Optimal Abatement of Nitrogen and Phosphorus Loading from Spring Crop Cultivation

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Discrete dynamic optimization is applied to examine the difference between socially and privately optimal fertilization patterns and to develop an incentive mechanism for efficient simultaneous nitrogen (N) and phosphorus (P) loading management. The problem formulation accounts for the causal interactions between P and N fertilization, crop yield, P carry-over, and P and N loading into waterways. Our analysis shows that the balance between private and social shadow values of the P carry-over is an essential feature for the design of the input tax-subsidy scheme for both N and P. Numerical analysis carried out for spring barley on clay soils and current damage costs in Southern Finland suggests that the difference between privately and socially optimal steady-state fertilization levels is substantial. The economic losses for the producer from the tax-subsidy scheme internalizing the damage costs are in the range of 18-32% of the profits, even at simultaneously adjusted N and P fertilizer inputs. Our sensitivity analysis indicates that other abatement measures, such as catch crops, are often competitive to fertilizer input reductions. For the producer, the computed break-even level of a subsidy for catch crops is well in line with the current subsidy levels applied in Finland.

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Keywords: **[**phosphorus loss, nitrogen loss, catch crop, carry-over, steady-state, incentive mechanism, structural uncertainty**]**

1. Introduction

Phosphorus (P) and nitrogen (N) are the two most important nutrients in terms of plant growth, but also a major cause of the eutrophication of water systems (e.g. Carpenter et al., 1998; Shortle et al., 2001). Globally agriculture is the largest contributor of the nutrient loads to aquatic ecosystems (e.g. Cao et al., 2014). Nutrient loading from agricultural land is particularly crucial in catchment areas that drain their waters to vulnerable rivers, lakes, estuaries, shallow bays or brackish seas such as Chesapeake Bay or Baltic Sea.

To efficiently manage nutrient losses from agricultural land, it is necessary to understand the causal interactions between N and P fertilization, soil processes and crop growth (e.g. Khaiter and Erechtchoukova, 2007). For example, ignoring the dynamic feedbacks have been shown to lead to suboptimal management decisions (e.g., Sharpley, 1995; Martínez and Albiac , 2004). A research tradition of studying the optimal fertilization in dynamic framework dates back to Loftsgard and Heady (1959). The effect of soil P carry-over has been studied since Kennedy et al. (1973), and more recently by Thomas (2003), who focused on N carry-over.

More generally, the economic theory of nonpoint pollution control dates back to Griffin and Bromley (1982) and Shortle and Dunn (1986). Xepapadeas (1992) proposed general dynamic framework for determination of the incentive schemes for nonpoint-source pollution. Larson et al. (1996) developed the welfare-theoretic approach for comparing second-best single-tax policies using a crop yield and pollution production model. Schnitkey and Miranda (1996) condcuted a steady state analysis for controlling P runoff from livestock producing farms with crop production. Hart and Brady (2002) and Hart (2003) studied the impacts of alternative policy goals and time-lags of abatement due to upstream and downstream measures.

Despite of the extensive literature, to best of our knowledge, there are no studies that explicitly combine the interactions between the soil P dynamics, simultaneous N and P fertilization responses, P and N loss processes and management. The objective of this study is to improve our understanding regarding the bio-economic interactions that drive rational fertilization decisions and nutrient loss control. The special feature of our approach is simultaneous treatment of N and P inputs in bio-economic modelling. This paper extends the approach by Lambert et al. (2007), who studied spatial and dynamic management of P and N in a crop rotation model, but neglected N and P losses and the linkage between STP and P response, and the approach by Iho (2010), Iho and Laukkanen (2012), who considered the interactions between crop growth, STP and P losses, but neglected N. Our numerical analysis is based on an agricultural system model developed by Sihvonen et al. (2017).

In this study, we focus on following questions: (1) how does the socially optimal fertilization patterns differ from the privately optimal fertilization; (2) which are the key characteristics that drive the incentive mechanism in case of simultaneous input decisions in a dynamic framework; (3) how are the socially optimal fertilization patterns and the incentive mechanism affected by the possibility of additional abatement measures.

2. Agronomic model components

Each period $t \in [1, T]$ of the planning horizon in agricultural production is characterized by a state, which is described with a current soil phosphorus (STP) level, denoted by x_t. The STP is an important factor governing the agricultural system because most of the effect of P fertilization stems from the long run steering of soil P fertility, whereas the immediate effect of P fertilization on crop growth is minor (McLaughlin et al., 1988; Hooda et al., 2001) and negatively associated with the STP-level (e.g., Mallarino and Prater, 2007; Valkama et al., 2011). The crop yield in absolute terms, in the other hand, is positively associated with the STP-level (e.g., Barrow, 1980; Dodd and Mallarino, 2005). The dynamic behavior of the state variable is described with the carry-over equation. The P fertilization has a positive direct effect on current period STP if the marginal effect of P fertilization on P balance is positive. Instead, N fertilization is assumed to have a negative indirect impact on STP via increased crop-uptake.

The production process generates also externalities, which are described by leaching functions. The annual N leaching has been described as a function of N-balance (e.g., Korsaeth and Eltun, 2000; Blicher-Matheisen et al., 2014), or just of an annual N fertilization (e.g., Bechmann et al., 2014). The P leaching function includes both P fertilization and STP because P-loss to surface waters consists of two forms: (1) the leaching of dissolved reactive phosphorus (DRP) and (2) the loss of particulate phosphorus (PP) (e.g. Sharpley, 1994). The level of STP mostly determines DRP (e.g., Sharpley, 1995; Pote et al., 1996; Schroeder et al., 2004). Both nutrient loads are negatively associated with the crop-uptake; only the inputs that are not removed from the soil via annual yield contribute to negative externalities of the production.

We also consider a catch crop, as a measure to reduce N losses at the source. According to Valkama et al. (2015), non-legume catch crops (mainly ryegrass species), can reduce N leaching loss by 50% on average and grain yield by 3%. Moreover, catch crop has a positive indirect effect on STP via the reduction in crop-uptake (Lemola et al., 2000). We define the cultivation of catch crops as a binary control variable, the measure is implemented fully or not at all. In addition, we assume that the choice of cultivation of a catch crop can be made annually, since catch crops are typically killed after a production period and cultivated again next spring if the decision of continuing the practice is made (Aronsson et al., 2016). In contrast to N loss, less is known about the effect of catch crops on P loss (Liu et al., 2014), and therefore, we omit the function for P losses.

To link the agronomic model components, describing the nutrient losses, to economic model, social damages due to N and P water pollution are defined. We describe the monetary damage associated with nutrient pollution as a function of P and N loading. We consider a typical constant damage function and assume that the damage function is additively separable.

3. Social optimum

From the society's point of view the decision problem is to maximize the discounted net present value of the annual rewards from production and the monetary value of environmental damage caused by nutrient loading into waterways. Thus, we define a social welfare maximum problem as follows:

$$max_{\{\boldsymbol{v}_t\}_{t=0}^{\infty}} SW = \sum_{t=0}^{\infty} \beta^t \left\{ p_t^y y(\boldsymbol{v}_t) - \boldsymbol{p}_t^{\boldsymbol{u}} \boldsymbol{u}_t - \left(d\left(e_t^P(P_t, x_t)\right) + d\left(e_t^N(N_t, \varphi_t)\right) \right) \right\} \qquad (1)$$

subject to

$$x_{t+1} \leq \vartheta\big(x_t, P_t, y(\boldsymbol{v}_t)\big), \qquad (2)$$

$$x_0 \text{ given}, \qquad (3)$$

$$x_t, P_t, N_t \geq 0, \text{ and } \varphi_t \in \{0,1\} \qquad (4)$$

where $\boldsymbol{v}_t = (P_t, N_t, \varphi_t, x_t)$ is an input vector, $\boldsymbol{u}_t = (P_t, N_t, \varphi_t)$ is a decision input vector, φ_t is catch crop, $y(\boldsymbol{v}_t) = \varphi_t 0.97 f(P_t, N_t, x_t) + (1 - \varphi_t) f(P_t, N_t, x_t)$ gives an annual average barley yield, f is a yield response function, p_t^y is a price of the barley yield , $\boldsymbol{p}_t^{\boldsymbol{u}} = \left(p_t^P, p_t^N, p_t^{\varphi}\right)$ is an input price vector, $e_t^P = e^P(P_t, x_t)$ is a leaching function for P, $e_t^N = \varphi_t 0.5 e^N\left(N_{bal,t}, N_t\right) + (1 - \varphi_t) e^N\left(N_{bal,t}, N_t\right)$ is a leaching function for N, $d(e_i) = \mu^i e^i$ with $i \in [N, P]$ is a damage function, μ^i with $i \in [N, P]$ is a marginal damage estimate, $\beta = \left(\frac{1}{1+\rho^s}\right)$ is a discount factor, ρ^s denotes a social discount rate (SDR), $x_{t+1} = \vartheta\big(x_t, P_t, y(\boldsymbol{v}_t)\big)$ is a carry-over equation describing the development of the STP and ϑ is a transition function.

