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Habits and Autocorrelation in the Almost Ideal Demand System Applied to Food

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ABSTRACT

A complete system of budget share equations for U.S. food consumption was estimated using alternative specifications of the Almost Ideal Demand System (AIDS) and four food groups. The specifications accounted for habits, through a systematic change in one of the parameters, and first-order autocorrelation. The theoretical structure of the AIDS enabled the utility maximization framework to remain intact, satisfied the theoretical properties pertaining to demand systems, and provided a way to ascertain the amount that habits affected current consumption. Likelihood ratio tests indicated that both habits and autocorrelation were present.

Keywords: Demand systems, habits, autocorrelation, food.

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INTRODUCTION

A recent contribution to the estimation of complete systems of consumer demand equations has been the introduction of the Almost Ideal Demand System (AIDS) by Deaton and Muellbauer (1980a, 1980b). These authors maintain that the AIDS preserves the generality of the Rotterdam model (Theil, 1965, 1976; Barten) and the translog model (Christensen et. al., 1975a; Jorgenson et. al.) yet is considerably better for the following reasons:

> "The AIDS gives an arbitrary first-order approximation to any demand system; it satisfies the axioms of choice exactly; it aggregates perfectly over consumers without invoking parallel linear Engel curves; it has a functional form which is consistent with known household budget data; it is simple to estimate, largely avoiding the need for nonlinear estimation; and it can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters" (p. 312, Deaton and Muellbauer, 1980a).

Deaton and Muellbauer discuss the theoretical specifications of the AIDS, estimate the model for postwar British data, and test the homogeneity and symmetry restrictions. They find that "the imposition of homogeneity generates positive serial correlation in the errors" and feel that the "standard rejection of homogeneity in demand analysis may be due to insufficient attention to the dynamic aspects of consumer behavior."

This report presents full information maximum likelihood (FIML) estimates of the AIDS for the static case and for the case incorporating habits under the alternative assumptions of autocorrelation and no autocorrelation. The FIML estimates of the AIDS parameters are based on annual U.S. consumption of four major categories of food for the 1948-78 period. Civilian consumer expenditure data for domestic farm food products are adjusted and aggregated into four categories: meats, fruits and vegetables, cereals and bakery products, and miscellaneous foods.

THE ALMOST IDEAL DEMAND SYSTEM

In this demand system, consumer's preferences are represented by the following cost function: (1)

$$\log C(u,p) = \alpha_o + \sum_k \alpha_k \log p_k + 1/2 \sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_o \prod_k p_k^{\beta_k}$$

where α_i, β_i and γ^*_{ij} are the parameters, p is price, and u is a specified utility level. Hotelling's theorem 1/ applied to a cost function results in quantities demanded. Applying this theorem and converting to elasticity form yields:

1/ Hotelling's theorem states: if the cost function is continuous, differentiable, and satisfies the preference ordering axioms, then the derivative of the cost function with respect to prices results in the optimal (Hicksian) quantity-demand function:

$$\partial C(u,p)/\partial p_i = q_i(u,p) = q_i \, .$$

This is a fundamental property of the cost function.

$$\frac{\partial \log C(u,p)}{\partial \log p_i} = \frac{\partial C(u,p)}{\partial p_i} \cdot \frac{p_i}{C(u,p)} = \frac{p_i q_i(u,p)}{C(u,p)} = w_i(u,p) \quad . \qquad (2)$$

Since total cost, $C(u,p)$, is the same as total expenditure, Y, then the cost elasticity is equivalent to the budget share of commodity i, w_i, in terms of utility and prices. Equation (2) is, then, a compensated budget share equation.

Equivalently, converting equation (1) results in:

$$w_i(u,p) = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u \beta_o \prod_k p_k^{\beta_k} \qquad (2')$$

where $\gamma_{ij} = 1/2(\gamma^*_{ij} + \gamma^*_{ji})$. And, through a straightforward inversion of the cost function, the indirect utility function in terms of prices and income can be determined. Inversion of $\log C(u,p) = \log Y$ results in the indirect utility function:

$$u = V(p,Y) = \frac{\log Y - \alpha_o - \sum_k \alpha_k \log p_k - 1/2 \sum_{kj}\sum \gamma^*_{kj} \log p_k \log p_j}{\beta_o \prod_k p_k^{\beta_k}} \quad . \qquad (3)$$

Substituting equation (3) into equation (2') results in the ordinary budget share equation in terms of prices and income

$$w_i(p,Y) = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i(\log Y - \alpha_o - \sum_k \alpha_k \log p_k - 1/2 \sum_{kj}\sum \gamma^*_{kj} \log p_k \log p_j) \qquad (4)$$

Hereafter, equation (4) is referred to as the AIDS. Deaton and Muellbauer define

$$\log P = \alpha_o + \sum_k \alpha_k \log p_k + 1/2 \sum_{kj}\sum \gamma^*_{kj} \log p_k \log p_j \qquad (5)$$

so that P is a price index. Thus, equation (4) reduces to a more manageable form:

$$w_i(p,Y) = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(Y/P) \qquad (6)$$

If P is known, then equation (6) is linear in the parameters, α, γ, and β, and estimation can be done by ordinary least squares (OLS). This equation is derived from the maximization procedure with P unknown, and then assumed to be known for estimation purposes. It is not directly derived from the maximization procedure but assumes that "demands are continuous functions of the budget and of prices" (Deaton and Muellbauer, 1980a, p. 315).

From the simplified form (equation (6)), one can see that the α parameter acts as an intercept. In the absence of changes in relative prices and real expenditure, α_i is the average amount of the i-th budget share. The γ_{ij} parameter measures the change in the i-th budget share corresponding to a percentage change in p_j holding all other variables, in this case (Y/P) or real expenditure, constant. Similarly, β_i measures the change in the i-th budget share corresponding to a percentage change in real expenditure (Y/P). If β_i is negative, the i-th budget share decreases with the percentage change in real expenditure. Similarly, if β_i is positive, the i-th budget share increases with the percentage change in real expenditure. In this latter case, β_i determines that the i-th commodity is a luxury item. In the former case, β_i is negative for necessities.

