A RATIONAL EXPECTATIONS MODEL OF INVENTORY HOLDING

UNDER RISK AND UNCERTAINTY*

by

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Abstract

In this paper a model of inventory holding under risk and uncertainty is developed and introduced into a model of the U.S. soybean market. Through simulation experiments the effects of risk on price levels and stability are measured and the welfare effects on producers, consumers and inventory holders are discussed.

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i. Introduction

Rational private inventory holders base their storage decisions on information concerning future market conditions. Since these conditions are uncertain inventory holders are exposed to price risk. Such uncertainty affects how much risk-averse inventory holders store, which, in turn, affects intertemporal price relationships.

While recent studies (cf. Chavas 1985; Glauber and Powers 1985; Glauber 1986) have analyzed how risk affects the storage decisions of the individual firm, little attention has been focused on how such storage decisions, in the aggregate, affect market prices. The growing literature on commodity storage (cf. Gardner 1979, 1983; Wright and Williams 1982, 1984; Plato and Gordon 1983, 1984; Helmberger and Akinyosoye 1984; Lowry, Glauber, Miranda and Helmberger 1985) has analyzed the effect of risk-neutral storage behavior on intertemporal price relationships. This paper attempts to further this research by examining the effect of risk-averse storage behavior on intertemporal price relationships.

The paper is organized as follows: In section two, behavioral equations for the risk-averse inventory holder are developed. In the following section, these relationships are introduced into a market model for U.S. soybeans. In section four, solution algorithms are developed to simulate the effect of various risk preferences on inventory levels, consumption, production, prices and welfare. Policy implications and conclusions are drawn in the final section.

ii. Inventory Holding Under Price Uncertainty

At the beginning of each period the firm can purchase the storable commodity at a given price, P_t, and sell it at a price, P_{t+k}, k periods later. The firm's revenue in period t is given by $P_t(x_{t-1}-x_t)$, where x_t is the level of inventory carried out of period t. If $x_t > x_{t-1}$, then the firm has purchased inventory and this appears as a cost. Likewise, if $x_t < x_{t-1}$ the firm has sold inventory and this appears as revenue. A storage cost, $C(x_t)$, is incurred to store the commodity from one period to another. The cost function is assumed to be continuous and twice differentiable with respect to x_t such that $C'(x_t) > 0$ and $C''(x_t) \geq 0$. The net proceeds earned for the storage service in period t is thus:

$$(1) \quad \pi_t^c = P_t\,(x_{t-1}-x_t) - C(x_t)$$

We assume the firm maximizes the certainty equivalence of the present value of an additive stream of discounted net proceeds by choosing the level of carryouts from period t to period T -- the planning horizon of the inventory holder:

$$(2) \qquad \max_{x_i} L_t = (E_t(\Theta) - R^h/2(V_t(\Theta)))$$

$$i = t, t+1, \ldots, T-1, T;$$

$$\text{s.t. } x_i \geq 0, \forall i; \qquad L_t \geq 0;$$

$$\Theta = \sum_{i=t}^{T} B^{i-T}\pi_i^c;$$

where E and V denote the mean and variance, Θ is profit, R^h is the risk-aversion coefficient of the inventory holder, and B is one plus the rate of interest.

The firm is assumed to make an inventory decision (x_t) at the beginning of each period to maximize (1). The restrictions on $C(x_t)$ assure the existence of optimal x_t's for risk-neutral or risk-averse firms. For convenience we assume that $C'(x_i)$ and B are nonstochastic and that the risk preferences of the inventory holder are unchanged over the planning horizon. It is also assumed that the inventory holder is not a consumer of the commodity and hence the convenience yield of storage--the so-called demand for pipeline stocks--is zero (Working 1949).

The price at t, P_t, is exogenous and the prices for future periods are randomly distributed about a known distribution. The distribution of prices is conditional on the market information available to the firm at period t.