Solving the first order conditions and evaluating those at the steady-state we obtain the following optimal steady-state conditions (see Appendix, A1):

$$MVP_N + \left(\frac{\mathrm{MVP}_x - \mathrm{MVD}_x}{\rho^s}\right)\left(\frac{\partial\vartheta(\cdot)}{\partial y(v)} MP_N\right) \leq MC_N + \mathrm{MVD}_N \qquad (5)$$

$$MVP_P + \left(\frac{\mathrm{MVP}_x - \mathrm{MVD}_x}{\rho^s}\right)\left[\frac{\partial\vartheta(\cdot)}{\partial P} + \frac{\partial\vartheta(\cdot)}{\partial y(v)} MP_P\right] \leq MC_P + \mathrm{MVD}_P \qquad (6)$$

$$MVP_\varphi + \left(\frac{\mathrm{MVP}_x - \mathrm{MVD}_x}{\rho^s}\right)\left(\frac{\partial\vartheta(\cdot)}{\partial y(v)} MP_\varphi\right) \leq MC_\varphi + \mathrm{MVD}_\varphi \qquad (7)$$

$$x - \vartheta(\cdot) = 0 \quad (8)$$

where MVP_v (marginal value product) implies the market value of an additional unit of the input to the producer, MP (marginal product) implies the contribution to the total output which an additional unit of the given input would generate, $\text{MC}_u = p^u$ (marginal cost) implies the price per unit of a given input, and MVD_v (marginal value damage) implies the expected damage value of an additional unit of the input to society (which depends on the unknown leaching function).

The term $(\text{MVP}_x - \text{MVD}_x)(\rho^s)^{-1}$ in equations (5)-(8) defines the shadow value of the STP in steady-state equilibrium. The shadow value of STP is positive only if $\text{MVP}_x > \text{MVD}_x$. Hence, the shadow value of STP is an increasing function of the marginal value product of STP and a decreasing function of marginal damages of STP. If μ^P is high enough, $\text{MVD}_\text{x} > \text{MVP}_\text{x}$, and the shadow value of STP is negative. Therefore, STP is a direct source of social benefits as well as damages. It is also clear that smaller the ρ^s, the higher the shadow value of STP. This implies that if $\text{MVP}_x > \text{MVD}_x$, the positive effect of a shadow value is a higher for lower SDRs, whereas if $\text{MVP}_x < \text{MVD}_x$, the negative effect of a shadow value is a higher for lower SDRs. Thus, there is a threshold value for μ^P, denoted by $\hat{\mu}^P$, where the effect of shadow value changes from positive to negative. At the same point, the effect of SDR changes from negative to positive.

The conditions (5)-(7) balance the effects of current period's choice of fertilizer inputs on immediate and future profits and damages. As shown by (5) and (6), the optimal steady-state demand for N or P is reached when the MVP of the fertilization and the next period marginal effect of the carry-over evaluated with the shadow price is less or equal to the steady-state price of the fertilizer and the marginal value damage of the fertilization. The equation (7) is similar to (5) with the exception that the MVP and MP of a catch crop are both negative. However, also the MVD of catch crop is negative. Therefore, catch crop demand is an increasing function of MVD_φ. The private optimum steady-state conditions are identical to the social optimum with the exception of the omission of the marginal damage terms. Therefore, the socially optimal demands for N and P fertilizers are lower compared to private demands. In addition, there is no incentive to implement catch crop since $\text{MVD}_x = 0$ and $\text{MVD}_\varphi = 0$.

4. Incentive mechanism: optimal tax-subsidy scheme

The social planner acknowledges the externalities of the crop production and aims to internalize those in the price system. Without direct control over production and abatement levels, the regulator must rely upon the tax mechanism to direct producers towards the socially optimal levels. Taxes increase the price of inputs and the annual profits of the private producer take the following form:

$$\pi_{private\ ,t} = p_t^y y(\boldsymbol{v}_t) - (\boldsymbol{p}_t^{\boldsymbol{u}} + \tau_t^{\boldsymbol{u}})\boldsymbol{u}_t. \quad (9)$$

With the help of this expression, we define the following steady-state tax-subsidy scheme:

$$\tau^N \leq \text{MVD}_N + \left(\frac{MVP_x}{\rho^c} - \frac{MVP_x - MVD_x}{\rho^S}\right)\left(\frac{\partial\vartheta\,(\cdot)}{\partial y\,(v)} MP_N\right) \quad (10)$$

$$\tau^P \leq \mathrm{MVD}_P + \left(\frac{MVP_x}{\rho^c} - \frac{MVP_x - MVD_x}{\rho^S}\right)\left[\frac{\partial\vartheta\,(\cdot)}{\partial P} + \frac{\partial\vartheta\,(\cdot)}{\partial y\,(v)}\frac{\partial y\,(v)}{\partial P}\right] \tag{11}$$

$$\tau^\varphi \leq \mathrm{MVD}_\varphi + \left(\frac{MVP_x}{\rho^c} - \frac{MVP_x - MVD_x}{\rho^S}\right)\left[\frac{\partial\vartheta\,(\cdot)}{\partial y\,(v)} MP_\varphi\right] \tag{12}$$

where $\tau^{\boldsymbol{u}}$ is an input tax for a given input and ρ^c is a private discount rate (PDR). The SDR is assumed to be lower than the PDR, i.e. $\rho^s \leq \rho^c$, because the social planner appreciates more the welfare of the future generations (e.g., Caplin and Leahy, 2004).

The equations (10)-(12) show that derived input tax takes into account the direct effect of a given input on associated nutrient loss as well as the indirect effect of a given input on P carry-over, weighted with the difference between private and social shadow values of P carry-over. The direct effect increases the N and P input taxes and the subsidy on catch crop. The indirect effect is more complicated. The term $\left(\frac{MVP_x}{\rho^c} - \frac{MVP_x - MVD_x}{\rho^S}\right)$ in equations (10)-(12) indicates the difference between private and social shadow values. The term reflects the difference in private and social valuation of the P carry-over. The case where $\rho^s < \rho^c$ reflects the case of a temporal externality, which implies that the private producer will have a suboptimal input control trajectory in terms of social perspective (Griffin and Bromley, 1982). When $\rho^c > \rho^s$, the term $\left(\frac{MVP_x}{\rho^c} - \frac{\mathrm{MVP}_x}{\rho^s}\right) < 0$. The intuition is that private producers do not appreciate the carry-over as much as the social planner, and therefore the private STP stock is too low from a social perspective. However, also the marginal value damage of STP affects the optimal level of STP stock. Therefore, we divide the examination of the incentive mechanism into following cases:

Case 1: PDR=SDR

In this case there is no temporal externality. The expression for τ^P reduces to the following: $\tau^P \leq \mathrm{MVD}_P + \left(\frac{\mathrm{MVD}_x}{\rho}\right)\left[\frac{\partial\vartheta\,(\cdot)}{\partial P} + \frac{\partial\vartheta\,(\cdot)}{\partial y\,(v)}\frac{\partial y\,(v)}{\partial P}\right]$, which shows that $\tau^P = 0$ if $\mu^P = 0$. The expression for τ^N, in the other hand, reduces to the following: $\tau^N \leq \mathrm{MVD}_N + \left(\frac{\mathrm{MVD}_x}{\rho}\right)\left(\frac{\partial\vartheta\,(\cdot)}{\partial y\,(v)} MP_N\right)$, which shows that, if $\mathrm{MVD}_N = 0$, an optimal tax on N is a subsidy, rather than a tax, since the second RHS term is negative.

