United States
Department of
Agriculture

Economic
Research
Service

**Natural
Resource
Economics
Division**

Is the Phillips Curve Stable?

A Time-Varying Parameter Approach

Roger K. Conway
Gurmukh Gill

Is the Phillips Curve Stable? A Time-Varying Parameter Approach.
Roger K. Conway and Gurmukh S. Gill, Natural Resource Economics Division,
Economic Research Service, U.S. Department of Agriculture, ERS Staff Report
No. AGES861209.

ABSTRACT

We derive two empirical Phillips curve models based on Robert Gordon's reduced form specification of conventional wage and price equations of a more complete structural model of the U.S. economy. One is a stochastic coefficients model and the other is a conventional fixed coefficients model. We used a stochastic coefficients empirical model to investigate the volatility of the Phillips curve relationship hypothesized by many economists during the 1970's. The visual evidence of the time-varying parameter plots suggests there has been variation in the shortrun Phillips curve. Comparative forecasting shows the stochastic coefficients model dominates the fixed coefficients model, further supporting the hypothesis of volatility.

Keywords: Phillips curve, Natural rate hypothesis, Macroeconomic policy, stochastic coefficients

ACKNOWLEDGMENTS

The authors thank Lorna Aldrich, Dan Larkins, Bill Lin, Tom Lutton, John Kitchen, Robbin Shoemaker, Jack Triplett, Frank de Leeuw, and Kan Young for comments, and Thelma Pearson and Nadine Loften for their expert preparation of this manuscript.

1301 New York Avenue, NW.
Washington, DC 20005-4788

May 1987

CONTENTS

Is the Phillips Curve Stable?

A Time-Varying Parameter Approach

Roger K. Conway
Gurmukh Gill

INTRODUCTION

Almost 30 years ago, A. W. Phillips discovered an inverse relationship between British unemployment and inflation rates. His empirical finding became known as the Phillips curve and gave rise to the notion that policymakers could trade off inflation with unemployment and thus achieve an "optimal" combination of both. The importance of his result as a policymakers' tool motivated numerous empirical studies attempting to test this finding for other periods and countries.1/

The view that a stable and permanent tradeoff between unemployment and inflation exists seemed unsupported during the turbulent 1970's, when a positive correlation between unemployment and inflation developed. In the early 1970's, Phelps (1967) and Friedman (1968) argued that in the long run there was only one rate of unemployment--the "natural" rate of unemployment--that could be sustained with a steady rate of inflation. Further, Friedman stated that there was no stable longrun relationship between inflation and unemployment, but only a shortrun relationship between unanticipated inflation and unemployment. Friedman's reasoning was as follows. Initially during an inflationary period, labor is misled by money illusion into believing that the higher nominal wages offered are really higher real wages. As a result, labor offers more services and the unemployment rate drops. Once labor discovers that it misjudged the inflation rate, it withdraws extra services, and the unemployment rate returns to its natural rate. A shortrun tradeoff between inflation and unemployment does exist because expectations of inflation adjust slowly. However, since firms and labor cannot be permanently fooled, there is no longrun relationship between inflation and unemployment. Unemployment can be kept below the natural rate only by accelerating inflation. Policy options still exist, but the tradeoff must be made between gains from temporary employment and losses from permanently higher inflation rates. Therefore, instead of a unique, invariant relationship, the natural-rate hypothesis argues for a series of shifting shortrun Phillips curves, each corresponding to different underlying conditions and expectations in the labor and product markets.

Conway is section leader, Productivity and Public Policy Section, Natural Resource Economics Division, Economic Research Service, U.S. Department of Agriculture. Gill is division chief, Current Business Analysis Division, Bureau of Economic Analysis, U.S. Department of Commerce. The views expressed are those of the authors and do not necessarily reflect the views of the U.S. Department of Agriculture or the U.S. Department of Commerce.

1/ See Humphrey (1985) and Santomero and Seater (1978) for excellent surveys of the Phillips curve literature.

Some rational-expectations theorists go further and argue that no systematic error in forecasting inflation rates will occur because individuals learn from experience and improve their forecasts. Any deviation of the unemployment rate from its natural rate will be the result of random errors. Mistakes in forecasting inflation are possible, but they cannot be systematic. Acquisition of new information renders Phillips curves unstable and their empirical estimation useless. The most powerful expression of this viewpoint was the famous polemic by Lucas and Sargent (1978).

Nevertheless, Phillips curve econometrics has been revived by its success in tracking the 1981-83 disinflation. Papers by Eckstein (1983), Englander and Los (1983 a,b), Perry (1983), Blanchard (1984), and Gordon (1982, 1984, 1985 a,b) now contend that there is little volatility in the Phillips curve. Since the Phillips curve is viewed as an important "rule of thumb" by policymakers, the volatility issue is a significant one. If the Phillips curve has been shifting, then policy analysis and forecasting based on a fixed coefficients model may yield dubious results. This article examines the volatility hypothesis by applying stochastic-coefficients empirical procedures to a variant of the the basic single equation model developed by Robert Gordon--a leading practioner. His model is modified somewhat by using fixed-weighted price measures, rather than implicit price deflators, and by using de Leeuw's and McKelvey's expectations model, rather than adaptive-expectations distributed-lag framework. The results under this more general testing procedure conform to the widely held belief that the shortrun Phillips curve is volatile and dispute Gordon's and others recent findings of parameter fixity based on a questionable testing procedure. Results here do indicate that shifts in the longrun Phillips curve are negligible. However, the shortrun Phillips curve as well as the natural rate of unemployment has apparently experienced pronounced shifts, conditional in this specification.

MODEL SPECIFICATION

In a recent series of articles, Robert Gordon estimated a single reduced-form equation to explain the rate of inflation. Gordon's general specification was chosen as the empirical model for this analysis because of its explicit theoretical development and widely known and documented track record. Gordon obtains his equation by specifying separate wage and price-markup equations and then substituting the wage equation into the price-markup equation. He then hypothesizes that firms determine product prices through a fixed markup calculated on the basis of unit labor costs. The markup includes the normal industrywide profit margin and a provision for fixed capital depreciation.

This relationship is:

$$P_t = (1 + b) \frac{W_t N_t}{Q_t} \quad , \qquad (1)$$

where P_t is the price level, W_t is the money wage rate, N_t is employment, Q_t is real gross national product (GNP), b is the constant profit margin, and $(W_t N_t / Q_t)$ represents unit labor costs. This relationship can also be expressed as the ratio of the wage rate to labor productivity or $W_t/(Q_t/N_t)$. Substituting this expression into equation (1), taking natural logs of both sides of the equation, and differentiating with respect to time results in:

$$\dot{P}_t = \dot{W}_t - (\dot{Q_t/N_t}), \qquad (2)$$

where • denotes rate of change.

As a result of the price-markup behavior, the inflation rate equals the rate of growth in money wages minus the rate of growth in labor productivity. If productivity growth is zero and wage changes are negatively related to the unemployment rate, the reduced-form Phillips curve is easily derived by substituting (2) into the equation $W_t = f(U_t)$, where U_t is the unemployment rate.

Based on this general framework, Gordon attempts to explain U.S. inflation as a function of the expected rate of inflation, aggregate demand shifts (represented by the unemployment rate), Government intervention (in the form of wage and price controls), and external supply shocks (that include the influence of the relative prices of food and energy and import price changes). The resulting reduced form may be written in implicit form as:

$$\dot{P}_t = F(\dot{P}^e_t, U_t, Z_t), \qquad (3)$$

where $\dot{P}_t$ is the rate of inflation, $\dot{P}^e_t$ is the expected rate of inflation, U_t is the unemployment rate, and Z_t represents a set of variables accounting for "supply shocks." Variables accounting for changes in productivity can also be entered in the reduced form.

To examine the natural-rate hypothesis under this specification, a researcher must formulate and test a joint hypothesis. First, given a level of excess demand and consequent deviation in unemployment from its natural rate, will a rise in inflation expectations be accompanied by an equiproportionate rise in the actual rate of inflation? That is, will the magnitude of the coefficient on price expectations equal 1? A unit coefficient would imply that price expectations are completely incorporated into actual price changes and that economic agents are not subject to money illusion. A unit coefficient on price expectations would also suggest the complete absence of a tradeoff between inflation and unemployment in the long run when expectations are fully realized. Second, one hypothesizes a process by which the inflation expectations are formed. Gordon assumed an adaptive-expectations process and, in addition, restricted the coefficient on inflation expectations to 1.

