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Vector Autoregression Forecasting Models:
Suggested Improvements

by

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ABSTRACT

Two methods for building vector autoregression forecasting models are proposed. The first allows exclusion of intermediate lags; the second considers the effects of jointly entering lags from different series into an equation. Live hog market models are developed and out-of-sample forecasting results suggest both methods have merit.

Introduction

The univariate autoregressive moving average (ARMA) model has been shown to be quite successful in the forecasting of time series. The main problem with this type of model is that it fails to take into account information about other variables which may have a strong influence on the series of interest. Vector autoregression (VAR) models represent one approach towards incorporating this potentially pertinent information. However, researchers have typically found that unrestricted VAR models do not forecast well (e.g., Nerlove, et al, Kling and Bessler). One reason for this may be that they are over-parameterized. The subsequent variability associated with the parameter estimates contributes to the lack of precision in the forecasts.

Attempts to overcome this degrees-of-freedom problem fall into two broad categories: Bayesian estimation of the parameters (Litterman) and the exclusion of explanatory variables [Tiao and Box, Hsiao (1979), Caines, et al]. The first approach appears to be difficult to implement. Indeed, if a VAR is viewed as the direct estimation of a reduced form model, it may be argued the Bayesian approach is inappropriate. While economic theory may be used to form priors on structural coefficients, its value for reduced form coefficients is questionable.

The purpose of this paper is to suggest two new techniques for building VAR forecasting models, both of which belong in the exclusion of variables category. After detailing the proposed modeling procedures, applications are made to the live U.S. hog market. The final section of the paper offers some concluding remarks.

VAR Forecasting Models

Ignoring deterministic components (trends, constants, etc.), the unrestricted form of a VAR is given by

$$Y(t) = \phi(B)\ Y(t) + \varepsilon(t) \tag{1}$$

where $Y(t)$ is an $m \times 1$ vector of observations on the m series at time t, $\phi(B)$ is an $m \times m$ matrix of polynomials in the lag operator B [defined by $B^k\ X(t) = X(t - k)$], and $\varepsilon(t)$ is an $m \times 1$ vector of error terms. The model is unrestricted in that the orders of all of the polynomials in $\phi(B)$ are the same and none of the coefficients in the polynomials are set to zero prior to estimation.

If the polynomials in (1) are of order Q, each equation necessitates the estimation of mQ parameters. With limited sample sizes, even moderate values of m and Q may result in few degrees of freedom. Tiao and Box attempted to overcome this problem by deleting from each equation those variables with statistically insignificant coefficients. The model is then reestimated using the method of seemingly unrelated regressions (SUR). Potentially, variables are again deleted from equations and this process repeats until all of the explanatory variables have significant coefficients.

Applications of the Tiao-Box technique have failed to yield good out-of-sample forecasts (e.g., Brandt and Bessler, Kling and Bessler). One serious drawback to this method is that a set of variables may be statistically significant, but each of the individual variables in the set are not. This is especially likely in a VAR context, where multicollinearity is usually a factor.

A second approach to decreasing the profligacy of VAR models has involved treating each equation in the model individually. Right-hand-side variables are chosen based on the minimization of a criterion which is a function of the number of explanatory variables and the estimated variance of the error term. A problem with this approach is the number of regressions called for may be quite large. If there are m series and Q is the maximum lag length considered, 2^{mQ} regressions may have to be performed for each equation.

Hsiao (1979) suggested one way to overcome this problem is to sequentially determine the "best" lags associated with each series. The first step in his procedure involves ordering the series. At the start of the j-th stage in the procedure, regressions involving previously chosen right-hand-side variables and every one of the possible ways to choose lags of the j-th ordered series are performed. The stage terminates with the determination of which of the 2^Q regressions yielded the best criterion. The associated right-hand-side variables are then retained in the model and stage j + 1 begins. A total of $m2^Q$ regressions may have to be performed for each equation, representing a considerable savings over 2^{mQ}. The computational burden could be further eased by refusing to "zero out" intermediate lags, resulting in only mQ regressions per equation.