Case 2: PDR>SDR and $\frac{MVP_x}{\rho^c} - \frac{MVP_x - MVD_x}{\rho^S} > 0$

In this case there is a temporal externality and the private shadow value exceeds the social shadow value, indicating high damage cost for P losses. Therefore, P-tax is higher compared to that in the previous case if $\left[\frac{\partial\vartheta\,(\cdot)}{\partial P} + \frac{\partial\vartheta\,(\cdot)}{\partial y\,(v)}\frac{\partial y\,(v)}{\partial P}\right] > 0$ (if the marginal effect of P fertilization on P-balance is positive). In addition, the negative marginal effect of N on P carry-over decreases the tax on N. Thus, if the term MVD_N is low enough and the term MVD_x high enough, then $\tau^N < 0$ and the tax on N fertilization becomes a subsidy. In this case the second RHS-term of (12) is positive and thus $\tau^\varphi < 0$ only if the direct and significant-in-magnitude effect of catch crop on N loss dominates the indirect and relatively minor effect of catch crop on P carry-over.

Case 3: PDR>SDR and $\frac{MVP_x}{\rho^c} - \frac{MVP_x - MVD_x}{\rho^S} < 0$

In this case the social shadow value exceeds the private shadow value. Thus, if $\left[\frac{\partial \vartheta(\cdot)}{\partial P} + \frac{\partial \vartheta(\cdot)}{\partial y(v)} \frac{\partial y(v)}{\partial P}\right] > 0$, the positive marginal effect of P on P carry-over reduces the P-tax. If the terms MVD_P and MVD_x are low enough and discrepancy between PDR and SDR is high enough, then $\tau^P < 0$ and the P-tax becomes a subsidy. Further, the negative marginal effect of N on P carry-over increases the N-tax. Moreover, in this case the second RHS term of (12) would be negative, and hence we would have a subsidy on catch crop.

These results indicate that taxes for N and P fertilizers should take into account also the indirect effects that occur via the impacts of the nutrients on P carry-over, in addition to direct contributions to respected nutrient losses. The effect of P carry-over on taxes depend on the relative magnitudes of the private and social shadow values. When these effects are taken into account, there can be situations when it is socially optimal to subsidize either N or P fertilization.

5. Empirical model

We carried out an empirical analysis to test the validity of the theoretical ideas. We applied data regarding spring barley since it is the most widely cultivated cereal crop in Finland with 500 000 ha (25% of total cultivated area). In addition, we focused on clay soils since it is the most common soil texture in Southern Finland. We applied the agricultural system models determined by Sihvonen et al. (in press). Moreover, we applied three competitive functional forms (averaged from larger set of model specifications) for N-loss functions to investigate the effect of structural uncertainty and to provide richer variety in results. All the applied models and the parameters are provided in Appendix (A2). The applied marginal damage estimates are by Gren and Folmer (2003).

6. Numerical analysis of the social optimum

Table 1 shows the differences between private and social optimums for the cases of equal and unequal discount rates:

Case 1: PDR=SDR (Table 1, comparison of the results shown in columns 1-3 to those shown in columns 4-6)

In this case the private optimal steady-state N rate is approx. 1.5-2.2 times higher than the corresponding social N rate, depending on the applied N-loss function. Correspondingly, the private optimal steady-state P rate is approx. 4.1-7.1 times higher than the corresponding social P rate.

Case 2: PDR>SDR (Table 1, comparison of the results shown in columns 1-3 to those shown in columns 7-9)

Compared to the Case 1, the difference between private and social optimal steady-state P rates is more significant, because the applied estimate for the marginal damage of P-loss is such high that the effect of

SDR on the demand of P fertilization is positive. Moreover, the difference between private and social NPV is considerably lower in this case (private NPV is approx.0.9-1.1times social NPV) compared to that for Case 1 (private NPV is approx.1.7-1.9 times social NPV), because the NPV is a decreasing function of a discount rate.

Table 1 shows that the effect of discount rate on steady-state nutrient losses, STP-levels and yields is minor. Further, although the difference between private and social optimums is greater for steady-state P rates compared to that for steady-state N rates, the corresponding difference for P-losses is less significant compared to that for N-losses. This results from the fact that P-losses are a function of STP as well as of P fertilization, and STP-level is higher for lower N rates, since the crop-uptake is lower for lower N rates. Therefore, the reduction in P losses, resulting from lower P rates, is mitigated by reduced crop-uptakes. Thus, the impact of N fertilization on P-losses, although indirect via the crop-uptake, is crucial. This impact would not be captured if N and P management would be studied separately.

Table 1: The optimal steady-state values characterizing the private and social optimums for three different N loss functions and two different social discount rates (SDR)

	Private optimum (PDR=3.5%)			Social optimum (SDR=3.5%)			Social optimum (SDR=2.0%)		
	N-loss functions								
Variable	1	2	3	1	2	3	1	2	3.
N rate (kg ha^{-1})	131.7	131.7	131.7	63.2	61.2	88.6	63.6	61.2	88.7
P rate (kg ha^{-1})	8.46	8.46	8.46	1.2	2.08	1.56	1.11	1.96	1.41
Catch crop (0=no, 1=yes)	0	0	0	0	0	1	0	0	1
STP (mg l^{-1})	4.14	4.14	4.14	3.00	3.24	2.88	2.97	3.21	2.85
Yield (kg ha^{-1})	3690	3690	3690	2770	2760	3020	2770	2760	3010
Profit (€ ha^{-1})	306	306	306	195	176	163	195	176	163
NPV (€ ha^{-1})	8780	8780	8780	5314	5019	4623	9706	8790	8110
N-loss (kg ha^{-1})	19.8	24.6	26.3	9.25	11.7	9.68	9.27	11.7	9.69
P-loss (kg ha^{-1})	0.55	0.55	0.55	0.34	0.39	0.31	0.33	0.38	0.30

The significant differences between private and social optimums originate from the high estimates for the marginal damages of the N- and P-losses: MD_N-parameter (6.6€ kg^{-1}) is 7.3 times higher than the applied price for N (0.91€ kg^{-1}), whereas the applied MD_P-parameter (47€ kg^{-1}) is 23.6 times higher than the applied price for P (1.99€ kg^{-1}). Since these estimates can be considered highly uncertain (e.g., Keeler et al., 2016), it was of the interest to examine their impact on socially optimal fertilizer rates and nutrient losses.

Figure 1 illustrates the effect of SDR and marginal damage of N (denoted by μ^N) on social optimum for second N-loss function. The threshold μ^N-value, denoted by $\widehat{\mu^N}$, is where catch crop is implemented. Thus, we have a following condition:

$$\varphi = \begin{cases} 0, \text{when } \mu^N < \widehat{\mu^N} \\ 1, \text{when } \mu^N \geq \widehat{\mu^N} \end{cases}. \qquad (14)$$

At $\widehat{\mu^N}$, the optimal steady-state N rate jumps higher for every SDR, because at this point the externalities of N fertilization become so costly that it is optimal to implement catch crop to reduce N-losses. As a result of catch crop implementation, N-losses reduction is so significant, that is it optimal to increase the N fertilization rate. Nevertheless, optimal steady-state N rate is a decreasing function of μ^N(Fig.1a). The optimal steady-state P rate drops at $\widehat{\mu^N}$, which results from the fact that at this point it becomes optimal to apply relatively more N fertilization and less P fertilization. Moreover, optimal steady-state P rate is a decreasing function of μ^N. This, in the other hand, results from the fact that N fertilization cannot be compensated by increasing P rates even for higher damages associated with N fertilization. Figure 1g shows that there is a jump downwards at the threshold μ^N-value for decreasing steady-state N-losses, whereas the corresponding jump is upwards for decreasing optimal steady-state N rates (Fig.1a). This can be explained with the effect of crop-uptake; since the steady-state yield also jumps upwards at $\widehat{\mu^N}$ (Fig.1d), the corresponding N-losses jump downwards because N-losses are lower for higher crop-uptakes.

