Calibration of Agricultural Risk Programming Models Using Positive Mathematical Programming

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Abstract:

Beginning in the 1960s, agricultural economists used mathematical programming methods to examine producers' responses to policy changes. Today, positive mathematical programming (PMP) employs observed average costs and crop allocations to calibrate a nonlinear cost function, thereby modifying a linear objective function to a nonlinear one to replicate observed allocations. The standard PMP approach takes into account producers' risk aversion, which is not a very satisfying outcome because it intricately entangles the cost parameters and the producer's attitudes – biophysical aspects of production and human behavior are intertwined so that one cannot study the impact of policy on one in the absence of the other. Several approaches that calibrate both the risk coefficient and cost function parameters have been proposed. In this paper, we discuss two methods mentioned in literature – one based on constant absolute risk aversion (exponential utility function) and the other on decreasing absolute risk aversion (logarithmic utility function). We compare these methods to an approach that employs maximum entropy method. Then we use historical data from a region in Alberta's southern grain belt to compare the different outcomes to which the three approaches lead. We find that the latter approach is robust and easier to employ.

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Abstract: Beginning in the 1960s, agricultural economists used mathematical programming (MP) methods to examine producer responses to policy changes. Today, positive mathematical programming (PMP) employs observed average costs and crop allocations to calibrate the parameters of an assumed nonlinear cost function, thereby modifying a linear objective function to a nonlinear one to replicate observed crop allocations exactly. The standard PMP approach takes into account producers' risk aversion, which is not a very satisfying outcome because it intricately entangles the cost parameters and the decision maker's attitudes – biophysical aspects of agricultural production and human behavior are intertwined so that one cannot study the impact of policy on one in the absence of the other. Several approaches that calibrate both the risk coefficient and cost function parameters have been proposed by different researchers. In this paper, we discuss two methods mentioned in literature – one based on assumed constant absolute risk aversion (and exponential utility function) and the other on decreasing absolute risk aversion (logarithmic utility function). We compare these methods to a more standard approach that employs maximum entropy (ME) method. Then we use crop insurance and historical data from a region in Alberta's southern grain belt to compare the different outcomes to which the three approaches lead. We find that the latter approach is robust and easier to employ.

Key Words: Agricultural policy analysis; calibration of farm management models; expected utility; decreasing and constant absolute risk aversion.

JEL Categories: Q14, Q18, C61

1. Introduction

Business risk management (BRM) is the most important component of Canada's agricultural policy strategy; federal expenditures or planned spending on BRM has exceeded half of Agriculture and Agri-Food Canada's (AAFC) total farm program outlays of $2.0 billion to $2.4 billion since 2013 (AAFC 2016). Farmers face risk and uncertainty that threaten their livelihoods, the incomes of those working in input supply, transportation and downstream processing sectors, and, ultimately, Canada's trade balance. As Moschini and Hennessy (2001) point out:

> *"Many distinct sources of risk may exist, and many discretionary actions may be available to the decision maker. Decisions and realizations of randomness may occur at several points in time. Further, actions may influence the distributions of yet-to-be-realized random variables, while the realizations of random variables may alter the consequences of subsequent actions. To represent such an intricate network of interactions is analytically very difficult, but insights are possible by focusing on simpler stylized models. … [T]he main risks that a typical farmer faces are due to the fact that output prices are not known with certainty while production decisions are made and that the production process contains inherent sources of uncertainty" (pp.96-97).*

Governments attempt to protect farmers against risk and uncertainty through various stabilization and agricultural support programs, but these often have unintended consequences that could increase uncertainty. Agricultural policies in rich countries, such as the EU, U.S. and Canada originally guaranteed producers price floors and insurance against production risk (e.g., low crop yields), and/or production quotas with tariff and non-tariff trade barriers to prevent imports (Barichello 1995; Schmitz et al. 2010). Farmers responded by increasing production; excess grain, milk powder and other products were disposed of through low domestic prices (with import restrictions if the price was below world price), export subsidies, and/or storage programs, with only the cost to the public purse varying according to how programs were implemented.

Agricultural support programs incentivized production at the expense of the environment. Despite conservation compliance that required farmers to meet certain environmental standards to be eligible for program payments, and acreage reduction programs, government intervention changed farmers' behaviour, at least partially cancelling out the stabilization effect of the policies. Therefore, discussions are ongoing about how to provide BRM programs without distorting farmers' production incentives – how to decouple production and corresponding adverse environmental and trade distortion from agricultural support.

Many farm management models have been developed to study the efficacy of agricultural BRM policies in reducing farmers' exposure to risk, and the effect BRM programs have on land and input use, outputs and incomes. However, it has been challenging to calibrate models that maximize expected utility (EU) rather than expected gross margins when considering the impacts of risk explicitly. To be specific, farm management models often assume that a producer varies land uses or crop activities to maximize her expected utility, where utility is modelled as the expected gross margin minus its variance multiplied by a risk aversion parameter (denoted φ). Parameter φ is important for investigating farmers' economic decisions and evaluating the effectiveness of agricultural support programs, subject to technological and market constraints of course. Given its importance in these types of models, parameter φ must be calibrated along with the parameters of the cost functions before one can use the farm management model to examine the impact that a new policy might have (e.g., see Howitt 1995, 2005; Paris 2011).

