# TECHNICAL CHANGE, UNCERTAINTY AND INVESTMENT

Spiro E. Stefanou

Department of Agricultural Economics and Rural Sociology
The Pennsylvania State University
University Park, PA 16802



Selected paper presented at the annual meetings of the American Agricultural Economics Association, July 27-30, 1986 in Reno, Nevada.

## Technical Change, Uncertainty and Investment

The generation of new technology is a necessary development in order to increase agricultural output. The development of new technologies in agriculture is a growth industry with both the private and public sectors making great strides in pushing back the frontiers of knowledge. Already efforts are underway to apply emerging biotechnologies in agriculture in both plant and animal production. In plant production examples include improving existing plant varieties to inhibit the effects of herbicides and pests, and developing new plant varieties that can resist bacteria and disease and can biologically fix nitrogen. In animal production examples of new biotechnologies include bovine growth hormone, embryo transfer, and twinning.

The literature in agricultural economics on the innovation, dissemination and adoption of new technologies acknowledges the role of accumulated knowledge in increasing output (Hiebert; Kislev and Shchori-Bachrach; Feder and O'Mara; Feder and Slade). The processes of innovation and adoption are modeled within various information processing regimes: passive vs. active accumulation of knowledge, Bayesian vs. ad hoc learning frameworks. Although some of the studies mentioned above address the dynamic aspects of the innovation and adoption processes, none have explicitly developed their analyses within an intertemporal model of investment decision making. Feder and Slade treat the acquisition of information as an investment by the firm with a cost of aquiring information similar to the cost of adjusting capital stock; their analysis maintains static behavioral assumptions. Other work on uncertain future technology has focused on the case of a new, well-defined technology that is expected to become available at some unknown future date. Dasgupta and Heal, Hoel (1978, 1979), Kamien and Schwartz (1978) and Bhattacharya have focused on the extraction of an exhaustible resource that has a non-exhaustible substitute produced by a "backstop technology" or that allows the new technology to render the existing technology obsolete.

Models of stochastic technical change in the literature, as well as this work, must necessarily abstract from reality in order to glean some general behavioral conclusions. Mirman considered the case of a random element in the production function that represents the possibilities of stochastic technical change which does not exhibit a systematic evolution over time. He goes on to characterize the distribution of possible states of a one-sector growth model and the sector's long-run behavior. Kamien and Schwartz (1972) consider the scenario where a firm makes a one-time, irreversible decision to adopt a new, well-defined technology. Their analysis suggests that uncertainty about the time availability of this improved technology is available tends to delay the adoption decision. Balcer and Lippman focus on the more general case where more than one well-defined technological innovation is anticipated with expectations about the likelihood of such innovations being updated as time passes. Their analysis suggests that the adoption of an innovation is not necessarily delayed when many technological innovations are stochastically anticipated.

Mathematical programming formulations such as Wicks and Guise and Paris and Easter can incorporate stochastic technology within an operational framework. This class of models considers the coefficients of the technology matrix to be stochastic and sacrifices realism (e.g., linear or quadratic objective functions, linear production technologies) for computability.

Investment behavior under uncertainty for risk-neutral firms facing adjustment costs has been addressed by Hartman, Pindyck (1982, 1984) and Abel (1983, 1984). Abel (1983) corroborates Hartman's finding that uncertainty about future prices leads to an increase in the current rate of investment under constant returns to scale technology.[1]

This article examines the expected investment dynamics of a risk-neutral competitive firm in the presence of uncertainty concerning the future evolution of prices and technical change. Focusing on the investment dynamics provides insight into the firm's accumulation (or disinvestment) of capital which can be either an expansion of the current technological processes being practiced or a shift to an improved, more capital intensive technology. Production facilities may be available in more than a few widely differing sizes. Variations in capacity utilization rates may be accompanied by a wide range of technological adjustments. Rather than focus on a firm stochastically anticipating one or more well-defined, improved technologies in the future, a state of technology concept is modeled as a stock variable which evolves in a stochastic manner. Although the discovery of a specific innovation is a discrete event, once these innovations are ready for dissemination the flow of operational technical knowledge to the firm can be considered continuous. Thus, focusing on the impact of uncertain technical change on capital accumulation is not especially restrictive. The next section develops a model of the firm that is characterized by quasi-fixed factor-augmenting technical change that is taken exogenously by the firm, allowing for costs of adjusting the stock of capital. The following section considers the effect of uncertainty on investment and the last section provides some concluding comments.