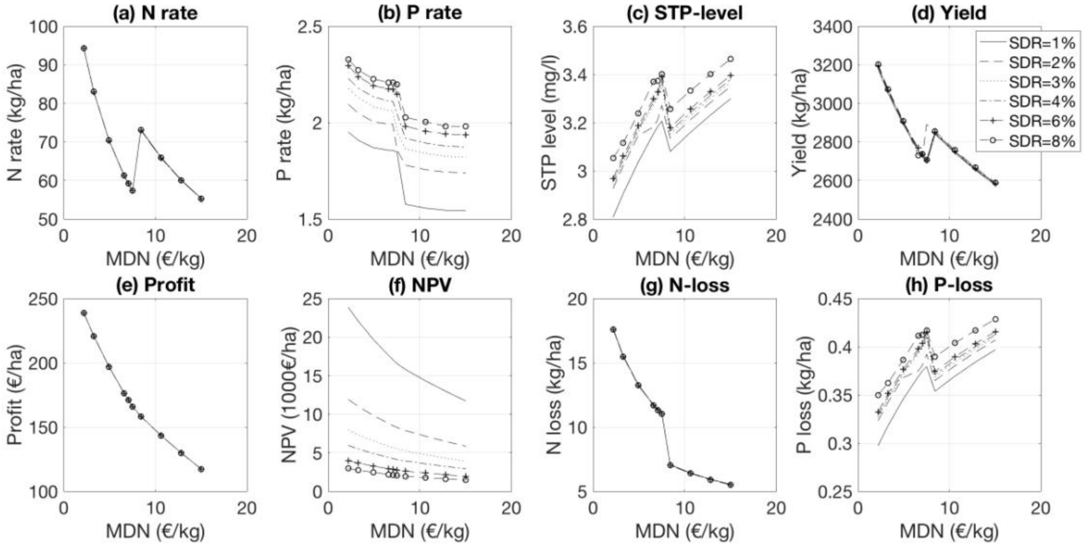

Fig.1. The effect of SDR and MD_N on social steady-state optimum

We also examined the effect of the estimate for the marginal damage of P-loss (μ^P). Figure 2 illustrates the effects of SDR and μ^P on social optimum for second and third N-loss functions. Figure 2 shows that there indeed is a threshold level for the marginal damage of P-loss ($\hat{\mu}^P$), after which the steady-state equilibrium changes. Figures 2a and 2b show that the optimal steady-state N rate is first a decreasing function of μ^P. After $\hat{\mu}^P$ is reached, the optimal steady-state N rate is an increasing function of μ^P. As was shown analytically in Section 3, the $\hat{\mu}^P$ is a point where the shadow value of STP becomes negative, which actually increases the demand for N for higher μ^P-values, whereas when $\mu^P < \hat{\mu}^P$, the shadow value of STP is positive and the demand for N is a decreasing function of μ^P-values. The optimal steady-state P rates are clearly lower for higher μ^P-values (Fig.2c, d). Instead, the optimal steady-state P rates are higher for lower SDRs up to the $\hat{\mu}^P$, after which the effect of a SDR converts: the optimal steady-state P rates are higher for higher SDRs. Figure 2m shows that N-losses are higher for higher μ^P-values in the case of second N-loss function, although the optimal steady-state N rates are not increasing for lower μ^P-levels. This results from the fact that, in the case of second N-loss function, the N-losses are a decreasing function of annual yields. Thus, as the annual yields are lower for higher μ^P-values, the N-losses are actually higher for higher μ^P-values. Instead, in the case of third N-loss function, which is not a function of annual yields, the steady-state N-losses follow strictly the optimal steady-state N rates (Fig.2n).

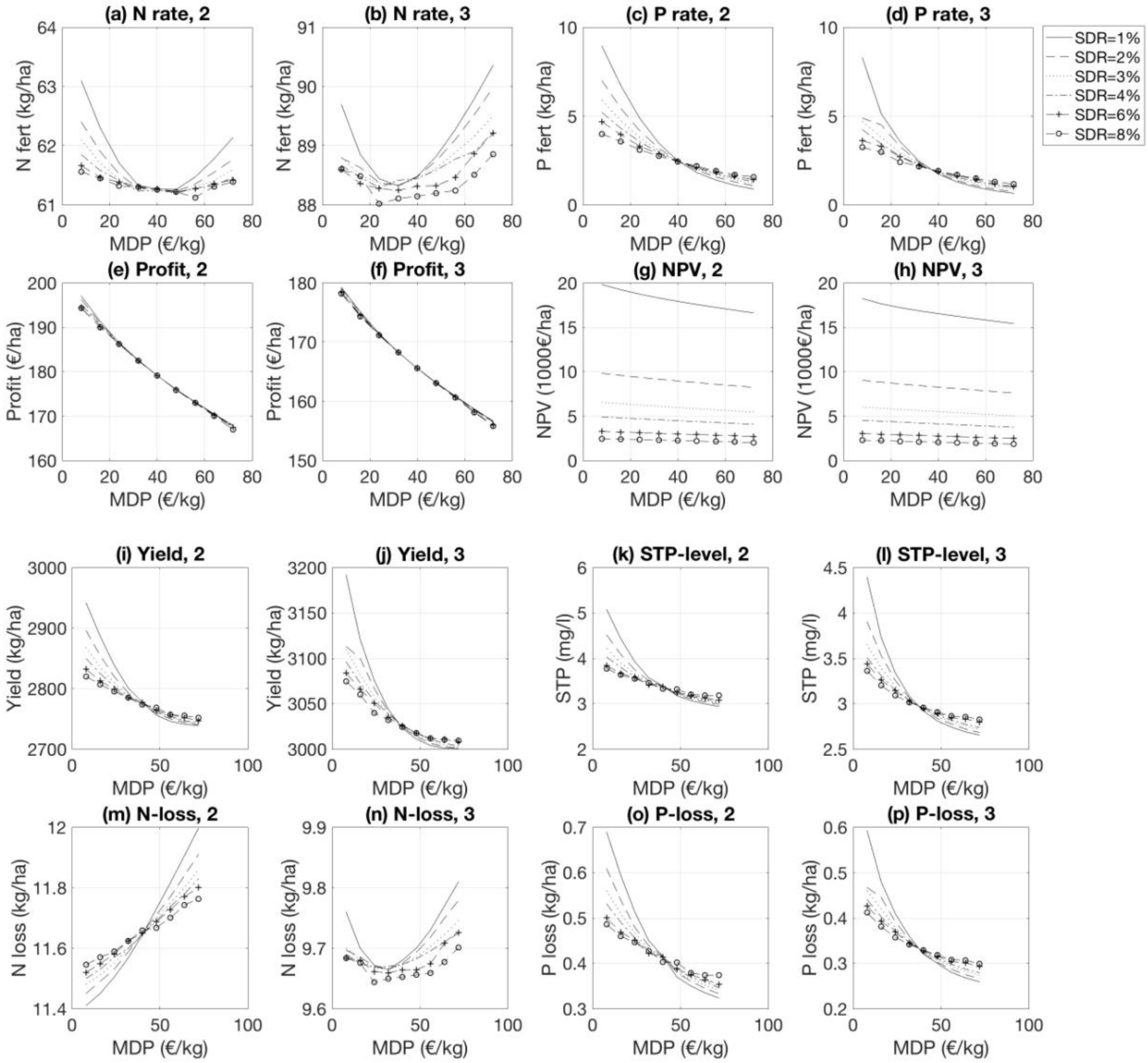

Fig.2. The effect of SDR and MD_P on social steady-state optimum. The number (2 or 3) indicates the applied N-loss function

Figures 1 and 2 show that N and P fertilization affect each other via indirect causal interlinks, which emphasizes the importance of the examination of the simultaneous utilization of N and P fertilization and the dynamic feedbacks of the agricultural system.

Since the choice of the catch crop implementation is sensitive to the realization of the marginal cost of catch crops (denoted by MC_{φ}), we determined the threshold value for the increasing MC_{φ}, denoted by $\widehat{MC_{\varphi}}$, after which the measure is too costly to be implemented. Thus, we have the following condition:

$$\varphi = \begin{cases} 1, \text{when } \mathrm{MC}_{\varphi} < \widehat{MC_{\varphi}} \\ 0, \text{when } \mathrm{MC}_{\varphi} \geq \widehat{MC_{\varphi}} \end{cases} \tag{13}$$

Figure 3 shows that at $\widehat{MC_{\varphi}}$ the change in steady-state equilibrium is quite significant, particularly for optimal steady-state N rates and yields. Figure 1c shows that the effect of the functional form of the N-loss function on the realization of the $\widehat{MC_{\varphi}}$ is crucial; the $\widehat{MC_{\varphi}}$ is lowest for the first N-loss function because the associated N-losses are lowest, whereas the $\widehat{MC_{\varphi}}$ is highest for the third N-loss function because the associated N-losses are highest. Figure 3a shows that the steady-state N rate drops when the $\widehat{MC_{\varphi}}$ is reached, since after this point it becomes optimal to reduce the N losses via N fertilization reductions. Figure 3b shows that depending on the applied N loss function, the optimal steady-state P rate may decrease or increase at $\widehat{MC_{\varphi}}$. The reason for this is that in the case of second N-loss function the optimal steady-state N rate decreases to the lowest level and therefore it is optimal to apply more P fertilization to minimize the reduction in annual yields. Regardless of this phenomenon, the optimal steady-state STP-level is higher when $\varphi = 0$. This results from the fact that once the $\widehat{MC_{\varphi}}$ is reached, the optimal steady-state N rate drops considerably and as a result the annual yield also drops (Fig.3e). Consequently, the annual crop-uptake drops, which implies that the STP-level increases.

