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COMMODITY FUTURES PRICE CHANGES: NORMALITY AND IMPLICATIONS FOR OPTION PRICING

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Futures trading

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(ABSTRACT)

The distribution of day-to-day price changes for wheat, soybean, and live cattle futures contracts was examined for the period from January 1973 through December 1982. The results demonstrate a move toward independence and normality, suggesting option pricing formulae which assume normality may provide accurate representations of option values.

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The nature and distribution of commodity futures changes has been widely researched and continues to be of interest. Specifically, the distribution of commodity futures price changes is important in testing the efficient market hypothesis since it affects both the selection of appropriate statistical methods and the interpretation of their results. With the initiation of trading in commodity futures options and the reliance on the normal distribution assumption in options pricing formulae, the question of the distribution of futures prices becomes increasingly important.

Empirical evidence (Houthakker, Rocca, Stevenson, and Bear, Mann and Heifner, and Grauer) suggests that the distribution of future price changes is not normal. The general finding of these studies is that the distribution of price changes tends to have more weight around the midpoint and in the tails than would be expected with a normal distribution (i.e., it is leptokurtic). However, this literature is somewhat dated, with most studies considering data prior to 1972. Only the research of Grauer includes data from the period of increased price volatility during the 1970s. In addition, the conclusions of Mann and Heifner are suspect, as the authors have since reported computational errors (Leuthold and Tomek, p. 43). A notable exception is the recent work of Gordon which examined futures price changes for eight agricultural commodities for contracts maturing from January 1979 through May 1984.

The objective of the current study is to examine the distribution of futures price changes during the period from January 1974 through December 1982 for wheat, soybeans, and live cattle. The methods employed are similar to those of Mann and Heifner in order to allow an assessment of the impact of the reported computational errors. In addition, the time period of analysis will allow examination of the impact of the increased price volatility during the 1970s on the distribution of price changes for these commodities, a longer historical period than that provided by the Gordon study. Finally, since options are currently being traded for these commodities, the results will provide insight into the appropriate pricing formulae to use in computing distant option premiums for use in producer decision models.

The paper is structured as follows. A brief discussion of the background and of previous research is provided in the next section. Data and methods are briefly discussed in the third section. The fourth section presents the results of the analysis in some detail. The paper concludes with a discussion of the implications of the results.

Background and Previous Research

The theoretical foundations for the analysis of the distribution of futures price changes are tied to the efficient market theory suggested by Working. Specifically, in efficient markets, prices fully reflect all available information. Since truly new information flows randomly into the market, price changes would be expected to be random. Bachelier, and later Working, argued therefore, that price changes could be viewed as the sum of a large number of independent and identically distributed random effects with a finite variance. Further, if the effects are distributed uniformly over time, price changes would, by the central limit theorem, have a normal distribution.

As noted above, however, empirical evidence suggests that the distribution does not appear normal, but is actually leptokurtic. Mandelbrot argued that this leptokurtosis was due a failure of the assumption that the individual effects composing a price change are of finite variance. Thus, the central limit theorem does not apply. Mandelbrot therefore argued that the distribution of price changes was a stable Paretian. Rocca, in contrast, suggested that the distribution is a mixture of two normals with different variances, as the market goes through periods of volatile and tranquil price movements. Grauer provided further evidence against the stable Paretian hypothesis. In comparing the stable Paretian distribution to a normal distribution with changing variance, Mann and Heifner were unable to rule out either, although the evidence favored the stable Paretian hypothesis.

Clark questioned the use of the central limit theorem in such analyses, suggesting that it does not apply since the daily price change is a sum of a random number of independent events. Thus, he argued that price changes are normally distributed over transaction to transaction intervals and not over time intervals. Helms and Martell supported this notion in their investigation of transaction-to-transaction price changes, concluding that futures prices changes are closer to the normal than they are to any other member of the Pareto family of distributions.

Finally, Kamara suggested and Neftci later demonstrated the importance of accounting for the impacts of limit move restrictions in analyses of futures price change distributions. Lee and Leuthold considered this notion, suggesting that the distributions of futures price changes appear more normal if price changes are computed over period lengths of several days.

Data and Methods

Daily closing prices were recorded for each wheat (Chicago Board of Trade), soybean, and live cattle contract maturing between January 1, 1973 and December 31, 1982, excluding January cattle contracts 1977-81. A total of 180 contracts were available for the analysis (50 wheat, 70 soybean, and 60 live cattle). Tests for normality and independence were performed on the first differences of the natural logarithms of the daily closing prices over the life of each contract.