## The Model

The firm uses capital, K(t), and labor, L(t), to produce output, y(t) where labor represents a flexible factor and capital represents a quasi-fixed factor. Assuming the presence of capital-augmenting technical change, output can be expressed as

$$(1)\quad y(t) = F(L(t), B(t)K(t))$$

where F( ) is assumed to be a differentiable quasi-concave function. The expression B(t)K(t) is referred to as the stock of effective capital where B(t) is an index denoting the stock of accumulated technical knowledge associated with capital-augmenting technical change. This representation of capital-augmenting technical change implies that the firm draws upon a stock of freely available technical knowledge. In agriculture, the availability of the services of extension personnel and materials to the operator suggests that this assumption is plausible.

However, the rate of growth in the stock of technical knowledge is assumed to evolve in a continuous stochastic manner[2] according to

$$(2)\quad dB = (\beta_1 B + \beta_2 K + \beta_3 I)dt + \sigma_B B dW_B,\ B(0) = B_o$$

where I(t) is the rate of investment in time t, $\beta_i$, i = 1, 2, 3, $\sigma_B$ and $B_o$ are known parameters, and $W_B(t)$ is a Weiner process where

$$(3)\quad E\,\{dW_B(t)\} = 0,\ E\{(dW_B(t))^2)\} = dt.$$

The rate of growth in capital-augmenting technical change is composed of deterministic and stochastic drift components. The coefficient $\beta_1$ is interpreted as the expected proportional rate of growth in capital-augmenting technical knowledge that can be passively accumulated by the farm operator independent of capital and investment. The coefficients $\beta_2$ and $\beta_3$ are interpreted as the change in the rate of expected change in technical knowledge for a change in the capital stock and investment levels, respectively. Observed rates of adoption suggest that adoption of new technologies is not instantaneous and may be hampered by farm size (Feder, Just and Zilberman). These coefficients describe the impact of farm size on the rate of accumulation of technical knowledge by the operator. It is assumed that these coefficients are non-negative, implying that larger farm operators can increase their expected change in the stock of technical knowledge as least as fast, if not faster, than smaller farm levels of capital and investment.

Another form of uncertainty concerns the evolution of prices in the future. The nominal wage normalized by output price, R, is assumed to behave according to

(4) $dR/R = \sigma_R dW_R,\ R(0) = R_0$

where $R_0$ is assumed to be a known constant and

(5) $E\{dW_R(t)\} = 0,\ E\{(dW_R(t))^2)\} = dt$ and $E\{dW_B(t)dW_R(t)\} = \sigma_{BR}dt,$

where $\sigma_{BR}$ is the contemporaneous correlation coefficient between the stochastic processes in (2) and (4). The stochastic specifications in (2) and (4) imply that B(t) and R(t) are lognormally distributed and these random variables will always take on positive values.[3]

The firm is also constrained by the capital accumulation equation

(6) $dK = (I - \delta K)dt,\ K(0) = K_0$

where I is the level of investment, $\delta$ is the constant rate of depreciation, and $K_0$ is known.

Since the current state of technical knowledge and prices are known, the maximization of short-run unit profits in time $t_o$ involves choosing $L(t_o)$ to

(7) $\max\ [F(L(t_o),\ B(t_o)K(t_o)) - R(t_o)L(t_o)].$

Assuming an interior solution, the short-run profit maximization condition is

(8) $F_L = R(t_o)$

which implies that the maximizing short-run unit profit function can be expressed as $\pi[R(t_o),\ B(t_o)K(t_o)]$, which is convex in real wage, $R(t_o)$, and concave in the effective capital stock, $B(t_o)K(t_o)$.

The firm's intertemporal investment decision problem involves choosing an investment plan to maximize the expected sum of discounted cash flow, which at time t is $\pi[R(t),\ B(t)K(t)] - C(I(t))$, subject to starting conditions and constraints on the movement of the state variables. The term $C(I(t))$ is the cost of adjusting the capital stock and by assumption is characterized by $IC_I > 0$ and $C_{II} > 0$. The introduction of adjustment costs leads to the sluggish adjustment in the capital stock only if the cost of adjustment is convex (Brechling, Chapter 4).

