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Joint Products in the SWOPSIM Modeling Framework

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ABSTRACT

A consideration of joint products is important in the trade modeling currently underway at the Economic Research Service of the U.S. Department of Agriculture. Joint products are especially important in the modeling of the dairy and oilseed sectors. This report explains how recent work in multioutput production theory can be used to model these sectors in the SWOPSIM modeling framework. The advantage of applying theory is improved consistency of model parameter values for evaluating trade liberalization scenarios.

Keywords: economic model, production theory, international trade, trade liberalization

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INTRODUCTION

There are two interpretations of jointness in production theory. Both deal with multiproduct production processes. The first interpretation refers to jointness in input usage. If the cost of producing a commodity is affected by the level of production of other commodities, the technology is said to be joint in inputs. The second interpretation refers to a single production process which yields more than one output. In this case, quantities of two or more outputs are technically interdependent.

An example of the first type of jointness would be the farm in which the farmer produces more than one crop utilizing the same factor inputs such as farm machinery or farm labor. The farmer's profitability of producing wheat may depend on the level of barley he produces. In formal economic modeling, the wheat supply equation will include the price of barley as an argument. Examples of the second type of jointness would be the production of soybean oil and meal or butter and skim milk. If one of the products is produced (oil or butter), then the other product (meal or skim milk) must as a consequence be produced as well. It is possible to think of the second type of jointness as a special case of the first. The technically related products are produced from a raw agricultural product: in the example, either soybeans or dairy milk. The related products share a common input. The cost of producing one cannot be separated from the cost of producing the other.

Joint products are important in the trade modeling currently underway at the Organization for Economic Cooperation and Development (OECD) and the U.S. Department of Agriculture (USDA). OECD is currently trying to incorporate an input sector into its Ministerial Trade Mandate (MTM) model (9).[1] Researchers are building on the work of Ball, Hertel, Lopez and others to expand the MTM model (1,4,5). Although much of the current USDA work has been focused on determining support levels and incorporating them into the model, joint product relationships have been examined closely to yield better (that is, more consistent) parameter estimates upon which the model is built. Applications of multioutput production theory have been particularly useful to the modeling of the dairy and oilseed

[1] Underscored numbers in parentheses refer to sources listed in the References.

sectors. The purpose of this report is to show how these joint product relationships have been handled, and how the USDA model can be expanded to include factor inputs.

TRADE ANALYSIS IN SWOPSIM FRAMEWORK

SWOPSIM is a framework used to create static world policy simulation models (6). The models created by the SWOPSIM procedure are located in spreadsheets and are modified and solved as spreadsheets. SWOPSIM models are designed to simulate the effects of policy changes on production, consumption, and trade. The framework allows the construction of single-commodity or multicommodity world trade models.

SWOPSIM models are characterized by an economic structure that includes constant elasticity supply and demand equations and summary policy measures. Trade is the difference between supply and demand. Models in the SWOPSIM framework use two measures to incorporate the effects of government policies on agricultural producers and consumers. The producer subsidy equivalent (PSE) is a measure of the level of the subsidy that would be necessary to compensate producers in terms of income for removing government support under current programs. The consumer subsidy equivalent (CSE) is a measure of the level of subsidy that would have to be paid to consumers to compensate them for removing agricultural programs.

Within the Economic Research Service (ERS) of the U.S. Department of Agriculture, researchers have constructed a world policy model of trade (the Trade Liberalization or TLIB model) in the SWOPSIM framework (7). The purpose of the model is to analyze probable effects of liberalization resulting from the Uruguay Round of multilateral trade negotiations under the auspices of the General Agreements on Tariff and Trade (GATT). The TLIB model consists of 22 commodities and comprises 36 countries/regions. The livestock commodities include: beef and veal, pork, mutton and lamb, poultry meat, poultry eggs, milk, butter, cheese, and dairy (skim) powder. The crops include: wheat, corn, rice, other coarse grains, soybeans, other oilseeds, cotton, sugar, and tobacco. Other commodities include: soybean meal, soybean oil, other oilseed meals, and other oilseed oils.

