THE EFFICIENCY OF OPTIONS COMPARED TO FIXED PRICE CONTRACTS FOR SHIFTING REVENUE RISK IN CROP PRODUCTION

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Abstract

Stochastic simulation is used to compare the probability distributions of revenues from soybean production for pricing with put options and fixed-price contracts. By applying safety first, expected utility, and stochastic dominance criteria, fixed-price contracts are shown to be superior to options for shifting risks under many, but not all, conditions.

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presented at AAEA, 1986

# THE EFFICIENCY OF OPTIONS COMPARED TO FIXED PRICE CONTRACTS FOR SHIFTING REVENUE RISK IN CROP PRODUCTION

The farmer´s choice between using put options and fixed-price contracts (including futures) for pricing growing crops is in essence a choice between alternative probability distributions for revenue. Since neither options nor fixed-price contracts can be expected to increase the farmer´s revenue on the average in an efficient market, the choice depends upon characteristics of the revenue distributions other than their means--particularly their dispersion and skewness. In this paper we describe these other characteristics for representative cropping situations and explore the implications for farmers´ decisions. Underlying the study is the question of whether commodity options offer crop producers any significant advantages over fixed-price contracts.

This paper goes beyond previous work, such as Plato and Heifner, by exploring how the distributions of revenues change as variation and covariation in prices and yields change, and by applying expected utility and stochastic efficiency criteria to compare distributions. Coefficients representative of soybean price and yield variation and covariation in the Cornbelt and Southeast are used in modeling the revenue distributions.

Asymmetry in the revenue distributions rules out methods such as mean-variance analysis for analyzing options trading decisions. Stochastic simulation is used here to allow for non-standard distributions. Although an element of approximation is involved, we believe the results are sufficiently strong and general to be of considerable interest to decision-makers.

## Characteristics of the Revenue Distributions

This analysis focuses upon risks for a single cropping enterprise over a single growing season. Three strategies for pricing the crop are considered: (1) pricing at harvest; (2) selling a specified percentage of the expected output for a fixed price at planting; and (3) buying an at-the-money put option at planting time covering the expected output. Risk is measured in terms of deviations in revenues realized at harvesttime from planting time expectations. Revenue is defined as price times yield per acre plus or minus any gains or losses from forward contracting. The sources of revenue variations are price and yield variations which are assumed to have known probability distributions.

In the absence of yield uncertainty revenue could be fixed at planting by entering fixed-price contracts. In contrast, buying put options sets a lower limit on revenue, at the strike price minus the option premium, and allows revenue to vary above this minimum. In choosing between these two pricing strategies, the farmer chooses between a known revenue and a random revenue with a lower bound. Options are the more risky strategy.

When yield uncertainty is present revenue risk cannot be completely eliminated by fixed-price forward contracting. Revenue varies not only because the quantity available for sale varies, but also because the quantity produced generally does not match the quantity contracted. For example, a farmer who sells futures contracts at planting time and then experiences a crop failure may lose from buying back futures contracts at a higher price as well as from not having a crop to sell.[1] The option buyer avoids such double losses. This analysis compares the two approaches in terms of the probability distributions of revenues after option premiums have been paid.

For each case analyzed 5000 pairs of correlated random yields and harvesttime prices were generated using the parameters shown in table 1. Price changes from planting to harvest were generated as lognormal variables with zero expectations. Yields were drawn from normal distributions truncated at the point in the lower tail where price times yield was less than harvesting cost.[2] Standard deviations and correlations in prices and yields were based upon estimates made by D. Grant using county data.

Revenues for each of the alternative selling strategies were calculated as follows for each yield-price pair.

$$(1)\quad V = YP + H\hat{Y}(\hat{P}-P) + G\hat{Y}(T-M), \quad \text{where } T = S - P \text{ if } P<S$$
$$T = 0 \text{ otherwise}$$

The variables are defined as follows: V = revenue per acre, Y = realized yield, P = price at harvest, H = proportion of the crop contracted at the forward price, $\hat{Y}$ = yield expected at planting time, $\hat{P}$ = forward price at planting time, G = proportion of crop covered by put options, T = value of put option at harvest, M = premium for put option at planting time, and S = strike price for put option. The minimum for YP was set at \$10 which was assumed to represent a saving in harvesting costs when the value of the crop would not pay for harvesting. Option premiums were calculated using the procedure described by Black.