The firm's intertemporal decision problem is formally stated as

(9) $J(k,\ r,\ b) = \max_{I}\ E_t\{\int_t^{\infty} e^{-\rho s}[\pi(R,\ BK) - C(I)]ds\}$

subject to

$$dK = (I - \delta K)dt, \; K(t) = k$$

(10) $$dR = \sigma_R R dW_R, \; R(t) = r$$

$$dB = (\beta_1 B + \beta_2 K + \beta_3 I)dt + \sigma_B B dW_B, \; B(t) = b$$

where $\rho$ is the constant rate of time discount and $E_t$ denotes the expectation operator starting at time t. Assuming that J(k, r, b) exists and is differentiable,[4] the dynamic programming equation (DPE) for this Weiner driven process is

(11) $$\rho J(k, r, b) = \max_I [\pi(r, bk) - C(I) + (I - \delta k)J_k + (\beta_1 b + \beta_2 k + \beta_3 I)J_b + (1/2)\sigma_B^2 b^2 J_{bb} + (1/2)\sigma_R^2 r^2 J_{rr} + \sigma_B \sigma_R \sigma_{BR} br J_{br}]$$

where subscripts indicate partial differentiation.

An alternative characterization of (11) can be developed using the Ito, or stochastic, calculus (Schuss). By Taylor expanding J(k, r, b), dividing through by dt and taking the expectation starting at time t as dt -> 0 (see Mangel, 1985, Chapter 2) one can develop an expression for the expected change in the value function as

(12) $$(1/dt)E_t\{dJ\} = (I - \delta k)J_k + (\beta_1 b + \beta_2 k + \beta_3 I)J_b + (1/2)\sigma_B^2 b^2 J_{bb} + (1/2)\sigma_R^2 r^2 J_{rr} + \sigma_B \sigma_R \sigma_{BR} br J_{br}.$$

The DPE can now be more compactly written in terms of the expected change in the value function as

(13) $$\rho J(k, r, b) = \max_I \; [\pi(r, bk) - C(I) + (1/dt)E_t\{dJ\}].$$

Thus, (13) suggests that the opportunity cost of the stock of technical knowledge and capital, $\rho J(k, r, b)$, is equal to the instantaneous cash flow, $\pi(r, bk) - C(I)$, plus the instantaneous expected change in the value of the stock of technical knowledge and capital, $(1/dt)E_t\{dJ\}$. Assuming an interior solution,

$$(14)\quad C_I(I) = J_k + \beta_3 J_b.$$

This states that, along the optimal path, the marginal cost of adjusting the capital stock is equal to the marginal valuation of the capital stock plus the marginal growth in expected capital-augmenting technical knowledge arising from an increase in investment, $\beta_3$, times the marginal valuation of the stock of capital-augmenting technical knowledge, $J_b$. Using the optimized level of investment, $I^* = I^*(k, r, b)$, the DPE is the second order non-linear partial differential equation

$$(15)\quad \rho J(k, r, b) = \pi(r, bk) - C(I^*) + (I^* - \delta k)J_k + (\beta_1 b + \beta_2 k + \beta_3 I^*)J_b + (1/2)\sigma_B^2 b^2 J_{bb} + (1/2)\sigma_R^2 r^2 J_{rr} + \sigma_B \sigma_R \sigma_{BR} br J_{br}.$$

Along the optimal trajectory, (14) must necessarily hold, and suggests that

$$(16)\quad dC_I(I^*) = d(J_k) + \beta_3 d(J_b).$$

Since the differentials in (16) are based on the evolution of stochastic processes, the expected marginal cost of adjustment dynamics can be expressed as

$$(17)\quad E_t\{dC_I(I^*)\} = E_t\{d(J_k)\} + \beta_3 E_t\{d(J_b)\}.$$

The evolution of the marginal cost of adjustment, $dC_I(I^*)$, is determined by stochastically differentiating $C(I^*)$ as follows

$$(18)\quad dC_I(I^*) = C_{II}dI^* + (1/2)C_{III}(dI^*)^2 + o(dt)$$

where $\lim_{dt\to 0} o(dt)/dt = 0$. The evolution of investment, $dI^*$, is determined by stochastically differentiating $I^* = I^*(k, r, b)$ as follows, recognizing that $I^*$ holds for all starting values,

$$(19)\quad dI^* = I_k^*dK + I_r^*dR + I_b^*dB$$
$$(1/2)I_{rr}^*(dR)^2 + (1/2)I_{bb}^*(dB)^2 + I_{br}^*dBdR + o(dt).$$