In this paper, we compare several methods of effectuating such a calibration. We begin in the next section by reviewing methods of model calibration and recent efforts to calibrate the risk aversion coefficient in farm BRM programming models using positive mathematical programming (PMP). In section 3, we discuss the models that we use to compare outcomes of three methods for calibrating the risk aversion coefficient. In particular, our objective is to compare specifications for calibrating risk aversion coefficients using an example from Canadian agriculture. Then, in section 4, we present the results of our comparative analysis using representative arable farms with mixed crop portfolios in different Alberta regions. Some conclusions follow in section 5.

2. Calibrating Agricultural Business Risk Management Models: Background

The complexity of the programming problem poses a number of challenges. The main one relates to the calibration of agricultural BRM models. One early approach to calibration is referred to as the historic mixes approach (McCarl 1982; Önal and McCarl 1989, 1991). This method does not find the explicit economic cost function, but, rather, constrains future crop allocations so it resembles the historic mix. It assumes that observed past crop choices are optimal or else they would not have been chosen. Because solutions occur at extreme points or corners (viz., a simplex algorithm for solving LP problems), a linear combination of observed mixes is also optimal. A mathematical programming (MP) model would then take historical choices into account by constraining the current decision to be a weighted average of past decisions, with the weights determined endogenously within the MP model and the sum of the weights constrained to equal 1. Chen and Önal (2012) suggest an important extension of this approach to include new crops that have not previously been planted by adding synthetic (or simulated) mixes of the decision variables to the historical mixes. The optimization procedure then chooses the weights, which are constrained so the sum of the historic and synthetic weights equals 1. In this case, farmers' risk attitudes are implicitly addressed because the observed optimal crop portfolio chosen by farmers does not consist solely of a single crop – the one with the largest gross margin.

Positive mathematical programming is now the preferred approach for calibrating farm management models. PMP was first developed by Howitt (1995) to address land-use allocation problems in agriculture (e.g., Röhm and Dabbert 2003), but has increasingly been adapted for use in trade modeling and other resource management settings (Weintraub et al. 2007; Paris et al. 2011; Heckelei et al. 2012; Mérel and Howitt 2014). PMP can be used to estimate crop-specific marginal cost functions and, thereby, replicate farmers' observed crop allocation decisions (Mérel and Bucaram 2010;

Mérel et al. 2011).

While the calibration of crop-specific cost functions using PMP is generally considered to be straightforward, significant challenges remain (Heckelei and Wolff 2003; Heckelei et al. 2012). PMP usually requires specification of a strictly diagonal quadratic cost matrix, implying that there are no substitutionary or complementary effects among crops. Clearly, the assumption of a diagonal cost matrix may not be realistic. Thus, Heckelei and Wolff (2003) argue that, in some cases, PMP is inconsistent, because the derived marginal costs will not converge to the true MCs. They introduce a generalized maximum entropy (GME) approach in which the shadow prices associated with the calibration constraints of PMP and the parameters of the cost function are estimated simultaneously using mathematical programming, something they refer to as econometric programming. In essence, the method employs a standard Lagrangian with econometric criteria applied directly to the Karush-Kuhn-Tucker (KKT) conditions. This permits prior information to influence the estimation results even in situations with limited data while ensuring computational stability. The PMP method has been extended by employing information theory and the principle of maximum entropy (ME) to obtain parameter estimates for the entire cost matrix (Paris and Howitt 1998; Buysee et al. 2007). The ME approach can be used in conjunction with PMP methods to reconstruct the parameters of the agricultural production function so as to duplicate the crop mixes historically observed, or the historic bilateral trade flows.

A major issue with agricultural BRM models relates to the calibration of both crop-specific cost functions and a risk aversion coefficient for the decision maker if risk attitudes are to be explicitly included in the analysis. The challenge is to estimate the risk aversion coefficient and cost parameters simultaneously within the PMP calibration framework. Several approaches are used in the literature for the calibration of φ; these can be categorized into two groups based on different assumptions about the utility function (Louhichi et al. 2010; Jeder et al. 2011, 2014; Petsakos and Rozaki 2011, 2015; Severini and Cortignani 2010, 2012).

The first assumes that wealth W is normally distributed and that the utility function is a negative exponential function of W as follows: $U(W)=1–e^{-\varphi W}$. For this functional form, the constant absolute risk aversion coefficient (CARA) can simply be derived as $-U''(W)/U'(W) = \varphi$ (McCarl and Spreen 2003). Then, maximizing the expectation of the negative exponential utility function is approximately equivalent to maximizing the certainty equivalent (CE) subject to technical constraints, where CE = μ – ½ $[U''(W)/U'(W)]\ \sigma^2 = \mu - ½\ \varphi\sigma^2$, and μ and σ^2 are the mean and variance of the distribution of wealth. The second option assumes a logarithmic utility function with a decreasing absolute risk aversion (DARA) parameter that is a concave function of wealth.