The optimal inventory level for each period can be found using dynamic programming (See Chavas 1985 and Glauber 1986). Proceeding backwards we solve for x_i in periods i = T, T-1, T-2, etc., and arrive at the current period (i=t). In general, the intertemporal arbitrage conditions for period t are:

$$(3)\quad \begin{aligned} &[B^{-1}E_t(P_{t+1})-(P_t+C'(x_t))-R^h(V_t(P_{t+1})(x_t-E_t(x^*_{t+1})) + \\ &\qquad \textstyle\sum_{i=2}^{T-t} B^{1-i}COV_t(P_{t+1},P_{t+i})E_t(x^*_{t+i-1}-x^*_{t+i}))] \leq 0 \\ &x_t[\qquad\qquad] = 0 \end{aligned}$$

where $E_t(x^*_{t+i})$ is the expected optimal carryout level in period t+i and $COV_t(P_{t+1},P_{t+i})$ is the covariance between price in the next period and price in subsequent periods. Note that the sign of the marginal risk-premium depends on whether there will be a net sale or purchase of inventory in future periods. If we assume that the covariance between P_{t+1} and prices in subsequent periods is negligible (e.g., between the old and new crop years), the marginal risk premium reduces to $R^h(V_t(P_{t+1})(x_t - E_t(x^*_{t+1}))$. If there is an expected net sale $(x_t > E_t(x^*_{t+1}))$ the marginal risk-premium is positive. Since the price in period t+1 is uncertain, inventory holders who expect to sell their inventory will require a premium above the physical costs of storage to induce them to hold grain. Conversely, if there is an expected net purchase $(E_t(x^*_{t+1}) > x_t)$ the marginal risk-premium is negative. The risk-averse firm is willing to pay more for stocks purchased in the current period at a known price, P_t, than it would to purchase all of its inventory in period t+1 at an uncertain price, P_{t+1}. Lastly, when the expected net sale is zero $(x_t = E_t(x^*_{t+1}))$ the marginal risk-premium is zero and the expected price equals the current price plus the carrying charge.

The arbitrage conditions in (3) reflect the effect of subsequent storage decisions on the storage decision made in the current period. The decision to store is based indirectly on expected prices and variances in all periods. At the aggregate level these results suggest that there may exist a positive risk premium (i.e., $E_t(P_{t+1}) > (P_t + C'(x_t))B$) whenever stock levels are declining, and a negative premium when stocks are increasing. Thus we would expect that for discontinuously stored commodities such as potatoes and onions a positive risk premium would exist throughout the storage season. For continuously stored commodities such as corn and soybeans, we have shown that is possible to have positive carryout of the old crop into the new crop year despite prices that show an inverse carrying charge between the old and new crop year.

iii. The Storage Model

Consider year t which begins with a historically predetermined supply of the agricultural commodity in question

(4) $S_t = X_{t-1} + H_t$

composed of the amount carried over from the preceding year and into the present, X_{t-1}, and the current harvest H_t. The amount harvested depends on Y_{t-1}, the expected production established during the preceding year's planting season, and $\tilde{w}_{t-1}$, a stochastic term to represent the weather conditions that prevailed during the growing season:

(5) $H_t = h_t(Y_{t-1}, \tilde{w}_{t-1})$.

X_t is the amount stored in the current year and carried over to the following period and the remaining supply is consumed. Thus, the market clearing price is:

(6) $P_t = p_t(S_t - X_t)$.

Retaining the notation from the previous section: k is the constant unit storage cost, B is the discount factor, $E_t(P_{t+1})$ is the price expected to prevail next year, $V_t(P_{t+1})$ is the variance of that price, and $E_t(X_{t+1})$ is the expected carryout from the next period.

Assuming that the covariance between prices in subsequent crop years is negligible, and rewriting (3) it is the case that the discounted expected future price of the commodity must not exceed the opportunity cost of storing one unit of the commodity:

(7) $$P_t + k + R(V_t(P_{t+1}))(X_t - E_t(X_{t+1})) \leqq BE_t(P_{t+1})$$

Otherwise, equally risk-averse expected-profit maximizing arbitrageurs would exploit profit opportunities by purchasing stocks. On the other hand, there is a disincentive to hold speculative stocks if the opportunity cost of storing exceeds the discounted expected futures price:

(8) $$[P_t + k + R(V_t(P_{t+1}))(X_t - E_t(X_{t+1})) - BE_t(P_{t+1})]X_t = 0$$

Producers make their production decisions based on the expected future price:

(9) $$Y_t = q_t(E_t(P_{t+1}))$$

The supply available at the beginning of the year t+1 will be the sum of current carryout and next period's harvest, which depends on the current production plans and the intervening weather conditions:

(10) $$S_{t+1} = X_t + h_{t+1}(Y_t, \tilde{w}_t).$$

We assume that private economic agents believe price in period t+1 is determined by the supply available at the beginning of that period, but that they possess only probalistic knowledge of the distribution of future weather; thus

(11) $$E_t(P_{t+1}) = f_t(X_t + h_{t+1}(Y_t, \tilde{w}_t))$$

(12) $$V_t(P_{t+1}) = v_t(X_t + h_{t+1}(Y_t, \tilde{w}_t))$$

(13) $$E_t(X_{t+1}) = x_t(X_t + h_{t+1}(Y_t, \tilde{w}_t))$$

where $f_t(S_{t+1})$ is the price they expect in period t+1 conditional on the supply at that time, $v_t(S_{t+1})$ is the variance, and $x_t(S_{t+1})$ is the expected carryout from period t+1.

We assume that producers and inventory holders are rational (Muth 1961); that is, their expectations are consistent with those of the model. The actual relation between expectations and the information available when production and storage decisions are made, however, is not known _a_ _priori_ and, futher, is not expressable in closed form. We therefore estimate the rational expectations functions (equations 11-12) numerically using a successive approximation scheme based on piecewise-linear splines. Details of the solution methods are developed further in Miranda (1985).

iv. The Effect of Risk and Uncertainty on Market Equilibrium

The data used in the simulation experiments which follow were obtained from an annual model of the U.S. soybean market estimated by Glauber (1984). The model assumes log-linear supply and demand equations with multiplicative disturbance terms. To obtain the reduced form estimates for the equations exogenous variables have been fixed at their 1977 values. We also have assumed a storage cost of \$0.36 per bushel per year and an annual rate of interest of eight percent

Simulation experiments are performed using various estimates for the coefficient of absolute risk-aversion (R). Since this parameter is dependent on the magnitude of income care must be taken in choosing a range large enough to encompass plausible values for it. In these experiments values for R range from zero (risk-neutrality) to 20 (strong risk-aversion). The upper bound was found to be sufficiently large to effectively shut off interyear carryout. Risk-preferring behavior is not considered in this study.

For a given value of R and using 1977 values for the exogenous variables, equations (11-13) are derived numerically using contraction mapping techniques (Miranda 1985). Once derived, simulations are performed and the steady state values for the endogenous variables are obtained. The means and coefficients of variations for these variables and for various values of R are presented in Table I.

The results in Table I show that as inventory holders become more risk-averse they are inclined to hold less, _ceterus paribus_, resulting in a concomitant increase in the average annual price. At high levels of risk-aversion (R=20) carryout is reduced to six million bushels, down from alomst 60 million bushels in the risk-neutral case (R=0). The decrease in carryout is offset somewhat by a small increase in production (responding to the increase in expected price); nonetheless, total supply falls by more than 42 million bushels. The effect of decreased supplies on price is small; even at R=20 the mean annual price is only four cents higher than the annual price when inventory holders are risk-neutral. Consumption over the same range actually shows a slight increase (result of the nonlinear demand specification and because price variability increases more rapidly over the same range).

Risk-aversion has a more substantial effect on price variability.

Increasing the risk parameter to R=20 causes the coefficient of variation for price to increase to 24.5 percent compared with only 20.3 percent for the risk-neutral case. This is a result of lower supply with risk-aversion. Since mean carryout levels at R=20 are negligible, price expectations are based primarily on planned production; hence, the coefficients of variation for expected price and, concomitantly, production decline as inventory holders become more risk-averse. Similar results have been shown for the effects of increased storage costs on production variability (cf. Wright and Williams 1982; Lowry et al. 1985). The coefficient of variation for total supply remains essentially unchanged as the decreased variation in production is offset by the increased variation in carryout.

