EXPECTED BENEFITS DERIVED FROM THE USE OF MORE SOPHISTICATED IRRIGATION TECHNOLOGIES*

by

Luis R. Zavaleta

Ronald D. Lacewell

C. Robert Taylor **

* Technical article no. 15307 of the Texas Agricultural Experiment Station.

** Luis R. Zavaleta is Assistant Professor of Section of Economic Entomology and Department of Agricultural Economics, University of Illinois, Urbana, Illinois; Ronald D. Lacewell is Professor of Agricultural Economics, Texas Agricultural Experiment Station, Texas A&M University, College Station, Texas; and C. Robert Taylor is Associate Professor of Agricultural Economics, Texas Agricultural Experiment Station, Texas A&M University, College Station, Texas.

Presented at AAEA meeting, Pullman, Washington, July 31 - Aug 3, 1979.

EXPECTED BENEFITS DERIVED FROM THE USE OF MORE SOPHISTICATED IRRIGATION TECHNOLOGIES

by

Luis R. Zavaleta and Ronald D. Lacewell

ABSTRACT

To measure the expected benefits that may be derived from a sophisticated irrigation technology, a stochastic open-loop feedback control policy was built into a grain sorghum growth simulation model. The control policy operated under the basis of constant revision of the expectations generated at every starting point for each of the production periods.

EXPECTED BENEFITS DERIVED FROM

THE USE OF MORE SOPHISTICATED IRRIGATION TECHNOLOGIES

Since 1939 the development of irrigation has expanded the agricultural output of several regions in Texas. This expansion has been reflected in the increased volume of production in established farming areas as well as in the incorporation of regions not previously under cultivation. At the present time, from the different agricultural regions of Texas, more than 60 percent of the cash value of crops is generated on irrigated land (Knutson, et al.).

One of the regions experiencing an expansion of its agricultural output is the High Plains of Texas. The development of irrigation in this area, however, has brought with it the threat of the economic depletion of the Ogallala aquifer which, constitutes the source for the irrigation water used. Although physical exhaustion of the stocks of groundwater is not foreseeable, water table levels may become depleted to such extent that the economics of continuous irrigation is not feasible. This has relevant implications for the economy of the region which is primarily supported by the agricultural sector.

Much of the economic growth in this region is associated with the introduction and development of irrigation on crops like cotton, wheat and grain sorghum. The latter is the principle irrigated crop grown in the High Plains, constituting about 2 million acres or 36 percent of the total irrigated acreage (Shipley and Regier).

Sprinkler and gravity flow or furrow methods of irrigation are both employed on the Texas High Plains depending on soil type, slope and availability of irrigation water. The major drawbacks of these systems is that they are subject to the existence of water losses due to evaporation as well as due to run-offs. Irrigation water is not applied efficiently in the sense that cannot be delivered at once uniformly across all fields missing, in most parts, the application where the marginal physical productivity from any additional unit of water is at its highest point. Three or four irrigations on grain sorghum are normally applied to meet the water use requirements during the growing season, but they may vary from only one preplant to as many as five or six postplant waterings (Shipley and Regier). In areas with very limited or no groundwater, grain sorghum is produced with no irrigation. The general consensus of water use studies (i.e., Swanson and Thaxton; Jensen and Musick; Shipley, Regier and Wehrly), however, emphasize the need for coordinating irrigation with water requirements of the plant during critical stages of development that affect grain production. The yield response attributed to a specific irrigation depends upon several factors, including (1) amount of soil moisture available at the time of irrigation or amount of rainfall immediately following irrigation, (2) stage of plant development, where moisture requirements and the effects of moisture stress differ, and (3) interaction effect from previous or subsequent irrigations, or both, which reduce or eliminate moisture stress conditions.

The declining water table, the existent moisture availability at different times and stages of development of the plant and the inability to provide the crops with the required amount of irrigation water at the precise moments thus emphasize the need for more effective irrigation strategies as well as more sophisticated delivery systems.

To determine the expected benefits that could be derived from introducing advanced irrigation technologies, a stochastic open-loop feedback control policy was built into a grain sorghum growth simulation model. The computational model optimized the amount of irrigation water to be applied during specified periods of the production process. The control policy operated under the basis of constant revision of the expectations generated at every starting point for each of the production periods. If discrepancies between the expected and the realized values existed, then, based on current conditions, a re-evaluation of the control variable, irrigation water, was made and the decision for the first period adopted. This process continued throughout each period of the growing season. Within the stochastic policy designed, the values for the control variable were obtained by numerical search.