Most approaches are developed using an exponential utility function and thereby the CARA assumption. Within this framework, one method for deriving the risk aversion parameter is that employed in the EU's Farm System SIMulator (FSSIM). The approach used in FSSIM is to vary φ in an iterative fashion until the simulated land allocation comes closest to duplicating the observed crop allocation. If the calibration in the first step is not exact (which is highly unlikely to be the case because then only risk attitude explains crop choice), the value of φ determined in the first step is used to calibrate the cost function in a second step in the same way as the PMP method with elasticity adjustment (Louhichi et al. 2010; Jeder et al. 2011, 2014). However, because the marginal crop from the utility perspective may not be the least profitable crop in the expected utility framework, directly applying the standard PMP method cannot guarantee the perfect recovery of the observed land allocation (see Liu et al. 2018).

Severini and Cortignani (2010, 2012) extend an ME approach proposed by Heckelei and Wolff (2003) to calibrate simultaneously all the parameters needed within a PMP framework, including the parameters of the quadratic cost functions and CARA coefficients. In their method, the objective function is based on the error terms on the observed land allocations. The main drawback is that their approach requires prior information about the land supply elasticities for all crops. We do not consider their approach here because we lack a suite of supply price elasticities of land in crops for Alberta or even Canada, and the methods we discuss below overlap with those of Severini and Cortignani.

Arata et al. (2014, 2017) propose to make use of the primal and dual specifications of the farmer's expected utility maximisation problem. They combine the 1st and 2nd steps of the standard PMP approach to derive the calibrated objective function and the constraints. Instead of ME estimation, their procedure includes a least squares estimator that is based on the errors on the marginal cost functions. Then they simultaneously calibrate the CARA coefficient, shadow prices of land and the parameters of the cost functions. This is explained further in the next section (see equations 10 through 14 below).

Contrary to other approaches that seek to estimate a CARA coefficient, Petsakos and Rozakis (2011, 2015) apply an alternative ME method within the PMP framework by assuming linear cost functions and a logarithmic utility function, which leads to a decreasing absolute risk aversion (DARA) parameter that is a concave function of wealth. One drawback is that they require knowledge of the initial level of wealth so that the DARA parameter changes in response to the farmer's cropping choice. Further, the assumption that the cost functions are linear implies that the variability in land uses is entirely attributed to the farmer's risk attitude and is not affected by other factors, such as technology.

In the remainder of this paper, three alternative PMP approaches for dealing with risk attitudes are compared using real-world data. First, we follow Petsakos and Rozakis's (2015) method and assume DARA, linear cost functions and a logarithmic utility function, and derive the value of the DARA coefficient and the values of the parameters of the unobserved part of the linear cost functions via the method of ME. Next, we assume an exponential utility function, CARA and quadratic cost functions. We then apply Arata et. al.'s (2017) approach to calibrate the risk aversion coefficient and the parameters for quadratic cost functions. Finally, retaining the same underlying assumptions, we propose to calibrate cost functions and CARA via Howitt's (1995) ME approach within the PMP framework. We conclude with a discussion of the results derived from the three models, the ability of these approaches to reflect decision makers' behaviour, and their limitations and applicability to different situations.

3. Agricultural Business Risk Management Modeling

In this section, we provide three models that we employ for addressing risk aversion on the part of agricultural decision makers and put them into two groups based on their assumptions about utility function and risk attitude.

Logarithmic Utility Function and DARA

The logarithmic utility function, $U(W) = \ln (W^0 + \sum_{k=1}^{K} R_k)$, has the DARA property although relative risk aversion remains invariant to wealth. For the logarithmic utility function, $\varphi = \frac{-U''(W)}{U'(W)} = \frac{1}{W^0 + E\left[\sum_{k=1}^{K} R_k\right]}$. Petsakos and Rozakis (2011, 2015) start with the nonlinear excepted utility maximization problem that is approximated by the following MP:

$$\text{Maximize CE} = W^0 + E\left[\sum_{k=1}^{K} r_k\right] - \frac{1}{2} \frac{\sum_{k=1}^{K} \sum_{i=1}^{K} \left[x_k \times V(r_k, r_i) \times x_i\right]}{W^0 + E\left[\sum_{k=1}^{K} r_k\right]} \quad . \qquad (1)$$

$$\text{Subject to: } \sum_{k=1}^{K} x_k \leq \overline{X} \quad [\lambda] \qquad (2)$$

$$x_k \leq x_k^o + \varepsilon \; [\lambda_k], \; \forall k, \text{ and} \qquad (3)$$

$$x_k \geq 0, \; \forall k. \qquad (4)$$

E is the expectations operator and $E[r_k]$ is the farmer's expected overall gross margin ($/ac) from planting crop k, and $E\left[\sum_{k=1}^{K} r_k\right] = E\left[\sum_{k=1}^{K} (p_k y_k - c_k) x_k\right]$. There are K crops that can be planted in any period; x_k denotes the number of acres allocated to produce crop k, and $\overline{X}$ represents the total area (acres) the farmer allocates to crop production. Further, p_k and y_k represent, respectively, the output price and yield for crop k; and c_k is the observed per-unit-area variable cost of producing crop k. $V(r_k, r_i)$ refers to the variance-covariance matrix of returns at the regional level, with r_k and r_i the realized per-acre gross margin from crops k and i, respectively. The optimal allocation of land to crops is endogenously determined. Constraint (2) restricts the farmer's cultivated area to that which is available. In constraint (3), x^o is a vector of observed crop plantings and ε is added to the calibration constraints to prevent degeneracy that could occur because constraints (2) and (3) are related. The shadow prices associated with the constraints are indicated in square brackets. Once the model is calibrated, observed land uses are reproduced by solving the problem given by (1), with the variance-covariance matrix of returns at the farm level subject to (2) and (4), that is, without the calibration constraint in (3).