This article modifies Gordon's reduced-form specification for empirical work. First, for price variables, a fixed-weight price index was used. Some studies, such as Englander and Los and McElhattan, use implicit price deflators as the measure of inflation. Gordon and McElhattan also use implicit deflators to describe the supply "shocks." Implicit price deflators, in their construction, use shifting current-period weights, which leads to a confounding of the changes in prices with changes in the mix of output (see the Survey of

Current Business, 1985). In a quarter marked by a sharp change in a particular category of nominal spending, the value of the deflator may rise or fall due to a shift in weights, even if none of the individual prices change. Heterogeneity induced by shifting aggregation weights can also be a potential source of parameter instability in econometric models.

The price-expectations variable in this article is based on the specification used by de Leeuw and McKelvey (1981) rather than the distributed-lag structure posited by Gordon. The expectations specifications used by de Leeuw and McKelvey are based on an adaptive-expectations model in which the expected rate for the current period is determined as a function of the preceding period's expected and actual rates. Expected inflation rate, import prices, and food and energy prices were constructed by use of the following equation:

$$\dot{P}^e_t = B_1 + B_2T + B_3[(\dot{P}^e_{t-1} + \dot{P}_{t-1})/2] , \qquad (4)$$

where $\dot{P}^e_t$ represents the expected change in the fixed-weight measure, $\dot{P}_t$ is the actual change, and T is a time trend. The coefficients are estimated from a regression of $\dot{P}_t$ on $\dot{P}_{t-1}$ and time. $\dot{P}^e_t$ and $\dot{P}_t$ are equated in the initial period, then $\dot{P}^e_t$ is calculated sequentially. Unlike Gordon's model, the price-expectation coefficient is not restricted to 1. Relaxing this restriction allows one to see whether the longrun tradeoff has varied over time. The price expectations coefficient can be greater than 1 in certain time periods. Indeed, Friedman (1977) noted that the longrun Phillips curve may become positively sloped in its upper ranges as higher inflation leads to greater inflation volatility that, in turn, raises the natural rate of unemployment.

Since the real economy is known to be fairly sluggish, a simple assumption maintained in Phillips curve empirical models is that the current value of the unobservable natural rate may be forecasted by extrapolating past values of actual unemployment. Gordon follows this procedure. The excess demand effect is captured by following Gordon and using the current and lagged values of the "official" unemployment rate.2/

The inflation-expectations variable has often been the only shift variable in Phillips curve empirical research, reflecting the view that changing price expectations are the predominant cause of observed shifts in the Phillips curve. Gordon's contribution is to add supply-shock shift variables that add considerably to the explanatory power of the equation. Supply-shock variables in this article follow the general specification of Gordon and McElhattan, except that they are represented by the change in the relative fixed-weight price of food and energy, measured as the difference between the changes in the fixed-weight PCE (personal consumption expenditures) price with and without expenditures on food and energy. Also included is the change in the relative price of imports, measured as the ratio of the fixed-weight import price to

---

2/ Possible explanations of the dependence of the unemployment rate on its lagged values are studied by Lucas (1975) who considers the influence of previous investment on the current capital stock, and by Blinder and Fischer (1978) who discuss a similar mechanism operating through inventory accumulation.

the fixed-weight GNP price. The GNP fixed-weight measure excludes the prices of imports. However, changes in the price of imported items may be correlated with changes in the GNP fixed-weight measure to the extent that changes in foreign prices, through competitive pressures, lead to changes in the U.S. prices of traded products produced in the United States. Therefore, a fixed-weight index of import prices was used in this study: Since it is generally accepted that one does not observe perfectly flexible prices over short time periods, some positive correlation between movements in supply-side price shocks and the aggregate price level appears likely.

FIXED COEFFICIENTS EMPIRICAL RESULTS

The estimated empirical model is the following:

$$\dot{P}_t = \beta_0 + \beta_1 \dot{P}^e_t + \beta_2 \dot{P}^e_{imt} + \beta_3 \dot{P}^e_{fet} + \sum_{j=0}^{4} \gamma_j \, UR_{t-j} + \text{D-ON} + \text{D-OFF} + e_t \qquad (5)$$

where $\dot{P}_t$ is the fixed weight GNP price index, $\dot{P}^e_t$ is the expectation of $\dot{P}_t$, $\dot{P}^e_{imt}$ is the expectation of the fixed-weight import price, $\dot{P}^e_{fet}$ is the expectation of the fixed-weight food and energy price, UR_{t-j} is the unemployment rate in time t-j, D-ON and D-OFF represent wage-price control dummies, e is an error term, and • is the rate of change. All price changes are expressed as percentage changes at annual rates. The empirical work covers the timespan 1960:I-1984:IV. All Bureau of Economic Analysis (BEA) source data are precomprehensive benchmark revisions effected in December 1985.

Table 1, row 1, shows the fixed-coefficient empirical results. The price-expectations variable has a value not significantly different from 1. This result indicates some support for the natural-rate hypothesis of Friedman and Phelps that there is no longrun tradeoff between inflation and unemployment. The import-price expectations variable is statistically significant at the 1-percent level. The variable for food and energy does have the correct sign, but is statistically insignificant; nevertheless, inclusion of the variable did improve the post-sample forecasting accuracy of the model uniformly, so it was kept in the equation.

Wage and price controls were applied in several stages from August 1971 through 1974, when they were removed entirely. Dummy variables constructed by Gordon are added to reflect the implementation of the price controls (1971:III-1972:III) imposed by the Nixon administration and their cancellation (1974:II-1975:I). The "on" effect is represented by the dummy variable D-ON which is unity in (1971:III-1972:III) and zero elsewhere, and the "off" effect by the dummy variable D-OFF which is unity in (1974:II-1975:I) and zero elsewhere. Controls, according to the estimates in row 1, tended to reduce the measured inflation rate about 0.11 percentage point in 1971-72 and their removal tended to increase inflation about 1.6 percentage points in 1974-75. Other studies have also found that prices increased more when controls were removed than they decreased when they were imposed. This asymmetry suggests that an inflationary price bubble developed when controls were lifted. Still, such estimates should be viewed cautiously, because a number of economic events influenced the economy in 1974 and 1975.

An F test of the explanatory variables shows the relationship is significant at well beyond the 1-percent level. The additional supply-shock variables reduce the absolute value of the unemployment coefficient and produce a less steep shortrun Phillips curve when they are included in the estimated model. As Gordon notes, reduced-form Phillips curve equations should most profitably "be viewed as a convenient characterization of the data rather than an attempt to describe structural behavior. Because the underlying structure may shift, the coefficients in the estimated equation may shift, so that any such single-equation approach should pay special attention to tests of the stability of coefficients across sub-intervals within the sample period," Gordon (1985a, p. 88). Following the recommendation of Gordon, we conducted Chow tests with a breakpoint in 1972:II (thereby splitting the data in half) and 1980:IV (when the disinflationary period began). The Chow tests conducted by us for both periods failed to reject the null hypothesis of stability.

Out-of-sample performance statistics are shown in table 2 for the period 1980:I-1984:IV. This equation uniformly dominated other equations without the supply "shock" variables suggested by Gordon. Turning point errors occur for 7 out-of-sample periods out of 15 and the Theil U2 statistic is 1.081. The mean absolute percent error is 30 percent for 16 observations.

STOCHASTIC COEFFICIENTS MODELS

The use of stochastic coefficients estimation is relatively new in econometrics, yet there are a number of theoretical and empirical reasons for specifying a stochastic coefficients model, especially for the problem at hand (see the appendix). A stochastic coefficients model is an alternative empirical approach that permits one to deal with any, including continually occurring, instabilities in economic relationships without excessive prior informational requirements. Stochastic coefficients procedures may be superior to fixed coefficients procedures for at least six reasons.

