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COMPARING RANDOM PROFIT FROM 'OPTIMAL' INPUT RECOMMENDATIONS

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Comparing Random Profit from 'Optimal' Input Recommendations

Since publication of the classic work of Heady and Pesek, the estimation of production functions from experimental data and use of these functions to derive profit-maximizing or cost-minimizing input levels have become common exercises for agricultural economists. Such analysis generally comprises four steps: (1) conceptualization of the manner in which one or more inputs affect a particular output; (2) statistical testing of alternative specifications of the input-output relationship; (3) adoption of a final specification; and (4) calculation of economic optima assuming particular prices.

Step (1) includes conceptualization in terms of both functional form and variable specification[1] Neither the true functional form nor the true specification can be known. Exact parameter values and, therefore, exact economic optima cannot be determined. Seldom are these inherent deficiencies of the modeling process explicitly recognized when optimal input recommendations are reported. Although uncertainty attached to point estimates of optima from a single model may be quantitatively estimated, presentation of such measures is rare. A method of estimating how closely a selected model approximates the true relationship has not been devised, and evaluation of the quality of results from Steps (1) and (3) has, in the few instances when attempted, centered on estimating the cost of selecting one model when another is assumed to be "true." This

[1] Griffin, Rister, Montgomery, and Turner illustrate that these are not the same process: i.e., for a particular functional form and a particular set of inputs, more than one model may be specified.

procedure has been conducted with point estimates of optima, the variability of which, again, has not been explicitly recognized. This paper demonstrates the effect of considering optima precision when estimating the cost of alternative decisions based on estimated functions. Tentative evidence is that, when the stochastic nature of optima is taken into account, Steps (1), (2), and (3) have less impact on determination of optima than may be generally thought.

Following a review of pertinent literature, we describe how the concepts of variance and of opportunity cost may be combined to provide practical information about modeling uncertainty to both the researcher and research client. An empirical example is presented to illustrate the procedure. The literature reviewed and empirical example presented involve profit-maximizing crop response to fertilizer, though the method described should be applicable to other types of economic studies involving production functions estimated by least squares regression.

Relevant Literature

Some researchers have estimated the uncertainty present in economic optima derived from functions assumed to correctly represent a production relationship. Others have estimated the cost of operating at suboptimal levels or optima derived from "incorrect" production models. A few analysts have combined these two procedures.

R. L. Anderson; Doll, Jebe, and Munson; and Fuller have estimated the uncertainty in derived economic optima. Anderson presented a formula for calculating confidence limits for the optimal level of an input derived from a single-input quadratic production function. Referring to the procedures of Anderson, but using a quadratic production function relating crop yield to two inputs, Doll, Jebe, and Munson fixed one input level at its experimental treatment mean and computed confidence limits for profit-maximizing levels of the remaining input. Fuller employed crop yield response functions of two different algebraic forms to demonstrate procedures for constructing confidence regions for optimum quantities of two inputs, but no attempt was made to compare these regions.

Researchers interested in the opportunity cost of choosing (or rejecting) a particular model, but disregarding variability of the economic optima derived from those models, have included J. R. Anderson, Lanzer and Paris, and Bay and Schoney. Opportunity cost, in this context, is defined to be the reduction in net revenue due to application of a suboptimal input level. For example, J. R. Anderson estimated a quadratic production function relating crop yield to three fertilizer inputs and calculated maximum profit assuming particular prices. The opportunity cost was then obtained by comparing this figure with profits accruing if input levels recommended from other sources were applied, given yields predicted from the estimated function. Similarly, Lanzer and Paris considered the difference in cost of fertilizer application required for nutrient maintenance determined by their models and by government agronomists. Although Lanzer and Paris reported the difference to be "significant," variability of the estimates was not explicitly considered. Bay and Schoney estimated three production functions of different algebraic forms and also employed computer graphics routines to generate a data surface plot of the input-output relationship. Each of the four models was, in turn, assumed to correctly specify the production process. Net returns resulting from substituting the optimum input combination of the "wrong" model into the "correct" model were deducted from the "true" returns to obtain the "cost of being wrong."