The model was applied to estimate optimal irrigation strategies and, therefore, possible benefits to be derived from a more sophisticated irrigation technology. Initially, optimal irrigation strategies were developed under the assumption of perfect knowledge to determine the amount of yields that could be generated with optimally applied irrigation water. Then, the assumption of perfect knowledge was relaxed to allow the model to perform under stochastic or real world conditions where the climatic environment was unknown. In the latter, as well as in the deterministic case, the optimal allocations depended much on the initial conditions. It was also observed, that with the open-loop feedback control, the results obtained for yields did not differ substantially from those obtained in the perfect knowledge case. The discrepancies among the two cases were primarily in the optimal amount of water applied and therefore in net returns. In the stochastic case, the use of irrigation water had a mean value approximately 25 percent more than in the case of perfect knowledge.

To summarize, improved irrigation distribution technology could result in increased yields and less irrigation water by simply having very close command on timing and quantity of water applied.

Irrigating Under a Dynamic and Stochastic Environment

Mathematically stated, the dynamic problem is that of choosing a time path for the control variable that will maximize the value of a given objective function. This value depends on the time path of the control and state variables (Intriligator). Because optimal allocation of irrigation water in the production process is a problem of the dynamic economic type it will be presented in the context of control theory. Of special interest is the part of control theory denoted as adaptive stochastic control. It recognizes that as the system progresses through time, more information is available which can be used to modify or reestimate the influence of alternative control variable settings on various performance measures. The formulation of a strategy in the use of the decision variable, irrigation water, at the beginning of the growing season is based on the expectations held by the decision maker on relevant variables such as rain and temperature. As the season evolves, the decision maker is not committed to keeping to the initially formulated plan, but rather, to altering it. In this case, a revision and reformulation procedure takes place in which the newly observed data are taken into account to reevaluate the effectiveness of the control variable. As a result, a new strategy is formulated for the remaining production periods.

The problem of optimal allocation of irrigation water in the production process can be represented by the following set of equations:

$$(1) \quad J(X_t, U_t, t) = \sum_{t=o}^{T-1} I(X_t, U_t, t) + F(X_T)$$

J is assumed to be a convex function made up of the summation of the discounted net returns from T-stage system operating under a deterministic environment: the intermediate function I(...) represents the discounted values for each period up to T-1, and F(·) is the ending value or terminal state function. The vector of the state variables X represents the values obtained by the system at each period t. The control variable at each instant t are completely described by the values of the vector U.

The maximization of the objective function (1) is subject to

$$(2) \quad X_{t+1} = f_t(X_t, U_t, T) \quad \text{and}$$

$$(3) \quad g(X_t, U_t) \leq b_t$$

where the planting date (t_o), the state of the system at t_o(X_o) and the date at which the physiological maturity of the plant is reached (T) are characteristics determined within the system.

Equation (2) represents the dynamic behavior of the system indicating that the change in the level of the state variable at any instant is a function of its present state, the decision taken, and the time period. Together, the equation of motion, equation (2), and the intermediate function, $I(X_t, U_t, t)$ reflect the problem of making decisions in a dynamic context (Dorfman) and imply that the decisions do, in fact, influence the level of production and returns in each time period.

The solution to the defined system can be approached by the optimality principle (Bellman) based on a first order difference equation. Other methods such as the Newton-Raphson, quasi Newton-Raphson, Golden Section, False Position and other gradient techniques bear the same kind of limitation faced by the Bellman's principle in that analytical solutions cannot be obtained.

A third type of approach, though initially developed by Pontryagin for the case of continouous-time type of problems, has been expanded by Holmes, among other authors, to include the discrete-time case.

Similar to the deterministic case, the problem of optimal allocation of irrigation water in a production process when stochastic conditions exist can be modeled as a control problem. Specifically, in those cases where uncertainties can be adequately modeled on stochastic processes, the problem of decision making can be approached by using adaptive control theory. In this context, the state of the system at time t is presumed to evolve according to

$$(4) \quad X_{t+1} = f(X_t, U_t, V_t, t) \qquad t = 0, 1, \ldots, T-1$$

with X_t and U_t being vectors of the state and control variables at time t and V_t being the process voice.

In this case, the choice of controls, which is a multistage decision process, constitutes the stochastic control problem and the ways of choosing them determines the class of control policies to be applied.

The various classes of stochastic control policies are defined according to the information on past and anticipated future observations available to the controller. This knowledge about the probabilities of future observations allows the controller to statistically anticipate the information to be obtained from subsequent observations and to be used in deciding the most desirable present action.