A difficulty with Hsiao's procedure lies in the ordering of the series. The lags associated with each series will, in general, depend on when they are introduced into the equation. Caines, et al suggested a procedure for automatically ordering the series. First, for every pair of series (X and Y) they construct a bivariate AR model using P lags. P is chosen to minimize the multivariate final prediction error (MFPE)

$$MFPE(P) = [(1 + a/n)\ (1 - a/n)]^m \det(\hat{\Sigma}_P\ ), \tag{2}$$

where a is the number of explanatory variables in each equation, n is the sample size, m is the number of equations in the model, and $\hat{\Sigma}_p$ is the sample covariance matrix of the residuals from the model of order P. The causal relationships between X and Y are then established using a series of likelihood ratio tests.

The Caines, et al procedure then follows the lines suggested by Hsiao, with some minor modifications. For each series, X, the first stage consists of finding the "best" lags of X itself to include in the equation. This is done using the criterion (2), with m = 1. Lags of any series found to cause X are then introduced into the equation, one series at a time, again using (2) to determine the "best" lags. The causal series are introduced in decreasing order of the reciprocals of their MFPEs. Once every equation in the model has gone through this process, the entire system is reestimated using SUR.

The Caines, et al approach suffers from several shortcomings. First, the criterion given by (2) is the multivariate version of Akaike's Final Prediction Error (Akaike, 1969), which Shibata has shown to be an inconsistent estimator of the lag length for an AR process. Second, the determination of causality in a multivariate framework is an extremely difficult problem [see Hsiao (1982)]. Empirically established causal relationships between variables may change as more variables are added to the model. This brings into question the appropriateness of using bivariate AR models to determine whether lags from one series should be used as right-hand-side variables in a regression of the other series. Third, as already noted, the lags chosen for a particular series depend on the order in which the series is introduced into the equation. Using the reciprocal of MFPE to establish the ordering is ad hoc. In fact, noting that COV (aX, bY) = a b COV (X,Y), it is easily seen that the MFPE ordering is unit dependent. Finally, the sequential manner in which series are introduced into the equation may cause problems.

Proposed Alternatives

Most of the criticisms of earlier efforts could be met by treating each equation individually and minimizing, with respect to p_{ij}, the criterion

$$\ln(\hat{\sigma}^2_{p_{11}, \ldots, p_{Q1}, \ldots p_{1m}, \ldots, p_{Qm}}) + (\sum_{j=1}^{m} \sum_{i=1}^{Q} p_{ij}) \ln(n)/n, \quad (3)$$

where $p_{ij} = 1$ if lag i of series j appears as an explanatory variable in the equation, else $p_{ij} = 0$ ($i = 1, \ldots, Q$; $j = 1, \ldots, m$).

Two advantages to using (3) as opposed to earlier criterions should be noted. First, the criterion in (3) is the multivariate version of the Bayesian Information Criterion proposed by Akaike (1977), which has been shown to yield asymptotically consistent estimates of the orders of an ARMA process (Hannan and Rissanen). Second, the minimization of (3) explicitly takes into account the joint effects of variables.

As previously noted, the problem with (3) is that it could entail a large number of regressions (2^{mQ}). Two compromises are proposed in this paper. The first approach specifically addresses the idea that there may be problems associated with the failure to allow for the possibility of excluding intermediate lags. At the start of each stage of this approach, the right-hand-side variables chosen from prior stages remain in the equation. The remaining series are then searched to find the one series (and its associated lags) which gives the most improvement in the criterion (3). While this approach still has the problems associated with the sequential way series enter an equation, the ordering of the series is determined by the magnitudes of their effects on the criterion (3). This appears to offer an improvement over the Caines, et al approach to the ordering question.

The feasibility of the suggested approach depends on the computational burden. At the start of stage j there are r = m - (j -1) series which have not entered the model. The maximum number of regressions during stage j is then given by $r(2^Q - 1)$, and the maximum number of regressions for one equation is $(2^Q - 1) \sum_{i=1}^{m} i$.