Squaring $dI^*$ yields

$$(20)\quad (dI^*)^2 = I_r^{*2}(dR)^2 + I_b^{*2}(dB)^2 + I_r^*I_b^*dRdB + o(dt).$$

Taking the expectation of (18) starting at time t, using (19) and (20), and dividing through by dt as dt -> 0 yields

$$(21)\quad (1/dt)E_t\{dC_I(I^*)\} = C_{II}(1/dt)E_t\{dI^*\}$$
$$+ (1/2)C_{III}(\sigma_B^2b^2I_b^{*2} + \sigma_R^2r^2\ I_r^{*2} + 2\sigma_B\sigma_R\sigma_{BR}br\ I_b^*I_r^*).$$

Differentiating (15) with respect to k and using $\kappa = bk$ yields

$$(22)\quad \rho J_k = b\pi_\kappa - [C_I(I^*) - J_k - \beta_3J_b]I_k^* - \delta J_k + (I^* - \delta k)J_{kk} + \beta_2J_b$$
$$+ (\beta_1b + \beta_2k + \beta_3I^*)J_{bk} + (1/2)\sigma_B^2b^2\ J_{bbk}$$
$$+ (1/2)\sigma_R^2r^2J_{rrk} + \sigma_B\sigma_R\sigma_{BR}brJ_{brk}.$$

Stochastically differentiating $J_k(k, r, b)$ yields

$$(23.1)\quad d(J_k) = J_{kk}dK + J_{rk}dR + J_{bk}dB$$
$$+ (1/2)J_{rrk}(dR)^2 + (1/2)J_{bbk}(dB)^2 + J_{brk}(dB)(dR) + o(dt),$$

and then taking the expectation of $d(J_k)$ starting at time t and dividing through by dt as dt -> 0 yields

$$(23.2)\quad (1/dt)E_t\{d(J_k)\} = (I^* - \delta k)J_{kk} + (\beta_1 b + \beta_2 k + \beta_3 I^*)J_{bk} + (1/2)\sigma_B^2 b^2 J_{bbk} + (1/2)\sigma_R^2 r^2 J_{rrk} + \sigma_B\sigma_R\sigma_{BR} brJ_{brk}.$$

One can obtain an expression for the instantaneous expected change in the marginal value of capital by using (14) and (22) in (23.2) to obtain

$$(24)\quad (1/dt)\ E_t\{(J_k)\} = (\rho + \delta)J_k - b\pi_\kappa - \beta_2 J_b.$$

In a similar fashion, one can determine that by differentiating (15) with respect to b and using (12) and (14) that

$$(25)\quad (1/dt)E_t\{d(J_b)\} = (\rho - \beta_1)J_b - k\pi_\kappa - \sigma_B^2 bJ_{bb} - \sigma_B\sigma_R\sigma_{BR} rJ_{br}.$$

Using (21), (24) and (25), (17) can be rewritten in terms of the expected investment dynamics

$$(26)\quad (1/dt)E_t\{dI^*\} = (1/C_{II})[(\rho + \delta)J_k - b\pi_\kappa - \beta_2 J_b] + (\beta_3/C_{II})[(\rho - \beta_1)J_b - k\pi_\kappa - \sigma_B^2 bJ_{bb} - \sigma_B\sigma_R\sigma_{BR} rJ_{br}] - (1/2)(C_{III}/C_{II})(\sigma_B^2 b^2 I_b^{*2} + \sigma_R^2 r^2\ I_r^{*2} + 2\sigma_B\sigma_R\sigma_{BR} br\ I_b^* I_r^*].$$

**Uncertainty and The Expected Dynamics of Investment**

The expected dynamics of investment, $(1/dt)E_t\{dI\}$, involves three major components. The first is the presence of uncertainty elements represented by the variances and covariances of the random price and technical change processes ($\sigma_B$, $\sigma_R$, $\sigma_{BR}$). The second major component measures the curvature of the value function and the cost of the adjustment and investment demand functions ($J_{bb}$, $J_{br}$, $C_{II}$, $C_{III}$). The third involves the effect of farm size on the absorption of additional capital-augmenting technical knowledge ($\beta_2$, $\beta_3$).