To calibrate the model, Petsakos and Rozakis (2011, 2015) first consider the FOCs associated with the above MP at the regional level:

$$\left(p_k y_k - c_k^o\right) - \frac{\sum_{i=1}^{K}\left[V(r_k, r_i) \times x_i^o\right]}{W^0 + E\left[p_k y_k x_k^o - c_k^o x_k^o\right]} + \frac{1}{2}\frac{\sum_{k=1}^{K}\sum_{i=1}^{K}\left[x_k^o \times V(r_k, r_i) \times x_i^o\right]}{\left[W^0 + E\left(p_k y_k x_k^o - c_k^o x_k^o\right)\right]^2} - \lambda_k - \lambda = 0, \ \forall k. \qquad (5)$$

Next, consider the cost and variance-covariance matrix at the farm level, denoted $c\left(x_k^o\right)$ and $S\left(r_k, r_i\right)$, respectively, that lead precisely to the observed crop allocation. Since the model calibrates for those values, the FOCs equivalent to (5) are:

$$\left[p_k y_k - c\left(x_k^o\right)\right] - \frac{\sum_{i=1}^{K}\left[S(r_k, r_i) \times x_i^o\right]}{W^0 + \left[p_k y_k - c\left(x_k^o\right)\right]x_k^o} + \frac{1}{2}\frac{\sum_{k=1}^{K}\sum_{i=1}^{K}\left[x_k^o \times S(r_k, r_i) \times x_i^o\right]}{\left[W^0 + \left(p_k y_k - c\left(x_k^o\right)\right)x_k^o\right]^2} - \lambda = 0, \ \forall k. \qquad (6)$$

Upon setting (5) and (6) equal to each other because they both lead to the same solution according to the PMP procedure, we obtain (Petsakos and Rozakis 2015, p.539):

$$\left(p_k y_k - c_k^o\right) - \frac{\sum_{i=1}^{K}\left[V\left(r_k, r_i\right) \times x_i^o\right]}{W^0 + E\left[p_k y_k x_k^o - c_k^o x_k^o\right]} + \frac{1}{2}\frac{\sum_{k=1}^{K}\sum_{i=1}^{K}\left[x_k^o \times V\left(r_k, r_i\right) \times x_i^o\right]}{\left[W^0 + E\left(p_k y_k x_k^o - c_k^o x_k^o\right)\right]^2} - \lambda_k = \left[p_k y_k - c\left(x_k^o\right)\right] - \frac{\sum_{i=1}^{K}\left[S\left(r_k, r_i\right) \times x_i^o\right]}{W^0 + \left[p_k y_k - c\left(x_k^o\right)\right]x_k^o} + \frac{1}{2}\frac{\sum_{k=1}^{K}\sum_{i=1}^{K}\left[x_k^o \times S\left(r_k, r_i\right) \times x_i^o\right]}{\left[W^0 + \left(p_k y_k - c\left(x_k^o\right)\right)x_k^o\right]^2}, \forall k. \quad (7)$$

Then a maximum entropy approach is applied as described by Petsakos and Rozakis (2011, 2015) to obtain the values of the parameters that represent the unobserved part of the farm-level variable costs.

To get the values of the parameters of the linear cost function and the DARA coefficient for one particular representative farm, the initial wealth level of the farm, and time series data of regional-level prices, yields and accounting variable costs for each crop are required. The calibrated model can only recover the observed land allocation across crops when the expected farm-level prices and yields derived from the calibration process, instead of original regional-level prices and yields, are used in the objective function (1).

Expected exponential utility maximization and CARA

A dual approach

Arata et al. (2014, 2017) assume a farmer seeks to:[1]

$$\text{Maximize CE} = E\sum_{k=1}^{K}\left(p_k x_k y_k - \alpha_k x_k\right) - \frac{1}{2}\sum_{k=1}^{K}\sum_{i=1}^{K}\left[x_k \times Q_{k,i} \times x_i\right] - \frac{\varphi}{2}\sum_{k=1}^{K}\sum_{i=1}^{K}\left[x_k \times S(r_k, r_i) \times x_i\right] \quad (8)$$

[1] In the model described by Arata et al., the variance-covariance (VC) matrix S is based on farm-level prices, and x_k represents the total yield of the k^{th} crop, which equals the crop yield per acre times the number of acres. The implication is that a farmer can choose each crop's output level. For comparison purposes, we adjust the model used here and define S as the VC matrix based on crops' gross margins per acre. Hence, a farmer chooses how to allocate land to maximize her utility. However, the calibration results differ for different S (based on prices versus based on gross margins) and related objective functions.

