TOWARDS A MORE GENERAL DYNAMIC ECONOMIC MODEL OF THE OPTIMAL ROTATION OF FORESTS

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The Faustmann model has played a key role in the determination of optimal forest rotations. Faustmann (1968) introduced a simple and deterministic competitive economic model, the objective of which was to maximize the present value of perpetual returns to the fixed factor of production, a unit of timber land. The optimal rotation problem, as viewed by him, was a timber management problem abstracting from the important multiple use characteristics of today's forest stand. Samuelson (1976) took note of the problem. Hartman (1976), Berck (1981) and Strang (1983) developed a modified Faustmann model where the forest resource stock 'per se' is assumed to have consumptive value in the form of "recreation", a general term used to capture non-timber forest uses (e.g. wildlife habitat, flood control, viewing, and hunting).

However, an important issue with bearing on the problem of optimal forest rotation remains yet to be explored. Hartman correctly pointed out that in any realistic model, regeneration costs and the costs of producing and making recreational services accessible to users must be explicitly considered. The required management decision would then be based on net values. Consequently, recreational as well as timber values should be considered net of their costs of generation and maintenance.

This paper attempts to provide an alternative model formulation that includes maintenance of recreational facilities (which involves costs) as an explicit choice variable. It includes costs of maintaining recreational facilities, a fixed regeneration cost of tree population and implicitly considers costs of making recreational services accessible to users. Initially we assume that the maintenance policy remains the same during each period and develop a rotation model following the classical Faustmann formulation. The model is then extended to allow the maintenance policy to vary over time. These two alternative model formulations are solved for the optimal rotation cycle. The solution derived is then compared with the Faustmann and the Hartman-Strang results.

The Model: Faustmann Formulation

In this forestry problem, the forest resource is assumed to be owned by a hypothetical competitive firm operating in an environment of certainty. The forest land is considered to be a source of timber that can be sold in a competitive market when harvested and a source of a flow of recreational values that can be captured and marketed when kept standing. Production of timber and recreational services involve regeneration inputs, inputs required for preparing campgrounds, trail passes, view points, maintaining mountain rescue teams, generating wildlife habitat improvement programs, and providing program administration. Maintenance activity involves inputs related to the preservation of the flow of services of a standing forest in addition to protecting the stock of trees.

Here an initially bare given plot of land is considered, G(t), with all the trees harvested simultaneously. The value of tree growth assumed in this analysis is shown in Figure 1. Individual trees are assumed to be identical and even-aged. The forest manager responsible for regenerating, maintaining, and harvesting a forest stand as well as providing recreational services is faced with the problem of choosing a sequence of time or rotation cycles for successive forest stands that will maximize the net returns that can be made from harvesting and maintaining a forest resource. It is assumed that the objective of the resources manager is to maximize the present discounted value of all net returns calculated over the infinite chain of rotation cycles. Rotation cycle is defined to be the length of time between two regenerations and is denoted by the variable T.

A growing forest along with other cooperating inputs yields a positive value of recreational services, i.e., $F(t) > 0$, net of all costs except of maintenance of flow of recreational services and regeneration of trees. F(t) is assumed to be bounded with respect to t (Figure 2). With the aging process some forests, in some locations (like Redwoods of California or other forests elsewhere endowed with unique species) provide high non-timber values. For them, F(t) is likely to be non-decreasing with respect to t. For some others it is plausible to assume that to an extent old growth trees are subject to "wear out", defined as the decline in the quality of recreational value or quality of the standing forest attributable to the normal forest aging process. This flow of value ceases when the forest is harvested. F(t) depends upon the maintenance activity and the forest site characteristics represented by the resources stock (biomass of trees) as well as the age of the forest,

$$F(t) = F[t, M, X(t)] \tag{1}$$

where

M = current year's rate of maintenance effort.

$X(t)$ = A state variable that represents the tree population at time t.

t = number of years since the initiation of the tree regeneration process.