Meal and oil produced from soybeans and other oilseeds are joint products. They are processed from a raw agricultural product that can be traded internationally. Butter and skim milk are joint products. They are processed from fluid milk. In addition, cheese and other dairy products depend on raw milk as an input, but there is not the same type of technical relationship between them as between butter and skim milk. The dairy sector therefore has both types of jointness in its structure.

Jointness involves two types of relationships. The first is between the base product (such as soybeans) and the derived products (meal and oil). The second type of relationship is between the derived products. The first relationship can be illustrated graphically. It is essentially a relationship between a good that is an intermediate input into the production of another good. The second relationship is slightly more involved: it will be dealt with in the next section.

Figure 1 illustrates the relationship between a base product and a processed derivative product. In this example, the country exports both the base product X and a derived product Y. It is assumed that the country has market power in trading X, but has none in trading Y[2]. Both producing sectors receive government subsidies (PSE's), but consumers are not directly subsidized. The subsidy-laden supply curves are drawn as dashed curves parallel to the true (solidly drawn) supply curves. The distance between the solid and dashed curves represents the initial unit subsidy that each producing sector receives from the government. The quantity supplied is determined at the point at which the subsidy-laden curve intersects the world price curve (p_w^0 for X and $\hat{p}_w$ for Y). The domestic producer price is the world price plus the unit value of the subsidy (p_p^0 for X and $\hat{p}_p$ for Y). Consumers purchase at the world price. Trade is the difference between production and consumption. It is represented by T_x^0 and T_y^0 for X and Y, respectively.

Producer subsidies are removed in both sectors. In market Y where the country has no world market power, the domestic producer price drops to the world level. The lower producer price reduces the quantity of Y supplied to the market, and consequently reduces the derived demand for the base product X. Its demand curve shifts from D_0 to D_1 . As the subsidy is removed to producers of X, the producer price falls and the quantity supplied is reduced. For convenience, it is assumed that the reduction in quantity demanded due to the demand curve shift is less than the reduction in quantity supplied. Hence the net effect is to reduce the country's excess supply on the world market. Because this country is assumed to have world market power, the world price of X increases to p_w^1. This increase means that the domestic producer price does not drop by the full amount of the subsidy, and that domestic consumers of X must pay a higher purchase price. Because X is the primary input in the production of Y, the higher consumer price of X shifts Y's supply curve from S_0 to S_1. New trade amounts for X and Y are shown as T_x^1 and T_y^1, respectively.

Letting p_c and $\hat{p}_c$ represent the respective consumer prices of X and Y, the supply and demand equations can be represented as follows:

$$D_x = D_x(p_c, \hat{p}_p) \qquad (1)$$

$$S_x = S_x(p_p) \qquad (2)$$

$$D_y = D_y(\hat{p}_c, \text{income}) \qquad (3)$$

$$S_y = S_y(\hat{p}_p, p_c) \qquad (4)$$

In the original version of SWOPSIM, equations 1 and 4 were misspecified. They were as follows:

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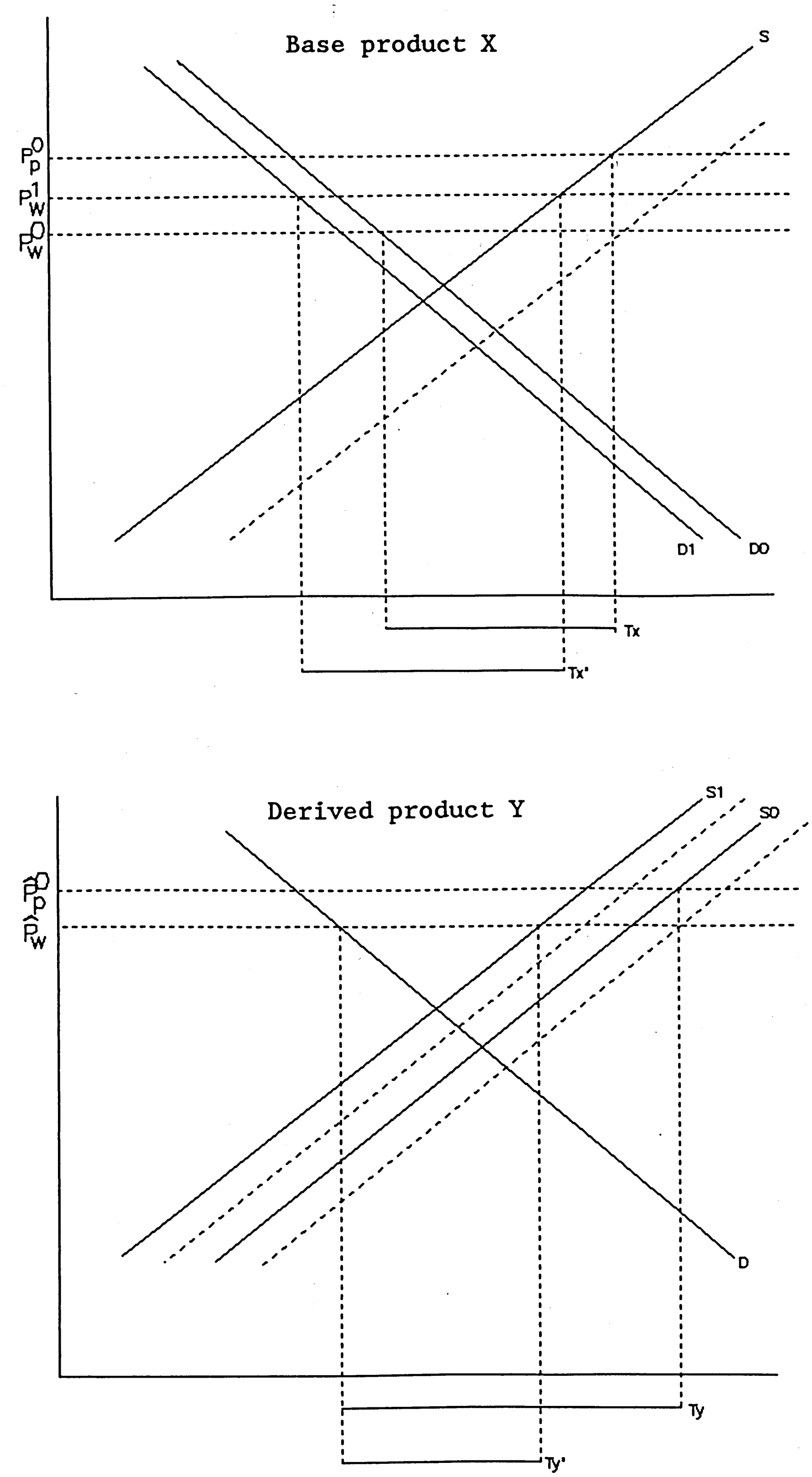
[2] Producers of X are assumed not to collaborate to optimize joint profits based on a recognition of world market power. Each producer acts as a price-taker, although collectively, producers influence the world price.

Figure 1 -- PSE removal for joint products



$$D_x = D_x(\, p_c,\ \hat{p}_c)$$

$$S_y = S_y(\, \hat{p}_p,\ D_x) \quad \text{and} \quad \frac{\delta S_y}{\delta D_x} = \frac{S_y}{D_x}$$

The first misspecification resulted when the cross price elasticity was entered into appropriate space in the demand-elasticity block of the country spreadsheet. Without additional information, the SWOPSIM program assumed that X and Y were substitutes in consumption instead of serving in an input-output relationship. The second misspecification is not so readily apparent. The quantity demanded of the base product enters as an argument instead of the consumption price of the base. The elasticity with respect to this quantity is unity. The problem with the specification is that it puts restrictions on implied price elasticities which are inconsistent with production theory. The elasticity of derived product supply and base product demand with respect to the base product consumer price are assumed to be equal. This specification will be shown to be theoretically incorrect in the next section.