The second approach proposed in this paper addresses the idea that the chosen lag lengths may vary if two or more series are introduced at the same time. At the start of each stage of this approach, the right-hand-side variables chosen from prior stages remain in the equation. Those series not already included are searched to find the subset of series and the lag (K) such that entering these series with lags 1, ..., K gives the most improvement in the criterion (3).

At the start of stage j there are r series which have no lags appearing as right-hand-side variables. For every lag length (1, ..., Q), 2^r-1 subsets of series have to be checked. Thus, stage j requires at most $(2^r-1)Q$ regressions, and the maximum number of regressions for one equation is $Q \sum_{i=1}^{m} (2^i-1)$.

The feasibility of this latter procedure is due to the constraining of additional series to enter the equation with the same number of lags during each stage. However, this could cause severe problems. Consider the case of two series which in the "true" model have lags 3 and 4 present. There is a good chance this procedure would enter these series jointly with either both having 3 or both having 4 lags. In order to address this problem, the above procedure is modified so that, if at any stage more than one series is entered into the model, the whole procedure is restarted from that point, constraining the number of series entering the equation at that stage to be no more than one less than was previously found.

It should be noted that the actual number of regressions required for the proposed techniques may be much less than the reported maximums. Knowledge of the current "best" criterion and the minimum possible $\hat{\sigma}^2$ (all possible variables used as explanatory variable) allows computation of the maximum number of variables to be considered. Further savings in computations can be achieved using the subset regression procedure of Hocking and Leslie.

Empirical Applications

The model building techniques proposed in this paper have been applied to the live U.S. hog market. Seven series were considered (see Kaylen for a discussion of the relevance of these series): sow farrowings (SF), hog slaughter (HS), live hog price (HP), feed price (FP), retail beef price (RB), retail pork price (RP), and the log of total disposable personal income (IN). The income and price series were deflated by CPI. Quarterly data was used; the estimation period covered 58-1 through 80-4 and 16 out-of-sample forecasts were generated for 81-1 through 84-4. The maximum lag length considered was eight quarters, corresponding to the maximum lag length Bessler and Binkley found for some of the series using univariate AR models.

The theory behind the criterion used to choose lags requires the series of interest to be stationary. To help ensure this, sow farrowings and hog slaughter were seasonally differenced and the other series were first differenced.

The lags included in each equation for both models are shown in Table 1. Two features stand out. First, for every series having a lag appearing in both equations for a dependent variable, VAR-1 (excluding intermediate lags) used higher-order lags. Second, for all of the dependent variables other than hog slaughter, VAR-1 involved more right-hand-side variables.

Table 2 reports summary out-of-sample forecasting statistics for the two models developed in this paper, an unrestricted, six lag VAR (Kaylen), and ARIMA models (Kaylen). The statistics are only given for the three variables of primary interest: sow farrowings, hog slaughter, and hog price. For the hog slaughter series, all three measures of performance rank the models from best to worst: VAR-1, VAR-2, VAR-3, ARIMA. For sow farrowings, three of the models can be ranked: VAR-1, VAR-2, ARIMA. While VAR-3 performed the worst in terms of cardinal measures [mean absolute deviation (MAD) and root mean square error (RMSE)], it performed the best using the ordinal measure (direction). The statistics for hog price forecasts yield mixed rankings. While the ARIMA model performed the best by cardinal measures, it performed the worst in forecasting direction. Of the VAR models, VAR-1 did the best in terms of MAD and RMSE, but the worst in terms of direction.

Table 3 compares hog prices forecasts generated by the models developed in this study with other forecasts which have appeared in recent literature. By all three measures of performance, the experts

(F3) did the best, although VAR-1 did as well in forecasting the direction of hog prices and had only slightly higher MAD and RMSE (the differences were only 0.6 and 0.3 percent, respectively). The VAR model developed along the lines suggested by Tiao and Box performed the worst using all three performance measures.

Concluding Remarks

This paper has proposed two new methods for building VAR forecasting models. Each method addresses a potentially severe shortcoming of the Caines, et al approach. The first technique allows for the exclusion of intermediate lags, while the second allows lags from different series to jointly enter an equation.