Our ability to assess <u>ceterus paribus</u> changes in the expected investment dynamics due to a perturbation in the uncertainty of a particular stochastic process depends on the signs of $C_{III}$, $\sigma_{BR}$, $I_b^*$, $I_r^*$, and the magnitude of the farm size effect $\beta_3$. With $\pi(r, \kappa)$ concave in $\kappa = bk$, $\pi(\ )$ is also concave in b and in k.[5] However, the curvature of the value function is more difficult to determine. Using (15), the short-run profit function can be expressed as

$$\text{(27)}\quad \pi(r, \kappa) = \rho J(k, r, b) + C(I^*) - (I^* - \delta k)J_k - (\beta_1 b + \beta_2 k + \beta_3 I^*)J_b - (1/2)\sigma_B^2 b^2 J_{bb} - (1/2)\sigma_R^2 r^2 J_{rr} - \sigma_B \sigma_R \sigma_{BR} br J_{br}.$$

Thus, $\partial\pi^2/\partial r^2 > 0$, $\partial\pi^2/\partial b^2 < 0$ and $\partial\pi^2/\partial k^2 < 0$ imply conditions (C.1) through (C.3), respectively,

$$\text{(C.1)}\quad (\rho - \sigma_R^2)J_{rr} > [(I^* - \delta k)J_{krr} + (\beta_1 b + \beta_2 k + \beta_3 I^*)J_{brr} + (1/2)\sigma_B^2 b^2 J_{bbrr} + 2\sigma_R^2 J_{rrr} + \sigma_R^2 r J_{rrrr} + 2\sigma_B \sigma_R \sigma_{BR} b J_{brr} + \sigma_B \sigma_R \sigma_{BR} br\ J_{brrr}],$$

$$\text{(C.2)}\quad (\rho - 2\beta_1 - \sigma_B^2)J_{bb} < [(I^* - \delta k)J_{kbb} + (\beta_1 b + \beta_2 k + \beta_3 I^*)J_{bbb} + 2\sigma_B^2 b J_{bbb} + (1/2)\sigma_B^2 b^2 J_{bbbb} + (1/2)\sigma_R^2 r^2 J_{rrbb} + 2\sigma_B \sigma_R \sigma_{BR} r J_{brb} + \sigma_B \sigma_R \sigma_{BR} br J_{brbb}],$$

$$\text{(C.3)}\quad (\rho + 2\delta)J_{kk} < [(I^* - \delta k)J_{kkk} + 2\beta_2 J_{bk} + (\beta_1 b + \beta_2 k + \beta_3 I^*)J_{bkk} + (1/2)\sigma_B^2 b^2 J_{bbkk} + (1/2)\sigma_R^2 r^2\ J_{rrkk} + \sigma_B \sigma_R \sigma_{BR} br J_{brkk}].$$

The evaluation of the <u>fourth order</u> derivatives is required to determine the convexity of $J(\ )$ in r and the concavity of $J(\ )$ in b and k.

In order to evaluate the sign of $I_b^*$ one notes from (14) that

$$\text{(28)}\quad C_{II} I_b^* = J_{kb} + \beta_3 J_{bb}.$$

The term $J_{kb}$ is interpreted as the change in the marginal valuation of the capital stock given a change in the stock of technical knowledge. With $C_{II} > 0$ and $\beta_3 > 0$, $I_b^*$ can be signed as follows

$$I_b^* \gtrless 0 \text{ for } J_{kb} \gtrless \beta_3 J_{bb}.$$

In order to evaluate the sign of $I_r^*$ one observes from (14) that

$$(29) \quad C_{II} I_r^* = J_{kr} + \beta_3 J_{br}.$$

Two cases arise in an attempt to determine the direction of $I_r^*$. The first case is when the shadow values of capital and technical knowledge, respectively, given a change in the real wage are in the same direction; i.e., $\text{sign}(J_{kr}) = \text{sign}(J_{br})$. In this case, $\text{sign}(I_r^*) = \text{sign}(J_{kr})$. The second case is when $J_{kr}$ and $J_{br}$ are of the opposite sign. In this instance