Some researchers have considered variability in derived optima while comparing models. Perrin estimated two models of the relationship between crop yield and soil nutrients. Based on a *t* test on paired differences, he found that one model showed soil test information to be of no value, while the other model indicated the value of this information to be "very high." Havlicek and Seagraves estimated quadratic and square root functions relating crop yield to applied nitrogen and a moisture deficiency index. Following the procedures presented by Box and Hunter, confidence intervals about optimum nitrogen levels were computed for two moisture levels. The researchers noted that the confidence intervals on the optimal nitrogen levels derived from the two models did not overlap at the mean experimental weather conditions, but that point estimates of expected net returns differed by less than one dollar per acre when the optimal level of nitrogen indicated by either model was substituted into the remaining model. As a result, Havlicek and Seagraves concluded that the optimal levels of nitrogen differed significantly "in a statistical sense but not in an economic sense," stating that since the dollar loss from using an incorrect model was "economically insignificant," "we can use whichever equation we like, or we can flip a coin in order to decide which equation to use" (pp. 162-3).

Three major subjects are raised in the literature reviewed: 1) on what variables should precision be measured; 2) how should the precision of variables of interest be used to compare alternative recommendations, especially if these come from alternative models; and 3) what should be done by the researcher if a significant difference between variables/models exists or does not exist. Most researchers have estimated variability of optimal input levels, although Perrin estimated variability in net returns. He used a t test to compare the variability of maximum returns derived from two models, while Havlicek and Seagraves considered whether confidence intervals on optimal input levels derived from alternative models overlapped. Perrin appears to indicate that there is no relative cost associated with using a model for which the economic optimum is not significantly different from that derived from an alternative model, though Havlicek and Seagraves conclude that there is no cost even when a significant difference does exist. The evidence from the literature is that if no cost exists, any one of the alternative models may be selected or optima reported.

Methodology

Considering the initial question raised in the literature, two stochastic variables appear to be of major interest: the optimal input level and the maximum profit. If researcher and client are concerned with a single input, precision of this variable may tractably be estimated. However, if a multiple-input production function is specified, confidence intervals become confidence regions, and region boundaries may be difficult to specify. Confidence regions may, in certain instances, be open (Box and Hunter). On the other hand, yield or profit provides a single variable of which the precision may be more easily measured and communicated. This information may be provided to the producer, who is undoubtedly in the best position to assess what is absolutely significant in an "economic sense."

The modeler, however, should be able to determine when the estimated cost of alternative recommendations is, in a statistical sense, significantly different from zero. While tests such as those conducted by Perrin and by Havlicek and Seagraves may be performed, a conclusion of no significant difference between optima derived from two different functions does not necessarily imply zero cost of choosing an alternative recommendation. Zero cost is implied if, when one model is assumed to be "true" and the input quantities recommended by the alternative source are substituted into the "true" model, no statistically significant difference between the resulting profit level and the maximum profit level derived from the "true" model is found. The significance of this difference may be tested using the *t* distribution.

If the difference between profits from alternative input recommendations is not statistically significant, then it may be concluded that no wrong decision can be made in model (recommendation) selection. While the cost of choosing one model over another may be statistically zero, this does not mean that one model should be selected and the other forgotten. Producers may find it useful to have information about the range of input levels (over the two models) which provide statistically equal profit. On the other hand, if differences in profits resulting from alternative input recommendations are statistically significant, relative estimated cost of model selection is positive and should be reported. Yet, with no criteria for model selection, presentation of reliability information for both models seems desirable.

If such reliability is to be estimated and the *t* test is to be conducted, it is necessary to estimate the variance of both calculated optima and suboptima. The remainder of this section describes procedures for calculating appropriate statistics. The proposed methodology is explained in terms of a function relating per acre crop response to fertilizer.