Following Deaton and Muellbauer, Stone's (1953) index, $\log P^* = \Sigma w_k \log p_k$ was utilized, where $P \approx \phi P^*$ is an approximation for P, and OLS estimation was applied (Deaton and Muellbauer, 1980a, p. 314). Equation (6) becomes:

$$w_i(p,Y) = \alpha_i^* + \sum_j \gamma_{ij} \log p_j + \beta_i \log(Y/P^*) \qquad (6')$$

where $\alpha_i^* = \alpha_i - \beta_i \log \phi$ and is referred to as the linear approximate AIDS.

The interpretation of the coefficients in equation (4) is not as straightforward. Here, β_i is the change in the i-th budget share corresponding to a percentage change in expenditure (not real expenditure) with all other prices held constant and α_i is the average budget share when the logarithm of all prices and expenditure is zero. 2/ However, the change in the i-th budget share corresponding to a percentage change in the price of the i-th good (holding expenditure constant) is $\gamma_{ij} - \beta_i\alpha_j - \beta_i \sum_k \gamma_{jk} \log p_k$ -- not solely γ_{ij}. Thus, γ_{ij} and α_j are more difficult to estimate in the AIDS than in the linear approximate AIDS.

The expenditure, own-price, and cross-price elasticity formulas, respectively, for the linear approximate AIDS are:

$$\eta_i = 1 + \beta_i/w_i, \qquad (7)$$

$$\varepsilon_{ii} = \gamma_{ii}/w_i - 1, \qquad (8)$$

and $$\varepsilon_{ij} = \gamma_{ij}/w_i \text{ (for } i \neq j). \qquad (9)$$

The expenditure, own-price, and cross-price elasticity formulas, respectively, for the AIDS are

$$\eta_i = 1 + \beta_i/w_i, \qquad (10)$$

$$\varepsilon_{ii} = [\gamma_{ii} - \beta_i(\alpha_i + \sum_k \gamma_{ik} \log p_k)]/w_i - 1, \qquad (11)$$

2/ The logarithm of price is zero in the base year.

and $\varepsilon_{ij} = [\gamma_{ij} - \beta_i(\alpha_i + \sum_k \gamma_{jk}\log p_k)]/w_i$ (for $i \neq j$). (12)

Imposition of the demand properties results in restrictions on the parameters of the AIDS. For the adding-up property ($\Sigma w_i=1$) to hold, α, γ, and β are restricted so that $\Sigma\alpha_i = 1$, $\Sigma\beta_i = 0$ and $\sum_i \gamma_{ij} = 0$. Homogeneity is imposed by restricting $\sum_j \gamma_{ij} = 0$. Finally, Slutsky symmetry holds if $\gamma_{ij} = \gamma_{ji}$ is enforced.

The major limitation of this approach for estimating demand systems is the number of structural parameters that must be estimated. The number of unrestricted, unknown structural parameters in equation (4) is $2n + n^2$ (n α's, n β's, and n^2 γ_{ij}'s). However, homogeneity provides n restrictions; symmetry provides $n(n-1)/2$; and adding-up provides two. 3/ These restrictions total to $(n^2 + n + 4)/2$ leaving $(n^2 + 3n - 4)/2$ unknown structural parameters to be estimated. Thus, even with the restrictions imposed, if there are 4 commodities, one needs to estimate 12 parameters; for 8 commodities, 42 parameters; and, for 12 commodities, 88 parameters. So, for progressively more refined commodity groupings, the number of parameters to be estimated is greatly increased and may result in serious estimation problems.

Habits are introduced utilizing the Manser (1976) specification which states that "any utility function can be extended to allow for habit formation simply by specifying that certain of the parameters depend on past consumption" (p. 880, Manser, 1976). Thus, we assume that α_i, a parameter in both the cost and indirect utility functions, depends linearly on previous consumption levels:

$$\alpha_i = a_i + d_i q_{i\,t-1} \quad (\text{for} = 1,\ldots,n) \tag{13}$$

where d_i is the habit parameter and $q_{i\,t-1}$ is the quantity of the i-th commodity consumed in the previous period. Here, the new average budget share is a certain linear relationship of the quantity consumed in the immediately preceding period, rather than a constant value as in the AIDS. This habit scheme is a myopic view of decisionmaking in that the consumer is assumed to not look to the future when making present decisions -- a reasonable assumption for food purchase decisions.

Utilizing equation (13), the budget share equation for the AIDS incorporating this habit concept (AIDHFS) is:

$$w_i(p,Y) = a_i + d_i q_{i\,t-1} + \sum_j \gamma_{ij} \log p_j + \beta_i \Big\{\log Y - \alpha_0 - \sum_k (a_k + d_k q_{k\,t-1}) \log p_k - 1/2 \sum_k\sum_j \gamma^*_{jk} \log p_k \log p_j\Big\}. \tag{14}$$

3/ The third adding-up restriction ($\sum_i \gamma_{ij} = 0$) by the combination of the homogeneity and Slutsky symmetry restrictions makes one restriction redundant. That is, $\sum_i \gamma_{ij} = \sum_i \gamma_{ji} = \sum_j \gamma_{ij}$.

Only one alteration in the parameter restrictions occurs. Now the adding-up property $\Sigma\alpha_i = 1$ is replaced by $\Sigma(a_i + d_i q_{it-1}) = 1$ to assure that budget shares sum to one. If $d_i = 0$, the AIDHFS reduces to the static AIDS.