Subject to $\sum_{k=1}^{K} x_k \leq \overline{X}$ and $x_k \geq 0, \forall k$. (9)

when land is the only resource constraint; α_k and $Q_{k,i}$ are the parameters of the quadratic cost function; and $Q_{k,i}$ is a symmetric matrix. The other variables are defined as previously.

To calibrate the model given by (8) and (9) to the base year, Arata et al. construct the objective function for calibrating parameters by combining the primal and dual objective functions of a farm-level risk model that uses observed variable costs for linear cost functions and assumes a farmer with CARA seeks to maximize her expected utility. The related calibration constraints are based on the FOCs derived from both the first- and second-step PMP equations.

While Petsakos and Rozakis use time-series regional-level data for the calibration, Arata et al. employ cross-sectional, farm-level data. The farms used for calibration share the same technology, which is represented by the $Q_{k,i}$ part in the quadratic cost function, but each farm has its own values of φ and α. Thus, the mathematical programming model needed to calibrate the parameters is as follows:

Minimize

$$\sum_{f=1}^{F} \lambda_f \overline{X}_f + \sum_{f=1}^{F}\sum_{k=1}^{K}\left[\frac{1}{2}\alpha_{fk}^2 + c_{fk} y_{fk} x_{fk}^o + \lambda_{fk}(y_{fk} x_{fk}^0 + \varepsilon_f) + \varphi_f \sum_{k=1}^{K} x_{fk}^o S(r_k, r_i) x_{fk}^0 - \sum_{k=1}^{K} E(p_k) y_{fk} x_{fk}^o\right] \quad (10)$$

subject to

$$\boldsymbol{c_f + \varphi_f V(r_k, r_i) x_f^o + \lambda_f + \lambda_{fK} \geq E(p_f)}\ [\boldsymbol{w_f}] \quad (11)$$

$$\boldsymbol{c_f + \lambda_f = Q_{k,i} \times x_f^o + \alpha_f}\ [v_f] \quad (12)$$

$$\boldsymbol{Q_{k,i} = LDL'} \quad (13)$$

$$\lambda_f, \lambda_{f,k}, \varphi_f \geq 0 \quad (14)$$

where the subscript f represents different farms; L and D are Cholesky decomposition matrices of $Q_{k,i}$; λ_f represents the land shadow price for f^{th} farm; and $\lambda_{f,k}$ is the shadow price for the k^{th} crop on the f^{th} farm.

$Q_{k,i}$, λ_f, $\lambda_{f,k}$, φ_f and α_f are to be simultaneously calibrated and implemented via the model described by equations (8) and (9).

An ME approach[2]

With this approach, the calibrated model for one representative farm will be used for future policy analysis. In this model, a farmer decides her land allocation by solving the following MP:

$$\text{Maximize EU } = \sum_{k=1}^{n}\left(p_k x_k y_k - \alpha_k x_k - \frac{1}{2}\beta_k x_k^2 \right) - \frac{1}{2}\varphi\sigma^2 \tag{15}$$

$$\text{Subject to } \sum_{k=1}^{K} x_k \le \overline{X} \text{ and } x_k \ge 0, \forall k. \tag{16}$$

To calibrate the model parameters, we implement the GME method to derive the values of the cost function parameters and CARA coefficient φ. Similar to Arata et al. (2017), we merge the first-order conditions from the 1st and 2nd steps of the standard PMP. The ME problem for estimation is:

$$\text{Maximize H} = -\sum_{\text{k}=1}^{\text{K}}\sum_{\text{z}=1}^{\text{Z}} \pi\alpha_{\text{k,z}} \ln(\pi\alpha_{\text{k,z}}) - \sum_{\text{k}=1}^{\text{K}}\sum_{\text{z}=1}^{\text{Z}} \pi\beta_{\text{k,z}} \ln(\pi\beta_{\text{k,z}}) \tag{17}$$

subject to

$$p_k y_k - \sum_{\text{z}=1}^{\text{Z}} \text{z}\alpha_{\text{k,z}}\pi\alpha_{\text{k,z}} - \sum_{\text{z}=1}^{\text{Z}} \text{z}\beta_{\text{k,z}}\pi\beta_{\text{k,z}} \times x_k - \lambda - \varphi \times \sum_{i=1}^{K} \text{cov}(x_k, x_i) \times x_i = 0, \forall k \tag{18}$$

$$c_k + \lambda_k = \sum_{\text{z}=1}^{\text{Z}} \text{z}\alpha_{\text{k,z}}\pi\alpha_{\text{k,z}} + \sum_{\text{z}=1}^{\text{Z}} \text{z}\beta_{\text{k,z}}\pi\beta_{\text{k,z}} \times x_k, \forall k \tag{19}$$

$$\sum_{\text{z}=1}^{\text{Z}} \pi\alpha_{\text{k,z}} = 1,\ \sum_{\text{z}=1}^{\text{Z}} \pi\beta_{\text{k,z}} = 1,\ \pi\alpha_{k,z},\ \pi\beta_{k,z},\ \sum_{\text{z}=1}^{\text{Z}} \text{z}\beta_{\text{kz}}\pi\beta_{\text{kz}} \ge 0 \tag{20}$$