The three alternative pricing strategies were compared for conditions representing Iowa, North Carolina, and a hypothetical high-yield-risk case. North Carolina differs from Iowa in having relatively more variable yields and a less negative price-yield correlation. The high-yield-risk case was constructed to test the sensitivity of the results to yield variability. Theoretical considerations suggest that individual farm yields are more variable and less closely correlated with price than county yields. Lacking data to estimate these farm level statistics, we experimented with doubling and quadrupling the county-level yield standard deviations. Doubling the standard deviations resulted in relatively modest changes in the shapes of the distributions. The quadrupling example is reported here as the high-yield-risk case to provide greater contrast.

The calculated revenue distributions are described in tables 2, 3 and 4 and displayed in figures 1 through 4. The distributions for alternative strategies exhibit the same basic relationships in all three cases, but the differences between strategies are less for the high-yield-risk case. Both fixed-price contracting and buying put options concentrates the distribution of revenues around its midpoint compared to selling at harvest. Compared to the distribution for fixed-price contracting, which is approximately symmetrical, the distribution for options is skewed to the right and has a lower mode. The standard deviation of revenue and the coefficient of skewness are larger for options than for fixed-price sales in all cases.

## Choosing Between Revenue Distributions

Although the choices between the different revenue distributions depend ultimately on the unique preferences of individual decision-makers, the application of more general decision criteria is of interest. Three types of criteria were applied: (1) safety first; (2) expected utility; and (3) stochastic dominance.

### Safety First Comparisons

Some farmers may wish to minimize the probability of a catastrophic outcome--default on a loan for example. This would involve choosing the distribution with the lowest probability of revenue below some critical level. Such probabilities can be read directly from the cumulative probability distribution. For example, in the Iowa case shown in figure 2, the probabilities of revenue below $150 per acre are .35, .05, and .16 respectively for selling at harvest, fixed-price forward selling, and buying put options. For a farmer who requires at least $150 per acre to pay off a loan, a fixed-price sale would be the safest strategy.

The cumulative density function for fixed price contracting generally lies below the cumulative density function for put options at moderately low revenue levels. This suggests that fixed-price contracts afford greater safety for many farmers. However, the distribution for fixed-price sales appears to lie above the distribution for options at extremely low returns. Farmers who are concerned only about these rare extremely low revenues may find that put options offer the greatest safety.

### Expected Utility Comparisons

The choice between alternative pricing strategies can be treated as a problem in maximizing expected utility. Expected utility was calculated for each strategy for three types utility of wealth functions:

(1) Constant relative risk aversion

$$U(W) = W^{1-R}/(1-R), \quad R = 1$$

$$U(W) = \log(W), \quad R = 1$$

(2) Constant absolute risk aversion

$$U(W) = -ke^{AW}$$

(3) Quadratic

$$U(W) = -(a-bW)^2$$

where $W = \bar{W} + V$, $\bar{W}$ is wealth before receiving the revenue from the crop, R is relative risk aversion and A is absolute risk aversion.[3] Constant relative risk aversion utility functions for R=1 and R=2 and constant absolute risk aversion utility functions for $A=1/\bar{W}$ and $A=2/\bar{W}$ were inserted into the simulation program to calculate expected utility for producers with 200 acres of soybeans and two levels of wealth, \$30,000 and \$100,000. The quadratic utility function was constructed so that marginal utility is zero at \$400,000. The calculated utility indexes were converted to certainty equivalents by applying the inverses of the respective utility functions. The calculated certainty equivalents are reported in tables 5, 6 and 7.

For Iowa and North Carolina all of the utility indicators are highest for fixed-price forward contracting. However, the differences, measured in certainty equivalents, are relatively small at the \$100,000 wealth level. Forward selling was preferred even in the high-yield-risk case for wealth at $100,000. For wealth at $30,000, however, the two utility functions representing greatest risk aversion indicated higher utility for options than for forward selling. This suggests that options may be the preferred alternative in some situations where risk aversion is high and yields are extremely variable.

Stochastic Dominance Comparisons

The concept of stochastic dominance provides rules for ranking income distributions when little is known about the decision-maker's preferences.[4] First degree stochastic dominance requires only that high incomes are preferred to low incomes. One income distribution dominates another if its cumulative density function lies entirely to the right of the cumulative density function for the dominated distribution. First degree stochastic dominance does not occur among the distributions considered here.

Second degree stochastic dominance requires that the decision-maker be risk averse in addition to preferring high incomes to low incomes. It occurs when the area under the cumulative density function for the dominant distribution is everywhere less than the area under the cumulative density function for the dominated distribution. For this analysis the required measures were calculated by cumulating the frequencies under the cumulative distributions. The criterion for second degree stochastic dominance of fixed-price contracting over options would have been met in the cases examined here if it were not for the extreme lower tails of the distributions.