The linear approximate form of the AIDHFS is:

$$w_i(p,Y) = a_i^* + d_i q_{it-1} + \sum_j \gamma_{ij} \log p_j + \beta_i \log (Y/P^*). \quad (14')$$

ESTIMATION METHODS

Because factors other than price and income influence consumer demand and have not been explicitly introduced into these demand equations and since random errors occur in the data, structural disturbance terms are assumed to enter additively into equations explaining budget shares. This is done to account for any errors of omission and to aid in empirical implementation. Equation (4) for the t-th observation is written in the form:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \log p_{jt} + \beta_i (\log Y_t - \alpha_o - \sum_k \alpha_k \log p_{kt} - 1/2 \sum_{kj}\sum \gamma_{kj}^* \log p_{kt} \log p_{jt}) + e_{it}. \quad (15)$$

Equation (14), the habit formation model, similarly, is:

$$w_{it} = a_i + d_i q_{it-1} + \sum_j \gamma_{ij} \log p_{jt} + \beta_i \{\log Y_t - \alpha_o - \sum_k (a_k + d_k q_{kt-1}) \log p_{kt} - 1/2 \sum_{kj}\sum \gamma_{kj}^* \log p_{kt} \log p_{jt}\} + e_{it}. \quad (16)$$

The error vector, e_t, is assumed to be characterized by a zero mean, is temporally uncorrelated, and has a contemporaneous variance-covariance matrix, Ω. That is, $E(e_t) = 0$; $E(e_t x_{t'}) = 0$; and $E(e_t e_{t'}) = \delta_{tt'}\Omega$, where $\delta_{tt'}$ is the Kronecker delta. Because of the adding-up criterion, summing e_{it} over i gives the restriction $\sum_i e_{it} = 0$. Also, since $E(e_t) = 0$ and $E(e_t e_{t'}) = \delta_{tt'}\Omega$, the contemporaneous variance-covariance matrix Ω is singular. This singularity of Ω must be considered in estimating the demand equations. This is typically handled by arbitrarily deleting the Nth equation in the system. 4/ The estimates will not depend on which equation is omitted when autocorrelation is not present. The resulting subsystem of N-1 equations has a positive definite variance-covariance matrix. The parameter estimates of the Nth expenditure equation as well as the full covariance estimator Ω are then obtained by applying the Engel aggregation conditions. Alternatively, the full system of N equations could have been estimated using a generalized inverse for the singular covariance matrix. 5/ The former procedure was used in AIDS and AIDHFS estimations.

4/ For more information, see Barten (1969) and Berndt and Savin (1975).

5/ See Theil (1971). Initial parameter estimates for the FIML procedure were obtained from OLS results.

However, if first-order autocorrelated disturbances are present, that is, if:

$$e_t = \rho e_{t-1} + v_t \tag{17}$$

where the e_t are considered as realizations of a first-order Markovian process, the residuals v_t are uncorrelated and homoscedastic, and ρ is the autocorrelation coefficient, then the variables in the system must be replaced by their first-order transforms. But, the ρ must be the same for each equation and there must be no serial correlated disturbances across equations. 6/

THE DATA AND COMMODITY GROUPINGS

The time period 1948 to 1978 was selected for analysis, resulting in 31 observations; data from 1947 to 1977 were utilized for the lagged specifications. Data necessary to estimate the parameters for the food commodity budget share equations include per capita expenditures, prices, and per capita food consumption.

Four commodity groups were used in the analysis: meats, fruits and vegetables, cereals and bakery products, and miscellaneous foods. 7/ Consumer expenditures for domestic farm food products bought by U.S. civilians were obtained from the U.S. Department of Agriculture (USDA) (1968, 1979, 1981). These food expenditure data are available for seven commodity groups: meat products, poultry and eggs, dairy products, fruits and vegetables, grain mill products, bakery products, and other food products. Expenditures on nonalcoholic and alcoholic beverages, as well as fish and imported foods, are excluded from this data. For estimation purposes, these data were adjusted and aggregated into the four commodity groupings. 8/

6/ See Berndt and Savin (1975).

7/ The adjustments and group selections follow the approach utilized by Manser (1976) and Christensen et al. (1975b). Consideration was given to the number of parameters in the estimated equations, as discussed in section III, in the number of commodity groups selected. Data used in the analysis are presented in Blanciforti (1982).

8/ The analysis assumes the consumer allocates total expenditures in two stages. Expenditures are allocated to food, a broad group of commodities, at the first stage, and food expenditures are allocated to the subgroup commodities (meats, fruits and vegetables, cereals and bakery products, and miscellaneous foods) at the second stage. Weak separability in the first stage is both necessary and sufficient for the second stage of two-stage budgeting. Thus, decisions at each stage can be thought of as corresponding to a utility maximization problem of its own. (See Deaton and Muellbauer, 1980b, pp. 122-124.) The focus of this study is the second stage of the decisionmaking and not the separability issue *per se*. However, a way to determine the relationship of the food group expenditure elasticities to total expenditure elasticity is discussed below, where strong separability is assumed to be in the first stage.

The meats series was adjusted to include fish and poultry. It contains expenditures for beef and veal, pork, fish, and poultry. Per capita quantity data for each of these meat items are available in USDA (1968, 1981). Consumer price indexes, originally published by the U.S. Bureau of Labor Statistics (USBLS) and available in USDA (1968, 1979, 1981), corresponding to the four categories were converted to a 1972 base period. Constant dollar expenditures for the entire meats group were obtained by multiplying the quantity per capita for each meat item by its 1972 consumer price index. Quantity indexes for the aggregate meats group equivalent to the total constant dollar expenditure were used to represent aggregate quantity. The implicit price deflator used to represent aggregate price for the meats group was defined by current dollar expenditure divided by constant dollar expenditure. Population data used for per capita specifications were for midyear (July 1) U.S. resident civilian population from the U.S. Department of Commerce (various issues).