These problems are solved by requiring the insertion of an additional column in the SWOPSIM master file to the right of the rest-of-world country column. Figure 2 shows a demonstration master file with this column under the "F" heading. An "IN" in the appropriate commodity row indicates that the commodity is a base product from which joint products are derived. An "OU" indicates the corresponding commodity is a joint product derived from a base product in the model. The information in this column guarantees the specifications shown in equations 1 and 4.

Figure 2 -- Master file for SWOPSIM demonstration model

	A	B	C	D	E	F	G
1	DEMO		SWOPSIM				|
2							
3		US	EC	RW	-	-	-
4							
5	BF	D	D	D	IU		
6	DM	D	D	D	IU	IN	NT
7	DB	1	1	1		OU	
8	DC	1	1	1		OU	
9	DP	1	1	1		OU	
10	CN	1	1	1	I		
11	CG	1	1	1	I		
12	SB	1	1	1		IN	
13	SM	1	1	1	I	OU	
14	^	.	.	.			+

THEORY OF JOINT PRODUCTION

Economic theory that deals with multiproduct output can be useful in guaranteeing parameter consistency in multicommodity agricultural trade models like TLIB. This section describes some salient features of the theory as they relate to modeling in the SWOPSIM framework. The primary source of this theory is Sakai (8). Chambers provides an excellent general discussion of the issues as well (2). In the next section, the theory will be applied in examples from the dairy and soybean complex sectors in a SWOPSIM demonstration model.

It is necessary to define new notation in this section. The firm uses "n" inputs and produces "m" outputs. The following symbols are defined:

$x = (x_1, x_2, \ldots, x_n)$, an input bundle;

$y = (y_1, y_2, \ldots, y_m)$, an output bundle;

$w = (w_1, w_2, \ldots, w_n)$, an input price vector;

$p = (p_1, p_2, \ldots, p_m)$, an output price vector.

Input and output bundles are assumed to be nonnegative vectors of real numbers, and the price vectors are positive vectors of real numbers. The firm has a production possibility set "T" which is the set of all input-output bundles compatible with the technology available to it. T is assumed to be nonempty, closed, bounded above, and strictly convex. (See Sakai and/or Chambers for discussion.) The multiproduct profit function for the firm is defined as:

$$\chi(p,w) = \text{Max } \{p{*}y - w{*}x : (x,y) \in T;\ w,p > 0\} \qquad (5)$$

Chambers lists the following properties of the multiproduct profit function:

1. $\chi(p,w) \geq 0$;
2. if $p \geq p^1$, then $\chi(p,w) \geq \chi(p^1,w)$ (nondecreasing in p);
3. if $w \geq w^1$, then $\chi(p,w) \leq \chi(p,w^1)$ (nonincreasing in w);
4. $\chi(p,w)$ is convex and continuous in its arguments;
5. $\chi(tp,tw) = t\chi(p,w)$, $t > 0$ (positive linear homogeneity); and
6. there exist fixed vectors $(\bar{y},\bar{x})$ and $(\hat{y},\hat{x})$ such that

   $\chi(p,w) \geq p{*}\bar{y} - w{*}\bar{x}$ and $\chi(p,w) \leq p{*}\hat{y} - w{*}\hat{x}$.