$$I_r^* \gtrless 0 \text{ for } J_{kr} \gtrless \beta_3 J_{br}.$$

An increase in uncertainty concerning technical change is characterized by an increased value of $\sigma_B$ and an increase in future price uncertainty is characterized by an increased value of $\sigma_R$. When there is a zero correlation between a change in the real wage and a change in the stock of technical knowledge, $\sigma_{BR} = 0$, an increase in $\sigma_B$ has the following impact on $(1/dt)E_t\{dI^*\}$

$$- 2(\beta_3/C_{II})\sigma_B b J_{bb} - (C_{III}/C_{II})\sigma_B\, b^2 I_b^{*2}.$$

The first effect depends on the curvature of the value function with respect to b. Specifically, if J( ) is concave in b, $J_{bb} < 0$ and the rate of expected change in investment increases as $\sigma_B$ is increased. The second effect of an increase in $\sigma_B$ is to decrease (increase) $E_t\{dI^*\}$ as $C_{III} > 0$ ($< 0$). An increase in $\sigma_R$ influences $(1/dt)E_t\{dI^*\}$ according to

$$- (C_{III}/C_{II})\sigma_R r^2 I_r^{*2}.$$

Thus, an decrease (increase) occurs in the rate of expected change in investment as $C_{III} > 0$ ($< 0$). The term $C_{III}$ can be viewed in terms of the marginal cost of adjustment. Namely, $C_{III} > 0$ ($< 0$) implies that the marginal cost of adjusting the capital stock is increasing at an increasing (a decreasing) rate. Economic theory is sufficiently vague so as to preclude an a priori specification of the sign of $C_{III}$.

When $\sigma_{BR} \neq 0$, an increase in $\sigma_B$ generates two additional effects that depend on the linkage of technical change to the evolution of the real wage. The first effect depends in the sign of $J_{br}$, while the second depends on the signs of $(I_b^* I_r^*)$ and $C_{III}$. An increase in price uncertainty has the same marginal impact as an increase in $\sigma_B$.

In the case of quadratic costs of adjustment, $(1/2)\phi I^2$, there are only two components in (26) that involve the variances and covariance of the stochastic processes. An increase in $\sigma_B$, or $\sigma_R$, leads to an increase in $E_t\{dI^*\}$ when $\sigma_{BR}$ and $J_{br}$ have opposite signs or when $\sigma_{BR} = 0$. Otherwise, one cannot make a definitive assessment about the impact of increasing uncertainty of technical change or future price changes on the expected investment dynamics of the risk-neutral competive firm.

These results critically depend on the farm size investment effect associated with the absorption of capital-augmenting technical knowledge; specifically, $\beta_3 > 0$. With no farm size investment effect on the accumulation of technical knowledge, $\beta_3 = 0$, increased fluctuations in the variation of the stochastic processes influence the expected investment dynamics via the term associated with $C_{III}$. With no farm size effects and quadratic costs of adjustment, the expected dynamics of investment are reduced to

$$(30)\quad (1/dt)E_t\{dI^*\}/I^* = (\rho + \delta) - b\pi_K /(\phi I^*)$$

which states that the proportional rate of expected change in investment is equal to the opportunity cost less the instantaneous change in net cash flow arising from a change in the capital stock per unit of capital invested. Thus, the expected investment dynamics is independent of the degree of uncertainty under quadratic costs of adjustment and without an investment farm size effect.

An increase in uncertainty can either increase or decrease the rate of expected investment at the firm level. Increased fluctuations in the stochastic processes can reduce the value of the effective capital stock. On the other hand, the variances of these processes are an increasing function of the stock of technical knowledge, creating an incentive for the firm to increase investment in order to reduce the variance in the future. The increased fluctuations also increase the expected adjustment costs as time goes on, thus creating an incentive to increase current investment. The decrease in $E_t\{dI^*\}$ principally depends on the farm size effect, the magnitude and direction of the correlation between the evolution of the real wage and the evolution of the stock of technical knowledge, and the curvature of the cost of adjustment function (in particular, the sign of $C_{III}$). The sign of $C_{III}$ provides some insight into how the penalty associated with larger and larger capital expansions (or contractions) changes at the margin. This is illustrated in figure 1. With convex costs of adjustment and $C_{III} < 0$, $C_I$ must asymptotically approach a constant $\underline{c}$ as $|I|$ becomes very large. That is, for very large changes in the capital stock, the marginal cost of adjustment is constant implying that there is no penalty to adjust the capital stock at the margin. With $C_{III} > 0$, $C_I$ grows without bound implying that, at the margin, the larger the change in the capital stock the greater the marginal cost of adjustment.