Both VAR model building techniques proved to be quite feasible. The first method could have entailed a maximum of 49,980 regressions, but only 7,686 were actually needed. Most of these involved a small number of right-hand-side variables, so the cost using Purdue University's CDC computers was low (under $3). In comparison, the second method was more expensive (about $16). For the large problem considered in this paper (seven series, a maximum of eight lags), the computational burdens appear reasonable.

The empirical results suggest both of the proposed model building techniques result in forecasting models which are competitive with other forecast generating mechanisms. In particular, allowing for the exclusion of intermediate lags appears to be the most promising avenue for improving the forecasting ability of VAR models.

TABLE 1. Lags Included in Hog Market Vector Autoregressions. [a]

Dependent variable	Model	SF	HS	HP	RP	FP	IN	RB
		Lags						
SF	VAR-1	1,3	-	-	-	7	-	4,8
SF	VAR-2	1,2	1,2	-				
HS	VAR-1	1,2	3,4,5	1,2,3,5,	1,2,4,5	-	-	8
HS	VAR-2	1,2	1,2,3,4,5	1,2	1,2,3,4,5	1	-	-
HP	VAR-1	1,3,5	-	5	1	1,6	-	-
HP	VAR-2	1	-	1,2	1	-	1	-
RP	VAR-1	1	5	1,3,4,5	1,2	6	1	4
RP	VAR-2	1	-	1,2,3,4,5	1	-	1	-
FP	VAR-1	-	7	4	4	-	-	6
FP	VAR-2	-	1	-	1	-	-	1
IN	VAR-1	-	-	-	-	4	-	4,6
IN	VAR-2	-	1	-	-	-	-	-
RB	VAR-1	-	-	-	-	1,6	-	5
RB	VAR-2	-	-	-	-	1	1	-

[a] The lags for VAR-1 were chosen using the first technique proposed in this paper (excluding intermediate lags). The lags for VAR-2 were chosen using the second technique (jointly entering series into an equation).

TABLE 2. Summary Out-of-Sample Forecasting Statistics [a]

Dependent Variable	Statistic [b]	VAR-1 [c]	VAR-2 [d]	VAR-3 [e]	ARIMA [f]
SF	MAD	4.24	4.61	4.91	4.87
	RMSE	4.91	5.73	6.33	5.80
	Direction	87.50	87.50	93.75	81.25
HS	MAD	2.28	2.74	3.14	4.05
	RMSE	2.85	3.48	3.82	5.07
	Direction	100.00	100.00	87.50	75.00
HP	MAD	8.99	11.44	10.06	7.61
	RMSE	10.50	12.26	14.77	9.05
	Direction	81.25	87.50	81.25	75.00

[a] All models were initially estimated using 58-1 through 80-4 data. The estimates were updated after each one-quarter-ahead forecast. Forecasts were generated for 81-1 through 84-4.

[b] The statistics are defined as follows:

MAD - Mean Absolute Deviation as a percentage of the actual mean for 81-1 through 84-4.

RMSE - Root Mean Square Error as a percentage of the actual mean for 81-1 through 84-4.

Direction - The percentage of times the model correctly forecast the direction of movement. If F_t denotes the forecast made at time t for time t + 1 and A_t denotes the actual value at time t, the direction is correctly forecast if

$$\mathrm{sgn}(F_t - A_t) = \mathrm{sgn}(A_{t+1} - A_t).$$

[c] The lags for this model were chosen using the first technique proposed in this paper (excluding intermediate lags).

[d] The lags for this model were chosen using the second technique proposed in this paper (jointly entering series into an equation).

[e] The forecasts for this model were generated by an unrestricted, six lag vector autoregression (Kaylen).

[f] These forecasts were generated by ARIMA models (Kaylen).