According to the amount of information used, four classes of stochastic control policies -- as defined by Bar-Shalom and Tse -- can be distinguished: Open-loop (OL), Open-loop Feedback (F), m-measurement Feedback (mF) and Closed-loop (CL). It should be noted that F, mF and CL have the same information about the past, and the only difference among them resides in the anticipation of future knowledge. Of these four policies, the optimal stochastic control belongs, in general, to the closed loop class (Bar-Shalom and Tse; Intriligator; Rauser). The optimal stochastic control is obtained by using Bellman's principle of optimality.

The inherent analytical difficulties in deriving the closed-loop control rule, though, suggest other policy classes should be used as an approximation. The analytical hardships can be reduced by decreasing the amount of information available to the decision maker. Because of the complexity involved in the modeling and analytical solution of the stochastic closed-loop policy, the stochastic open-loop feedback control was considered as an alternative. Albeit the limitation of finding an analytical solution could not be overcome for this control, the possibility of being modeled so as to obtain a numerical solution leaves it as the best viable alternative. To this extent, different iteratives numerical solution techniques can be utilized. The practical importance of these methods should by no means be overlooked. They often offer the simplest, most direct alternatives for obtaining solutions. An extended number of approaches to this important phase of optimization can be cited: Fibonacci, cubic and quadratic fit, steepest ascent, coordinate ascent methods, conjugate direction methods, etc. in which the sophistication of implementation varies directly to the speed of convergence. The selection of the method depends on the particularities of each problem analyzed. In the specific case of the problem faced in this study, professional judgement and ciphering facilities lead to the use of the theory involved in quasi-Newton methods.

Model Results and Policy Implications

This study adopted the computerized grain sorghum growth program developed by Arkin et. al. to address the issue of potential benefits derived from the use of more sophisticated irrigation technologies. The approach followed loosely defines "more sophisticated irrigation technologies" as those that would allow the possibility of applying up to eight postplant irrigations and would permit the application of exact as well as small amounts of water. Though the study considered a maximum of eight postplant irrigations, this number is by no means unique. The program is flexible enough to consider over 100 irrigation periods. However, to include so many possibilities has the disadvantage of rapidly running into--what is known in the literature --the "curse of dimensionality". Also, the model is able to evaluate the effects of introducing environmental and economic stochastic disturbances. These cases were analyzed by Zavaleta et. al. and Lacewell et. al. by introducing stochastic curtailments and price changes.

The possible effects of better irrigation technologies are shown in table 1. The set of data presented under the heading "stochastic" reflects the results obtained from the optimizing model. "Three Postplant Irrigations" reflects to a certain extent the possible outcomes that could be obtained from a "conventional" system which allows about three or four irrigations throughout the growing season. The statistics presented in the table seem to imply that a more frequent number of irrigations would have the effect of increasing yields as well as net returns: from 71.40 to 89.90 cwt. and from 57.40 to 99.40 dollars, respectively. The use of more frequent irrigations not only has a positive effect on net returns and yields, but also decreases their spectrum of fluctuation or risk involved. This implies that appropriate decisions on the amount by which to irrigate -- given the existing weather conditions and the expectations on climatic variables -- can be better formulated for shorter rather than longer periods of time. The data under the heading "Deterministic" (also in table 1) shows the effects that stochastic events have in the use of a resource such as irrigation water: though the mean value of the yields does not change, the average value of net returns shows a decline (from $109.50 to $99.40). The reason for the lower net returns is embedded in the higher mean value of the amount of water used. An excessive use of irrigation water equivalent to approximately 25 percent of the amount strictly needed (as given by the deterministic case) is the cost introduced by the lack of knowledge on weather factors.

Finally, table 2 presents the value of the amount of irrigation water applied in each period under deterministic as well as stochastic weather conditions. This information reveals that the optimization of net returns received from grain sorghum yields can be obtained by small applications of irrigation water, at each time.

In summary, the existence of more sophisticated irrigation technologies may benefit producers by means of increased yields, increased net returns and a reduced spectrum of fluctuation in yields as well as returns.

Table 1. Per Acre Net Returns, Yield and Water Use Under Alternative Scenarios and Alternative Irrigation Strategies for Grain Sorghum: Texas High Plains. a/

Item	Stochastic						Deterministic		
	Three Postplant Irrigations			Eight Postplant Irrigations			Eight Postplant Irrigations		
	Net Return b/	Yield	Water Used c/	Net Return b/	Yield	Water Used c/	Net Return b/	Yield	Water Used c/
	($)	(cwt.)	(in.)	($)	(cwt.)	(in.)	($)	(cwt.)	(in.)
Average	57.40	71.40	7.70	99.40	89.90	13.80	109.50	89.90	11.10
High	130.20	91.80	8.80	132.40	99.30	15.10	146.30	99.30	13.60
Low	8.20	57.40	5.50	61.70	79.40	11.20	75.20	79.40	9.00
$S_{\bar{x}}$	27.35	7.78	0.86	17.81	5.06	0.80	17.99	5.02	1.18

a/ Based on 30 simulations.

b/ Assumes price of Grain Sorghum at $4.07/cwt. and price of Natural Gas at $2.50/mcf.

c/ Irrigation water used. Does not include preplant irrigation. Water application is effective water to the root zone. Costs are based on pumping 1.6 acre feet for each acre foot of effective water.