**Concluding Comments**

Uncertainty has been modeled as a set of a Weiner processes, where changes in the variances and covariances of these processes suggest changes in the level of uncertainty. The certainty version of the investment dynamics in (26) is

$$(26') \quad (1/dt)dI^{*} = (1/C_{II})[(\rho + \delta)J_k - b\pi_K - \beta_2 J_b].$$

The investment dynamics do not depend on the sign of $C_{III}$. In the absence of the investment farm size effect ($\beta_3 = 0$) and in the presence of quadratic adjustment costs ($C_{III} = 0$), the uncertainty and certainty cases are equivalent. However, if $\beta_3 = 0$ and $C_{III} \neq 0$, the expected investment dynamics depend on the evolution of both stochastic processes. Conversely, if $\beta_3 \neq 0$ and $C_{III} = 0$, the expected investment dynamics depend on the stochastic evolution of the stock of technical knowledge and, only for $\sigma_{BR} \neq 0$, on the stochastic evolution of the real wage.

The exact nature of the exogeneous technical change information available to the firm is not specifically addressed. As additional technical knowledge is accumulated, capital and labor use adjusts instantaneously to the extent allowed by the costs of adjusting the capital stock. These results basically provide insight into the firm's decisions to acquire (or disinvest) capital which can be an expansion of the current technological processes employed or a shift to an improved technology. Since a wide range of technological production patterns are typically available to the firm, this interpretation of the results is not especially restrictive. In addition to the impact of price and technological uncertainty on the optimal investment trajectory, the curvatures of the cost of adjustment function and the value function play a role in how quickly investment changes over time. In the absence of uncertainty, equation (26) indicates that the curvature of C(I) influences the rate of change in investment. In particular, as the curvature of the convex cost of adjustment function becomes more extreme (as $C_{II} \to \infty$), expected changes in investment become very small.

## Footnotes

[1] The Abel (1983) and Hartman results suggest that price uncertainty influences the expected investment dynamics given quadratic costs of adjustment. Pindyck (1982) finds that for quadratic adjustment costs the future price uncertainty does not influence the expected investment dynamics. This discrepancy is attributed to the price uncertainty assumption. While Pindyck (1982) considers current prices to be known with certainty and the future movement of prices to be a random process, Abel (1983) and Hartman allow for current price uncertainty as well.

[2] Sharp, discontinuous change in the state of technical knowledge is another possible modeling approach. The discontinuous changes can be modeled by a Poisson process (see Mangel, pp. 22-27). The assumptions of the discontinuous processes model, also known as a jump process, implies that the evolution of the process depends only on the current state (i.e., a Markovian process), but that the exit time from the current state depends on the state.

[3] One should be aware that an infinite number of stochastic calculi exist, each with its own stochastic differentiation rules. While analysis using the Ito calculus is widely conducted in economics, the Stratonovich calculus is another poosibility. Stefanou and Mangel discuss how economic interpretations of fundamental equations in stochastic dynamic analysis change with the choice of calculus.

[4] Benveniste and Scheinkman identify the sufficient conditions to guarantee that J( ) is differentiable for deterministic dynamic optimization. Brock

and Magill presents some results on the properties of the value function for stochastic variational problems in economics.