[2] ME is a special case of generalized maximum entropy (GME), which can be used to derive the values of parameters. If there is prior information on the range of the targeted parameters, the prior information is specified in the form of support values and a related discrete uniform probability distribution. Then, to estimate the parameters, the objective is to maximize the Shannon entropy (equation 17) subject to the known constraints (equations 18 and 19) and the available data for calculating the probabilities. Where a probability distribution is available, the expected values of estimates of parameters are used (i.e., for αs and βs in expressions within summations); but parameters such as λ are simply determined upon solving the MP.

Following the steps of the standard ME method, for each crop, we define discrete support vectors $\mathbf{z\alpha_k}$ and $\mathbf{z\beta_k}$ for α_k and β_k, respectively. Accordingly, $z\alpha_{k,z}$ and $z\beta_{k,z}$ are the z^{th} element of the support vectors, whose values are defined based on crops' gross margins and accounting variable costs; $\pi\alpha_{k,z}$ and $\pi\beta_{k,z}$ are the endogenous z^{th} element of the corresponding discrete probability distributions for the above support vectors. The values of p_k, y_k, c_k, λ_k and x_k are from the pre-defined base-case data. Constraints (18) and (19) are derived based on the first order conditions of the Lagrange functions for the PMP's first and second step. After obtaining the values of all $\pi\alpha_{k,z}$ and $\pi\beta_{k,z}$, $\sum_{z=1}^{Z} z\alpha_{k,z}\pi\alpha_{k,z}$ and $\sum_{z=1}^{Z} zb_{k,z}\pi\beta_{k,z}$ are calculated as the expected estimation of α and β. No prior information regarding elasticity is required.

4. Agricultural Business Risk Management: An Application to Alberta

For the current application, we identify one representative arable grain farm in Vulcan County located in South Central Alberta (Figure 1). Seventy municipalities in Alberta have cropland, among which Vulcan County had 608 farms, 1.067 million acres of cropland, and a population of 3,984 (Government of Alberta 2017). This municipality has the largest area of cropland, the second largest number of cropland acres per person and the third largest average acres per farm. Farmers in Vulcan County mainly produce barley, durum, canola, peas and wheat, which cover the most important crops in Alberta. Therefore, it is well representative of large farms in Alberta.

To facilitate the presentations and discussion, we will refer to the Petsakos and Rozakis approach as model PRD (D referring to DARA), the model proposed by Arata et al. as model AC (C referring to CARA), and our proposed approach as BC. To calibrate the base-case crop allocation model for the representative farm, the following data are required: product prices, yields, production variable costs, land allocations and the variance-covariance matrix of realized returns per acre among crops.

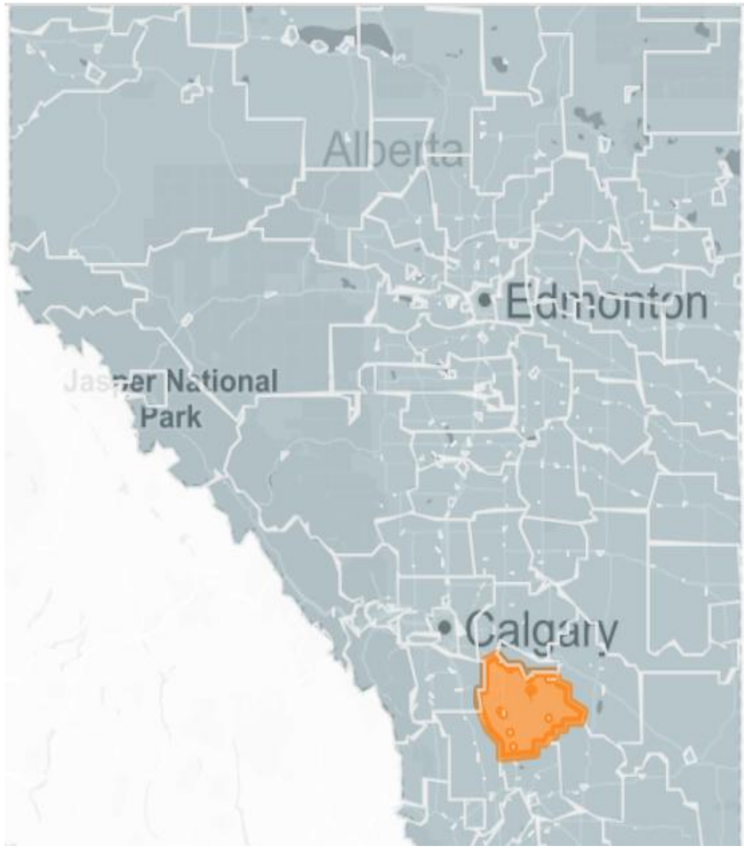

Figure 1: Vulcan County in Alberta, Canada
Source: Government of Alberta (2017)