First, the "true" coefficients themselves may be generated by a nonstationary or time-varying random process. Second, omitted variables, which exhibit nonstationary behavior and which are not orthogonal to the included variables, can induce variability in the coefficients. Third, it is a conventional econometric practice to use proxy variables in place of unobservable explanatory variables. As econometricians are aware, proxy variables only imperfectly capture changes in the economic behavior of the true variable and the relationship between the true variable and its proxy may change over time. Fourth, aggregation over microeconomic units can induce variation. It is highly restrictive to assume that the aggregation weights of microeconomic units will not change over time. Furthermore, the rationale for imposing the constraints suggested by microtheory on a constant-coefficient equation to be fitted at the aggregate level is typically rather weak. A more general theory of increased aggregation leads naturally to stochastic coefficients models. The error term due to aggregation is more likely to converge to zero and converge faster under a stochastic coefficients specification than under a constant coefficients specification (see Swamy, Barth, and Tinsley (1982, p. 134)). Fifth, coefficient variation may occur as a result of an incorrect functional form being imposed. As Rausser, Mundlak and Johnson (1982) note, "This approximation of highly nonlinear 'true' relationships by simpler functional forms, along with observations outside the narrow sample range provides perhaps the strongest motivation for a varying parameter structure." Finally, conventional econometric models may not be consistent with the dynamic economic theory of optimizing behavior. A change in economic or policy variables will result in a new environment that will, in turn, lead to new optimal decisions and new microeconomic and macroeconomic structures. This insight is the contribution of Lucas (1976). As Lucas and Sargent (1978) note further, this "[e]quilibrium theorizing ... readily leads to a model of how process nonstationarity and Bayesian learning applied by agents to the exogenous variables leads to time-dependent coefficients in agent's decision rules."

Earlier studies purporting to test for structural stability are not entirely satisfactory. Gordon (1984) uses a Chow test and Englander and Los (1983a) use a Lagrange Multiplier test as well as the Brown, Durbin, and Evans cumulated sum of squares (CUSUM) test. One problem with the Chow test is that it assumes that one "knows" the breakpoint. It also is based on the arbitrary assumption that any structural break occurred only once. The Lagrange Multiplier test, as noted by Chow (1983), is not a uniformly powerful test statistic with a known distribution in small samples. The power of the aforementioned tests for structural breaks cannot be determined without specification of a hypothesis alternative to the one under which the sample significance is computed.

The most general alternative to the hypothesis that coefficients are time-invariant constants is that all the coefficients are changing from period to period. However, Swamy and Tinsley (1980, p. 117) demonstrate that, under this most general alternative hypothesis, changes in coefficients are not consistently estimable and, therefore, the power of the classical tests for structural shifts do not tend to one as the sample size increases indefinitely. The Brown, Durbin, and Evans test is also subject to this criticism. As a result, standard tests for structural change have limited value and may prove to be misleading. Finally, Resler, Barth, Swamy, and Davis (1985) note that, "when testing between a hypothesis of no change in coefficient values at certain points in time and a specific alternative hypothesis, the specification only of probabilities of the type I and II errors, and of the decision reached, is often viewed as unsatisfactory... Kiefer (1977) feels the basis for this concern is the practical person's feeling that classical test procedures often assign the same decision and numerical measure of conclusiveness for two different sample values, when one actually seems intuitively much more conclusive than the other... The stochastic coefficient approach, by permitting the possibility of variation in any of the coefficients in any... [time period]... may temper the uneasiness that one might have with the classical test statement of error probabilities and of decision."

STOCHASTIC COEFFICIENTS EMPIRICAL RESULTS 3/

The only difference between the variable-coefficient specification employed here and the fixed-coefficient Phillips curve specification estimated earlier is the omission of wage and price control dummy variables. In this regard,

---

3/ The Swamy-Tinsley stochastic coefficients model used in this analysis is described in greater detail in the Technical Appendix.

Blinder and Newton (1981) make the interesting point that the use of dummy variables to capture the influence of controls is based on the restrictive assumption that the controls program does not change any of the parameters of the model except the constant term. Blinder's argument is buttressed by Jack Triplett's statement to the authors that, when he was on the Council of Wage and Price Stability, he and others were explicitly trying to influence inflationary expectations. A more general model would allow all parameters to be influenced by the controls program as stochastic-coefficients models do.

The stochastic coefficients empirical model based on Gordon's specification of the Phillips curve equation is as follows:

$$\dot{P}_t = \beta_{0t} + \beta_{1t}\dot{P}^e_t + \beta_{2t}\dot{P}^e_{imt} + \beta_{3t}\dot{P}^e_{fet} + \beta_{4t}\left(\sum_{j=0}^{4} \hat{\gamma}_j\ UR_{t-j}\right) \tag{6}$$

where for every j = 0, 1, ..., 4 and t = 1960-I,..., 1984-IV

$$\beta_{jt} = \overline{\beta}_j + \varepsilon_{jt} \tag{7}$$

and

$$\varepsilon_{jt} = \sum_{i=0}^{4} \varepsilon_{jt}\ \phi_{it-1} + u_{jt}$$

such that

$$E(u_{jt}) = 0$$

and

$$E(u_{it}u_{js}) = \begin{cases} \sigma_{ij} \text{ if } t = s \\ 0 \text{ if } t \neq s \end{cases}$$

In this model, each coefficient (β_{jt}) may vary about its own mean value (β_j) by an error term (ε_{jt}), which is assumed to be related to its own past value (ε_{jt-1}) as well as the previous past values of the error terms in other coefficients (ε_{it-1} for $i \neq j$). The error term (ε_{jt}) is assumed to contain a white-noise component (u_{jt}) which is contemporaneously correlated with the white-noise compoenents of the error term in other coefficients. The white-noise components (u_{jt}) represent a process formed of identically distributed random variables with zero mean and unit variance such that it is time invariant.

One encounters degree-of-freedom problems in using the Swamy-Tinsley package for estimating equations with many independent variables. To deal with this problem, we followed the advice of Swamy, Kennickell, and von zur Muehlen (1986), and, consequently, we weighted the distributed-lag of unemployment by the ordinary least squares parameters during the pertinent time period (1960:I to

1984:IV). Therefore, $\hat{\gamma}_j$ in equation (6) are the least squares estimates from equation (5).

Estimates reported here are produced by an iterative procedure. Because arbitrary values of the unknown parameters Δ_u and Φ were used as the starting values in the initial iteration, the limiting distribution of the estimates obtained after one iteration will depend on those arbitrary starting values. This is not true, however, of the estimates obtained after two iterations (see Swamy, Tinsley, and Moore (1982)). Eight iterations of the Swamy and Tinsley (1980) procedure were applied. This iterative procedure is discussed in Havenner and Swamy (1981) and Swamy and Tinsley (1980). The authors, following the recommendation of Swamy, Tinsley, and Moore (1982), compared the numerical accuracy of inverses used in these iterations, which suggested that results obtained for the third iteration were most accurate.

Table 1, row 2, shows the mean values of the stochastic coefficients along with their asymptotic t statistics. One may note there are some slight differences in coefficient values between the fixed-coefficients (row 1) and stochastic-coefficients models. The mean coefficient value for price expectations, using the stochastic-coefficients model, is not significantly different from 1. The expected price coefficients for imports and for food and energy have marginally smaller coefficients when the stochastic-coefficients model is used. Although not strictly comparable, the asymptotic t statistics for the mean values of the stochastic coefficients are almost uniformly larger than those associated with the fixed coefficients.

Table 3 shows estimates of the coefficients of variation of the coefficients. These results suggest that expected import price had the greatest level of variation, followed by the coefficients for expected inflation, the constant, weighted unemployment, and the expected food and energy price. However, none of these coefficients appear by this criterion to have dramatic levels of variation.

Table 4 shows the average decomposition of normalized variance. The variable with the largest influence on the variance of inflation is the expected inflation rate, followed by the expected import price, the constant, weighted unemployment, and the expected food and energy prices. This ranking differs somewhat from an evaluation of t statistics to determine the variable with the largest influence on the mean value of inflation. Under that criterion, the expected inflation rate would still be first, followed by the expected food and energy price, expected import price, weighted unemployment, and constant. The difference in ranking points out a virtue in using the stochastic-coefficients model because a variable may influence the variance of the dependent variable despite being statistically insignificant.

Charts 1 through 5 document the timepaths of the various coefficients. The movements of the coefficients do appear to coincide with some historical occurrences in the macroeconomy.

Chart 1 shows the time profile of the expected inflation rate coefficients. The mean values of the fixed- and stochastic coefficient models are very close and the individual time-varying coefficients change marginally over time. In general, the results indicate that the longrun Phillips curve is fixed over time and conforms to the natural rate hypothesis that the parameter value is one.