TABLE 3. Comparison of Hog Price Forecasting Mechanisms [a]

Date	Actual	VAR-1 [b]	VAR-2 [c]	F1 [d]	F2 [e]	F3 [f]	F4 [g]
81-1	15.63	18.53	17.88	18.15	17.04	19.19	17.90
-2	16.21	19.16	18.79	14.87	15.42	16.35	14.88
-3	18.22	20.14	19.78	17.18	15.81	18.97	16.72
-4	15.20	15.93	18.46	20.62	18.14	17.12	16.01
82-1	17.05	16.55	15.38	17.87	18.87	17.17	15.33
-2	19.69	19.87	18.63	17.50	18.74	17.79	17.80
-3	21.21	21.95	22.21	21.90	19.52	20.53	18.77
-4	18.82	20.92	20.94	22.38	20.82	20.83	20.49
MAD [h]		8.45	10.93	12.39	9.86	7.83	9.58
RMSE [i]		10.25	11.66	15.04	10.59	9.97	9.97
Direction [j]		87.50	87.50	50.00	62.50	87.50	75.00

[a] All reported forecasts are for one quarter ahead. The F1 through F4 forecasts were originally reported in nominal terms. They were deflated to facilitate comparison with the forecasts generated by the models developed in this study.

[b] The lags for this model were chosen using the first technique proposed in this paper (excluding intermediate lags).

[c] The lags for this model were chosen using the second technique proposed in this paper (jointly entering series into an equation).

[d] Forecasts from a VAR model using the Tiao-Box method (Brandt and Bessler).

[e] Forecasts from a single equation econometric model (Brandt).

[f] Expert forecasts (Brandt).

[g] Forecasts from a ARIMA model (Brandt).

[h] Mean Absolute Deviation as a percentage of the actual mean for 81-1 through 82-4.

[i] Root Mean Square Error as a percentage of the actual mean for 81-1 through 82-4.

[j] The percentage of times the model correctly forecast the direction of movement during 81-1 through 82-4. See footnote b of Table 2.

References

Akaike, H. "Fitting Autoregressive Models for Prediction." Annals of the Institute of Statistical Mathematics 21 (1969): 243-7.

________. "On Entropy Maximisation Principle." In Application of Statistics (P. R. Krishnaiah, ed.). Amsterdam: North Holland, 1977.

Brandt, Jon A. "Price Forecasting, Futures Markets, and Risk." Paper presented at the Midwest Economics Association Meeting. St. Louis, MO (April 7, 1983).

Brandt, Jon A. and David A. Bessler. "Forecasting with Vector Autoregressions versus a Univariate ARIMA Process: An Empirical Example with U.S. Hog Prices." North Central Journal of Agricultural Economics 6 (1984): 29-36.

Caines, P. E., C. W. Keng, and S. P. Sethi. "Causality Analysis and Multivariate Autoregression Modelling with an Application to Supermarket Sales Analysis." Journal of Economic Dynamics and Control 3 (1981): 267-98.

Hannan, E. J. and J. Rissanen. "Recursive Estimation of Mixed Autoregressive-Moving Average Order." Biometrika 69 (1982): 81-94.

Hocking, R. R. and R. N. Leslie. "Selection of the Best Subset in Regression Analysis." Technometrics 9 (1967): 531-40.

Hsiao, Cheng. "Autoregressive Modeling and Causal Ordering of Economic Variables." Journal of Economic Dynamics and Control 4 (1982): 243-59.

________. "Autoregressive Modeling of Canadian Money and Income." Journal of the American Statistical Association 74 (1979): 533-60.

Kaylen, Michael S. "Time Series Modeling of the Live Hog Market: Forecasts and Analysis of Dynamics." Ph.D. thesis, Purdue University, December 1985.

Kling, John L. and David A. Bessler. "A Comparison of Multivariate Forecasting Procedures for Economic Time Series." Paper presented at the Fourth International Symposium on Forecasting. London, England, July 1984.

Litterman, R. "A Bayesian Procedure for Forecasting with Vector Autoregressions." MIT Working Paper, 1980.

Nerlove, Marc, D. M. Grether, and J. L. Carvalho. Analysis of Economic Time Series: A Synthesis. New York: Academic Press, 1979.

Shibata, Ritei. "Selection of the Order of an Autoregressive Model by Akaike's Information Criterion." Biometrika 63 (1976): 117-26.

Tiao, G. C. and G. E. P. Box. "Modeling Multiple Time Series with Applications." Journal of the American Statistical Association 76 (1981): 802-16.