Results

Tests of Normality

Kurtosis. Four tests were performed to characterize futures price changes relative to the normal distribution, and all tests were conducted at the .01 level of significance. Table 1 summarizes the results of the kurtosis test. The normality assumption is rejected in about 47 percent of the contracts examined. Soybeans and live cattle show about the same proportion of rejection, 38 and 42 percent respectively, while wheat appears to be the least normally distributed with the null hypothesis being rejected 64 percent of the time. Examination of the individual contracts shows about the same number of positive and negative kurtosis values for soybeans and live cattle whereas wheat contains more positive kurtosis than negative ones. Positive values indicate more observations at the tails than expected theoretically. Most of the significant values were positive with almost all of the significant negative ones falling in the 1973-74 period.

These results suggest some degree of leptokurtosis (too many observations in the tails). However, there is a substantial move towards normality compared with the Mann and Heifner results who found non-normality in 96, 85 and 68 percent of their soybeans, live cattle and wheat contracts, respectively, utilizing data for 1959-71.

Ratio of the Range to the Standard Deviation (R/S). Distributions with an excessive number of extreme observations in either tail translates into a high R/S value. David, et al. tabulated the upper and lower percentage points of this ratio. Table 1 indicates that about two-thirds of the contracts are normal. As with kurtosis, wheat shows the most non-normality of the three commodities. This test may show more normality than the kurtosis test because daily price limits may prevent full price moves relative to the information, reducing the R/S ratio. Nevertheless, these contracts show more normality than those examined by Mann and Heifner who found 74 percent among the same commodities were non-normal.

Table 1. Kurtosis and R/S Tests

Commodity	Number of Contracts	Number (Percent) of Cases Significantly Different from Zero	
		Kurtosis	R/S[a]
Wheat	50	32(64%)	22(44%)
Soybeans	70	27(38%)	21(30%)
Live Cattle	60	25(42%)	21(35%)

[a] R/S is the ratio of the range to the standard deviation.

Characteristic Exponent. The characteristic exponent, α, of a stable distribution indicates the height of the distribution at the tails. An alpha equaling two corresponds to the normal distribution. When alpha is less than two, a stable paretian distribution with a tail fatter than the normal distribution is implied and the variance of the distribution is infinite. When alpha is less than one, the mean is a biased estimate of the location parameter.

A procedure devised by Fama and Roll was used to estimate alpha. This procedure assumes a symmetrical distribution. Skewness tests were performed on all the contracts and only 11 percent of the contracts reject symmetry, suggesting that the Fama and Roll procedure is appropriate.

Estimates of the characteristic exponents are summarized in table 2. Alpha is less than two in 75 percent of the contracts. Live cattle have the largest number showing a normal distribution, 30 percent of the cases. All the estimated alphas are greater than one implying that the mean is a valid estimate of the location parameter. However, the fact that the majority of the contracts have an estimated alpha less than two casts doubt on the validity of variance as a measure of variability. Nevertheless, when

Table 2. Characteristic Exponent Estimates

Commodity	Number of Contracts	Characteristic Exponent Estimates[a]			
		$\alpha = 1.0$	$1.0 < \alpha \leq 1.5$	$1.5 < \alpha < 2$	$\alpha = 2.0$
Wheat	50	0	7(14%)	31(62%)	12(24%)
Soybeans	70	0	8(11%)	47(67%)	15(22%)
Live Cattle	60	0	8(13%)	34(15%)	18(30%)

[a] The estimated characteristic exponent, α, can range from 0 to 2. The value of $\alpha = 2$ is associated with a normal distribution.

compared to the results by Mann and Heifner where the proportions of normal distributions were 20, 1, and 0 percent for wheat, soybeans, and live cattle, respectively, these results confirm the recent move toward normality as suggested by the kurtosis and R/S tests. These results relative to the characteristics exponent are partially confirmed in the Gordon study who used a smaller, but more recent data set. He did not find any alphas less than one, similar to here, but only 7 percent of his alphas for wheat, cattle and soybeans were equal to 2.0, less than our 30 percent.