If the forest is preserved up to age $t = T$, its salvage or stumpage value net of harvesting cost at age T is $G(T)$. We assume that $G'(t) \gtreqless 0$. Initially, the stumpage value, $G(t)$, increases at an increasing rate, then at a decreasing rate, reaches a maximum, starts falling and then may level off in a steady state (Figure 1).

Maintenance cost is

$$C(t) = C[F(t)] = C[F(t, M, X(t))] , \qquad (2)$$

where $C(t)$ is assumed to be nondecreasing.

Given these definitions, the quasi-rent which is revenue from providing recreational services $F(.)$ minus maintenance cost $C(.)$ is:

$$R[t, M, X(t)] = F[t, M, X(t)] - C[F(t, M, X(t))] . \qquad (3)$$

It is assumed that R is bounded with respect to t, and $R'(t) \gtreqless 0$ (Figure 2). Then the net return from a single rotation is given by

$$V_1 = \int_0^T R[t, M, X(t)]e^{-rt}dt + G(T)e^{-rT} - C_0^R \qquad (4)$$

where

r = the discount rate.

C_0^R = a fixed regeneration cost which is incurred at the beginning of rotation cycle.

Given that all rotations are alike, the net return from all future rotations is given by

$$V_\infty = V_1 + e^{-rT} V_1 + e^{-r2T} V_1 + \ldots.$$

or

$$V_\infty = A(T)\left(\int_0^T R[t, M, X(t)]e^{-rt}dt + G(T)e^{-rT} - C_0^R\right) \tag{5}$$

where $A(T) = (1 - e^{-rT})^{-1} = (1 + e^{-rT} + e^{-2rT} + \ldots.)$.

To solve for the optimal rotation time (T), Equation (5) is to be maximized with respect to replacment age T, subject to the law of motion of the state variable, which describes the evolution of the system through time.

The law of motion of the state variable is given by a differential equation of the form,

$$\frac{dx}{dt} = f(t, M, X(t)) . \tag{6}$$

Together with the initial conditions ($X(0) = X_0 = 0$), equation (6) determines the time paths of the state variable as a function of M. Let the solution to this initial value differential equation problem be given by

$$X(t) = \psi(t, M; X_0) . \tag{7}$$

Then the objective is to maximize equation (5) subject to equation (6) and the restrictions

$$X(T) = 0, \quad 0 \leq M \leq 1, \quad 0 \leq T \tag{8}$$

To analyze the problem, equation (7) is substituted into (5) yielding

$$V_\infty = A(T) \left[\int_0^T R(t, M, \psi(.))e^{-rt}dt + G(T)e^{-rT} - C_0^R\right] . \tag{9}$$

The necessary conditions for an internal solution (assuming it exists) with respect to T i.e., ($\partial V_\infty/\partial T = 0$) is satisfied if and only if

$$R[T, M, \psi(M, T; X_0)] + G'(T) - rG(T) - r V_\infty = 0 . \tag{10}$$

The optimal rotation criterion as given by (10), once rearranged implies that

$$R(.) + G'(T) = r[G(T) + V_\infty] . \tag{11}$$

It specifies that the forest should be maintained until the incremental net return from increasing T, resulting both from recreation R(.) as well as timber production (G'(T)), R(.) + G'(T), equals marginal opportunity cost, $r[G(T) + V_\infty]$, where opportunity cost represents interest on wealth realized from a T-year rotation cycle, i.e., the cost of preserving the forest for one more year over all the T-year rotation cycles.

The Model: Optimal Control Formulation

The assumption that the rate of maintenance will be the same during each time period is now relaxed, such that maintenance rate varies through time, M = M(t). This leads to a deterministic optimal control problem. We seek a maintenance policy, M, and harvesting time, T, to maximize the objective functional (5), subject to the initial stock of the state variable X(0) = 0, and the laws of motion of the system, equation (6). The control variable is M, the state is X, and the control parameter is T. Using Theorem 1 of Long and Vousden (1977), the Hamiltonian for the problem can be written as

$$H = A(T) [R(t, M(t), X(t))e^{-rt}] + \phi f(t, M(t), X(t)) , \tag{12}$$

where ϕ = the costate variable.