Table 2. Irrigation Water Used Per Period for 30 Years of Deterministic and Stochastic Weather Patterns: Texas High Plains. a/

	Deterministic								Stochastic							
	Days After Emergence															
Year	20	30	40	50	60	70	80	90	20	30	40	50	60	70	80	90
	Inches															
1	2.42	1.15	1.38	1.60	1.09	1.36	0.69	1.60	2.62	1.28	1.75	1.55	1.89	1.94	1.77	1.59
2	2.63	1.40	1.50	0.72	1.70	1.36	1.42	1.09	2.63	1.38	1.83	1.61	1.82	1.84	1.77	1.65
3	3.12	0.64	0.71	1.27	0.99	0.88	1.59	1.15	3.12	0.81	1.61	1.64	1.91	1.72	1.86	1.55
4	3.04	0.00	1.58	1.86	0.21	1.35	1.01	1.06	3.04	1.17	1.49	1.81	1.83	1.72	1.66	1.67
5	2.29	1.34	1.14	1.25	1.69	1.46	1.11	1.32	2.29	1.27	2.04	1.15	1.70	1.56	1.95	1.63
6	2.89	1.53	0.04	0.21	1.74	1.54	0.96	1.13	2.89	1.48	1.58	1.57	1.64	1.79	1.64	1.70
7	2.58	1.36	1.96	0.36	1.49	1.91	1.48	0.49	2.58	1.29	1.84	1.78	1.44	1.82	1.71	1.50
8	1.49	1.15	1.59	1.11	1.14	0.45	1.30	1.16	1.49	1.11	1.70	1.67	1.46	1.64	1.62	1.40
9	2.69	1.28	0.85	1.38	0.69	1.77	1.31	0.79	2.69	1.36	1.72	1.62	1.66	1.60	1.56	1.48
10	1.59	0.83	0.63	0.85	1.67	1.38	1.57	0.46	1.59	1.44	0.90	1.15	1.62	1.69	1.58	1.25
11	2.97	1.39	0.43	1.55	0.00	1.54	1.76	1.49	2.74	1.51	0.90	1.34	1.54	1.59	1.60	1.54
12	2.97	1.14	1.12	0.98	1.46	0.55	0.96	1.23	3.08	1.06	1.81	1.49	1.36	1.69	1.83	1.62
13	3.06	1.51	1.93	1.41	1.79	0.91	1.57	0.25	3.06	1.47	1.95	1.44	1.78	1.13	1.70	1.43
14	2.44	0.85	1.52	0.00	1.60	1.38	1.35	1.33	2.44	1.02	1.71	1.67	1.52	1.39	1.68	1.62
15	2.50	1.16	1.45	1.62	0.88	0.85	1.38	0.22	2.50	1.36	1.74	1.54	1.46	1.82	1.89	1.34
16	3.11	1.32	1.57	1.22	1.47	1.68	1.76	1.45	3.05	1.48	1.82	1.78	1.81	1.65	1.66	1.80
17	2.56	0.60	1.78	1.80	1.67	1.15	1.44	1.61	2.56	1.38	1.48	1.86	1.68	1.79	1.54	1.57
18	2.64	1.10	1.36	0.41	1.86	1.57	0.16	0.91	2.82	0.94	1.35	1.50	1.89	1.90	1.62	1.74
19	2.79	1.10	1.97	0.91	1.83	0.52	0.77	1.69	2.79	1.09	1.98	1.87	1.69	1.86	1.52	1.50
20	2.98	1.00	1.92	1.57	0.88	1.79	1.31	1.54	2.99	1.36	1.82	1.57	1.52	1.85	1.62	1.59
21	2.11	0.88	1.22	1.68	1.22	1.37	1.26	0.51	2.11	1.36	1.85	1.57	1.52	1.74	1.37	1.83
22	2.66	1.52	1.37	1.74	1.28	1.57	1.27	1.03	2.66	1.45	1.93	1.56	1.84	1.74	1.98	1.61
23	2.99	1.35	1.61	1.73	0.96	1.71	0.79	1.07	2.99	1.54	1.51	1.88	1.67	1.49	1.60	1.57
24	3.09	1.59	0.00	0.36	1.36	1.12	1.53	1.65	3.09	1.20	1.39	1.84	1.71	1.15	1.87	1.69
25	2.34	1.34	1.19	0.00	0.55	1.71	1.25	1.19	2.34	1.33	1.20	1.82	1.87	1.63	1.55	1.59
26	3.23	0.63	0.69	1.25	1.97	1.41	1.46	1.74	2.96	0.81	1.62	1.63	1.74	1.74	1.47	1.75
27	2.64	0.64	2.21	1.34	0.44	1.54	1.70	0.67	2.64	0.62	2.22	1.85	1.65	1.54	1.64	1.65
28	2.11	1.22	1.79	0.91	1.30	1.62	1.40	1.54	2.45	1.24	1.65	1.53	1.56	1.87	1.75	1.59
29	1.86	0.68	1.96	1.25	0.42	1.38	[illegible]	1.62	1.86	1.17	1.81	1.57	1.75	1.65	1.43	1.86
30	2.66	1.56	1.68	2.03	0.29	1.57	1.53	0.63	2.69	1.51	1.69	2.05	2.02	1.72	1.58	1.76