Charts 2 and 3 show the time profiles of the coefficients for the expected price of imports and food and energy. Their time-varying patterns are roughly similar to the expected inflation coefficients described above with considerably greater volatility and higher values over the 1976-78 span. One reason why the expected inflation rate coefficients do not vary a great deal may be because most of the price shocks from this time period came from the imports and food and energy sectors of the economy and are reflected in their coefficients.

Chart 4 shows the weighted unemployment rate coefficients. Between 1960 and 1979, there is a trend increase in the absolute values of the coefficients. Within this trend there are two peaks (decline in absolute value) in 1967 and 1978 and two troughs (increase in absolute value) in 1971-72 and 1979. From 1979 on, the demand parameters decline somewhat in absolute value to fluctuate around a new plateau. Overall, there is a slight trend toward a less favorable shortrun tradeoff.

A possible explanation for this pattern is that major determinants of the unemployment rate (and the natural rate of unemployment) are the composition of the labor force and legislation influencing the behavior of individuals engaged in job search. The composition of the labor force is important because there are persistent differences in unemployment rates across age-race-sex groups. When the percentage of groups with high unemployment rates rises in the labor force, the aggregate unemployment rate will also rise. From 1950-79, the percentage of teenagers in the labor force rose from 6.5 percent to 10 percent, while the proportion of married women in the labor force increased from 13 percent to 23 percent. In addition to the increase in the proportion of high-unemployment groups in the labor force, these high-unemployment groups experienced increasing unemployment rates relative to those for adult males.

Changes in the Fair Labor Standards Act may have also had some influence on the unemployment rate. The reason is that an effective minimum wage in low-wage sectors was set during a period of their most rapid growth. Amendments since 1970 have increased coverage to almost 70 percent in both the retail trade and service industry groups. These factors may be the cause for the trend increase in the absolute value of the unemployment coefficient from 1960 to 1979, indicating a steeper shortrun Phillips curve. However, after 1979, there is a somewhat more favorable trend that may be attributable to slowing growth in the working age population brought about by a decline in the number of teenagers of working age. This decline may be the result of the maturing of the post-World War II "baby boom" generation that increased the teenage population during the 1960's and early 1970's. Since 1978, the number of teenagers has been declining at a steadily increasing rate. This demographic trend has lowered the unemployment rate (and natural rate of unemployment) not only because it has slowed labor force growth but also because, as stated earlier, teenagers constitute a disproportionately large fraction of the unemployed, given their relative lack of skills and experience.

Chart 5 depicts the time profile of the constant incorporating omitted variables. Once again the 1976-79 period appears to be characterized by considerable parameter volatility.

One may obtain the natural rate of unemployment based on this specification. A natural rate series is generated by solving for that unemployment rate that arises when there are no inflationary surprises; that is, when actual inflation equals expected inflation.4/

Assume the Phillips curve equation is

$$\dot{P}_t = \sigma + \beta\dot{P}^e_t - \Sigma\gamma_i U_{t-i} + \Sigma\phi Z_{t-j}$$

Equating $\dot{P}_t$ to $\dot{P}^e_t$ and setting $\dot{P}_t$ to zero we have

$$0 = \frac{\alpha -}{1-\beta} \frac{\Sigma\gamma_i}{1-\beta} U_t + \frac{\phi}{1-\beta} Z_t$$

The estimated natural rate of unemployment is therefore

$$U_n = \frac{\alpha}{\Sigma\gamma_i} + \frac{\phi}{\Sigma\gamma_i}$$

Chart 5 shows the time profile for the natural rate of unemployment. There is a considerable amount of variation ranging from a high of 5.44 to a low of 1.66. Once again, there is a period of high volatility between 1976 and 1979, which dampens during the 1980's. An upward trend in the natural rate of unemployment occurs between 1960 through 1968, followed by a decrease and plateau during the early to mid-1970's. However, no real longrun trend is detectable over the entire sample period. Once again, one should be cautious in giving structural interpretations to parameter movements shown here since there are many possible causal factors.

MODEL COMPARISON

Table 2 compares out-of-sample forecast evaluation statistics for fixed and stochastic coefficients over the period 1981:I-1984:IV. The stochastic coefficients model performs considerably better than the fixed-coefficient model for all forecast criteria. This provides some useful evidence that the stochastic coefficients specification may be more appropriate and that there is some variability in the relationship between inflation and unemployment. This result would be undetected using a conventional fixed coefficients model and a Chow test as described earlier. The fixed-coefficients model provides mildly superior forecasts for three of the four quarters of 1981, while its forecast over the rest of the forecasting horizon compares considerably less favorably with that of the stochastic coefficients approach. This certainly suggests that a decisionmaker with a risk function that weights forecasts further out in the forecasting horizon may favor a stochastic-coefficients model.

4/ Another technique to estimate the natural rate follows a disagregated approach. See Antos, Mellow, and Triplett (1979) for a lucid survey of the technique and concomitant hazards.

SUMMARY

We have derived the familiar Phillips curve model that is the reduced-form of conventional wage and price equations of a more complete structural model of the U.S. economy. In addition to the unemployment rate, serving as a measure of excess demand, we also followed Gordon and McElhattan and introduced supply-side shocks into the conventional relationship between inflation and excess demand. These variables appear to have a significant influence in determining the rate of inflation. Furthermore, we used de Leeuw's and McKelvey's measure of price expectations formation that seems to improve out-of-sample forecasting results.

To investigate the variability of the Phillips curve relationship hypothesized by many economists during the 1970's, we used a stochastic-coefficients empirical model. Conditional on the specification used here, the visual evidence of the time-varying parameter plots suggests that there has been some variation in the shortrun Phillips curve. The price expectations coefficient is close to 1 throughout. The period between 1975 and 1979 seems to be one of heightened parameter volatility for the import price and food and energy price expectations variables as well as the intercept. The coefficients on the unemployment rate show a slight increase in their absolute value especially during the late 1970's to early 1980's. This result indicates a slightly less favorable short-run Phillips curve tradeoff. Examination of the coefficients of variation of coefficients suggests that these parameter changes are not large. Nevertheless, the comparative deterioration of the fixed-coefficients forecasts does suggest that assuming fixed parameters may not be appropriate, especially when long-term analysis is called for. In sum, conditional on this specification, the empirical evidence suggests that there has been some volatility in the shortrun Phillips curves. Capturing these shifts via a stochastic coefficients approach enhances considerably the forecasting prowess and the policy support capability of the specification.

TECHNICAL APPENDIX

Stochastic coefficients model

A first-order variant of the generalized ARIMA (autoregressive integrated moving average) stochastic coefficients process model, developed by Swamy and Tinsley (1980), was used to estimate the Phillips curve relation. Their model is a generalization of other stochastic coefficients models, such as the Kalman filter and the Cooley-Prescott procedure.

In vector notation, the time-varying models may be written as:

$$y_t = x_t' \underline{\beta}_t \tag{1}$$

To empirically implement this model, some structure must be imposed on $\underline{\beta}_t$ because there are only T observations. In this paper, the coefficients in (1) are driven about a fixed vector of mean values, $\overline{\underline{\beta}}$ by a stationary stochastic vector, ε_t. Thus,

$$\underline{\beta}_t = \overline{\underline{\beta}} + \varepsilon_t, \tag{2}$$

$$\varepsilon_t = \Phi\varepsilon_{t-1} + u_t \qquad \underline{u}_t \sim \text{ws}(\underline{0}, \Delta_u).$$

where u_t is a vector of white-noise innovations. Φ may or may not be diagonal and all the characteristic roots of Φ are less than one in absolute value. Δ_u is positive definite, it may or may not be diagonal.