Yearly crop prices in Alberta are obtained by taking the average of monthly crop prices available from Statistics Canada (2017a). Alberta's Agricultural Financial Services Corporation (AFSC 2016) provided municipal-level data on average yields, the number of farms and total insured acres of cropland, which are used to calculate yields per acre and the land allocations for all crops for each year. The time series data we use are for years 2008 through 2016 and are shown in Table 1. Total variable cost of production per acre is obtained from Alberta Agriculture & Forestry (2014) and assumed constant across time. Olympic averages of the prices and yields are used in the base case. The data for the base case for models PRD and BC are summarized in Table 2. Also for model PRD, the value of the initial wealth level W^0 is set at the average 2015 net worth for farms in the region, namely \$3,490,636 (Statistic Canada 2017b). Because cross-sectional farm-level data are not available, for model AC we use instead township-level data on yields and land allocations provided by AFSC (2016). Further, since durum is counted under wheat and not listed separately at the township level, weighted price and production costs caculated according to the land allocations of durum and wheat are used. For calibration, only the townships that have insured area data for all crops are included. The data required for the calibrating model AC are provided in Table 3.

Table 1: Time series data for prices and yields, 2008-2016

	Price ($/bu)					Yield (bu/acre)				
Year	**Barley**	**Canola**	**Durum**	**Peas**	**Wheat**	**Barley**	**Canola**	**Durum**	**Peas**	**Wheat**
2008	4.34	11.18	11.03	8.53	7.95	58.76	35.27	43.78	35.79	41.45
2009	3.33	9.93	6.69	6.16	5.97	43.85	28.63	34.43	21.68	31.39
2010	3.01	9.46	5.11	5.36	5.51	67.05	42.14	47.39	49.64	47.70
2011	3.80	12.02	6.49	7.74	6.38	71.21	40.78	48.33	51.76	46.35
2012	4.66	12.81	7.45	8.65	6.99	65.42	33.24	48.80	47.15	45.60
2013	5.21	13.03	6.85	8.30	7.38	80.04	42.80	55.13	54.68	56.17
2014	3.78	10.02	6.68	6.49	5.66	64.22	35.12	45.94	40.75	44.40
2015	4.76	10.40	9.48	8.36	6.06	68.06	40.34	45.83	36.93	47.76
2016	4.70	10.71	8.05	9.87	6.25	72.21	50.35	41.79	35.31	48.85

Table 2: Base-case information regarding the representative farm

	Barley	**Canola**	**Durum**	**Peas**	**Wheat**
Price ($/bu)	4.18	11.06	7.54	7.72	6.46
Yield (bu/acre)	65.65	38.74	45.71	41.52	45.52
Variable cost ($/acre)	110.49	172.70	138.15	135.63	138.15
Gross margin ($/acre)	163.67	255.88	206.36	184.80	155.96
Land (%)	17.7%	24.9%	11.0%	14.9%	31.6%
Land (acre)	311.2	436.6	192.6	261.2	554.4
Farm size (acre)			1756		
Region			South Central		
Soil Zone			Dark Brown		

Because models PRD and AC use different datasets, model BC will be applied to both for comparison. We derive farm-level average prices and yields based on the time-series data for model PRD and report them in Table 4; the base-case data are included for reference. The calibrated values of the parameters for models PRD and BC are provided in Table 5. For models AC and BC, it is difficult to directly compare the cost functions because PRD derives different intercept parts for each farm but employs the same cost matrix for all farms, while BC leads to different cost functions for each farm; therefore, we just report the values of the risk aversion coefficients and land shadow prices in Table 6.

Table 3: Township-level data of yields and land allocations in Vulcan County, 2016

	Yield (bu/acre)				Land allocation (%)				Farm size
No	**Barley**	**Canola**	**Peas**	**Wheat**	**Barley**	**Canola**	**Peas**	**Wheat**	**(acre)**
1	70.71	51.96	39.29	58.59	14.8%	20.6%	18.8%	45.7%	1828
2	38.37	53.53	33.52	38.57	9.4%	25.3%	19.8%	45.5%	2407
3	36.42	47.75	18.43	28.36	17.5%	20.7%	26.9%	34.9%	1635
4	60.54	50.94	37.56	47.66	8.9%	23.7%	23.5%	43.9%	1850
5	66.06	47.87	45.89	50.00	16.5%	42.2%	18.8%	22.5%	1696
6	105.18	59.62	49.13	51.31	26.6%	26.3%	32.1%	14.9%	1792
7	81.31	50.99	34.20	50.52	24.5%	34.2%	14.6%	26.7%	1396
8	70.61	52.99	50.97	57.72	19.7%	25.3%	20.0%	34.9%	1426
9	75.73	56.13	48.09	59.69	18.7%	31.9%	17.6%	31.7%	1334
10	73.65	56.61	43.47	49.30	10.5%	22.8%	19.5%	47.2%	2024
11	92.52	60.45	48.21	69.01	27.0%	23.4%	18.0%	31.6%	1183
12	88.90	51.92	54.46	66.47	17.6%	27.6%	19.0%	35.8%	2053
13	77.75	52.10	43.86	50.77	19.0%	24.0%	28.3%	28.7%	1420
14	68.62	43.74	41.70	50.07	10.7%	20.7%	26.3%	42.4%	1430
15	81.32	55.99	43.28	53.01	10.9%	24.1%	29.1%	35.8%	2266
16	83.80	48.07	46.82	54.00	18.9%	20.2%	12.3%	48.6%	2356
Price	4.18	11.06	7.72	6.74	($/bu)				
Cost	110.49	172.70	138.15	135.63	($/acre)				