With these conditions and assuming that the multiproduct profit function is differentiable, Hotelling's lemma defines m product supply functions and n derived input demand functions:

$$\frac{\delta \chi(p,w)}{\delta p_j} = y_j(p,w) \qquad (6)$$

$$\frac{\delta \chi(p,w)}{\delta w_i} = - x_i(p,w) \qquad (7)$$

Combining Hotelling's lemma with other properties of the profit function produces several useful results. Hotelling's lemma and the linear homogeneity of the profit function imply that the product supply and input demand equations are homogeneous of degree zero. Multiplication of all product and factor prices by a constant greater than zero leaves product supplies and factor demands unaffected. The convexity of the profit function implies that its Hessian matrix is positive semidefinite. This condition guarantees that output supply curves are upward sloping and that derived input demand curves are downward sloping. Young's theorem implies the following symmetry conditions:

$$\frac{\delta y_i}{\delta p_j} = \frac{\delta^2 \chi}{\delta p_i \delta p_j} = \frac{\delta^2 \chi}{\delta p_j \delta p_i} = \frac{\delta y_j}{\delta p_i} \qquad (8)$$

$$\frac{\delta y_j}{\delta w_i} = \frac{\delta^2 \chi}{\delta p_j \delta w_i} = \frac{\delta^2 \chi}{\delta w_i \delta p_j} = - \frac{\delta x_i}{\delta p_j} \qquad (9)$$

$$\frac{\delta x_i}{\delta w_j} = \frac{\delta^2 \chi}{\delta w_i \delta w_j} = \frac{\delta^2 \chi}{\delta w_j \delta w_i} = \frac{\delta x_j}{\delta w_i} \qquad (10)$$

These theoretical results can be very useful when placing restrictions on elasticity values in a multicommodity trade model, especially one which incorporates inputs. However, in the case of TLIB, the restrictions apply to the case of joint production of processed goods from a raw agricultural commodity like milk or soybeans.

APPLYING THE THEORY TO SWOPSIM

The application of the economic theory described above to the SWOPSIM framework can be best shown in terms of examples. Two cases of interest involve the modeling of the dairy and soybean complex industries.

Modeling of Dairy in SWOPSIM

TLIB's dairy sector includes butter, cheese, dairy (skim) powder, and fluid milk. Fluid milk is the base product, and the other three are derived products. Butter and skim powder are very close joint products in the technical sense. The production of one cannot be made without the production of the other. The model's sectoral composition parallels what is reported internationally by USDA's Foreign

Agricultural Service (FAS). The actual dairy sector is more complex than what is shown in TLIB. Figure 3 shows a more disaggregated view of the dairy sector. As can be seen, TLIB picks up only major product components. Manufactured products other than those explicitly modeled are lumped with whole milk as a residual.

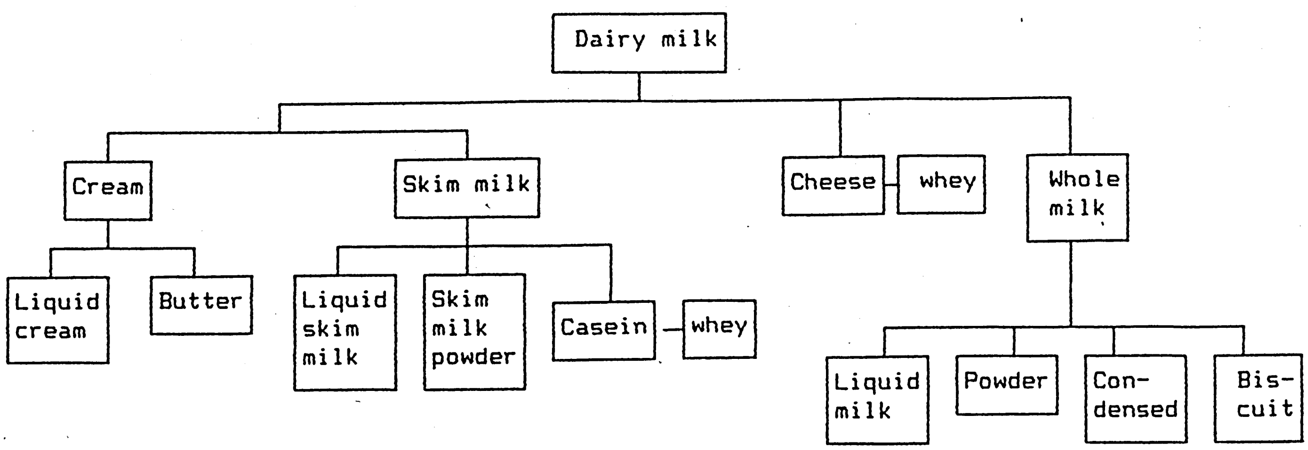
There are a set of supply and demand equations for each of the TLIB dairy products. The supply equations corresponding to the derived products include the consumer price of milk. The demand for fluid milk is twofold. There is the demand for manufacturing milk which is an input into the production of butter, skim powder, and cheese. Then there is the demand for what remains. This amount includes the demand for whole (drinking) milk and for other manufacturing uses not explicitly modeled in TLIB.