Given that an internal solution path ($0 \leq M(t) \leq 1$ and $0 \leq T$) exists, having initial condition X(0) = 0, the equation necessary for the determination of the optimal time horizon, T is given by the following transversality condition (Long and Vousden, 1977, PP. 16-17).

$$A(T)e^{-rT}\,[R(T, M(T), X(T)) + G'(T) - rG(T) + (e^{rT}/A(t))\,\phi(T)\, f(T, M(T), X(T)) - r\,V_{\infty}] = 0 \tag{13}$$

$$\phi(T) = 0 \quad \text{for } T < \infty.$$

To facilitate interpretation, define a transformed co-state variable as

$$\lambda = \phi\, e^{rt} \tag{14}$$

The ϕ's are shadow values of the state variable discounted to time zero, and λ is the spot or current period shadow value. The term $(e^{rT}/A(T)).\phi(T).f(T, M(T), X(T))$ is zero because the terminal time is finite and X(T) does not enter the salvage value term. In this case, the terminal condition for λ implied by its transversality condition is $\lambda(T)$ equals zero.

The transversality condition given by equation (13) is satisfied for a finite T if

$$R(T, M(T), X(T)) + G'(T) - r\,G(T) - r\,V_{\infty} = 0$$

or

$$R(T) + G'(T) = r[G(T) + V_{\infty}], \tag{15}$$

where V_{∞} is evaluated at t = T. The rotation period is extended to the point where the quasi-rent and the marginal return from harvesting during the Tth period is equal to the interest that could be earned on the wealth obtained by optimally utilizing the multiple-use forest resource over the T-year rotation period. The situation is also shown in Figure 3. Note that the generalized Faustmann formulation (11) has exactly this same necessary condition. If equation (15) is positive for all T > 0, then the terminal time is not to be bounded above and T is infinite (Kamien and Schwartz, 1981, p. 147). This implies, never to harvest the forest. For an initially barren forest land, an identical condition is required by the "never-to-cut a forest" rule developed by Hartman (1976) and Strang (1983) in their models without costs of recreation and regeneration. How plausible is this rule in the case considered here, i.e. in the presence of both type of costs?

An unbounded terminal time implies

$$\lim_{T \to \infty} [R(T) + G'(T) - r(G(T) + V_\infty(T))] > 0 \ . \tag{16}$$

Given that R and G are bounded and monotonic in the relevant time interval and assuming $\lim_{T \to \infty} G'(T) = 0$, inequality (16) implies

$$\frac{R(\infty)}{r} > G(\infty) + \int_0^\infty R(.)e^{-rt}dt - C_0^R \ . \tag{17}$$

The inequality (17) can be interpreted as follows: Since $R(\infty)/r = \int_0^\infty R(\infty)e^{-rt}dt$, the left-hand side is the accumulated discounted quasi-rent stream derived from starting with an infinitely old forest and never harvesting it. The right-hand side is the return obtained by starting with an infinitely old forest, harvesting it immediately to realize the stumpage value $G(\infty)$, replanting the forest immediately by incurring a regeneration cost C_0^R, without ever cutting it again to derive an accumulated discounted flow of quasi-rent $\int_0^\infty R(t)e^{-rt}dt$.

For some climax forests with very high non-timber values associated with them, such that R(t) is non-decreasing, it is plausible to assume that inequality (17) holds and (15) is positive. This implies that the maximum net return is obtained if the forest is never cut. Otherwise, given the general nature of the quasi-rent and the stumpage value functions with respect to age of a forest, and the above interpretation;

it seems plausible to assume that equation (15) holds. This implies that under the more general situation considered here, the maximum net return is obtained at a finite rotation age.