## Conclusions

Put options are less effective in shifting risks for soybean growers than fixed-price contracts under many, but not all, conditions. However, options appear to offer lower probabilities of extremely low returns--events that have very low probabilities in any case. The advantages of fixed-price contracts over put options are diminished or reversed when yields are extremely variable and the farmer is very risk averse. More information about the probability distributions of farm-level yields is needed to add precision to these conclusions. Options may, of course, be preferred to fixed-price contracting for reasons other than risk shifting. In particular, option buyers avoid the cash flow problems that may arise due to margin calls on futures positions, and some farmers may be willing to give up some downside risk protection to obtain revenue distributions that are skewed to the right.

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1. McKinnon, Grant, and others have derived or estimated risk-minimizing hedging levels to deal with this problem.

2. The standard normal random deviates used to calculate prices and yields were generated by a program written by D. Kahaner and J. Horlick, RNOR, Generate Normally Distributed Numbers and Place on a Disk File, National Bureau of Standards, Gaithersburg, MD 20899.

3. See Newbery and Stiglitz, pages 72-76 for a discusion of these utility functions.

4. See Anderson, Dillon, and Hardaker, pages 281-298, for an overview of stochastic dominance methods.

Table 1--Price-yield coefficients used for simulation

| Coefficient | Iowa | North Carolina | Hypothetical high yield-risk case |
|---|---|---|---|
| Forward price at planting, $/bu. | 5.20 | 5.20 | 5.20 |
| Std. dev. of price change, $/bu. | 1.83 | 1.83 | 1.83 |
| Expected yield, Bu./acre | 35.0 | 25.0 | 30.0 |
| Std. dev. of yield, Bu./acre | 3.5 | 3.5 | 14.0 |
| Price-yield correlation | -.30 | -.18 | -.10 |

Table 2--Probability density functions for revenues from alternative soybean pricing strategies, results from stochastic simulation, Iowa conditions

| | Pricing strategy | | |
|---|---|---|---|
| Revenue range $/acre | Sell at harvest percent | Sell 90% at planting percent | Buy put option at planting percent |
| 0-80 | 1.1 | 0 | 0 |
| 80-90 | 1.5 | .3 | 0 |
| 90-100 | 2.5 | 0 | 0 |
| 100-110 | 3.7 | .1 | 0 |
| 110-120 | 5.3 | .2 | .3 |
| 120-130 | 6.0 | .6 | 1.5 |
| 130-140 | 7.0 | 1.2 | 4.3 |
| 140-150 | 7.8 | 2.9 | 10.3 |
| 150-160 | 7.8 | 5.9 | 19.0 |
| 160-170 | 7.0 | 14.5 | 20.7 |
| 170-180 | 6.4 | 23.4 | 11.7 |
| 180-190 | 6.5 | 24.1 | 6.7 |
| 190-200 | 5.4 | 13.9 | 4.6 |
| 200-210 | 4.6 | 7.6 | 3.3 |
| 210-220 | 4.6 | 3.6 | 3.6 |
| 220-230 | 3.7 | 1.1 | 2.6 |
| 230-240 | 3.3 | .6 | 2.3 |
| 240-250 | 3.2 | .2 | 2.0 |
| 250-260 | 2.4 | 0 | 1.5 |
| 260-270 | 2.1 | 0 | 1.0 |
| 270-280 | 1.6 | 0 | .7 |
| Over 280 | 6.6 | 0 | 4.1 |
| Average, $/acre | 180.85 | 180.45 | 180.16 |
| Std. dev. $/acre | 60.89 | 18.64 | 42.68 |
| Skewness | .97 | -0.18 | 2.11 |
| Kurtosis | 1.45 | 1.55 | 6.23 |

Table 3—Statistics on revenues from alternative soybean pricing strategies, results from stochastic simulation, North Carolina conditions

| Statistic | Sell at harvest | Sell 90% at planting | Buy put option at planting |
|---|---|---|---|
| Average, $/acre | 129.55 | 129.15 | 129.07 |
| Std. dev., $/acre | 46.43 | 19.11 | 34.17 |
| Skewness | 1.03 | -0.01 | 1.91 |
| Kurtosis | 1.65 | 1.37 | 5.53 |