The imperfectly understood production process can be approximated by some theoretical model, or production function, such as

(1) $y = f(x;z)$

where y is crop yield, x represents decision variables, and z represents uncontrolled factors which can be measured. Profit maximization is the assumed objective with profit (π) defined as

(2) $\pi = p \cdot y - w' \cdot x$

where p is product price and w' is a vector of input prices. Once the production function is specified as, say, the linear stochastic relation

(3) $y = \beta_0 + \Sigma\beta_i x_i + \Sigma\beta_j z_j + \epsilon$

where the β's are parameters of the model and ϵ refers to the disturbance term, optimal levels of x, y, and π may be defined. Upon estimation of the production function parameters (B) using sample data, estimated optimal values ($\hat{x}^*$, $\hat{y}^*$, and $\hat{\pi}^*$) may be obtained for particular values of p, w, and z.

Thus, for fixed p and w, the estimated maximum value of profit may be written as

(4) $\hat{\pi}^* = h(\hat{B})$

where h is a nonlinear function. The variance of $\hat{\pi}^*$ is also a function of the parameter estimates. Unlike the variance of $\hat{\pi}_0$, which is derived from the estimated parameters of the production function and a selected determinant value of x, x_0, the variance of $\hat{\pi}^*$ is nonlinear due to the arguments in the profit equation which define $\hat{x}^*$ in terms of $\hat{B}$. The predicted mean value of profit, $\hat{\pi}_0$, given x_0, is a random variable with variance given by

(5) $\mathrm{Var}(\hat{\pi}_0) = p^2 \cdot \mathrm{Var}(\hat{y}_0)$

and, following Judge et al., the variance of the predicted yield is estimated by

(6) $s^2_{\hat{y}_0} = [\hat{e}'\hat{e}/(n-k)] \cdot (x_0'(X'X)^{-1}x_0)$

where e is the vector of residuals from the estimated production function, n is the sample size, k is the number of parameters, and X is the design matrix (pp. 139-45). This is the estimated variance of the predicted mean value of, in this case, per acre yield for given levels of the inputs x and may be easily written because $\hat{y}_0$ is a linear combination of the random variables $\hat{\beta}_0$, $\hat{\beta}_i$, and $\hat{\beta}_j$.

If the variance of $\hat{\pi}^*$ is to be approximated, the function h must be linearized. This may be accomplished by expanding the function around the true values of the parameters (B) according to Taylor's theorem. To begin, let $\hat{L}$ represent a vector of the derivatives of the function h with respect to $\hat{B}$. Expansion around B yields

(7) $\hat{\pi}^* = h(B) + \hat{L}' \cdot (\hat{B}-B)$ + higher order terms.

Provided the model is correctly specified, h(B) is the true value of profit, which allows substitution of π for this function such that

(8) $\hat{\pi}^* = \pi + \hat{L}' \cdot (\hat{B}-B)$ + higher order terms.

Rearranging terms,

(9) $\hat{\pi}^* - \pi = \hat{L}' \cdot (\hat{B}-B)$ + higher order terms.

The higher order terms tend to zero faster than $(\hat{B}-B)$ in a probability limit sense. It follows that

(10) $\text{Var}(\hat{\pi}^* - \pi) = \hat{L}' \cdot \text{Var}(\hat{B}-B) \cdot \hat{L}$.

Therefore, the limiting or asymptotic variance of the estimated maximum profit is given by

(11) $\text{Var}(\hat{\pi}^*) = \hat{L}' \cdot \text{Var}(\hat{B}) \cdot \hat{L}$

where

(12) $\text{Var}(\hat{B}) = \sigma^2 \cdot (X'X)^{-1}$

which may be estimated by

(13) $\hat{\text{Var}}(\hat{B}) = [\hat{e}'\hat{e}/(n-k-1)] \cdot (X'X)^{-1}$.

Substitution of (13) into (11) provides an estimator of the variance of $\hat{\pi}^*$, $s^2_{\hat{\pi}^*}$. The 100(1-a)% confidence limits for the predicted mean value of maximum profit, then, are given by

(14) $\hat{\pi}^* \pm t_{a/2,n-k-1} \cdot s_{\hat{\pi}^*}$

where $t_{a/2,n-k-1}$ is the 100(1-a/2) quantile of Student's t distribution with n-k-1 degrees of freedom.