a/ Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50.

REFERENCES

Arkin, G. F., R. L. Vanderlip, and J. T. Ritchie. "A Dynamic Grain Sorghum Growth Model." Mimeographed. Contribution of the Texas Agricultural Experiment Station.

Bar-Shalom, Y. and E. Tse. "Concepts and Methods in Stochastic Control," in Control and Dynamic Systems: Advances in Theory and Applications, C. T. Leondes (ed.). Academic Press, 12 (1976): 99-172.

Bellman, Richard. Adaptive Control Processes: A Guided Tour. Rand Corporation, Princeton University Press, Princeton, New Jersey, 1961.

Bellman, Richard. Dynamic Programming. Princeton University Press, Princeton, New Jersey, 1957.

Dorfman, R. "An Economic Interpretation of Optimal Control Theory." American Economic Review 59(1969): 817-831.

Holmes, W. L. "Derivation and Application of a Discrete Maximum Principle." Western Economic Journal 6(1968): 385-394.

Intriligator, M. D. Mathematical Optimization and Economic Theory. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1971.

Jensen, M. E. and J. T. Musick. "Irrigating Grain Sorghums." Leaflet No. 511, U.S. Department of Agriculture, June 1962.

Knutson, R. D., R. D. Lacewell, W. A. LePori, D. C. Hardin, E. A. Hiler, and C. W. Keese. "Analysis of the National Energy Plan: The Effects on Texas Agriculture." Texas Agricultural Experiment Station, MP 1331, June 1977.

Lacewell, R. D., Gary D. Condra, Daniel C. Hardin, Luis R. Zavaleta and James A. Petty. "The Impact of Energy Shortage and Cost on Irrigation for the High Plains and Trans Pecos Regions of Texas." TR-98, TWRI, Texas A&M University, September 1978.

Pontryagin, L. S., V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko. The Mathematical Theory of Optimal Process, trans. by K. N. Trirogoff. Interscience Publishing, John Wiley and Sons, Inc., New York, 1962.

Rausser, G. C. "Active Learning, Control Theory and Agricultural Policy." Paper presented at the Annual Meetings of the American Agricultural Economics Association, San Diego, California, July 1977.

REFERENCES (continued)

Shipley, J. and C. Regier. "Water Response in the Production of Irrigated Grain Sorghum, High Plains of Texas." The Texas A&M University System, Texas Agricultural Experiment Station, MP-1202, June 1975.

Shipley, J., C. Regier and J. S. Wehrly. "Soil Moisture Depletion Levels as a Basis for Timing Irrigations on Grain Sorghum." Texas Agricultural Experiment Station, Consolidated PR-2546-2555, June 1968.

Sorenson, H. W. "An Overview of Filtering and Stochastic Control in Dynamic Systems," in Control and Dynamic Systems: Advances in Theory and Applications, C. T. Leondes (ed.), Academic Press, 12(1976): 1-61.

Swanson, N. P. and E. L. Thaxton, Jr. "Requirements for Grain Sorghum Irrigation on the High Plains." Texas Agricultural Experiment Station Bulletin 846, January 1957.

Zavaleta, Luis R., Ronald D. Lacewell and C. Robert Taylor. "Economically Optimum Irrigation Patterns for Grain Sorghum Production: Texas High Plains. TR-100, TWRI, Texas A&M University, January 1979.