The observation vectors and matrices are

$$\underline{Y} = (Y_1, Y_2, \ldots, Y_T)', \tag{3}$$

$$X = (\underline{X}_1, X_2, \ldots, \underline{X}_T)', \tag{4}$$

$$D_x = \text{diag}(\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_T)' \tag{5}$$

The unobservables are

$$\underline{\varepsilon}_t = \overline{\underline{\beta}} + \underline{\varepsilon}_t \tag{6}$$

$$\underline{\varepsilon} = (\underline{\varepsilon}'_1, \underline{\varepsilon}'_2, \ldots, \underline{\varepsilon}'_T) \tag{7}$$

The variance-covariance matrices are

$$E(\beta_t - \overline{\underline{\beta}})(\beta_t - \overline{\underline{\beta}})' = E\underline{\varepsilon}_t\, \underline{\varepsilon}_t' = \Gamma_0 = \Phi\Gamma_0\Phi' + \Delta_u \tag{8}$$

$$E\underline{u}_t\underline{u}_t = \Delta_u$$

$$E\underline{\varepsilon}\ \underline{\varepsilon}' = \Sigma_\beta = \begin{bmatrix} \Gamma_0 & \Gamma_0\Phi' & \Gamma_0\Phi'^2 & \dots & \Gamma_0\Phi'^{T-1} \\ \Phi\Gamma_0 & \Gamma_0 & \Gamma_0\Phi'^3 & \dots & \Gamma_0\Phi'^{2\ T-2} \\ \vdots & \vdots & \vdots & & \vdots \\ \Phi^{T-1}\Gamma_0 & \Phi^{T-2}\Gamma_0 & \Phi^{T-3}\Gamma_0 & \dots & \Gamma_0 \end{bmatrix} \tag{9}$$

$$E(\underline{Y} - X\underline{\bar{\beta}}) = (\underline{Y} - X\underline{\bar{\beta}})' = \Sigma_y = D_x\Sigma_\beta D_x'$$

Following Anderson (1971, p. 196, Lemma 5.5.4) one can show that Σ_β is positive definite if the eigenvalues of Φ are less than 1 in absolute value and Γ_0 is positive definite if Δ_u is posititive definite.

Finally,

$$\underline{\beta} \sim ws(\underline{\bar{\beta}}, \Sigma_\beta) \text{ and } \underline{y} \sim ws(D_x\underline{\beta}, \Sigma_y).$$

Both the conditional expected value and variance of the dependent variable fluctuate with observations on the conditioning variables. One may decompose the variance in the dependent variables among its contributing factors. Permitting the independent variable to influence the variance of the dependent variable is important for it is possible for an independent variable to have a relatively large effect on the variance of the dependent variable, even though it has a relatively minor impact on the mean of the dependent variable. This decomposition is analogous to allocating the multiple R^2 among the explanatory variables in a conventional regression equation, as shown in Theil (1971, p. 168).

One may average over the sample period to make $\text{var}(y_t)$ unit-free:

$$1 = \frac{1}{T} \sum_{t=1}^{T} \left[\sum_{i=1}^{k} \sum_{j=1}^{k} x_{it}\, x_{jt}\, \Gamma_{ij} / \underline{x}_t'\ \underline{x}_t \right], \quad i, j=1, \dots, k$$

These results follow when the coefficient process is assumed to be stationary. When slope coefficients are fixed, both Δ_u and Φ will collapse to scalar characteristics of the intercept coefficients. One may obtain t-tests of the individual components by using an asymptotic approximation of the covariance matrix of the estimated column stack, $\text{vec}(\Delta u)$, to test the significance of the uncertainty allocations to slope coefficients.

The first regressor, x_{1t}, is usually a unit vector intercept with a stochastic component of its coefficient that serves as the analogue of the additive disturbance familiar to fixed-coefficient specifications. The stochastic coefficients model will have a total residual, u_t, that is a weighted sum of the stochastic elements of the coefficients of the intercept regressor and the time-varying regressors, where $u_t \equiv x_t'e_t$.

There is no necessary increase in the residual, u_t, by following a stochastic coefficients estimation approach instead of a fixed-coefficients approach. As shown by Tinsley, Swamy, and Garrett (1981), should ordinary least squares be a consistent estimator of the means of the coefficient vector, $\overline{\beta}$, then estimates of u_t (where $t=1, \ldots, T$) from the two estimators will converge as the sample size is increased.

Table 1--Phillips curve equation (1960:I-1984:IV)

Dependent variable $\dot{P}$	Constant	$\dot{P}^e$	$\dot{P}^e_{im}$	$\dot{P}^e_{fe}$	$\sum_{1=0}^{4} \hat{\gamma}_i \dot{UR}_{t-j}$	D-ON	D-OFF	$\bar{R}^2$	SEE 1/	D.W.
Fixed coefficients	0.595 (.559)2/	1.041 (9.166)	0.087 (3.025)	0.454 (.222)	-0.247 (4.436)	-0.107 (-.100)	1.561 (1.419)	0.88	1.086	2.25
Stochastic coefficients	.506 (1.457)	.997 (15.136	.060 (3.379)	.416 (5.021)	-.320 (3.006)4/	3/ 5/	---	---	---	---

where $\dot{P}$ = fixed weight GNP price index percent change at an annual rate

$\dot{P}^e$ = expectations of $\dot{P}$ using the de Leeuw-McKelvey method.

$\dot{P}^e_{im}$ = expectation of fixed-weight import price index percent change at an annual rate.

$\dot{P}^e_{fe}$ = expectation of the percentage change in the fixed-weight price for personal consumption expenditures minus the percentage change in the fixed-weight deflator for personal consumption expenditures net expenditures on food and energy

UR = official unemployment rate.

--- = Not applicable.

1/ SEE refers to standard error of the regression and D.W. refers to Durbin-Watson statistic.
2/ Values in parentheses are t-statistics.
3/ Not applicable.
4/ Asymptotic t statistics.
5/ Conditional on third iteration estimates of Δ_u and Φ.

Table 2--Forecast evaluation statistics (1981:I-1984:IV)

Mean	Mean absolute error	Mean absolute percent error	Mean squared error	Theil U_2
Fixed coefficients	1.155	30.043	2.499	1.081
Stochastic coefficients	.697	13.636	.638	.698

Year/quarter	Actual value of inflation rate	Fixed coefficients		Stochastic coefficients	
		Estimated	Absolute prediction error	Estimated	Absolute prediction error
1981:					
I	11.3	10.216	.184	10.671	.271
II	8.3	9.982	1.682	9.663	1.363
III	8.8	8.986	.186	8.282	.518
IV	8.1	8.426	.326	7.282	.818
1982:					
I	5.6	7.223	1.723	6.215	.615
II	4.7	7.370	2.670	5.926	1.226
III	5.8	5.532	.268	4.375	1.425
IV	4.6	5.338	.738	4.450	.150
1983:					
I	3.3	5.375	2.075	4.164	.864
II	4.1	5.735	1.635	4.350	.250
III	4.7	4.575	.125	3.605	1.095
IV	3.9	6.073	2.173	4.606	.706
1984:					
I	5.0	5.829	.829	4.486	.514
II	4.3	6.068	1.768	4.576	.276
III	4.0	6.179	2.179	4.556	.556
IV	3.6	5.780	2.180	4.104	.504

Table 3--Estimated coefficients of variation of coefficients 1/

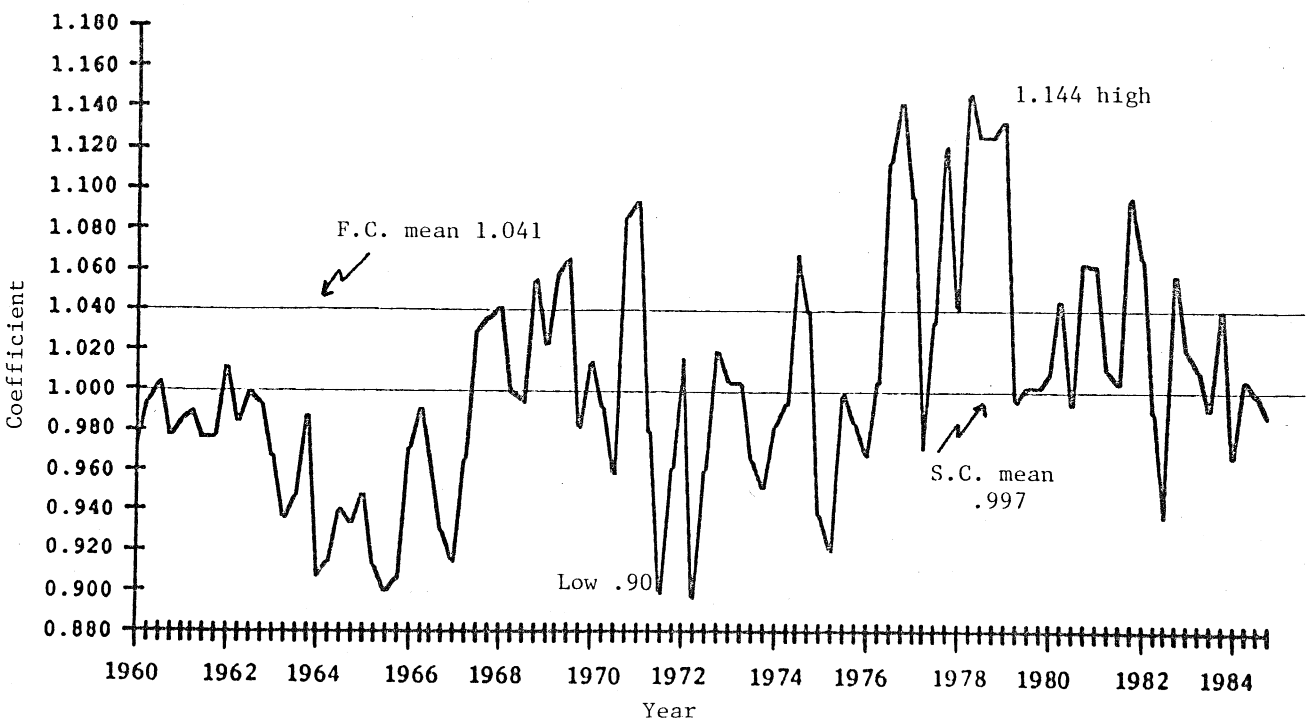
Span 1960:I-1984:IV				
Constant	$\dot{P}^e$	$\dot{P}^e_{im}$	$\dot{P}^e_{f}e$	$\sum_{i=0}^{4} \hat{\gamma}_i UR_{t-j}$
.00151	.003	.012	.00015	.0002