Estimates of the variance of $\hat{\pi}^*$ or $\hat{\pi}_0$ may be used to test the null hypothesis that two profit figures are the same (H_0: $\hat{\pi}_1 = \hat{\pi}_2$) against the alternate hypothesis that they are different (H_A: $\hat{\pi}_1 \neq \hat{\pi}_2$). If the two variances are unknown but presumed to be equal, the test statistic,

(15) $t = (\hat{\pi}_1 - \hat{\pi}_2) \;/\; [s_{pool} \cdot (1/n_1 + 1/n_2)^{1/2}]$

includes the pooled variance estimator

(16) $s^2_{pool} = [(n_1-1)s^2_{\hat{\pi}_1} + (n_2-1)s^2_{\hat{\pi}_2} + 2(n_1+n_2-2) \cdot \text{Cov}(\hat{\pi}_1, \hat{\pi}_2)]/(n_1+n_2-2)$.

and has a t distribution with n_1+n_2-2 degrees of freedom.

Empirical Problem

Use of the above methodology may be illustrated through application to previously reported work of Griffin, Rister, Montgomery, and Turner. They chose two functional forms and two input conceptualizations for analysis of 1920 observations of experimental response to weather, nitrogen application sequences (timings), and fertilization rates in Texas. Four production functions were identified by the following designations: (1) quadratic Split-N; (2) quadratic Total-N; (3) square root Split-N; and (4) square root Total-N. Each model was used to estimate optimal rates of nitrogen application for each of the five timings. The researchers compared the profit levels and optimal nitrogen rate (presented in Table 1), concluding that "the small ranges of maximum profit levels between timings within each of the four models suggest the need to explore the statistical significance of the economic results" (p. 166).

Within each of the four models, to determine whether the maximum profit for the timing calculated to have the largest profit is statistically greater than the maximum profit for another timing, a *t* test is conducted at significance level $\alpha = .05$. Due to the large number of observations used in the regressions, the degrees of freedom approach infinity. The critical value is $t_{.025,\infty} = 1.96$. If $t<1.96$, the profit levels are not (at the specified confidence level) statistically different; if $t>1.96$, the two levels are different, and the first timing may be judged superior to (more profitable than) the second, assuming that model is correct.

Between-model tests may be conducted between like timings or unlike timings, but are not as conclusive as within-model tests. Such a test first involves substituting the input level specified by one model into another model to obtain the predicted profit for that timing, and then considering whether this profit lies within the confidence interval on the

Table 1. Estimated Maximum Profit Per Nitrogen Timing For Rice Production, By Model Type, Texas, 1976-79[a]

Quadratic: Timing (ranked)	Quadratic: Profit[b] ($/acre)	Quadratic: Nitrogen (lb/acre)		Square root: Timing (ranked)	Square root: Profit[b] ($/acre)	Square root: Nitrogen (lb/acre)
			Split-N			
PP-PF	413.68	89.54		PP-PF	410.98	76.89
EP-PF-PD	409.14	98.50		PP-PF-PD	406.58	84.44
PP	408.48	109.15		PP	406.40	93.88
PP-PF-PD	406.70	98.46		EP-PF-PD	406.37	85.26
PP-PD	403.06	112.86		PP-PD	404.35	106.18
			Total-N			
PP	409.91	106.15		PP-PD	413.75	94.53
PP-PF	409.60	100.70		PP	408.95	100.93
PP-PF-PD or				PP-PF	408.85	94.53
EP-PF-PD	407.45	96.38		PP-PF-PD or		
PP-PD	405.78	100.70		EP-PF-PD	406.94	88.89

[a]Reproduced from Griffin, Rister, Montgomery, and Turner, p.165.
[b]Total revenue less nitrogen material and application costs.