How does the finite rotation period implied by equation (15) compare with the simple Faustmann formulation as well as Hartman-Strang finite time generalized Faustmann formulation? For this comparison, let us write equation (15) alternatively as

$$\frac{G'(T)}{G(T)} = \frac{1}{\rho} + \left\{ \frac{1}{\rho} \cdot \frac{\int_0^T R(t)e^{-rt}dt - C_0^R}{G(T)} - \frac{R(T)}{G(T)} \right\}, \tag{15'}$$

where $\rho = 1 - e^{-rT}/r = \int_0^T e^{-rt}dt$, is the present value of a dollar stream of return for T years. If $R(.) = C_0^R = 0$, i.e., when a forest has only stumpage value, equation (15') is converted into the simple Faustmann rule

$$G'(T)/G(T) = \frac{1}{\rho}, \tag{18}$$

where $G'(T)/G(T)$ is the growth rate of trees at the time of harvest. If $C_0^R = 0$ and there is no variable costs associated with recreational services, equation (15') turns out to be the generalized Faustmann rule of Hartman and Strang:

$$G'(T)/G(T) = \frac{1}{\rho} + \frac{1}{\rho} \cdot \frac{\int_0^T F(t)e^{-rt}dt}{G(T)} - \frac{F(T)}{G(T)}. \tag{19}$$

If the quasi-rent flow R(t) is constant over time, $\int_0^T R(t)e^{-rt}dt = \rho R(T)$, so that equation (15'), implies $G'(T)/G(T) = 1/\rho - C_0^R/\rho G(T)$. Since the effective discount rate is now lower this indicates a longer rotation period than that suggested by the simple Faustmann rule given by equation (18). Otherwise, the rotation period will depend on the sign of the term within the braces in equation (15'), or equivalently on whether $(\int_0^T R(t)e^{-rt}dt - C_0^R)/\rho - R(T) \gtreqless 0$. The first term of this expression is the net of regeneration cost discounted quasi-rent flow per period and the second term is the marginal quasi-rent derived at time T. If the former value is less than the latter value the effective interest rate will go down further and that will imply a further lengthening of the rotation period. Otherwise, the rotation period will be less than or equal to that implied by the simple Faustmann rule (18).

For comparison with (19), we decompose (15') as

$$\frac{G'(T)}{G(T)} = \frac{1}{\rho} + \frac{1}{\rho} \cdot \frac{\int_0^T F(t)e^{-rt}dt}{G(T)} - \frac{F(T)}{G(T)} + \left(\frac{C(T)}{G(T)} - \frac{1}{\rho} \cdot \left\{\frac{\int_0^T C(t)e^{-rt}dt}{G(T)} + \frac{C_0^R}{G(T)}\right\}\right). \qquad (15'')$$

Excepting the term within the parentheses, (15") is exactly the same as (19). Thus, based on the logic developed earlier (i.e., whether the discount rate is inflated or deflated or remain unchanged) the length of rotation implied by (15') or (15") compared to that implied by (19) will depend on whether the term of cost components within the parentheses is positive, zero, or negative, i.e., whether

$$C(T) - \frac{1}{\rho} \cdot \left[\int_0^T C(t)e^{-rt}dt + C_0^R\right] \gtreqless 0 \qquad (20)$$

Here C(T) is the marginal variable costs incurred for recreational services derived from the forest at time T. The second term is the per period present value of accumulated costs over T-years, associated with the regenerated forest. Hence, the difference between the finite rotation period suggested by Hartman-Strang formulation and the more generalized formulation developed here will depend crucially on the relative size of the marginal variable costs at time T and the present value of variable costs per period. The differences in costs will be reflected in the differences in "effective interest rate" and hence in the optimal rotation lengths.

Previous works for determining the optimal forest rotation ignored some important costs associated with having the forest. Costs are incurred in planting the trees. Costs are incurred in developing and maintaining the forest to obtain recreational services. The inclusion of these costs in the present paper lends it a more general character and significantly complicates the derivation and interpretation of the results. Despite the complexities, we obtain a relatively simple relationship (equation 15") to isolate the conditions under which the introduction of relevant costs leads to a shorter or longer optimal rotation. The present work also reduces the set of parameters under which the "never to cut" solution of Hartman and Strang would be valid.