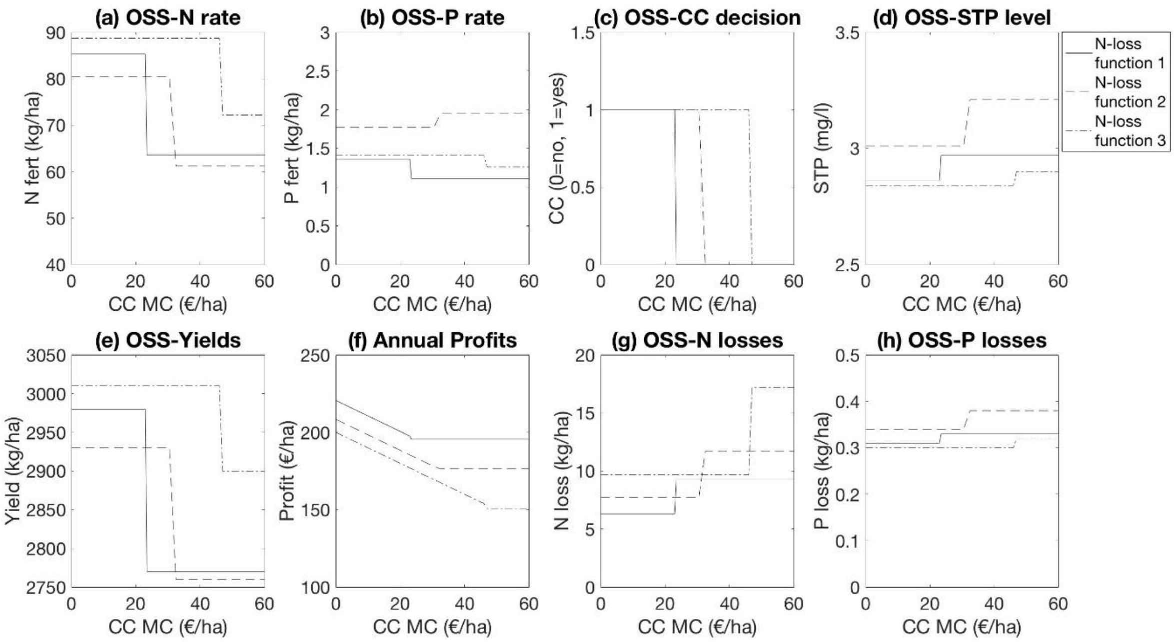

Fig.3. The effect of the catch crop marginal cost (MC) on social steady-state optimum (OSS implies optimal steady-state)

7. Numerical analysis of the incentive mechanism

As a final step in the study, we examined the incentive mechanism numerically. Table 2 shows the iteratively derived tax-subsidy scheme for the cases of equal and unequal discount rates and different N-loss functions, whereas Table 3 shows the associated economic losses for the private producer. Thus, for example, for the case of third N-loss function and equal discount rates, the social planner can induce the producer to apply socially optimal input rates by applying following tax-subsidy scheme: tax on N input is 0.4075€ kg^{-1}, tax on P input is 1.285€ kg^{-1}, and a subsidy on catch crop is 48€ kg^{-1}. This tax-subsidy scheme would reduce the profits from a producer by approx. 54€ ha^{-1}, whereas the second scheme would reduce the profits by approx. 98.8 € ha^{-1} (Table 3). It is apparent that the input taxes applied for the internalization of the externalities of the crop cultivation are substantial; when the N-tax (on average 0.80€ N kg^{-1}) is added to N input price, the resulting marginal cost for N fertilization is almost doubled. In the case of P-tax (on average 1.15€ P kg^{-1} and

1.06€ P kg^{-1}), the resulting MC for P fertilization is 1.6 and 1.5 times higher than the applied P price for unequal and equal discount rates, respectively. In addition, the subsidy for catch crop, 48€ ha^{-1} if applied, is 1.3 times higher than the MC of catch crop.

Table 2: The derived steady-state-tax-subsidy schemes for different N-loss function and for the cases where SDR=PDR and SRD<PDR*

	SDR=PDR			SDR<PDR		
N-loss function	1.	2.	3.	1.	2.	3.
N-tax (€ kg^{-1})	0.955	1.0325	0.4075	0.945	1.0325	0.4035
P-tax (€ kg^{-1})	1.255	0.64	1.285	1.4	0.64	1.415
Catch crop-tax (€ ha^{-1})	0	0	-48	0	0	-48

*Numbers 1, 2 and 3 indicate N-loss functions

Table 3: The losses for the private producer resulting from the derived steady-state tax-subsidy schemes and the associated required compensations for different N-loss function and for the cases where SDR=PDR and SRD<PDR

	SDR=PDR			SDR<PDR		
N-loss function	1.	2.	3.	1.	2.	3.
Economic loss (€ ha^{-1})	95.81	98.8	54	95.4	98.8	54.11

Following Cerrato and Blackmer (1990), we also examined the social deadweight loss resulting from the incorrect choice of N-loss function as a basis for tax-subsidy scheme. The examination was performed by inserting the taxes and/or subsidies from the above cases into private optimization problem and calculating the resulting input vectors for each N-loss function. Then we inserted these vectors back to the social optimization problem for the simulations of the optimal paths. Figure 4 shows matrices displaying the steady-state deadweight losses (positive values indicate losses and negative values profits) associated with each choice of N-loss function for the cases of equal and unequal discount rates.

The case of SDR=PDR

Applied N-loss function	Correct N-loss function: 1.	2.	3.
1.	0	19.3	45.9
2.	-18.4	0	28
3.	-20.6	-7.9	0

The case of SDR<PDR

Applied N-loss function	Correct N-loss function: 1.	2.	3.
1.	0	19.4	45.8
2.	-18.4	0	28.1
3.	-20.6	-7.8	0

Fig.4. Steady-state deadweight losses (positive values indicate losses and negative values profits) occurring from incorrect N-loss function choices

The matrices in Figure 4 show that the deadweight losses are almost identical for cases of equal and unequal discount rates when the tax-subsidy schemes are set in a socially optimal fashion. The matrices can be interpreted as follows: if for example, in the case of SDR=PDS, the social planner's prior belief is that the correct N-loss function is the second one, the planner introduces the associated tax-subsidy scheme and as a result the producer reacts by applying associated optimal amounts of inputs. Then, for example, it turns out that the correct N-loss functions was the third one. The resulting deadweight loss would be 28 € ha^{-1}. Assuming that the probabilities for each N-loss function to be the correct one would be equal, we may calculate the expected deadweight losses associated for different tax-subsidy schemes resulting from the basis of a particular N-loss function. The expected deadweight losses are 21.7 € ha^{-1}, 3.2 € ha^{-1}, and -9.5 € ha^{-}

[1] for the first, second and the third N-loss functions, respectively (for the both cases of equal and unequal discount rates). Thus, it is clear that the planner should apply the third N-loss function as a basis of the incentive mechanism, because this would generate expected profits instead of losses.

Last, the sensitivity of the incentive mechanism to the exogenous parameters is shown in Figure 5. The effects of a given exogenous parameter were determined by first calculating the associated social optimum and then iterating the tax-subsidy scheme for each parameter value. Since the calculations were computationally intensive, only the social optimum based on the third N-loss function was examined. Figure 5 shows that there indeed are situations where the taxes for P and N are zero or negative. Figure 5a shows that in *Case 1 (PDR=SDR),* and when $\mu^P = 0$, then $\tau^P = 0$. In addition, Figure 5b shows that when $\mu^N = 0$, then $\tau^N < 0$ (this hold also for *Case 2: PDR>SDR*). However, the subsidy for N fertilization quickly turns into a tax as μ^N rises. Further, in *Case 2*, when $\mu^P$is particularly low (less than 8 € kg^{-1}), then $\tau^P < 0$, indicating that P fertilization is subsidized.