Table 2. Confidence Limits On Estimated Maximum Profit Per Timing, By Model Type, α=.05

Quadratic: Timing	Limits on Profit Lower ($/acre)	Limits on Profit Upper ($/acre)	Variance ($/acre)	t		Square root: Timing	Limits on Profit Lower ($/acre)	Limits on Profit Upper ($/acre)	Variance ($/acre)	t
					Split-N					
PP-PF	406.10	421.26	14.95	0.00		PP-PF	401.93	420.02	21.31	0.00
EP-PF-PD	400.32	417.96	20.23	0.72		PP-PF-PD	398.45	414.71	17.20	0.58
PP	398.41	418.55	26.42	0.72		PP	397.08	415.72	22.63	0.60
PP-PF-PD	397.97	415.43	19.85	0.97		EP-PF-PD	397.02	415.72	22.77	0.64
PP-PD	392.50	413.62	29.05	1.73		PP-PD	395.16	413.54	22.00	1.06
					Total-N					
PP	402.35	417.47	14.86	0.00		PP-PD	406.10	421.40	15.23	0.00
PP-PF	404.37	414.83	7.12	0.05		PP	400.91	416.99	16.82	0.72
PP-PF-PD or						PP-PF	403.34	414.36	7.89	0.84
EP-PF-PD	401.42	413.48	9.46	0.46		PP-PF-PD or				
PP-PD	398.89	412.67	12.35	0.67		EP-PF-PD	400.56	413.32	10.59	1.14

[a]$t \leq 1.96$ implies no significant difference between profit estimates for this timing and for first timing listed in model.

Table 3. Predicted Profit For Particular Nitrogen Levels and Timings, With t Statistic (=.05), By Model Type

Nitrogen (lbs/acre)	Timing	Quadratic: Split-N Profit ($/acre)	Quadratic: Total-N Profit ($/acre)	Square Root: Split-N Profit ($/acre)	Square Root: Total-N Profit ($/acre)
89.54	PP-PF	413.68 (0.00)[a]	408.83 (0.18)[a]	409.86 (0.14)[a]	408.69 (0.89)[a]
106.15	PP	408.45 (0.88)[a]	409.91 (0.00)[a]	405.73 (0.80)[a]	408.82 (0.83)[a]
76.89	PP-PF	412.62 (0.14)[a]	406.49 (0.58)[a]	410.98 (0.00)[a]	406.92 (1.21)[a]
94.53	PP-PD	401.60 (2.34)[a]	405.51 (0.73)[a]	403.81 (1.19)[a]	413.75 (0.00)[a]

[a]$t \leq 1.96$ implies no significant difference between profit estimate for this timing and maximum profit for model.

optimal timing and level. This procedure is only an approximate test between models, because it involves a test between $\hat{\pi}*$ and $\hat{\pi}_0$. Not all variation in the alternate recommendation is taken into account. That is, $\hat{\pi}*$ (or $\hat{\pi}_1$ in the test statistic) includes variation associated with both $\hat{x}*$ and $\hat{y}*$ for the first model, whereas $\hat{\pi}_0$ (or $\hat{\pi}_2$) includes variation associated with $\hat{y}*$ for the second model but not that associated with $\hat{x}*$ for the second model. Unlike the within-model case, a significant difference between predicted profits among models will not result in a conclusion of superiority of one model or timing over another. However, the cost of selecting the "wrong" model, possibly useful information to research clientel, may be estimated.

Results

Point estimates of the mean values of maximum profit per acre ($\hat{\pi}*$) for each timing, as well as associated optimal nitrogen fertilization rates, are presented in Table 1. Confidence limits (α = .05) on the predicted mean value of maximum profit, estimates of the variance of this value ($s^2_{\hat{\pi}*}$), and calculated t values testing the null hypothesis that the profit is significantly different from the maximum profit for the most profitable timing within that model are presented in Table 2. Point estimates of mean maximum profit for each timing, within each model, lie within the confidence interval on the greatest maximum profit for that model, with the single exception of the PP-PD timing in the quadratic Split-N model. However, the upper bound of the interval on this estimate lies will within the interval on the maximal value (PP-PF timing). While the t statistic for a test between these two mean profit estimates is evaluated to be near the critical value, the test indicates no statistical difference at the designated significance level. Indeed, within each model no timing may be selected as superior to another.