5 $\pi(r, \kappa)$ concave in $\kappa$ implies that $\pi_{\kappa\kappa}(r, \kappa) < 0$. With $\kappa = bk$,

$$\partial\pi/\partial b = \pi_{\kappa\kappa}(\ )\ (\partial\kappa/\partial b) = \pi_{\kappa}(\ )k$$

and

$$\partial^2\pi/\partial b^2 = k\ \pi_{\kappa\kappa}(\ )\ (\partial\kappa/\partial b) = k^2\pi_{\kappa\kappa}(\ ).$$

By a similar manipulation, one can show that

$$\partial^2\pi/\partial k^2 = b^2\pi_{\kappa\kappa}(\ ).$$

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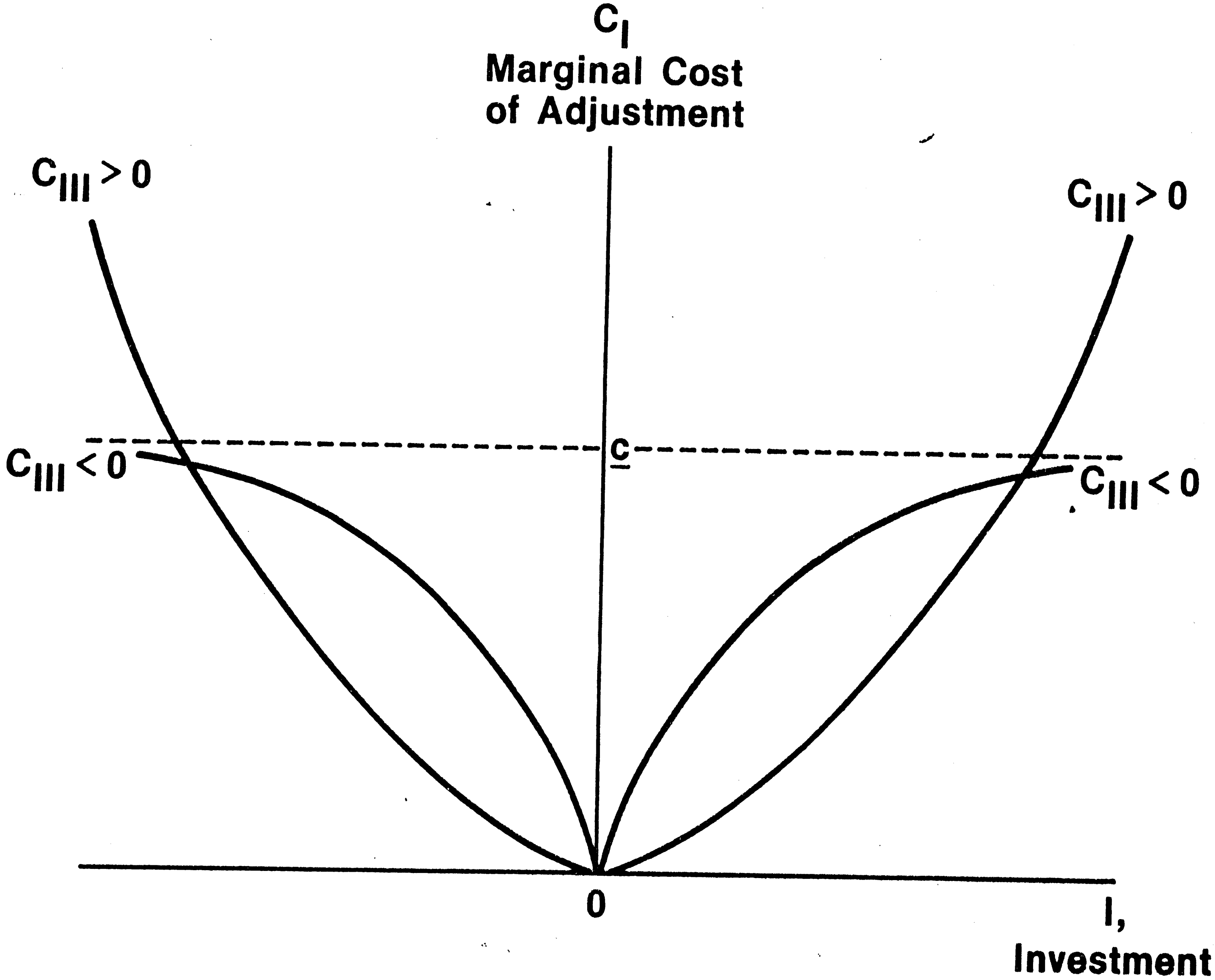
Figure 1. The impact of $C_{III}$ on the marginal cost of adjustment, $C_I$.


$C_I$
Marginal Cost
of Adjustment
$C_{III} > 0$
$C_{III} > 0$
$C_{III} < 0$
$\underline{c}$
$C_{III} < 0$
0
I,
Investment