Table 4--Statistics on revenues from alternative soybean pricing strategies, results from stochastic simulation, hypothetical high yield-risk case

| Statistic | Sell at harvest | Sell 90% at planting | Buy put options at planting |
|---|---|---|---|
| Average, $/acre | 155.74 | 155.20 | 155.16 |
| Std. dev., $/acre | 89.03 | 75.12 | 81.31 |
| Shrewness | .94 | -0.03 | .81 |
| Kurtosis | 1.37 | 0.93 | 1.76 |

Table 5--Certainty equivalent revenues from alternative pricing strategies, Iowa conditions, farmers with 200 acres of soybeans and different wealth levels and utility functions

| Wealth 1/ $1000 | Utility function 2/ | Sell at harvest $/acre | Sell 90% at planting $/acre | Buy put option at planting $/acre |
|---|---|---|---|---|
| 30 | R1 | 175.64 | 179.87 | 177.76 |
| | R2 | 170.75 | 179.33 | 175.66 |
| | A1 | 169.90 | 179.24 | 175.09 |
| | A2 | 161.10 | 178.04 | 171.42 |
| | Q | 174.05 | 179.76 | 176.80 |
| 100 | R1 | 178.25 | 180.14 | 178.92 |
| | R2 | 175.76 | 179.89 | 177.77 |
| | A1 | 177.28 | 180.06 | 178.44 |
| | A2 | 173.97 | 179.71 | 176.90 |
| | Q | 179.45 | 180.27 | 179.47 |

1/ Wealth excludes expected revenues from the crop.

2/ R1--Constant relative risk aversion = 1; R2--Constant relative risk aversion = 2; A1--Constant absolute risk aversion = 1/wealth; A2--Constant absolute risk aversion = 2/wealth; Q--Quadratic utility function with zero marginal utility at 4 times wealth.

Table 6--Certainty equivalent revenues from alternative pricing strategies, North Carolina conditions, farmers with 200 acres of soybeans, and different wealth levels and utility functions. 1/

| Wealth $1000 | Utility functions | Sell at harvest $/acre | Sell 90% at planting $/acre | Buy put option at planting $/acre |
|---|---|---|---|---|
| | | Pricing strategy | | |
| 30 | R1 | 125.98 | 128.49 | 127.21 |
| | R2 | 122.62 | 127.82 | 125.55 |
| | A1 | 123.05 | 127.93 | 125.66 |
| | A2 | 117.63 | 126.68 | 122.97 |
| | Q | 126.21 | 128.59 | 127.25 |
| 100 | R1 | 127.91 | 128.87 | 128.18 |
| | R2 | 126.33 | 128.57 | 127.37 |
| | A1 | 127.46 | 128.79 | 127.95 |
| | A2 | 125.50 | 128.42 | 126.92 |
| | Q | 128.77 | 129.02 | 128.64 |

1/ See footnote at bottom of Table 5.

Table 7--Certainty equivalent revenues from alternative pricing strategies, high yield-risk case, farmers with 200 acres of soybeans and different wealth levels and utility functions. 1/

| Wealth $1000 | Utility function | Sell at harvest $/acre | Sell 90% at planting $/acre | Buy put option at planting $/acre |
|---|---|---|---|---|
| | | Pricing strategy | | |
| 30 | R1 | 143.69 | 144.30 | 144.65 |
| | R2 | 132.31 | 126.79 | 134.05 |
| | A1 | 133.35 | 135.92 | 135.57 |
| | A2 | 116.56 | 114.50 | 119.05 |
| | Q | 152.79 | 153.11 | 144.15 |
| 100 | R1 | 150.03 | 150.77 | 150.31 |
| | R2 | 144.64 | 146.18 | 145.62 |
| | A1 | 148.23 | 149.54 | 148.81 |
| | A2 | 141.45 | 143.80 | 142.91 |
| | Q | 152.79 | 153.11 | 152.70 |

1/ See footnotes at bottom of Table 5.

Figure 1--Distribution of revenues from alternative soybean pricing strategies, Iowa conditions

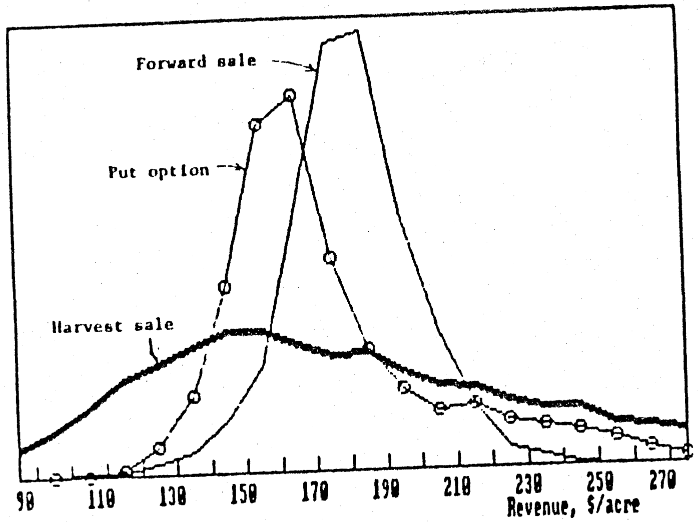



Figure 2--Cumulative distribution of revenues from alternative soybean pricing strategies, Iowa conditions