This move toward normality is even more dramatic if we analyze just the 1976-1982 period, removing 1973-1975 from the data set. Wheat is an exception with regard to the kurtosis test where only 26 percent of the contracts are normal. But, normality is accepted for over 74 percent of the soybean contracts during 1976-1982 while 67 percent of the live cattle contracts are normal. With respect to the ratio of the range to the standard deviation, normality is accepted over 69, 90 and 72 percent of the contracts during this period for wheat, soybeans and live cattle respectively. One explanation for these different results may be that the proportion of the price changes which were limit moves were much higher for all three commodities during 1973-1975 compared with 1976-1982.

An alternative to normality is the class of non-normal stable distributions. In order to find evidence of stability, a test proposed by Fama and Roll which consists of estimating the characteristic exponent for sums of non-overlapping observations is performed on 18 selected non-normal contracts. Although the results are difficult to generalize, there is reasonable evidence across the alphas computed to suggest non-stability. This suggests that the alternative hypothesis of stability may not describe accurately the futures price change distributions when normality is rejected.

Others have suggested that non-normality, especially leptokurtosis, is the result of heteroscedasticity. A Bartlett test for homogeneity of the variances is conducted on selected normal and non-normal contracts for the period 1976-1982. To conduct this test, each selected contract is divided into four subsets and the variance of each subset is calculated. The Bartlett test examines if these subset variances are equal within a contract. But first, the Bartlett test assumes normality of the samples, and tests show that about 85 percent of the subsets are normal, regardless whether the contract itself is normal or not. In general, those contracts tested displayed homogeneous variances 83 percent of the time, rejecting the hypothesis that non-normality is the result of heteroscedasticity. Gordon divided the contract data into 2 month periods and found the null hypothesis of normality could not be rejected. Seasonality and maturity may account for this. That is, futures prices may be normally distributed, but that the variance is changing periodically.

Tests of Independence

Turning Point. This test consists of comparing the number of turning points to the expected number in a random series. The test is performed on futures price changes, not actual futures prices as was done by Mann and Heifner. A Student's t-test can be used in the comparison.

In two-thirds of the cases, the number of observed turning points in less than the expected number, indicating that price changes are more likely to be characterized by trend rather than reversals. Nevertheless, table 3 shows that the hypothesis of randomness is rejected only 13 percent of the time. All but one of the non-random contracts had less turning points than expected. Also, 21 of the 23 non-random contracts were in the 1973-1975 period. These results indicate that futures prices generally adjust efficiently to new information, especially since 1975. The characteristic of highly random prices in recent data is confirmed by Gordon.

Table 3. Turning Point and Phase Length Tests

Commodity	Number of Contracts	Number (Percent) of Cases Significantly Different from Zero	
		Turning Point	Phase Length
Wheat	50	3(6%)	5(10%)
Soybeans	70	6(9%)	6(9%)
Live Cattle	60	14(23%)	15(25%)

Phase Length. This test compares the number of phases of given length to the expected number in a random series. Phase lengths of one, two and three are tested on the price changes using a chi-squared test. Table 3 shows that only 14 percent of the contracts exhibit non-random behavior. Again, a majority of the non-random contracts, 23 out of 26, were in the period 1973-1975. As with turning points, the number of observed phases is usually less than the expected number, indicating trends are more likely to be found than reversals in futures price change series. In any event, more recent contracts are highly characterized by randomness.

Implications and Conclusions

The above results demonstrate both a move toward normality and independence in recent years. While a significant level of leptokurtosis still exists in the data, the number of contracts exhibiting such characteristics have decreased substantially relative to the results reported by Mann and Heifner for the period 1959-1971. Also, this trend toward normality is even more apparent when we drop the data prior to 1976 from our sample set. Normality can also be achieved by disaggregating the contracts into subsets, and these subsets display homogeneous variances, especially since 1976.

The independence tests cannot be compared directly with Mann and Heifner since different procedures were followed in the two studies, but a high proportion of contracts exhibit random behavior, and almost all the non-random contracts fall in the 1973-1975 period. This shows that futures prices adjust efficiently to new information, especially from 1976 and beyond. One characteristic of the data during 1973-1975 relative to more recent times, 1976-1982, is that a much higher proportion of limit moves occurred during the earlier years. For wheat and soybeans, 7 percent of the price changes during 1973-1975 were limit moves, while only 1 percent of the price changes were limit moves in the latter period. For cattle, the drop was from 2 percent to nearly 0 percent. These changes in limit moves could help explain the observed trends toward normality and independence.