1/ The coefficients of variation for each coefficient is equal to 100 times the ratio of its standard deviation to its mean.

Table 4--Average decomposition of normalized variance

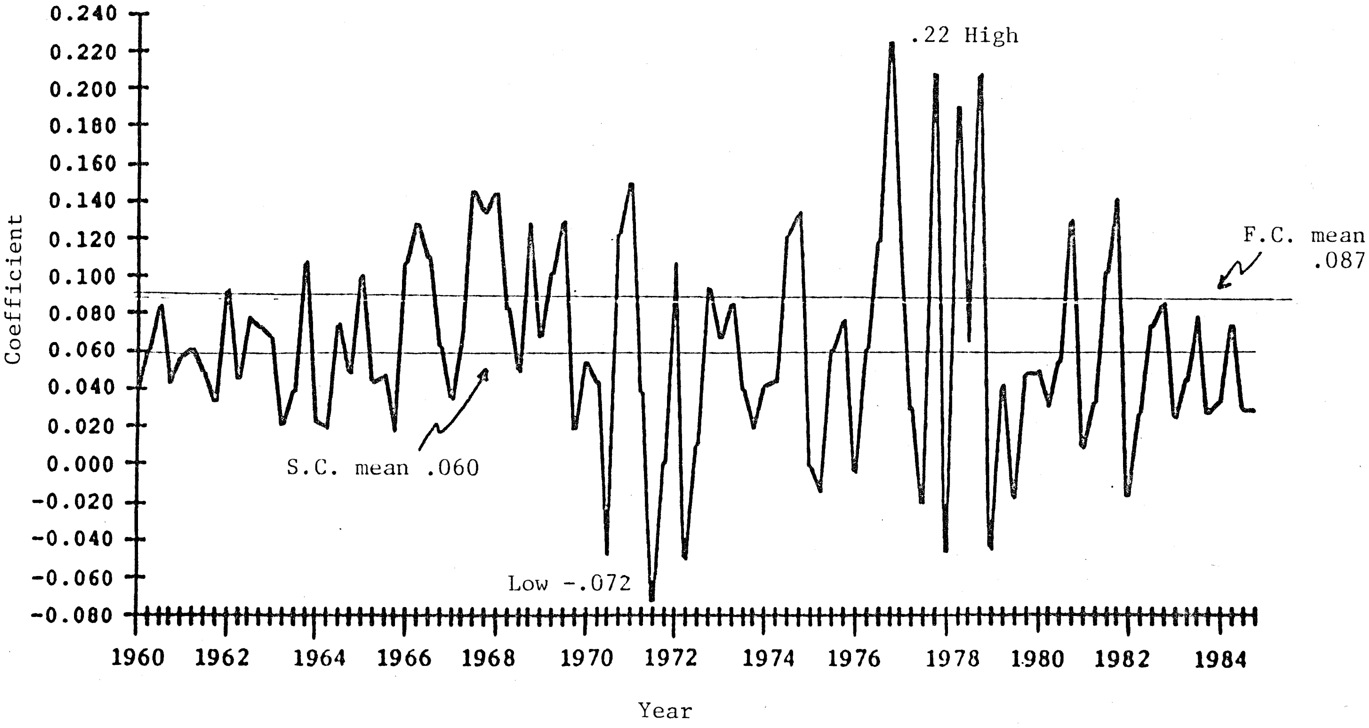
Item	Span 1960:I-1984:IV				
	Constant	$\dot{P}^e$	$\dot{P}^e_{im}$	$\dot{P}^e_{fe}$	$\sum_{i=0}^{4} \hat{\gamma}_i UR_{t-j}$
Constant	0.21315	0.02965	0.02750	-0.000406	0.000346
$\dot{P}e$	.02965	.367181	.152065	-.0045231	.0013302
$\dot{P}^e_{im}$	.02750	.152065	18533	.00221	-.0005232
$\dot{P}^e_{fe}$	-.000406	-.0045231	.00221	.0048703	.000269
$\sum_{i=0}^{4} \hat{\gamma}_i UR_{t-j}$	.000346	.001330	-.0005232	.0000269	.0053105
Net contributions	.270	.546	.310	.002	.012

CHART 1--Coefficients for Expected Inflation Rate



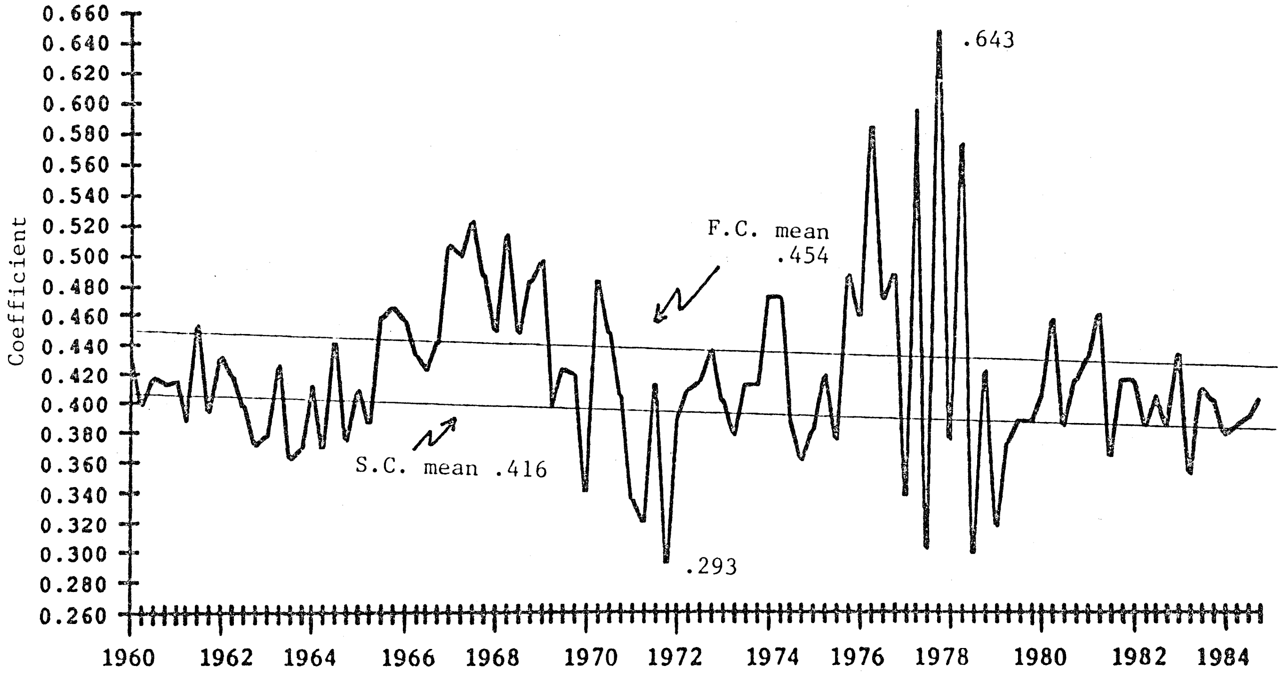
F.C. = Fixed coefficient S.C. = Stochastic coefficient