Expenditures for fruits and vegetables were taken directly from table 111 (USDA, 1968) for the years prior to 1960 and from table 88 (USDA, 1981) for 1960 and the years thereafter. The cereals and bakery products expenditures are the sum of the bakery goods and grain mill products series from the same USDA sources. In both the fruits and vegetables and cereals and bakery products categories, imports are a very small, almost negligible component, and were omitted. USBLS consumer price indexes were utilized for the price variable for these two series.

Miscellaneous foods contained the expenditure series for dairy products and other foods from the USDA (1968, 1981) consumer expenditure series plus expenditures for eggs (not included in the meat group) and sugar imports (a major import item). The data for these latter two series were obtained by following a procedure similar to that discussed above for meats.

The addition of expenditures on fish and sugar imports is an attempt to reconcile the USDA series with the commonly used U.S. Department of Commerce (USDC) personal consumption expenditure series on food consumption. The USDC series is available for major types of food, such as food at home and food away from home, but is not available for the categories of interest--namely, meats, fruits and vegetables, etc. Admittedly, this reconciliation is somewhat arbitrary but makes the best use of the available data. The USDC series (aggregating food at home and food away from home), the USDA series (the total of their seven commodity groups), and the present aggregate series (the sum of the four commodity groups utilized in this study) are presented for comparison in table 1.

THE RESULTS

Results for the Linear Approximate Case

The OLS estimates of equation (6'), the linear approximate AIDS, without and with habit formation, using the Stone index and not incorporating any constraints on the parameters are presented in tables 2 and 5. For the static linear approximate AIDS (table 2), the estimates of β classify meats as a relative luxury good and the rest of the commodities as relative necessities. 9/ Three out of the sixteen γ coefficients are significantly different from zero. Own-price elasticities are all less than one in absolute value, indicating that the price elasticity of demand is inelastic. $\sum_j \gamma_{ij}$ represents the total effect on the i-th budget share of a percentage change in all prices and total food expenditure. Under homogeneity, this total effect would be zero. Hence, a proportional increase in prices and total food expenditure will increase expenditure on meats and fruits and vegetables, decrease expenditure on miscellaneous foods, and leave cereals and bakery products essentially unchanged.

Tables 3 and 6 present the OLS parameter estimates for the linear approximate AIDS when homogeneity $\left(\sum_j \gamma_{ij} = 0\right)$ is imposed, without and with habit formation, respectively. In this case, equations (6) and (6') by substitution of this restriction become:

$$w_i = \alpha_i + \sum_{j=1}^{n-1} \gamma_{ij} \log (p_j/p_n) + \beta_i \log (Y/P) \tag{18}$$

and

$$w_i = \alpha_i^* + \sum_{j=1}^{n-1} \gamma_{ij} \log (p_j/p_n) + \beta_i \log (Y/P^*). \tag{18'}$$

In table 3, meats and fruits and vegetables are classified as relative luxury goods while cereals and bakery products and miscellaneous foods are classified as relative necessities. With the imposition of homogeneity on the static linear approximate AIDS an F-test comparing the residual sum of squares of equation (6') and (18') can be made. Referring to table 4, the first column gives an F-test for the validity of the homogeneity restriction for each equation of the static model. 10/ F = 4.24 is the critical value at the 5-percent level of significance. Homogeneity is rejected for meats and miscellaneous foods. For every commodity group where homogeneity is considered unacceptable by the F-value, its imposition leads to a sharp drop in the Durbin-Watson statistic. The implication is that the imposition of homogeneity generates positive serial correlation in the residuals. This suggests that the incorporation of habit formation in the static model and an analysis of autocorrelation should improve the estimates.

9/ Economists commonly classify commodities as necessities and luxuries on the basis of income elasticity. An income elasticity greater than one indicates the commodity is more or less a luxury and an income elasticity less than one indicates the commodity is a necessity. Since aggregate expenditure is used interchangeably with income and since β reflects food expenditure and not total expenditure, meat is a luxury good relative to food expenditure and not all expenditure.

10/ In table 2, the t-values for $\sum_j \gamma_{ij}$ are the square roots of the F-values in table 4.

Table 1--Personal consumption expenditures for food: U.S. Department of Commerce (USDC), U.S. Department of Agriculture (USDA), and present estimates

Year	USDC	USDA	Present
		Million dollars	
1947	40142	41937	44544
1948	42717	44805	47548
1949	41730	43371	46601
1950	42998	43992	47192
1951	48323	49252	52428
1952	50835	50932	54692
1953	51918	51013	55104
1954	53376	51140	54776
1955	55218	53127	56659
1956	57758	55548	58903
1957	61281	58293	61266
1958	63977	60994	64381
1959	66301	63118	66354
1960	68133	66881	69153
1961	70034	68672	70719
1962	71630	71317	73436
1963	73535	74044	76189
1964	77978	77504	78613
1965	83510	81114	82135
1966	90042	86923	87827
1967	92301	91620	91069
1968	99982	96789	96223
1969	106795	102623	102362
1970	115827	110590	109820
1971	119328	114627	113434
1972	127811	122192	121200
1973	143567	138817	139621
1974	163007	154617	160264
1975	180493	167020	173671
1976	195623	183301	183224
1977	212877	192298	190361
1978	235168	215961	213513

Table 2--The static linear approximate almost ideal demand system (nonhomogeneous)