If models are to be compared, Table 3 should be considered. This table presents profit ($\hat{\pi}_0$) obtained in each model if the optimal fertilization rate and timing suggested by the other three models is applied. Results of *t* tests of no significant difference (between $\hat{\pi}^*$ and $\hat{\pi}_0$) are presented within parameters. The only estimate (in Table 3) not falling within the confidence interval on an optimal level (in Table 2) is that obtained from the quadratic Split-N model when the recommendation of the square root Total-N model is applied. For example, suppose the square root Total-N model were selected by the researcher for analytic purposes. As a result, the PP-PD timing would be recommended at an application rate of 94.53 lbs/acre, with an estimated profit ($\hat{\pi}^*$) of $413.75/acre. If, in fact, the quadratic Split-N model comprised the true functional input-output relationship, the expected value of profit ($\hat{\pi}_0$) from an equivalent amount of nitrogen applied PP-PD would be only $401.60/acre. Conducting a a test between these two profit estimates, the *t* statistic is calculated to be 2.34, which does not allow rejection of the null hypothesis. Therefore, a statistically significant loss from using the "incorrect" model may occur in this case (and only in this case). Point estimates of the cost of this suboptimal fertilizer rate and schedule imply a loss of $413.68-$401.60, or $12.08/acre, which is a total of $4832.00 for a 400-acre operation.[2] However, caution should be exercised when interpreting the statistics in this manner due to lack of explicit

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[2] The average harvested rice area of farms growing this crop in Texas is about 400 acres (USDC).

recognition of variability in the substituted alternative input recommendation.

Estimates of confidence bounds, variances, and *t* tests based on the assumption that optimal input recommendations are determinant are presented in Table 4. These figures, which require much less effort to calculate than the exact estimates in Table 2, result in the same conclusions as these latter estimates.

Conclusions and Implications

Procedures for comparing two input recommendations where one or both such recommendations are derived from an estimated production function have been presented. Specifically, the alternative recommendation should be substituted into the model assumed to be true, and the resulting profit estimate should be statistically tested for equality with the estimate from the "true" model. Other methodologies presented in the literature and reviewed here either do not account for the stochastic nature of profit predictions, or do not include the substitution step. A combination of these concepts is required to properly estimate the cost of a "wrong" decision. The empirical example presented above shows that when optima variability is properly considered, quite different input recommendations and models may provide statistically equal profit. While this may result in a wide range of input recommendations being reported to the research client, to not provide such information is to impute greater precision to models and their recommendations than is warranted. As Paarlberg has recently commented, "the essence of inquiry is that it is not too sure of the answers."

Table 4. Confidence Limits On Estimated Profit Per Timing, Input Level Not Random, By Model Type, $\alpha = .05$

Model Type									
Quadratic					Square root				
	Limits on Profit		Variance	t		Limits on Profit		Variance	t
Timing	Lower ($/acre)	Upper ($/acre)	($/acre)		Timing	Lower ($/acre)	Upper ($/acre)	($/acre)	
Split-N									
PP-PF	406.67	420.69	12.78	0.00	PP-PF	403.27	418.69	15.47	0.00
EP-PF-PD	401.70	416.58	14.42	0.78	PP-PF-PD	399.13	414.03	14.44	0.66
PP	401.43	415.53	12.94	0.88	PP	398.67	414.13	15.55	0.70
PP-PF-PD	400.08	413.32	11.40	1.16	EP-PF-PD	398.28	414.46	17.05	0.71
PP-PD	396.18	409.94	12.30	1.95	PP-PD	397.05	411.65	13.87	1.09
Total-N									
PP	403.56	416.26	10.50	0.00	PP-PD	406.22	421.28	14.78	0.00
PP-PF	404.60	414.60	6.51	0.06	PP	402.24	415.66	11.71	0.79
PP-PF-PD or EP-PF-PD	401.98	412.92	7.78	0.48	PP-PF	403.50	414.20	7.44	0.86
PP-PD	399.06	412.50	11.74	0.74	PP-PF-PD or EP-PF-PD	401.14	412.74	8.77	1.18

[a]$t \leq 1.96$ implies no significant difference between profit estimate for this timing and maximum profit for model.

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