CHART 2--Coefficients for Expected Import Price



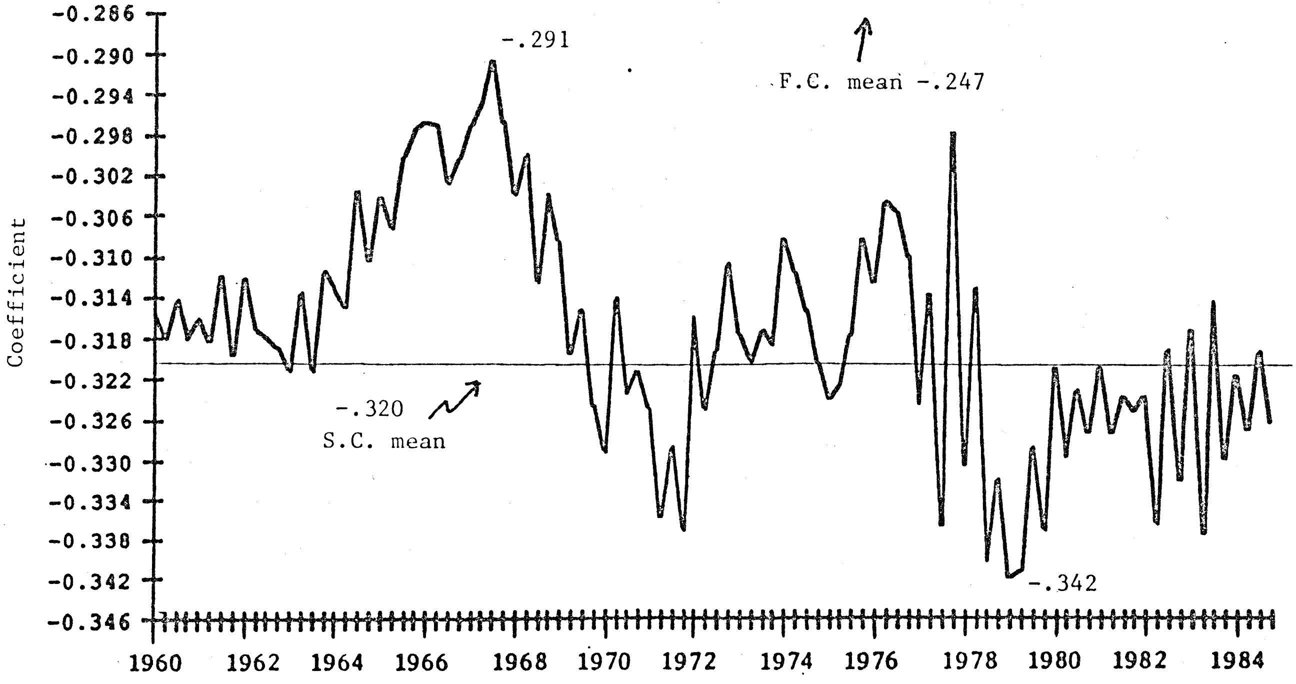
F.C. = Fixed coefficient S.C. = Stochastic coefficient

CHART 3--Coefficients for Expected Food and Energy Price



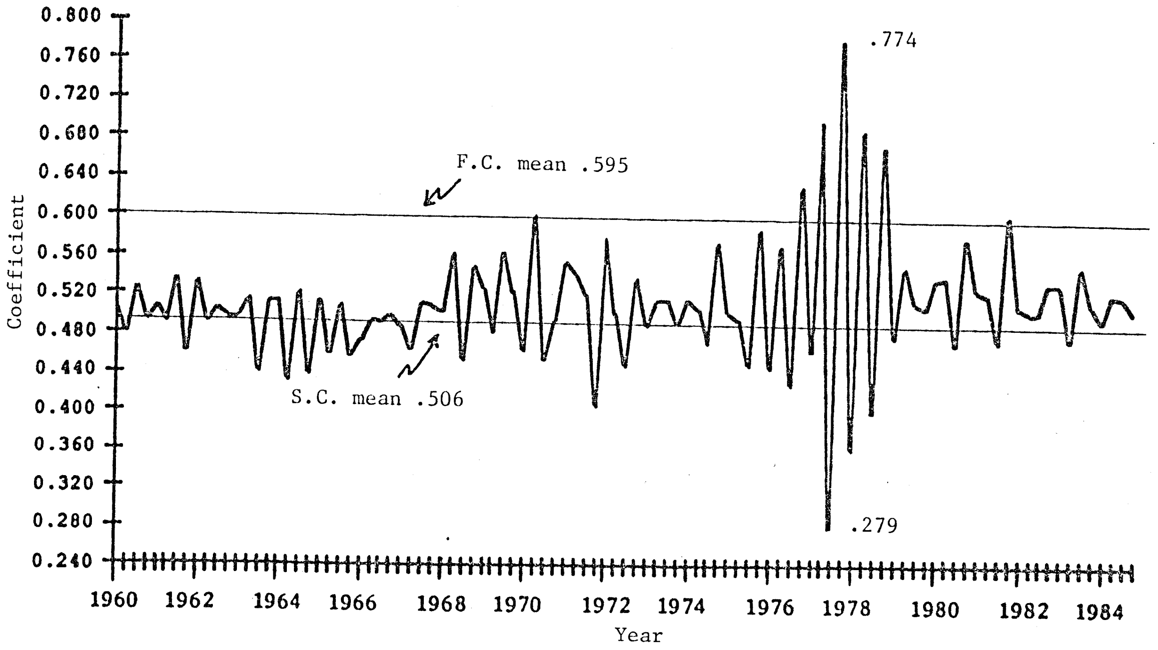
F.C. = Fixed coefficient S.C. = Stochastic coefficient