Food group i		Estimated coefficients a/							Summary statistics			Elasticities b/				
												Food expenditure	Uncompensated price			
		α_i^*	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	SSE (10^{-2})	R^2	DW		Meats	Fruits & vegetables	Cereals & bakery products	Miscellaneous foods
Meats	(1)	-0.564	0.140	0.120	-0.042	-0.056	0.010	0.032	0.684	0.816	0.976	1.45	-0.61	-0.14	-0.18	0.03
		(-1.5) c/	(2.4)	(6.6)	(-1.0)	(-1.6)	(0.4)	(4.2)								
Fruits and vegetables	(2)	0.230	-0.004	-0.127	0.153	0.028	-0.043	0.011	0.643	0.864	1.574	0.98	-0.63	-0.24	0.14	-0.21
		(0.7)	(-0.1)	(-7.4)	(3.7)	(0.9)	(-1.7)	(1.6)								
Cereals and bakery products	(3)	0.538	-0.064	-0.005	-0.026	0.032	-0.001	0.000	0.398	0.438	1.397	0.52	-0.04	-0.19	-0.77	-0.01
		(2.5)	(-1.9)	(-0.5)	(-1.0)	(1.6)	(-0.1)	(0.1)								
Miscellaneous foods	(4)	0.787	-0.070	0.011	-0.084	-0.003	0.032	-0.043	0.782	0.858	1.647	0.81	0.03	-0.24	-0.01	-0.91
		(1.9)	(-1.1)	(0.7)	(-1.7)	(-0.1)	(1.0)	(-5.0)								

a/ Coefficients are based on U.S. data 1948-78.
b/ Elasticities are calculated using equations 7 through 9 at the mean (1948-78) values.
c/ Coefficients in parentheses are t-values.

Table 3--The static linear approximate almost ideal demand system (homogeneous)

Food group i		Estimated coefficients a/							Summary statistics			Elasticities b/				
												Food expenditure	Uncompensated price			
		α_i^*	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	SSE (10^{-2})	R^2	DW		Meats	Fruits & vegetables	Cereals & bakery products	Miscellaneous foods
Meats	(1)	-1.763	0.328	0.106	0.118	-0.048	0.060	0.0	0.873	0.688	0.869	2.06	-0.66	-0.38	-0.16	0.19
		(-6.1)c/	(7.2)	(4.7)	(-2.3)	(-1.1)	(0.6)									
Fruits and vegetables	(2)	-0.191	0.062	-0.131	0.126	0.030	-0.025	0.0	0.661	0.851	1.531	1.31	-0.65	-0.37	0.15	-0.13
		(-0.9)	(1.8)	(-7.6)	(3.3)	(0.9)	(-0.4)									
Cereals and bakery products	(3)	0.553	-0.067	-0.005	-0.025	0.031	-0.001	0.0	0.390	0.438	1.401	0.51	-0.04	-0.18	-0.77	-0.01
		(4.3)	(-3.3)	(-0.5)	(-1.1)	(1.6)	(0.1)									
Miscellaneous foods	(4)	2.424	-0.328	0.030	0.020	-0.013	-0.037	0.0	1.082	0.717	1.160	0.08	0.08	0.06	-0.04	-1.10
		(6.8)	(-5.8)	(1.0)	(0.3)	(-0.2)	(-0.3)									

a/ Coefficients are based on U.S. data for 1948-78.
b/ Elasticities are calculated using equations 7 through 9 at the mean (1948-78) values.
c/ Coefficients in parentheses are t-values.

Table 4--Homogeneity test results for the linear approximate almost ideal demand system

Food group (i)		Static F-value criterion: F(1,25) = 4.24	Dynamic F-value criterion: F(1,24) = 4.26
Meats	(1)	a/17.40	a/4.49
Fruits and vegetables	(2)	2.43	.21
Cereals and bakery products	(3)	.01	.15
Miscellaneous foods	(4)	a/24.78	a/11.15

a/ Indicates rejection of the homogeneity hypothesis.

The results of the linear approximate AIDS with habit formation are shown in table 5. Meats are again classified as a relative luxury good and 6 out of the 16 γ coefficients are significantly different from zero. The change in the budget share corresponding to a change in lagged consumption is a small positive amount implying that habits have a positive effect on the budget share. All but the cereals and bakery products habit parameter are significantly different from zero. The price elasticity of demand is inelastic for the four groups. And, a proportional increase in prices and total food expenditure will increase expenditure on fruits and vegetables and decrease expenditure on the other three groups.

Table 6 shows the imposition of homogeneity on the linear approximate AIDS with habit formation results in meats as a relative luxury good and fruits and vegetables and cereals and bakery products as relative necessities. The habit parameter has a small positive effect on the budget share in every case. And, all but the cereals and bakery product habit parameter are significantly different from zero.

In the final column of table 4, homogeneity is again rejected for meats and miscellaneous foods. Notice that the Durbin-Watson statistic for these two categories does not change as it did before habit formation was imposed. The implication is that the inclusion of the habit formation parameter when homogeneity is imposed does not generate positive serial correlation in the residuals as it did in the static case. This is due to the fact that the habit parameter was an omitted variable in the static model.

Table 5--The dynamic linear approximate almost ideal demand system (nonhomogeneous)

Food group i		Estimated coefficients a/								Summary statistics			Elasticities b/				
													Food expend-iture	Uncompensated price			
		a_i^*	$d_i(10^{-3})$	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	SSE (10^{-2})	R^2	DW		Meats	Fruits & vege-tables	Cereals & bakery products	Miscel-laneous foods
Meats	(1)	-0.108 (-0.3) c/	0.423 (1.9)	0.055 (0.8)	0.129 (7.2)	-0.056 (-1.3)	-0.040 (-1.2)	-0.013 (-0.5)	-0.020 (2.1)	0.652	0.839	1.347	1.18	-0.58	-0.18	-0.13	-0.04
Fruits and vegetables	(2)	0.774 (2.4)	1.165 (3.6)	-0.111 (-2.1)	-0.076 (-3.8)	0.090 (2.4)	0.042 (1.6)	-0.054 (-2.5)	0.002 (0.5)	0.528	0.912	2.353	0.45	-0.37	-0.55	0.21	-0.27
Cereals and bakery products	(3)	0.550 (2.6)	0.630 (1.5)	-0.074 (-2.2)	-0.024 (-1.5)	-0.031 (-1.2)	0.063 (2.2)	-0.010 (-0.6)	-0.002 (-0.4)	0.485	0.485	1.789	0.45	-0.18	-0.23	-0.53	-0.07
Miscellaneous foods	(4)	1.367 (3.4)	0.572 (3.1)	-0.179 (-2.7)	-0.025 (-1.2)	-0.015 (-0.3)	-0.043 (-1.2)	0.054 (1.9)	-0.029 (-3.3)	0.899	0.899	2.328	0.50	-0.07	-0.04	-0.12	-0.85

a/ Coefficients are based on U.S. data for 1948-78.
b/ Elasticities are calculated using equations 7 through 9 at the mean (1948-78) values.
c/ Coefficients in parentheses are t-values.