The welfare effects of increased risk-aversion are presented in Table II. Consumer surplus (as measured by the area under the demand curve) decreases as R grows large, reflecting the increase in price. For the extreme case (R=20) consumer losses are estimated at 9 million dollars as compared to the risk-neutral case. Producer quasi-rents are calculated by subtracting the discounted costs of production (the area under the planned supply curve) from actual revenues received by the producers. Producer quasi-rents fall by more than 19 million dollars under the assumption that R=20. This decrease is explained by the fact that as average carryout decreases the covariance between price and quantity grows larger (as there are fewer stocks available to buffer the effects of a shortfall in supply). This negative effect on revenue offsets the relatively small increases in mean price and production.

Arbitrage profits represent the average net return obtained by the inventory holder for storing a portion of the old crop into the new crop year. Returns to inventory holders thus reflect the size of the risk premium. Under the assumption of risk-neutrality mean arbitrage profits are zero. As R increases, however, arbitrage profits increase, reflecting the increase in the risk premium, and then decrease because carryout levels decline. Arbitrage profits are at a maximum at R=3 (not shown in Table II).

Losses to consumers and producers total more than 27 million dollars for R=20. As demonstrated above, producers are more affected than consumers. Producer losses due to risk-averse storage behavior account from seventy to eighty percent of the total welfare loss. Losses on a per capita basis would further accentuate these differences. When arbitrage profits are considered, total deadweight loss is diminished. At the extreme (R=20) net social loss is about twenty million dollars.

v. Conclusions and Implications for Further Research

The analysis has presented a model of rational inventory holding under the assumption of risk and uncertainty. By introducing risk-averse storage behavior into a model of the U.S. soybean market we have measured the effect of risk on price levels and stability. The results show that risk-averse storage behavior increases mean annual price and price variability. While net losses to consumers and producers are small, the results indicate that price stabilization schemes (e.g., buffer stocks) may provide additional, albeit small, benefits to society. The exact magnitude of these gains and the costs of operating such a scheme are unfortunately beyond the scope of this research.

The study could be expanded in several ways. First, contingent claims markets, especially futures markets, could be included in the model. As Glauber and Powers (1985) have shown, hedging reduces the magnitude of the risk premium of the inventory holder. Reduction of the risk premium would thus offset some of the effects of risk. Secondly, the long run effects of risk-averse inventory holding have not been considered. Positive arbitrage profits would encourage less risk-averse arbitrageurs to enter the market, thus increasing the aggregate level of carryout and reducing mean annual price. Thirdly, the study has focused on interyear carryout in the context of an annual model. Recent research by Lowry _et al_. (1985) has shown that the effect of increased storage costs may have a more profound effect on _intra_seasonal storage, suggesting that the cost of risk-averse storage behavior may be greater than the figures reported here.

While beyond the scope of this study these topics merit further attention. They should prove fruitful avenues for future research.

Table I. Mean values and coefficients of variation (in parentheses) for selected endogenous variables for alternative levels of absolute risk aversion.

Item	Coefficient of Absolute Risk Aversion 0	2	5	10	20
	dollars per bushel				
Price	5.715 (20.3)	5.730 (21.9)	5.745 (23.3)	5.752 (24.1)	5.758 (24.5)
Expected Price	5.715 (3.5)	5.730 (2.5)	5.745 (1.3)	5.752 (0.7)	5.758 (0.4)
	billion bushels				
Consumption	1.811 (11.1)	1.816 (12.5)	1.820 (13.7)	1.822 (14.4)	1.823 (14.8)
Production	1.812 (15.6)	1.816 (15.4)	1.820 (15.3)	1.822 (15.3)	1.823 (15.2)
Carryout	.059 (172.0)	.038 (186.4)	.019 (190.5)	.011 (190.0)	.006 (189.4)
Total Supply	1.871 (15.0)	1.854 (15.0)	1.839 (15.1)	1.833 (15.2)	1.829 (15.2)

Table II. Changes in welfare for alternative levels of absolute risk aversion as compared to the risk-neutral case (R=0).

Item	Coefficient of Absolute Risk Aversion 0	2	5	10	20
	billion dollars				
Consumer Surplus	0.000	-0.003	-0.006	-0.008	-0.009
Producer Quasi-rent	0.000	-0.013	-0.019	-0.019	-0.018
Net Loss	0.000	-0.016	-0.025	-0.027	-0.027
Arbitrage Profits	0.000	0.014	0.013	0.009	0.006
Net Deadweight Loss	0.000	-.002	-0.012	-0.018	-0.021

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