Table 4: Derived average farm-level prices and yields from Model PRD vs base-case data

	Price ($/bu)		Yield (bu/ac)	
	Farm-level price	**Base case**	**Farm-level yield**	**Base case**
Barley	4.19	4.18	67.14	65.65
Canola	11.07	11.06	38.81	38.74
Durum	7.55	7.54	46.12	45.71
Peas	7.74	7.72	42.16	41.52
Wheat	6.50	6.46	46.96	45.52

Table 5 Calibrated parameters from PRD and BC models

	PRD	BC
Risk aversion coefficient	DARA	CARA
	2.66E-07	6.805E-08
Farm-level variance	1.65E+10	1.39E+10
Land shadow price	154.26	155.40
Cost	Linear cost function	Diagonal cost matrix
Barley	$=128.61\,x$	$=102.7\,x + ½\,0.049\,x^2$
Canola	$=273.08\,x$	$=91.54\,x + ½\,0.414\,x^2$
Durum	$=190.87\,x$	$=87.83\,x + ½\,0.523\,x^2$
Peas	$=169.48\,x$	$=106.69\,x + ½\,0.221\,x^2$
Wheat	$=149.69\,x$	$=61.39\,x + ½\,0.138\,x^2$

Table 6: Calibrated λ and φ from AC and BC models[a]

	Risk aversion coefficient		Land shadow price	
Farm	**AC**	**BC**	**AC**	**BC**
1		2.28E-08	165.04	165.06
2	6.70E-07	2.03E-08	14.77	49.71
3		1.18E-07	4.06	3.75
4		2.86E-08	130.11	142.32
5	3.03E-05			165.64
6		1.42E-07	203.87	209.66
7	2.17E-05	8.52E-08	41.40	120.44
8		2.40E-07	184.41	183.69
9		6.11E-07	205.80	203.78
10		2.35E-08	156.95	196.60
11		1.67E-07	260.65	233.50
12	3.40E-05	1.71E-07		260.39
13		1.12E-07	200.31	199.98
14		3.76E-08	176.10	176.05
15	1.09E-05	5.03E-08	57.75	195.68
16	9.81E-06	1.93E-08	58.44	223.00

[a] A blank cell indicates that no value was obtained.

Comparing the data from model PRD with the base-case data in Table 4, we find that all derived prices and yields are larger than the original data, as is the related variance (see Table 5). As indicated in Table 6, we cannot always guarantee that we can obtain a value for the risk aversion coefficient in models AC and BC. It might mean that a farmer is risk neutral in some cases, although further investigation into why this might be the case is required. Furthermore, the values of φ derived from model BC are obviously smaller than those of the other two models, which may be sensitive to the data and the choices of support vectors used for calibration.

5. Discussion and Conclusion

In this study, three approaches for explicitly calibrating the risk aversion coefficient for an agricultural BRM model were compared. The mathematical, farm management programming model seeks to maximize expected utility subject to biophysical and economic constraints, and, once calibrated, could be used for analyzing the effect of new policy initiatives. When applied to farmers in southern Alberta, Canada, the results from the three models led to several observations. First, Petsakos and Rozakis (2011, 2015) used a logarithmic utility function (and thus DARA) and linear cost function – model PRD, while that of Arata et al. (2017) employed an exponential utility function (CARA), quadratic cost function and primal/dual approach – model AC. The calibrated PRD model provided a better fit to the available regional-level time-series data than the AC model, which performed better when farm-level, cross-sectional data were available.

In contrast, our PMP approach, which used a maximum entropy method to estimate the parameters of a quadratic cost function and a CARA coefficient, relied on a representative farm and base-case data (namely, information averaged across farms for a base year) – the BC model. Model PRD and AC have the advantage that they use historical information in the calibration process, while model BC's advantage is that it can be applied to time-series data and cross-sectional data on a case-by-case base. As to the limitations, the PRD model assumes linear cost functions, which is not realistic because it implies that a crop's margin cost always equals its average cost. In addition, the PRD model cannot calibrate back to the observed land allocation with regional-level data, while the derived farm-level average prices and yields are not real farm-level data. The AC model, on the other hand, can lead to different calibrated values of the CARA coefficient and the intercept component of the cost functions for the observed farms even though they share the same technology; the differences are hard to interpret, which makes it difficult to set up a representative farm for a region. The BC model also has a major drawback: it restricts the cost matrix to a diagonal form, which is not very realistic; moreover, it employs just one observation for calibration and the values of CARA coefficients are quite small. Further, research into the BC model is required to determine how this 'diagonal restriction' on the cost matrix might be addressed, and how one would incorporate more historical data for calibration in the future.

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