The dairy elasticities used in TLIB are based on the OECD's MTM model. One problem is that there is not an exact correspondence in the two models' structures. Appropriating a set of elasticities suitable for one model may introduce biases and/or inconsistencies when used in simulations in another model. One way to ensure consistency in parameter values (short of their actual reestimation) is to select a subset of elasticities that seem suitable for TLIB use and generate the remaining elasticities using production theory and base year quantity and price data. The reasonableness of the elasticities can be examined, and sensitivity analysis can be performed.

The following is an example of the procedure mentioned above. The example uses the theory described in the preceding section to model the supply (S) of butter (DB), skim powder (DP), and cheese (DC), and the derived demand (D) of manufacturing milk (DM). The production of the derived products is represented by an aggregate profit function which includes manufactured milk as the only input into production:

$$\chi = \chi(p_{DB}, p_{DC}, p_{DP}, p_{DM}) \qquad (11)$$

Figure 3 -- Dairy sector



Hotelling's lemma implies the following supply and demand specifications:

$$\frac{\delta \chi}{\delta p_{DB}} = S_{DB}(p_{DB}, p_{DC}, p_{DP}, p_{DM}) \tag{12}$$

$$\frac{\delta \chi}{\delta p_{DC}} = S_{DC}(p_{DB}, p_{DC}, p_{DP}, p_{DM}) \tag{13}$$

$$\frac{\delta \chi}{\delta p_{DP}} = S_{DP}(p_{DB}, p_{DC}, p_{DP}, p_{DM}) \tag{14}$$

$$\frac{\delta \chi}{\delta p_{DM}} = - D_{DM}(p_{DB}, p_{DC}, p_{DP}, p_{DM}) \tag{15}$$

Each of these functions is homogeneous of degree zero under the assumptions placed on the profit function:

$$\frac{\delta S_{DB}}{\delta p_{DB}} \frac{p_{DB}}{S_{DB}} + \frac{\delta S_{DB}}{\delta p_{DC}} \frac{p_{DC}}{S_{DB}} + \frac{\delta S_{DB}}{\delta p_{DP}} \frac{p_{DP}}{S_{DB}} + \frac{\delta S_{DB}}{\delta p_{DM}} \frac{p_{DM}}{S_{DB}} = 0 \tag{16}$$

$$\frac{\delta S_{DC}}{\delta p_{DB}} \frac{p_{DB}}{S_{DC}} + \frac{\delta S_{DC}}{\delta p_{DC}} \frac{p_{DC}}{S_{DC}} + \frac{\delta S_{DC}}{\delta p_{DP}} \frac{p_{DP}}{S_{DC}} + \frac{\delta S_{DC}}{\delta p_{DM}} \frac{p_{DM}}{S_{DC}} = 0 \tag{17}$$