Figure 5a also shows that the P-tax is lower when SDR<PDR, compared to P-tax when SDR=PDR, up to the point where $\mu^P = \hat{\mu}^P$. After this point, the P-tax is higher when SDR<PDR, because, as it has been shown, the steady-state-demand for P is higher for higher SDRs after the $\hat{\mu}^P$ is reached. Moreover, Figure 5a shows that $\frac{\partial \tau^P}{\partial \mu^P} > 0$ and $\frac{\partial \tau^N}{\partial \mu^P} < 0$ (the effect of μ^P on N-tax is almost negligible), whereas Figure 5b shows that $\frac{\partial \tau^P}{\partial \mu^N} = 0$ and $\frac{\partial \tau^N}{\partial \mu^N} > 0$. It can also be observed that the STP have only a minor effect on N-tax (Fig.5a, b). As far as the PDR is concerned, Figure 5c shows that $\frac{\partial \tau^P}{\partial \rho^C} < 0$ and $\frac{\partial \tau^N}{\partial \rho^C} > 0$ (the effect of PDR on N-tax is minor). Further, Figure 5d shows that $\frac{\partial \tau^P}{\partial \rho^S} < 0$ and $\frac{\partial \tau^N}{\partial \rho^S} > 0$. It must be noted that in this case the μ^P is fixed at the level where $\mu^P > \hat{\mu}^P$ (μ^P=47 € kg^{-1}), which implies that socially optimal steady-state P rate is an increasing function of SDR. Thus, we examined also the situation where $\mu^P < \hat{\mu}^P$ (μ^P=24 € kg^{-1}). In such a case, socially optimal steady-state P rate is a decreasing function of SDR, and therefore the P-tax is an increasing function of SDR. Thus, we have the following condition:

$$\frac{\partial \tau^P}{\partial \rho^S} > 0\Big|_{\mu^P < \widehat{\mu^P}} \text{ and } \frac{\partial \tau^P}{\partial \rho^S} < 0\Big|_{\mu^P \geq \widehat{\mu^P}}. \qquad (15)$$

Hence, the numerical sensitivity analysis verifies the analytical results, according to which taxes for both inputs can be subsidies. Further, the marginal damage costs of P, in addition to discount rates, have only minor effect on optimal N-taxes. Instead, P-tax is clearly sensitive to the arguments of the private and social shadow values. Instead, the P-tax was not affected by the marginal damage costs of N fertilization.

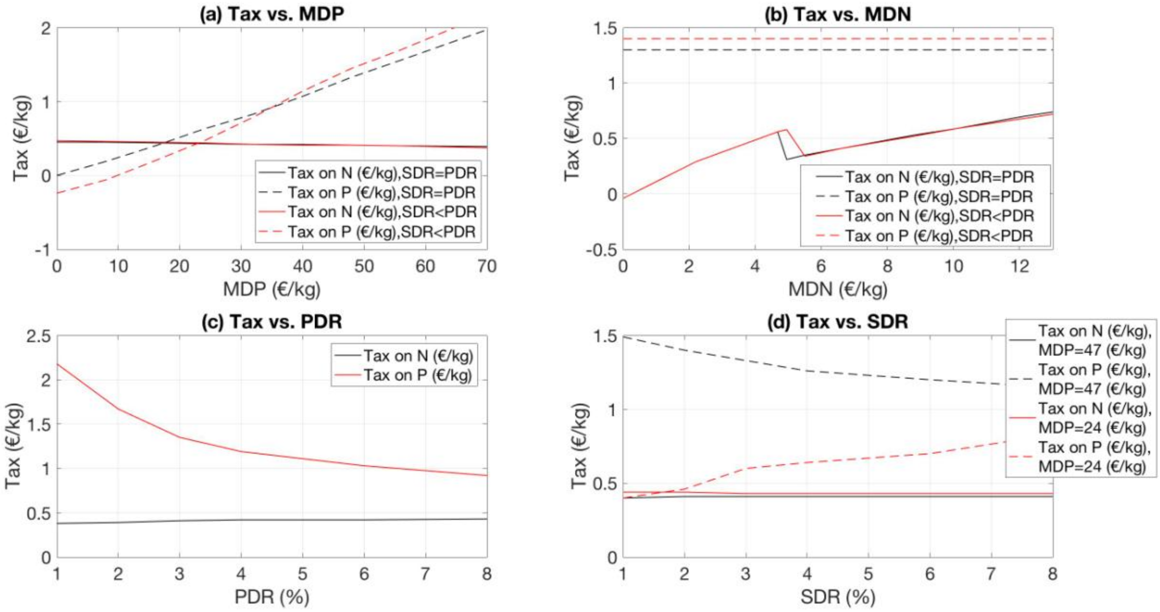

Fig.5. The effects of marginal damage parameters and discount rates on steady-state taxes for N and P

8. Discussion

The objective of the study was to develop an understanding of the bio-economic interactions that drive rational fertilization decisions and agricultural nutrient loss control by applying bio-economic modelling where N and P inputs are treated simultaneously and P carry-over is taken into account. Our results indicate, most importantly, that the balance between private and social shadow values of the P carry-over is an essential feature for the design of the input tax-subsidy scheme for both N and P. Thus, the derived taxes internalize the direct effect of each nutrient on respected nutrient losses, and also the indirect effects that occur via the impacts of the nutrients on P carry-over, weighted with the difference between private and social valuation of the P carry-over (i.e. the shadow value of the P carry-over). These findings verify and extend the findings by Hilden et al. (2007), who noted that the impacts of the P-tax could be indirect, and Iho (2010), who showed that the optimal P-tax should take into account also the indirect effects of the P carry-over. When these effects are taken into account, there can be situations when it is socially optimal to subsidize either N or P fertilization. The possibility for a negative tax on P was also discovered by Iho (2010). In practice, for example, governments in developing countries frequently subsidize the usage of fertilizers to achieve variable goals, including restoration and improvement of the soil fertility (e.g., Ricker-Gilbert et al., 2011).

Our numerical results indicated that the difference between private and social optimums was significant. Consequently, the input taxes, applied for the internalization of the externalities of the crop cultivation, were substantial (relative to price). In EU, input taxes have been applied and discussed as solutions to agricultural leaching problems. For example, in Sweden, there has been an input tax since 1995, which corresponds approx. 20% of the input prices. However, based on the assessment carried out in 2003, the taxes should be at least 6-8 times higher, in order to achieve the environmental targets (Hilden et al., 2007). In Denmark, there has been proposed that a tax on N input should raise the price of a N input 150-200% in order to reach the environmental targets in early 90s (Dubgaard, 1991). Thus, the taxes derived in this study are in the same order of magnitude, and mostly lower, than the input taxes discussed in EU. In practice, the input taxes must be set at very high rates (relative to price) to generate a significant effect (in terms of quantity reductions), because the price elasticity of the fertilizers is typically very low (Pearce and Koundouri, 2003).

It must be noted, however, that in this study, only limited number of measures was examined. If a larger set of measures would have been available, the taxes would have been lower, because there would have been more options for nutrient loss reduction in the source. If only taxes were applied for reaching the environmental goals, it would be very expensive for the producer: the resulting economic loss for the private producer would be in the range of 18-32% of the annual profits. In addition to input reductions, we explored the implementation of catch crops as a measure to reduce N loading, because the measure is shown to be a promising tool for nutrient reduction in Finland (Aronsson et al., 2016). Our findings indicate that catch crops are often competitive abatement measures to fertilization reductions. Whether or not this is the case, however, depends on the nutrient transfer mechanism (i.e. the N-loss function) applied for the basis of the decision-making, and the economic parameters, including marginal costs of the catch crop implementation and marginal damage estimates of the N-losses. If the social planner would prefer to introduce a tax-subsidy scheme that would maintain the profits of a producer unchanged, while still inducing the socially optimal fertilization patterns, the break-even level of a subsidy for catch crop would have to be approx. 102€ ha^{-1}. This corresponds to current subsidy in Finland (100€ ha^{-1}) introduced by Finnish Agri-Environmental Program (Ministry of Agriculture and Forestry, 2014).

