_1 and ϵ_2 are the income elasticities of the i th and j th goods, respectively.