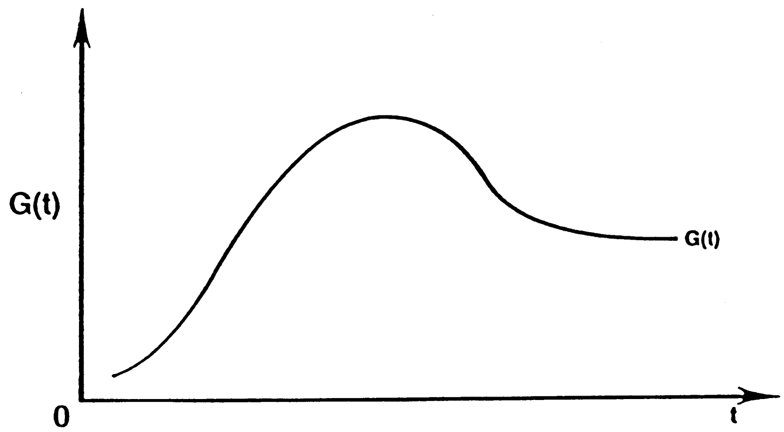

Figure 1

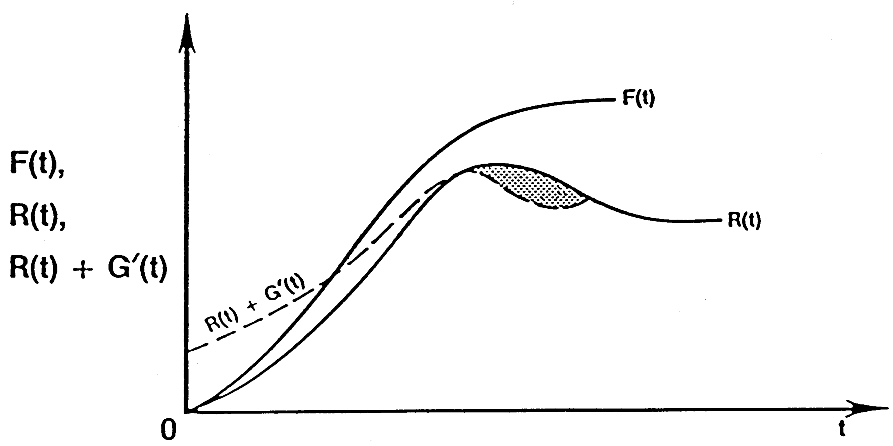

Figure 2

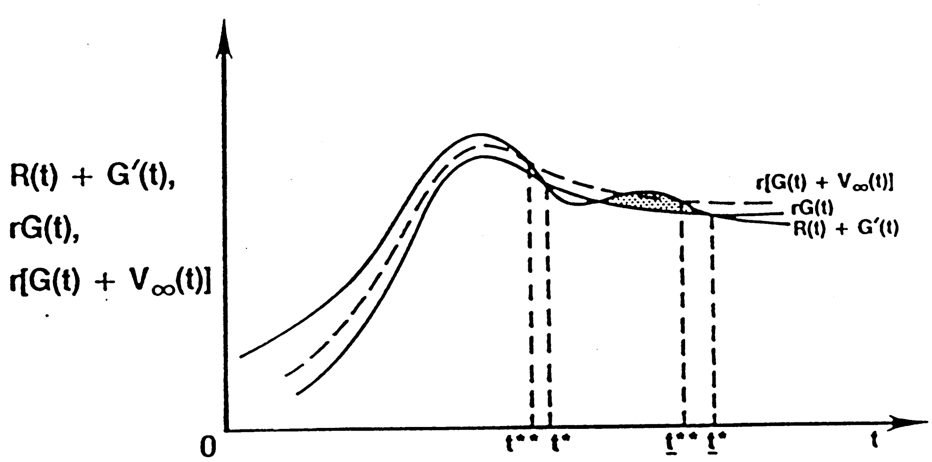

Figure 3

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