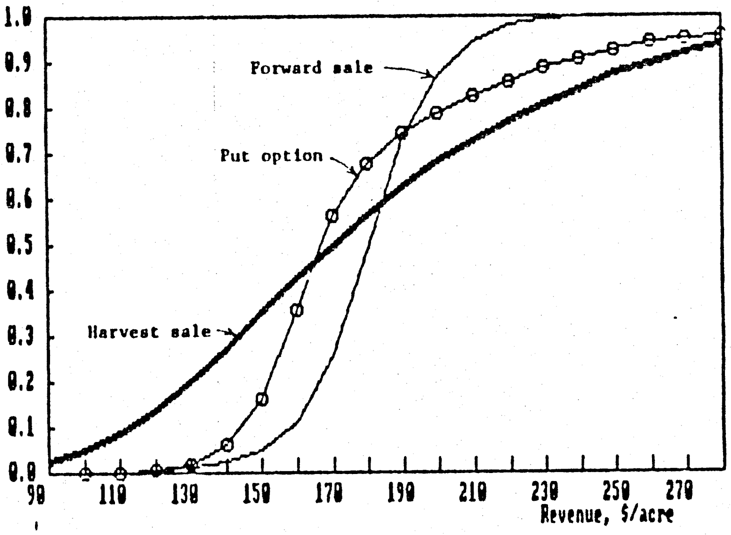



Figure 3--Distribution of revenues from alternative soybean pricing strategies, North Carolina conditions

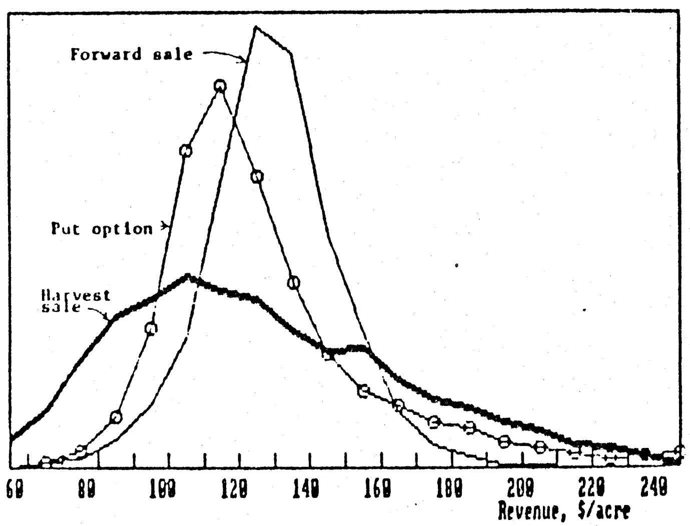
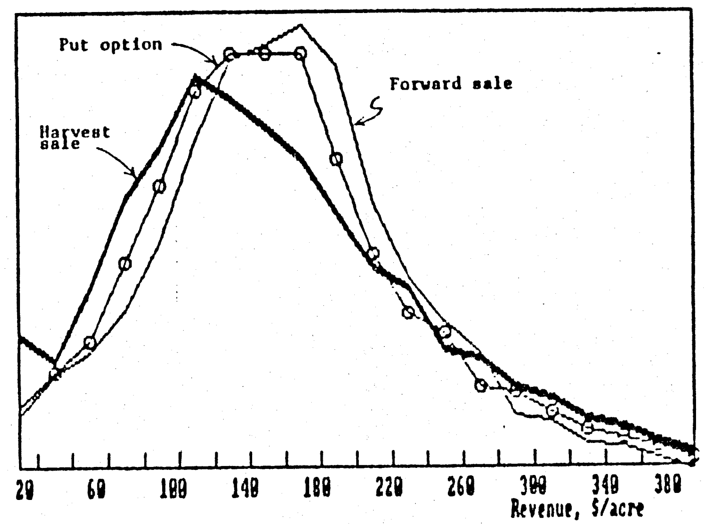



Figure 4--Distribution of revenues from alternative soybean pricing strategies, high yield-risk case



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