The Full Information Maximum Likelihood (FIML) Results

Application of demand properties implies restrictions on the parameters of the AIDS equation. To summarize, the restrictions are:

$$\sum_i \alpha_i = 1 \qquad \sum_i \gamma_{ij} = 0 \qquad \sum_i \beta_i = 0 \tag{19}$$

$$\sum_j \gamma_{ij} = 0 \tag{20}$$

$$\gamma_{ij} = \gamma_{ji} \tag{21}$$

the adding-up, homogeneity, and Slutsky symmetry properties, respectively. When these three properties hold, the AIDS represents a system of demand functions that aggregate to total expenditure ($\Sigma w_i = 1$), are homogeneous of degree zero in prices and total expenditure, and satisfy Slutsky symmetry. Tables 7 and 8 present the estimates of equations (15) and (16), the static AIDS, and the AIDS with habit formation, using FIML techniques. The FIML technique applied to the AIDS captures the correlation between equations as opposed to the OLS estimation of the linear approximate AIDS where each equation is treated separately.

Table 7 shows the AIDS estimates. The β coefficients classify meats and fruits and vegetables as relative luxuries and cereals and bakery products and miscellaneous foods as relative necessities. Four out of the 16 γ coefficients are significantly different from zero. One of the γ's, $\gamma_{34} = \gamma_{43}$, is very close to zero and could probably be eliminated from the system. The own-price elasticities are all less than one in absolute value, indicating that the price elasticity of demand is inelastic.

Table 8 contains the FIML results for the AIDS with habit formation. Meats and miscellaneous foods are classified as relative luxury goods; fruits and vegetables and cereals and bakery products are relative necessities. Again, own-price elasticities imply inelastic demand. The habit coefficient has a positive effect for all but the miscellaneous food category. The habit parameter is significantly different from zero for all but the cereals and bakery products group.

The static AIDS with autocorrelation (table 9) classifies all but cereal and bakery products as relative luxury goods and own-price elasticities as inelastic. The autocorrelation parameter is 0.907 and is significantly different from zero.

With habit formation and autocorrelation (table 10), miscellaneous foods is the only relative luxury good, and its own-price elasticity is elastic. The autocorrelation parameter is again strong at 0.985 and is significantly different from zero.

To test for habit effects, the likelihood ratio test is used where $\lambda = \max L_w / \max L_n$. That is, the λ statistic is the ratio of the maximum value of the likelihood function (max L) with

Table 6--The dynamic linear approximate almost ideal demand system (homogeneous)

Food group i		Estimated coefficients a/								Summary statistics			Elasticities b/				
													Food expenditure	Uncompensated price			
		a^*_i	$d_i(10^{-3})$	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	SSE (10^{-2})	R^2	DW		Meats	Fruits & vegetables	Cereals & bakery products	Miscellaneous foods
Meats	(1)	-0.216	0.732	0.063	0.131	-0.094	-0.026	-0.011	0.0	0.696	0.809	1.568	1.20	-0.58	-0.30	-0.08	-0.04
		(-0.5) c/	(4.0)	(0.8)	(6.8)	(-2.3)	(-0.1)	(-13.1)									
Fruits and vegetables	(2)	0.704	1.218	-0.101	-0.074	0.081	0.043	-0.050	0.0	0.520	0.911	2.367	0.50	-0.37	-0.60	0.21	-0.25
		(2.5)	(4.1)	(-2.1)	(-3.8)	(2.5)	(1.6)	(-2.07)									
Cereals and bakery products	(3)	0.610	0.598	-0.083	-0.022	-0.027	0.061	-0.012	0.0	0.383	0.481	1.773	0.39	-0.16	-0.20	-0.55	-0.09
		(4.6)	(1.4)	(-3.6)	(-1.4)	(-1.2)	(2.2)	(-18.7)									
Miscellaneous foods	(4)	2.500	0.889	-0.367	-0.036	0.074	-0.071	0.032	-0.0	0.800	0.852	2.270	-0.03	-0.10	0.21	-0.20	-0.91
		(9.5)	(4.8)	(-8.6)	(-1.4)	(1.6)	(-1.7)	(11.3)									

a/ Coefficients are based on U.S. data for 1948-78.
b/ Elasticities are calculated using equations 7 through 9 at the mean (1948-78) values.
c/ Coefficients in parentheses are t-values.

Table 7--The static almost ideal demand system (zero autocorrelation)

Food group i		Estimated coefficients a/							Elasticities b/						Budget share 1948-78 mean value
									Expenditure		Uncompensated price				
		α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	Total c/	Food	Meats	Fruits & vegetables	Cereals & bakery products	Miscellaneous foods	
Meats	(1)	0.327 (111.8)d/	0.328 (8.7)	0.110 (4.6)	-0.140 (-9.3)	-0.013 (-1.2)	0.042 (1.1)	-0.0	0.90	2.06	-0.99	-0.67	-0.18	-0.22	0.309
Fruits and vegetables	(2)	0.209 (88.2)	0.052 (1.3)	-0.140 (-9.3)	0.160 (4.4)	-0.004 (-0.2)	-0.016 (-0.3)	-0.0	0.55	1.26	-0.78	-0.26	-0.05	-0.17	0.202
Cereals and bakery products	(3)	0.129 (82.3)	-0.078 (-4.1)	-0.013 (-1.2)	-0.004 (-0.2)	0.017 (1.0)	-0.001 (-0.0)	0.0	0.18	0.42	0.10	0.09	-0.80	0.19	0.134
Miscellaneous foods	(4)	0.336 (72.9)	-0.302 (-4.5)	0.042 (1.1)	-0.016 (-0.3)	0.001 (-0.0)	-0.023 (-0.3)	0.0	0.06	0.15	0.40	0.13	0.11	-0.79	0.355

a/ Coefficients are based on U.S. data for 1948-78.
b/ Elasticities are calculated using equations 10 to 12 at the mean (1948-78) values.
c/ Based on the static first stage expenditure elasticity for food of 0.435.
d/ Values in parentheses are asymptotic t-statistics.