Our results regarding the structural uncertainty showed that the choice of a functional form had a great effect on decision-making, particularly on the choice of catch crop implementation. From a policy perspective, a modelling process should include a systematic examination and responsible reporting of the uncertainty surrounding the model outcome and the decision being addressed (Lowell, 2007). Our results suggest that expected deadweight losses from basing the incentive mechanism incorrectly are in the range of 21.7 € ha^{-1}, -9.5 € ha^{-1}, when there are three alternatives for N-loss functions. Thus, the planner should apply the third N-loss function as a basis of the incentive mechanism, because this would generate expected profits instead of losses.

Finally, we recognized that a natural extension for this study would be the inclusion of the mechanism linking N dynamics to N-application decisions into the analysis. In addition, in this study, the retention effects were ignored from the analysis. If the distance between a production area and a waterbody would be greater, the marginal damages would be lower (e.g., Shortle et al., 2001; Ilnicki, 2014). Considering this fact, it is clear that a uniform tax rate is unlikely to be economically efficient (Brannlund and Krtistrom, 1999). Thus, the analysis in this study gives appropriate description only of the agricultural production right next to a waterbody. A natural extension for the study, therefore, would be the inclusion of the spatial dimension in the analysis. This extension would enable the examination of the instruments that would be targetable in time and space. Such instruments are considered as economically and environmentally desirable (Braden and Segerson, 1993).

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Appendix

A1: Derivation of the social optimum

We solved the dynamic optimisation problem in three steps with a method of Lagrange. The deterministic social welfare maximum problem was defined as follows:

$$max_{\{\boldsymbol{v}_t\}_{t=0}^{\infty}} SW = \sum_{t=0}^{\infty} \beta^t \left\{ p_t^y y(\boldsymbol{v}_t) - \boldsymbol{p}_t^{\boldsymbol{u}} \boldsymbol{u}_t - \left(d\left(e_t^P(P_t, x_t)\right) + d\left(e_t^N(N_t, \varphi_t)\right)\right)\right\}$$

subject to

$$x_{t+1} \leq \vartheta\big(x_t, P_t, y(\boldsymbol{v}_t)\big),$$
$$x_0 \text{ given,}$$
$$x_t, P_t, N_t \geq 0, \text{ and } \varphi_t \in \{0,1\} \tag{1}$$

The Lagrange function for the problem is the following:

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ p_t^y y(\boldsymbol{v}_t) - \boldsymbol{p}_t^{\boldsymbol{u}} \boldsymbol{u}_t - \left(d\left(e_{P,t}(P_t, x_t)\right) + d\left(e_{N,t}(N_t, \varphi_t)\right) \right) - \lambda_t \left[x_{t+1} - \vartheta\big(x_t, P_t, y(\boldsymbol{v}_t)\big) \right] \right\} \tag{2}$$

In a first step, we derived the Kuhn-Tucker conditions for the optimum:

$$\frac{\partial L}{\partial N_t} = MVP_{N,t} + \lambda_t \frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{N,t} - MC_{N,t} - MVD_{N,t} \leq 0 \;\; (= 0 \text{ if } N_t > 0) \tag{3}$$

$$\frac{\partial L}{\partial P_t} = MVP_{P,t} + \lambda_t \left[\frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial P_t} + \frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{P,t} \right] - MC_{P,t} - MVD_{P,t} \leq 0 \;\; (= 0 \text{ if } P_t > 0) \tag{4}$$

$$\frac{\partial L}{\partial \varphi_t} = MVP_{\varphi,t} + \lambda_t \frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{\varphi,t} - MC_{\varphi,t} - MVD_{\varphi,t} \leq 0 \; (= 0 \text{ if } N_t > 0) \tag{5}$$

$$\frac{\partial L}{\partial \lambda_{t+1}} = \beta \left[MVP_{x+1,t+1} + \lambda_{t+1} \frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(v_{t+1})\right)}{\partial x_{t+1}} - MVD_{x+1,t+1} \right] - \lambda_t \geq 0 \;\; (= 0 \text{ if } \lambda_{t+1} > 0) \tag{6}$$

$$x_{t+1} - \vartheta\big(x_t, P_t, y(\boldsymbol{v}_t)\big) \leq 0, \; \left[x_{t+1} - \vartheta\big(x_t, P_t, y(\boldsymbol{v}_t)\big)\right]\lambda_t = 0, \; \lambda_t \geq 0, t = 0,1, \dots \tag{7}$$

In the second step, we solved the first order conditions of the control variables for present and forwarded costate value in order to obtain optimality conditions depending on known variables only. From (3) we get the following expressions:

$$\lambda_t = \frac{MC_{N,t} + MVD_{N,t} - MVP_{N,t}}{\frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{N,t}} \text{ and } \lambda_{t+1} = \frac{MC_{N,t+1} + MVD_{N,t+1} - MVP_{N,t+1}}{\frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(\boldsymbol{v}_{t+1})\right)}{\partial y(\boldsymbol{v}_{t+1})} MP_{N,t+1}} \tag{8}$$

From (4) we get the following expressions:

$$\lambda_t = \frac{MC_{P,t} + MVD_{P,t} - MVP_{P,t}}{\left[\frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial P_t} + \frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{P,t} \right]} \text{ and } \lambda_{t+1} = \frac{MC_{P,t+1} + MVD_{P,t+1} - MVP_{P,t+1}}{\left[\frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(\boldsymbol{v}_{t+1})\right)}{\partial P_{t+1}} + \frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(\boldsymbol{v}_{t+1})\right)}{\partial y(\boldsymbol{v}_{t+1})} MP_{P,t+1} \right]} \tag{9}$$

From (5) we get the following expressions:

$$\lambda_t = \frac{MC_{\varphi,t} + MVD_{\varphi,t} - MVP_{\varphi,t}}{\frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{\varphi,t}} \text{ and } \lambda_{t+1} = \frac{MC_{\varphi,t+1} + MVD_{\varphi,t+1} - MVP_{\varphi,t+1}}{\frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(\boldsymbol{v}_{t+1})\right)}{\partial y(\boldsymbol{v}_{t+1})} MP_{\varphi,t+1}} \tag{10}$$

In a third step, we insert these present and forwarded costate variables one by one into (6), and assuming that interior solution exists, we obtain following optimality conditions for input utilisation:

$$MVP_{x+1,t+1} + \frac{MC_{N,t+1} + MVD_{N,t+1} - MVP_{N,t+1}}{\frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(\boldsymbol{v}_{t+1})\right)}{\partial y(\boldsymbol{v}_{t+1})} MP_{N,t+1}} \frac{\partial \vartheta\left(x_{t+1}, P_{t+1}, y(v_{t+1})\right)}{\partial x_{t+1}} - MVD_{x+1,t+1} \geq \frac{1}{\beta} \frac{MC_{N,t} + MVD_{N,t} - MVP_{N,t}}{\frac{\partial \vartheta\left(x_t, P_t, y(\boldsymbol{v}_t)\right)}{\partial y(\boldsymbol{v}_t)} MP_{N,t}} \tag{11}$$

for N utilisation and

$$MVP_{x+1,t+1}+\frac{MC_{P,t+1}+MVD_{P,t+1}-MVP_{P,t+1}}{\left[\frac{\partial\vartheta\left(x_{t+1},P_{t+1},y(\boldsymbol{v}_{t+1})\right)}{\partial P_{t+1}}+\frac{\partial\vartheta\left(x_{t+1},P_{t+1},y(\boldsymbol{v}_{t+1})\right)}{\partial y\,(\boldsymbol{v}_{t+1})}MP_{P,t+1}\right]}\frac{\partial\vartheta\left(x_{t+1},P_{t+1},y(v_{t+1})\right)}{\partial x_{t+1}}-MVD_{x+1,t+1}\geq$$

$$\frac{1}{\beta}\frac{MC_{P,t}+MVD_{P,t}-MVP_{P,t}}{\left[\frac{\partial\vartheta\left(x_t,P_t,y(\boldsymbol{v}_t)\right)}{\partial P_t}+\frac{\partial\vartheta\left(x_t,P_t,y(\boldsymbol{v}_t)\right)}{\partial y\,(\boldsymbol{v}_t)}MP_{P,t}\right]} \tag{12}$$