$$\frac{\delta S_{DP}}{\delta p_{DB}} \frac{p_{DB}}{S_{DP}} + \frac{\delta S_{DP}}{\delta p_{DC}} \frac{p_{DC}}{S_{DP}} + \frac{\delta S_{DP}}{\delta p_{DP}} \frac{p_{DP}}{S_{DP}} + \frac{\delta S_{DP}}{\delta p_{DM}} \frac{p_{DM}}{S_{DP}} = 0 \tag{18}$$

$$\frac{\delta D_{DM}}{\delta p_{DB}} \frac{p_{DB}}{D_{DM}} + \frac{\delta D_{DM}}{\delta p_{DC}} \frac{p_{DC}}{D_{DM}} + \frac{\delta D_{DM}}{\delta p_{DP}} \frac{p_{DP}}{D_{DM}} + \frac{\delta D_{DM}}{\delta p_{DM}} \frac{p_{DM}}{D_{DM}} = 0 \tag{19}$$

There are six symmetry conditions:

$$\frac{\delta S_{DB}}{\delta p_{DC}} = \frac{\delta S_{DC}}{\delta p_{DB}} \qquad \frac{\delta S_{DC}}{\delta p_{DP}} = \frac{\delta S_{DP}}{\delta p_{DC}} \qquad \frac{\delta S_{DB}}{\delta p_{DP}} = \frac{\delta S_{DP}}{\delta p_{DB}} \tag{20}$$

$$\frac{\delta S_{DC}}{\delta p_{DM}} = - \frac{\delta D_{DM}}{\delta p_{DC}} \qquad \frac{\delta S_{DB}}{\delta p_{DM}} = - \frac{\delta D_{DM}}{\delta p_{DB}} \qquad \frac{\delta S_{DP}}{\delta p_{DM}} = - \frac{\delta D_{DM}}{\delta p_{DP}}$$

Butter and skim powder are joint products in the technical sense: they are produced in fixed proportions to each other. A change in the supply of butter or skim powder due to any particular price change causes the other joint product to change by the amount of the first commodity's change times the proportionality factor. These relationships can be expressed as:

$$\frac{\delta S_{DB}}{\delta p_{DB}} = \frac{S_{DB}}{S_{DP}} \frac{\delta S_{DP}}{\delta p_{DB}} \qquad \frac{\delta S_{DP}}{\delta p_{DP}} = \frac{S_{DP}}{S_{DB}} \frac{\delta S_{DB}}{\delta p_{DP}} \qquad \frac{\delta S_{DB}}{\delta p_{DC}} = \frac{S_{DB}}{S_{DP}} \frac{\delta S_{DP}}{\delta p_{DC}} \qquad (21)$$

There are 16 elasticities in the dairy specification described so far. There are 13 equations: four homogeneity conditions, six symmetry conditions, and three technical conditions. Three exogenous elasticity values are needed to fully specify the model.[3]

Table 1 shows the initial quantity and price data for a SWOPSIM dairy example. The quantity of milk refers to the portion used as an input into butter, powder, and cheese production. Table 2 shows the complete set of dairy elasticities. As indicated in the table, three of the elasticities were set exogeneously: butter supply with respect to the prices of butter and cheese, and cheese supply with respect to the price of cheese. All the other elasticities were generated using the homogeneity, symmetry, and technical conditions described in this report.[4] A quick check of the signs shows the expected effects of changes of prices on quantities supplied and demanded. The milk elasticity in the table is not entirely correct, however. The reported elasticity accounts for manufacturing milk. The problem is that the difference between total milk supply and the amount used in butter, skim powder, and cheese is unaccounted for. The easiest way to adjust for this problem is have an own price elasticity that accounts for changes in the residual amount. This elasticity, multiplied by the ratio of the residual amount to total milk supply, should be added to the manufactured own price elasticity for insertion into the model. If the residual constitutes 50 percent of milk and its own price elasticity is - 0.20, then the aggregate own price elasticity is [-0.08 + .5*(-0.20)] or - 0.18.

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[3] Convexity of the profit function implies that the Hessian matrix of the profit function (that is, the matrix of second derivatives) should be positive semi-definite. Checking of this condition on the TLIB parameters has not been pursued at this time.