Table 8--The dynamic almost ideal demand system (zero autocorrelation)

Food group i		Estimated coefficients a/								Elasticities b/						Budget share 1948-1978 mean value
										Expenditure		Uncompensated price				
		a_i^*	$d_i(10^{-3})$	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	Total c/	Food	Meats	Fruits & vege- tables	Cereals & bakery products	Miscel- laneous foods	
Meats	(1)	0.181	0.767	0.046	0.132	-0.093	-0.024	-0.015	0.0	0.68	1.15	-0.62	-0.33	-0.10	-0.10	0.309
		(3.7) d/	(2.9)	(0.5)	(3.8)	(-5.1)	(-1.3)	(-0.3)								
Fruits and vegetables	(2)	0.085	1.011	-0.079	-0.093	0.132	0.004	-0.044	0.0	0.36	0.61	-0.34	-0.27	0.07	-0.08	0.202
		(2.0)	(3.0)	(-1.0)	(-5.1)	(2.9)	(0.1)	(-0.6)								
Cereals and bakery products	(3)	0.102	0.349	-0.092	-0.024	0.004	0.042	-0.023	0.0	0.19	0.32	0.04	0.17	-0.60	0.08	0.134
		(2.9)	(0.7)	(-3.6)	(-1.3)	(0.1)	(1.1)	(-0.3)								
Miscellaneous foods	(4)	0.633	-1.420	0.125	-0.015	-0.044	-0.023	0.081	0.0	0.80	1.35	-0.15	-0.19	-0.11	-0.90	0.355
		(7.5)	(-3.9)	(1.0)	(-0.3)	(-0.6)	(-0.3)	(0.8)								

a/ Coefficients are based on U.S. data for 1948-78.
b/ Elasticities are calculated using equations 10 to 12 with the substitution of equation (13) at the mean (1948-78) values.
c/ Based on the dynamic first stage expenditure elasticity for food of 0.591.
d/ Values in parentheses are asymptotic t-statistics.

Table 9--The static almost ideal demand system with autocorrelation a/

Food group i		Estimated coefficients b/: α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\sum_j \gamma_{ij}$	Elasticities c/ — Expenditure: Total d/	Expenditure: Food	Uncompensated price: Meats	Uncompensated price: Fruits & vegetables	Uncompensated price: Cereals & bakery products	Uncompensated price: Miscellaneous foods
Meats	(1)	0.329	0.034	0.110	-0.042	-0.036	-0.031	0.0	0.48	1.11	-0.68	-0.16	-0.13	-0.14
		(13.4) e/	(0.3)	(3.1)	(-1.8)	(-2.2)	(-0.6)							
Fruits and vegetables	(2)	0.219	0.014	-0.042	0.072	-0.022	-0.009	0.0	0.47	1.07	-0.23	-0.66	-0.12	-0.07
		(15.1)	(0.1)	(-1.8)	(2.0)	(-0.8)	(-0.1)							
Cereals and bakery products	(3)	0.124	-0.085	-0.036	-0.022	0.067	-0.009	0.0	0.16	0.37	-0.06	-0.02	-0.42	0.14
		(7.0)	(-1.2)	(-2.2)	(-0.8)	(1.6)	(-0.1)							
Miscellaneous foods	(4)	0.328	0.037	-0.031	-0.009	-0.009	0.049	0.0	0.48	1.10	-0.12	-0.05	-0.04	-0.89
		(11.9)	(0.3)	(-0.6)	(-0.1)	(-0.1)	(0.1)							

a/ The autocorrelation coefficient equals 0.9068 with a t-value of 10.8.
b/ Coefficients are based on U.S. data for 1948-78.
c/ Elasticities are calculated using equations 10 to 12 at the mean (1948-78) values.
d/ Based on static first stage expenditure elasticity for food of 0.435.
e/ Values in parentheses are asymptotic t-statistics.

Table 10--The dynamic almost ideal demand system with autocorrelation a/

Food group i		Estimated coefficients b/								Elasticities c/							Budget share 1948-78 mean value
										Expenditure d/		Uncompensated price					
		a_i^*	$d_i(10^{-3})$	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	$\Sigma_j \gamma_{ij}$	Total	Food	Meats	Fruits & vegetables	Cereals & bakery products	Miscellaneous foods	
Meats	(1)	0.530	-0.126	-0.068	0.097	-0.057	-0.055	0.015	-0.0	0.46	0.78	-0.57	-0.10	-0.14	-0.04	0.309
		(1.3)e/	(-0.7)	(-1.0)	(2.5)	(-1.4)	(-2.1)	(0.8)								
Fruits and vegetables	(2)	0.399	-0.230	-0.068	-0.057	0.556	-0.035	0.037	-0.0	0.40	0.67	-0.11	-0.60	-0.11	0.04	0.202
		(1.4)	(-0.5)	(-1.0)	(-1.4)	(1.7)	(-1.2)	(0.9)								
Cereals and bakery products	(3)	0.186	-0.207	-0.086	-0.055	-0.035	0.045	0.045	0.0	0.21	0.36	-0.08	-0.02	-0.55	0.06	0.134
		(1.2)	(-0.4)	(-1.3)	(-2.1)	(-1.2)	(0.5)	(0.5)								
Miscellaneous foods	(4)	0.116	-1.588	0.221	0.015	0.037	0.045	-0.097	0.0	0.96	1.62	-0.27	-0.13	0.02	-1.01	0.355
		(-1.4)	(-1.1)	(2.4)	(0.8)	(0.9)	(0.5)	(-0.1)								

a/ The autocorrelation coefficient, ρ, equals 0.9845 with a t-value of 40.6.
b/ Coefficients are based on U.S. data for 1948-78.
c/ Elasticities are calculated using equations 10 to 12 with the substitution of equation 13 at the mean (1948-78) values.
d/ Based on the dynamic first stage expenditure elasticity for food of 0.591.
e/ Values in parentheses are asymptotic t-statistics.