for P utilisation and

$$MVP_{x+1,t+1}+\frac{MC_{\varphi,t+1}+MVD_{\varphi,t+1}-MVP_{\varphi,t+1}}{\frac{\partial\vartheta\left(x_{t+1},P_{t+1},y(\boldsymbol{v}_{t+1})\right)}{\partial y\,(\boldsymbol{v}_{t+1})}MP_{\varphi,t+1}}\frac{\partial\vartheta\left(x_{t+1},P_{t+1},y(v_{t+1})\right)}{\partial x_{t+1}}-MVD_{x+1,t+1}\geq\frac{1}{\beta}\frac{MC_{\varphi,t}+MVD_{\varphi,t}-MVP_{\varphi,t}}{\frac{\partial\vartheta\left(x_t,P_t,y(\boldsymbol{v}_t)\right)}{\partial y\,(\boldsymbol{v}_t)}MP_{\varphi,t}} \tag{13}$$

for catch crop implementation. Evaluating these at the steady state we get the following steady state equations:

$$MVP_N+\frac{(MVP_x-MVD_x)}{\left(\frac{1}{\beta}-\frac{\partial\vartheta\,(x,P,y(v))}{\partial x}\right)}\frac{\partial\vartheta\,(x,P,y(v))}{\partial y\,(v)}MP_N\leq MC_N+MVD_N \tag{14}$$

for N

$$MVP_P+\frac{MVP_x-MVD_x}{\left(\frac{1}{\beta}-\frac{\partial\vartheta\,(x,P,y(v))}{\partial x}\right)}\left[\frac{\partial\vartheta\,(x,P,y(v))}{\partial P}+\frac{\partial\vartheta\,(x,P,y(v))}{\partial y\,(v)}MP_P\right]\leq MC_P+MVD_P \tag{15}$$

for P and

$$MVP_\varphi+\frac{MVP_x-MVD_x}{\left(\frac{1}{\beta}-\frac{\partial\vartheta\,(x,P,y(v))}{\partial x}\right)}\frac{\partial\vartheta\,(x,P,y(v))}{\partial y\,(v)}\leq MC_\varphi+MVD_\varphi \tag{16}$$

for catch crop. It must be noticed that $\frac{1}{\beta}-\frac{\partial\vartheta\,(x,P,y(v))}{\partial x}=(1+\rho^S)-\frac{\partial\vartheta\,(\cdot)}{\partial x}$. Further, since at the steady the condition (7) must be binding, we have that $x=\vartheta\big(x,P,y(\boldsymbol{v})\big)$. Taking the derivative w.r.t. x from both sides we have that $1=\frac{\partial\vartheta\,(x,P,y(v))}{\partial x}$. Therefore, at the steady state the shadow value is $\frac{MVP_x-MVD_x}{\rho^s}$.

A2: The empirical models and the parameters

Table 1: Parameter estimated for the applied model specifications*

Model		Parameter	Estimate	Source
Yield response model	$\bar{y}=y_{P0}\omega_P\omega_N$			Sihvonen et al. (in press)
First model element in yield response model	$y_{P0_t}=\theta_{1,1}\left(x_t^{\theta_{1,2}}+1\right)\left(1+\sqrt{x_t}\right)^{-1}$	$\theta_{1,1}$	3233 (251.7)	
		$\theta_{1,2}$	0.5903 (0.03576)	
Second model element in yield response model	$\omega_{P_t}=\left(\theta_{2,1}\sqrt{P_t}+\theta_{2,2}\right)\left(1+\exp(\theta_{2,3}x_t)\right)^{-1}+0.96$	$\theta_{2,1}$	0.0375 (0.00597)	
		$\theta_{2,2}$	0.0999 (0.01976)	
		$\theta_{2,3}$	0.0759 (0.02107)	
Third model element in yield response model	$\omega_{N_t}=\left(\theta_{3,1}\sqrt{N_t}+\theta_{3,2}N\right)\left(\theta_{3,3}{Y_{N0}}^2+1\right)^{-1}$	$\theta_{3,1}$	0.197 (3.741e-02)	
		$\theta_{3,2}$	-0.005299 (2.336e-03)	
		$\theta_{3,3}$	9.322e-08 (4.336e-08)	
Transition model for clay soils	$\vartheta=x_t+\delta_1+\delta_2P_{bal,t}+\delta_3P_{bal,t}x_t-\delta_4x_t$	δ_1	0.12738 (0.0511)	Uusitalo et al., (2016)
		δ_2	6.7745e-03 (2.05e-03)	

		δ_3	0.3225e-03 (0.106e-03)	
		δ_4	0.023591 (0.00270)	
P balance model	$P_{bal,t} = P_t - (\beta_1 \log(x_t) + \beta_2) y_t$	β_1	0.000186	Iho and Laukkanen (2012)
		β_2	0.003	
DRP loss function for clay soils	$e_{DRP,t} = \zeta_1 [\zeta_2 \text{ x} (0.021(x_t + 0.01P_t)) - 0.015] /100$	ζ_1	0.5	Puustinen et al., 2010
		ζ_2	270	Ekholm et al. (2005)
PP loss function for clay soils	$e_{PP,t} = \zeta_3 \zeta_4 \{250 \ln[x_t + 0.01P_t] - 150\} 10^{-6}$	ζ_3	1.8	Puustinen et al., 2010
		ζ_4	1200	Ekholm et al., 2005
N loss function for clay soils	$e_{N,t} = \xi_1 N_{bal,t} + \xi_2$	ξ_1	0.27	Turtola et al. (2017)
		ξ_2	9.8	
N loss function for clay soils	$e_{N,t} = \xi_1 N_{bal,t} + \xi_2$	ξ_1	0.19	Salo and Turtola (2006)
		ξ_2	7.2	
Simmelsgaard N loss function for clay soils	$e_{N,t} = \xi_1 \exp\left\{0.71 \left[\frac{N_t}{\bar{N}_t} - 1\right]\right\}$	ξ_1	21	Simmelsgaard and Djurhuus (1998) Helin et al., (2006)
N balance function	$N_{bal,t} = N_t - \frac{12.6\ (\%)}{a} y_t \frac{0.85}{100} (\%)$	a	6.25	Valkama et al. (2013)
N balance function	$N_{bal,t} = \iota_1 N_t + \iota_2$	ι_1	0.51	Salo et al. (2013)
		ι_2	-33.5	
N balance function	$N_{bal,t} = \iota_1 N_t + \iota_2$	ι_1	0.83	Salo et al. (2013)
		ι_2	-45	
Averaged N loss functions for clay soils	$e_{N,t} = \xi_1 N_{bal,t} + \xi_2$	$\xi_{1,avg}$	0.23	Averaged parameter estimates based on Turtola et al. (2017) and Salo and Turtola (2006)
		$\xi_{2,avg}$	8.5	
Averaged N balance function	$e_{N,t} = \xi_1 N_{bal,t} + \xi_2$	$\iota_{1,avg}$	0.67	Averaged parameter estimates based on Salo et al. (2013)
		$\iota_{2,avg}$	-39.25	

*In brackets are the standard deviations for the estimates

Table 2: Exogenous parameter estimates

Parameter	Symbol	Estimate	Source
N price (€ kg^{-1})	p^N	0.91	LUKE 2015
P price (€ kg^{-1})	p^P	1.99	LUKE 2015
Catch crop MC (€ ha^{-1})*	p^φ	44	Stjernholm (2012)
		29-38	Alhvik et al. (2014)
		23.7	Schou et al. (2006)
Barley price (€ kg^{-1})	p^y	0.12	K-maatalous 2017
Private discount rate (%)	ρ^c	3.5	N.A.
Social discount rate (%)	ρ^s	2.0	N.A.
Initial STP level for clay soils (mg l^{-1})	$x_{0,clay}$	4.5	N.A.

* The price for catch crop is considerably high and it consists of the price for seeds (24 € 10 kg-1 ha-1) and sowing costs (5 € ha-1) and glyphosate treatments costs (15 € ha-1) (Stjernholm, 2012). Therefore, we applied also alternative estimates for the cost of catch crops: we applied the average of the marginal costs regarding the Finnish data: 36.5 € ha^{-1} (sources: Stjernholm (2012) and Alhvik et al. (2012)).

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