CHART 4--Coefficient for Weighted Unemployment Rate



F.C. = Fixed coefficient S.C. = Stochastic coefficient

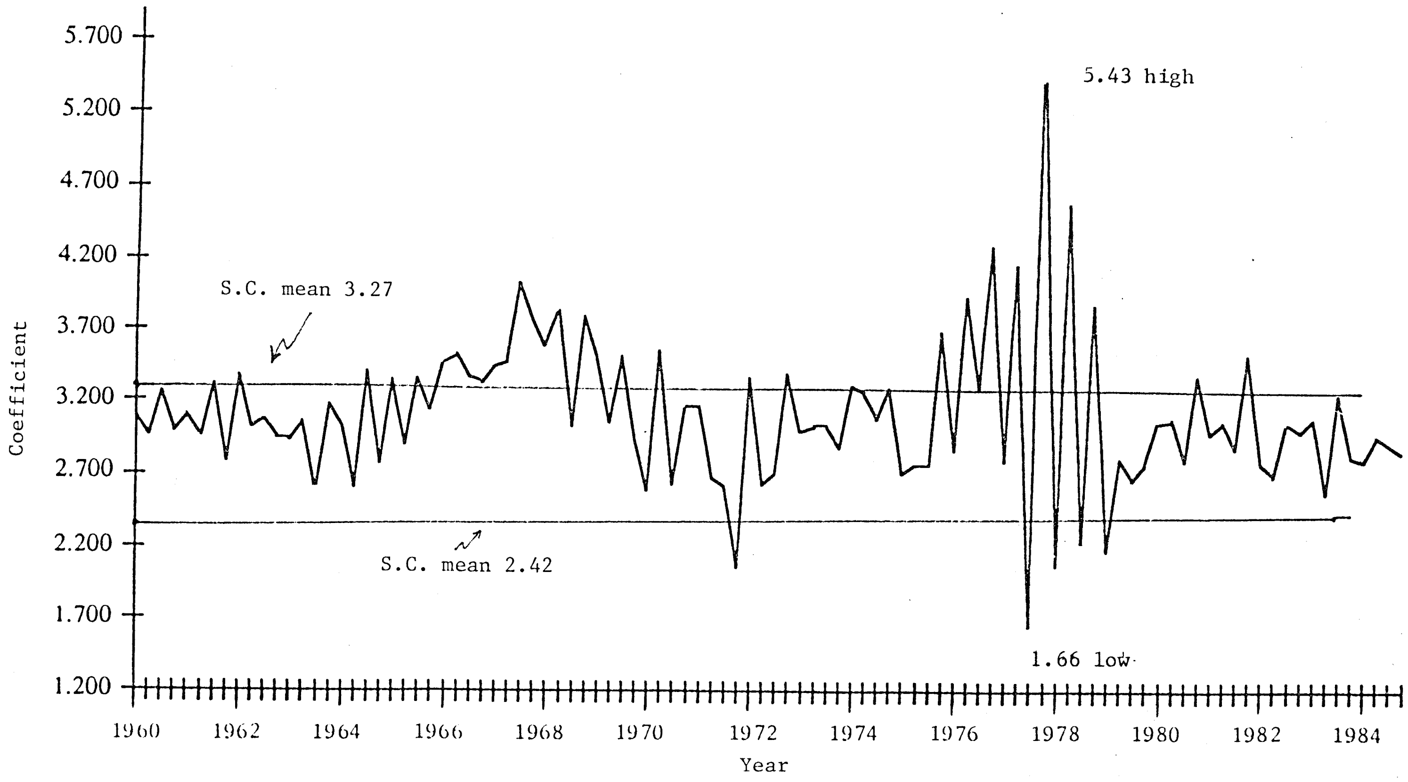
CHART 5--Coefficients for Constant Term



F.C. = Fixed coefficient

S.C. = Stochastic coefficient

CHART 6--Natural Unemployment Rate



F.C. = Fixed coefficient

S.C. = Stochastic coefficient

REFERENCES

"A Note on the Implicit Price Deflator," Survey of Current Business, 65 (May 1985), 6.

Anderson, T. W. (1971) The Statistical Analysis of Time Series, John Wiley & Sons, New York.

Antos, Joseph, Wesley Mellow, and Jack E. Triplett (1979) "What is a Current Equivalent to Unemployment Rates of the Past?" Monthly Labor Review, March, 36-46.

Blanchard, O. (1984) "The Lucas Critique and the Volcker Deflation," American Economic Review, Papers and Proceedings, 74, 211-215.

Blinder, A. S., and W. J. Newton (1981) "The 1971-1974 Controls Program and the Price Level: An Econometric Post-Mortem," Journal of Monetary Economics, 8, 1-23.

Blinder, A. S., and S. Fischer (1978) "Inventories, Rational Expectations, and the Business Cycle," Journal of Monetary Economics, 8, 277-304.

Chow, Gregory C. (1983) "Econometrics," McGraw-Hill, New York.

de Leeuw, F., and M. J. McKelvey (1981) "The Realization of Plans Reported in the BEA Plant and Equipment Survey," Survey of Current Business, 61, October, 28-37.

Eckstein, O. (1983) "Disinflation," in William D. Nordhaus, ed., Inflation: Prospects and Remedies, Alternatives for the 1980's, No. 10, Washington Center for National Policy.

Englander, A. S., and C. A. Los (1983a) Robust Phillips Curve, Natural Rate of Unemployment and Productivity Growth. Research Paper No. 8305, Federal Reserve Bank of New York.

Englander, A. S., and C. A. Los (1983b) The Stability of the Phillips Curve and Its Implications for the 1980's. Research Paper No. 8303, Federal Reserve Bank of New York.

Friedman, M. (1977) "Nobel Lecture: Inflation and Unemployment," Journal of Political Economy, 85(3), 451-72.

Friedman, M. (1968) "The Role of Monetary Policy," American Economic Review, 58(1), 1-17.

Frye, J., and R. J. Gordon (1980) "The Variance and Acceleration of Inflation in the 1970's: Alternative Explanatory Models and Methods," National Bureau of Economic Research Working Paper No. 551.

Gordon, R. J. (1985a) "The Natural Rate of Unemployment and the Growth of Potential Output in the 1980's," Brookings Papers on Economic Activity, Brookings Institution, Washington, DC.

Gordon, R. J. (1985b) "Understanding Inflation in the 1980's," Brookings Papers on Economic Activity, Brookings Institution, Washington, DC.

Gordon, R. J. (1984) "Commentary on 'Estimated Tradeoffs Between Unemployment and Inflation' by R. C. Fair," in Price Stability and Public Policy, Federal Reserve Bank of Kansas City, Kansas City, MO.

Gordon, R. J. (1982) "Inflation, Flexible Exchange Rates, and the Natural Rate of Unemployment," Workers, Jobs, and Inflation, M. N. Baily, ed., Brookings Institution, Washington, DC.

Havenner, Arthur, and P. A. V. B. Swamy (1981) "A Random Coefficient Approach to Seasonal Adjustment of Economic Time Series," Journal of Econometrics, 15, 177-210.

Humphrey, Thomas M. (1985) "The Evolution and Policy Implications of Phillips Curve Analysis," Federal Reserve Bank of Richmond Economic Review, 3-22.

Kiefer, J. (1977) "Conditional Confidence Statements and Confidence Estimators (with discussion)." Journal of the American Statistical Association, 72, 789-827.

Lucas, R. E., Jr., and T. J. Sargent (1978) "After Keynesian Macroeconomics," After the Phillips Curve: Persistence of High Inflation and High Unemployment, Conference Series No. 19, Federal Reserve Bank of Boston.

Lucas, R. E., Jr. (1976), "Econometric Policy Evaluation: A Critique. The Phillips Curve and Labor Markets," Carnegie-Rochester Conference Series on Public Policy, Vol. I, eds. Karl Brunner and Allen Meltzer. Supp. to Journal of Monetary Economics.

Lucas, R. E., Jr. (1975) "An Equilibrium Model of the Business Cycle," Journal of Political Economy, 83, 1113-1144.

McElhattan, Rose (1985) "Inflation, Supply Shocks and the Stable-Inflation Rate of Capacity Utilization," Federal Reserve Bank of San Francisco Economic Review, 45-63.

Perry, George (1983) "What Have We Learned About Disinflation?" Brookings Papers on Economic Activity, 14, 25-289.

Phelps, E. S. (1967) "Phillips Curves, Expectations of Inflation and Optimal Unemployment Over Time," Economica, 34, 254-81.

Rausser, Gordon C., Yair Mundlak, and S. R. Johnson (1982) "Structural Change, Updating, and Forecasting," New Directions in Econometric Modeling and Forecasting in U.S. Agriculture, Gordon C. Rausser, ed., North-Holland Publishing Co., Amsterdam.

Resler, D. H., J. R. Barth, P. A. V. B. Swamy, and W. D. Davis (1985) "Detecting and Estimating Changing Economic Relationships: The Case of Discount Window Borrowings," Applied Economics, 17, 509-529.

Santomero, Anthony M., and John J. Seater (1978) "The Inflation-Unemployment Tradeoff: A Critique of the Literature," Journal of Economic Literature, XVI, 499-544.

Sargent T. J. (1971) "A Note on the 'Accelerationist' Controversy," Journal of Money, Credit, and Banking, 3, 721-725.

Swamy, P. A. V. B., A. B. Kennickell, and P. von zur Muehlen (1986) Forecasting Money Demand with Econometric Models. Special Studies Paper 196, Federal Reserve Board, Washington, DC.

Swamy, P. A. V. B., J. R. Barth, and P. A. Tinsley (1982) "The Rational Expectations Approach to Economic Modeling," Journal of Economic Dynamics and Control, 4, 125-147.

Swamy, P. A. V. B., and P. A. Tinsley (1980) "Linear Prediction and Estimation Methods for Regression Models with Stationary Stochastic Coefficients," Journal of Econometrics, 12, 103-142.

Swamy, P. A. V. B., P. A. Tinsley, and G. Moore (1982) An Autopsy of a Conventional Macroeconomic Relation: The Case of Money Demand. Special Studies Paper 167, Federal Reserve Board, Washington, DC.

Theil, Henri (1971) Principles of Econometrics, John Wiley & Sons, Inc. New York.

Tinsley, Peter A., P. A. V. B. Swamy, and Bonnie Garrett (1981) The Anatomy of Uncertainty in a Money Market Model. Federal Reserve Board Working Paper, Federal Reserve Board, Washington, DC.

☆U. S. GOVERNMENT PRINTING OFFICE:1987- 180-916:40671/ERS