restrictions (w) to the maximum value of the likelihood function, no restrictions (n) are present. The likelihood ratio test statistic may be written in terms of the log likelihood function LL as:

$$-2 \ln \lambda = 2(\max_n LL - \max_w LL) = 2(\ln|\Sigma_n| - \ln|\Sigma_w|) \quad (22)$$

can be shown under the null hypothesis to be distributed asymptotically as χ^2 with the number of degrees of freedom equal to the number of restrictions to be tested and where Σ_w is the restricted estimator of the covariance matrix and Σ_n is the unrestricted estimator. 11/ The null hypothesis for no habit effects is H_0: $d_i = 0$. The static AIDS, equation (15), is a restricted form, $d_i = 0$, of the linear habit formation model, equation (16). Table 11 presents the appropriate chi-square test statistics and the critical chi-square values at the 99-percent confidence level. Referring to table 11, the computed value of $-2 \ln \lambda$ is larger than the critical $\chi^2_{.005,3}$ value. Thus, the null hypothesis is rejected and habit formation is present.

Equations (15) and (16) were estimated with the incorporation of equation (17) to account for autocorrelation. The results are shown in tables 9 and 10.

Table 11--Test statistics for alternative restrictions on the almost ideal demand system

Restriction	Degree of freedom	χ^2	$\chi^2_{.005}$
Habits	3	42.63	12.84
Autocorrelation:			
Static	1	54.75	7.88
Dynamic	1	21.83	7.88
Habits and autocorrelation	4	64.46	14.86

As shown in table 11, test for autocorrelation in both the static AIDS and the AIDS with habit formation a likelihood ratio test with one degree of freedom can be used. This is equivalent to an asymptotic t-test. Both of these cases result in rejection of the hypothesis that the autocorrelation parameter is equal to zero. This implies that autocorrelation is

11/ See pp. 396-397 of Theil (1971).

present. Finally, a likelihood ratio test of the most simple model with no habit formation and no autocorrelation versus the most general model with habit formation and with autocorrelation also rejects the hypothesis that habits and autocorrelation are equal to zero. The calculated $-2 \ln \lambda = 64.46$ and the criterion value is $\chi^2_{.005,4} = 14.9$. There are four restrictions since there are three habit parameters (one is deleted for the adding-up condition) and one autocorrelation parameter. This implies that both habit formation and autocorrelation are present.

Total Expenditure Elasticities

Elasticities are generated using equations (7) to (9) for the linear approximate AIDS and equations (10) to (12) for the AIDS. All elasticities are calculated for the mean values of the budget shares and prices over the 1948 to 1978 period. The food expenditure elasticities give the elasticity of demand for the four food types with respect to total food expenditure. In order to obtain a rough estimate of the elasticity of demand with respect to total expenditure, the food expenditure elasticities must be multiplied by an estimate of the elasticity of demand for food with respect to total expenditure. 12/

That is:

$$\eta_{iY} = \frac{\partial q_i Y}{\partial Y\, q_i} = \frac{\partial q_i Y_f}{\partial Y_f q_i} \cdot \frac{\partial Y_f Y}{\partial Y\, Y_f} = \eta_{iY_f} \cdot \eta_{Y_f Y} \quad (23)$$

where Y_f is total food expenditure and Y is total expenditure, η_{iY_f} is the food expenditure elasticity, η_{iY} is the total expenditure elasticity, and $\eta_{Y_f Y}$ is the elasticity of food expenditure with respect to total expenditure. A positive value of $\eta_{Y_f Y}$ results in η_{iY} having the same sign as η_{iY_f}.

Therefore, if a good is not inferior with respect to the total food expenditure elasticity, it will not be inferior with respect to total expenditure elasticity. Blanciforti (1982) estimates the average elasticity of food expenditure with respect to total expenditures for 1948 to 1978 to be 0.435 and 0.591, respectively, based on the static and dynamic linear expenditure systems in budget share form. Applying this to the conditional elasticities implies that the four food types are necessities with respect to total expenditure (they range from a low value of 0.16 to a high value of 0.96). That is, the demand for these food items is expenditure inelastic.

CONCLUSION

In this paper, alternative versions of the AIDS, a theoretically based demand system, have been investigated with an application to four food commodity groups for the United States for the period 1948 to 1978. Expenditure and price elasticities were calculated using the parameter estimates of these systems.

12/ See p. 887 of Manser (1976).

F-values and likelihood ratio tests were used to test the various hypotheses. F-values for the linear approximate AIDS indicated that the incorporation of habit formation in the static model and an analysis of autocorrelation would improve the estimates. Habits were embodied by the introduction of a systematic change in one of the parameters. This embodiment enabled the utility maximization framework to remain intact, satisfied the theoretical properties pertaining to demand systems, and provided a way to ascertain by what amount last period's consumption affected current consumption. The incorporation of habits in the linear approximate AIDS resulted in reduction in the serial correlation of the residuals. Likelihood ratio tests on the AIDS estimated by FIML indicated that habit formation and autocorrelation were present for the food items. The theoretical structure of the AIDS has proven to be both flexible and general in terms of application to this analysis of food demand.

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