One explanation for non-normality existing could be heteroscedasticity in the futures price changes. The small amount of heteroscedasticity found is not sufficient to explain exclusively the non-normality, nor can the theory of stable distributions. In fact, the results support Neftci's hypothesis that normality may be related to the number of limit moves. Neftci demonstrated that the limit move restriction prevents all the relevant information from being incorporated into the futures prices and destroys the martingale nature of futures price changes. Consequently, since the normality assumption in futures price changes arise from the central limit theorem which assumes randomness in the underlying variables, one would expect that when the market is inefficient futures price changes will be non-normal. This is confirmed in this study where during 1976-1982, a period of few limit moves, normality was in general achieved. Non-normality can also be reduced by shortening the sample periods.

The departure from normality may not always be due to leptokurtosis as commonly found. Non-normality can emerge from the presence of lesser observations at the tail than expected, called platykurtosis. This could be associated with limit moves which prevent prices from changing the full extent relative to the information generating the move.

These results have implications for option pricing. Black's option pricing formula assumes log-normality and constant variance. The recent move toward normality in commodity price changes suggest that such formulas may adequately represent the actual value of the commodity traded on option markets. Also, hedgers who rely on the portfolio theory of hedging typically maximize revenue given risk (variance). These results suggest that the variance exists and is finite, allowing portfolio models to be used optimally.

Finally, the move toward more independent or random movements of futures prices may suggest that technical analysis of futures prices has become less effective. If futures price changes follow a random walk, as suggested by the recent data, one cannot consistently use past prices to predict future price changes accurately. Technical analysis schemes based on price trends and periodic price behavior will become less effective for trading.

Literature Cited

Bachelier, L. "Theory of Speculation." in The Random Character of Stock Market Prices. P. H. Cootner, Ed., MIT, Cambridge, MA, 1964.

Black, F. "The Pricing of Commodity Contracts." Journal of Financial Economics. 3(1976):167-179.

Clark, P. K. "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices." Econometrica. 41(1973):135-155.

David, H. A., H. O. Hartley, and E. S. Pearson. "The Distribution of the Ratio, in a Single Normal Sample Range to Standard Deviation." Biometrika. 41(1954):482-493.

Fama, E., and R. Roll. "Parameter Estimates for Symmetric Stable Distributions." Journal of the American Statistical Association. 66(1971):331-338.

Gordon, D. J. "The Distribution of Daily Changes in Commodity Futures Prices." USDA, Economic Research Service, Technical Bulletin Number 1702, July 1985.

Grauer, L. F. "Equilibrium in Commodity Futures Markets: Theory and Tests." Ph. D. dissertation, Stanford University, 1977.

Helms, B. P., and T. F. Martell. "An Examination of the Distribution of Commodity Price Changes." Working Paper Series, Columbia University Center for the Study of Futures Markets, CFSM-76, 1984.

Houthakker, H. "Systematic and Random Elements in Short Term Price Movements." American Economic Review. 51(1961):164-172.

Kamara, A. "Commodity Futures Markets: A Survey." Journal of Futures Markets. 3(Fall 1982):261-294.

Lee, C. F., and R. M. Leuthold. "Investment Horizon, Risk, and Return in Commodity Futures Markets: An Empirical Analysis With Daily Data." Quarterly Review of Economics and Business. 23(1983)6-18.

Leuthold, R. M., and W. G. Tomek. "Developments in the Livestock Futures Markets." in Livestock Futures Research Symposium. R. M. Leuthold and P. M. Dixon, Eds. Chicago Mercantile Exchange, Chicago, 1979, pp. 39-69.

Mann, J. S., and R. G. Heifner. "The Distribution of Short-Run Commodity Price Movements." USDA, Economic Research Service Technical Bulletin 1536, March 1976.

Mandelbrot, B. "The Variation of Certain Speculative Prices." Journal of Business. 36(1963):394-419.

Literature Cited (Continued)

Neftci, N. S. "Some Econometric Problems in Using Daily Futures Price Data." Working Paper Series, Columbia University Center for the Study of Futures Markets, CFSM-55, 1982.

Rocca, L. H. "Time Series Analysis of Commodity Futures Prices." Ph.D. dissertation, University of California, Berkeley, 1969.

Samuelson, P. A. "Proof That Properly Anticipated Prices Fluctuate Randomly." Industrial Management Review. 6(1965):41-49.

Stevenson, R. A., and R. M. Bear. "Commodity Futures: Trends or Ransom Walks." Journal of Finance. 25(1970):65-81.

Working, H. "A Theory of Anticipatory Prices." American Economic Review - Papers and Proceedings. 48(1958):188-199.