[4] It is important to keep in mind that the elasticities themselves are not symmetric. The adjustment of the elasticities using model data means that symmetry cannot hold at every data point. In TLIB, base period data are used when applying symmetry conditions. It could be argued that the mean value of the data over which the parameters were estimated should be used instead. However, because TLIB is a synthetic model (that is, based on a number of econometric studies), this latter procedure is not practical.

Modeling of the Soybean Complex

Modeling of the soybean complex is similar to the dairy example. In fact it is somewhat simpler because there are only 9 elasticities instead of 16. The two joint products are soymeal and soyoil, and the base product is soybeans.

Table 1 -- Base values for dairy example

Commodity	Quantity	Price
	1,000 mt	Dol./mt
Dairy milk	61,439	287
Butter	500	3,453
Skim powder	526	2,518
Cheese	2,120	3,965

Table 2 -- Elasticities for dairy example

Commodity	With respect to change in price of:			
	Milk	Butter	Skim powder	Cheese
Milk	- 0.08	0.01	0.01	0.06
Butter	- .13	.50*	.38	- .75*
Skim powder	- .13	.50	.38	- .75
Cheese	- .13	- .15	- .12	.40*

* indicates exogeneously set value.

There are three homogeneity conditions, three symmetry conditions, and two technical relationships. There are, therefore, eight equations and nine unknowns. It is necessary to exogenously specify at least one elasticity. Tables 3 and 4 show a soybean complex example.

As in the case of milk, there may be demand for soybeans apart from crush demand from the meal and oil sectors. In this instance, the same sort of adjustment to the own-price soybean demand elasticity should be made to account for the non-crush demand.

CONCLUSION

The elasticities typically used in the SWOPSIM framework come from several sources. In general, selection of elasticities should be a careful procedure when building an economic model. In the selection process, there is no guarantee that all the elasticities will provide a consistent picture of the myriad number

Table 3 -- Base values for soybean complex example

Commodity	Quantity	Price
	1,000 mt	Dol./mt
Soybean	17,792	282
Soymeal	22,252	197
Soyoil	5,202	725

Table 4 -- Elasticities for soybean complex example

Commodity	With respect to change in price of:		
	Soybean	Soymeal	Soyoil
Soybean	- 1.54	0.83	0.71
Soymeal	- .95	.51*	.44
Soyoil	- .95	.51	.44

* indicates exogeneously set value.

of relationships involved in what is being modeled. Insights from theory should be used when possible to provide consistency. Once the model has been constructed, extensive sensitivity analysis should be performed to judge whether the elasticities are reasonable.

The modeling of multiproduct production processes are important in the SWOPSIM framework. The dairy, soybean, and other oilseed sectors all involve the joint production of more than one product derived from a raw agricultural commodity also modeled in the same framework. It is important to model the input-output aspect as well as the joint nature of production.

Recent production theory offers insights into the modeling of production processes where outputs are joint in input usage and profit maximization is assumed. Homogeneity, symmetry, and convexity conditions associated with the multiproduct profit function place restrictions on model elasticities which contribute to greater model consistency. In the case where outputs are produced in fixed proportions to each other (soymeal/soyoil and butter/skim powder), further restrictions on model elasticities can be applied.

This report has reviewed some of the salient features of recent production theory and has applied them to modeling in the SWOPSIM framework. As mentioned, the theory is particularly relevant to the modeling of dairy and oilseed sectors. As more work is done to incorporate factor inputs into the TLIB model, many of the theoretical insights already examined will be important to consider. It is now generally recognized that the production of many agricultural commodities use the same factor inputs. The level of production of one commodity depends on the level of production of another commodity. As the modeling work proceeds, it will be important to judge cross-commodity relationships in the context of their joint input usage. The payoff will be models which offer policymakers better insights into the tradeoffs involved in policy design.

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