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Multiple Crop Supply and Factor Demand Component of the World Grains, Oilseeds, and Livestock Model

by

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ABSTRACT

This paper reviews the crop supply component of the world grains, oilseeds, and livestock (GOL) model and attempts to develop an improved conceptual framework for specifying the multiple crop supply and input demand relationships in the GOL model. As a basis for examining and revising the crop supply component, the theoretical foundations for a multiple product production system and empirical studies related to agricultural commodity supply response were reviewed. The revised specification of the crop supply equations consists of a nonlinear equation system of area, yield and production. The major emphasis on the revision of the crop supply component is to ensure consistent acreage allocation among crop alternatives, to more realistically capture cross-price effects or substitution possibilities between alternative crops and to include policy variables to reflect the effects of government farm policies on crop supply response.

Key Words: multiple crop supply, GOL model, area, yield, production.

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MULTIPLE CROP SUPPLY AND FACTOR DEMAND COMPONENT OF THE WORLD GRAINS, OILSEEDS, AND LIVESTOCK MODEL

by Karen Liu

I. Introduction

The purpose of this paper is to review the crop supply component of the world Grains, Oilseeds, and Livestock (GOL) model and to develop an improved conceptual framework for specifying the multiple crop supply and input demand relationship in the GOL model.

The GOL model is the principal analytical tool for global long-run projections and intermediate-run policy analysis of the International Economics Division (IED), ERS, USDA. It is a multiple-commodity and multiple-region (12-commodity and 28-region) agricultural trade model of the world grain-oilseed-livestock economy. Model equations were developed to reflect: (1) important technical input-output relationship, (2) the economic behavioral pattern of the world grain-oilseed-livestock complex, and (3) the institutional environment (50). 1/ The model consists of about 1000 simultaneous linear equations. The coefficients used within the GOL model are synthesized from existing studies and analyses. The base period for the model is 1969-1971.

The GOL model was built in 1973-74 to produce analysis for the U.S. contribution to the 1974 World Food Conference held in Rome. The model was intended to generate projections of the world food supply-demand balance under alternative scenarios. Recently, the model has also been used as a policy analysis tool. 2/ The model may be useful for making long-range

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1/ Underlined numbers in parentheses refer to items in Reference at the end of this paper.

2/ The GOL model has been used to examine a broad range of issues for various agencies. Examples of uses are the Global 2000 study, the FAO AT-2000 projections, analysis of the potential impact of alternative assistance programs for the U.S. Agency for International Development, etc.

projections. However, the structure of the original GOL model has severe limitations for use in policy analysis. In order to improve the usefulness of the GOL model as an analytical tool for global long-run projections and intermediate-run policy analysis, the Trade Policy Branch, IED, has reviewed the model, critiqued its shortcomings and defined a set of revision and updating activities. 3/

Four major problems concerning the GOL model are discussed here. (1) The present GOL model is a system of linear equations, for which there is no guarantee against negative values in the model solutions. The linear function implies the constant slope coefficients and can only be expected to perform well in the neighborhood of the base period observation point. Because of the constant slope coefficients in the model, elasticities may change through time. The theoretical restrictions such as the homogeneity and the adding-up conditions imposed on parameters of demand and supply equations may not be able to attain over time. (2) The present GOL model lacks policy distortions. There are no nontariff barriers to agricultural trade in the model, except for a few instances in which the volume of imports is exogenously fixed. Also, no domestic policy variables are included in the supply equations to reflect the effects of government farm policies on commodity supply response. Excluding the policy distortions from the model severely limits its use for policy analysis. (3) Because of data limitations for many GOL countries/regions, all supply and demand relations in the model are synthesized. This means that the model was constructed by starting with the observed price-quantity data in all markets in all countries/regions and assuming that all markets were in equilibrium. Then, based on available

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3/ Thompson has given a critical review of the GOL model and recommendations for restructuring it for purposes of medium-term policy analysis (61).

empirical evidence, expert opinion and judgment of the researchers, long-run own and cross-price elasticities of supply and demand were selected for each commodity in each region, and hyperplanes were passed through points assumed to have the selected elasticities at the base year observation. By this way the vertical intercept of each supply and demand relations was calculated (61). (4) The model is getting out of date. The base period for the model is 1969-71 before the convulsions that occurred in the world grain markets in the early 1970's. Elasticity estimates in the model are outdated too.

These problems represent the major concerns with the present GOL model and provide justifications for revising and updating the model. A set of revision and updating activities has been defined. These activities include: rebasing the model to 1975-77 from 1969-71, introducing non-linear equations into the model and reviewing the demand and supply components of the model. As a part of the revision and updating activities, this paper examines the crop supply component of the GOL model, reviews the theoretical restrictions and empirical studies related to multiple output production system and develops an improved specification for the multiple output supply relations in the GOL agricultural trade model.

The following paper will be organized into seven parts. Section II briefly describes the information set addressed in the original GOL model. Section III discusses problems in modeling a multiple output supply system. Section IV reviews the theoretical foundations for a multiple-output production system. Section V contains a cursory review of literature related to supply studies (both at the aggregate level and partial equilibrium single commodity level). Sections V and VI describe the shortcomings in the current GOL crop supply equations and planned modifications in the crop supply system. And the last Section (VIII) discusses several longer-term considerations.

II. Information Set Addressed in the GOL Model

The GOL agricultural trade model consists of supply, demand and trade sectors for 12 commodities of grains, oilseeds and livestock products. The 12 commodities are wheat, rice, coarse grains, oilseeds, oilmeal, beef and veal, pork, poultry, mutton, milk, butter and cheese, where each commodity is assumed to be homogenous. Within the GOL model, 28 regions are represented - 8 regions of developed countries, 3 regions of centrally planned countries, and 17 regions of developing countries (50). The grouping of countries in the model is based on similar geographical, political and economic conditions within each region. Through its comprehensive commodity and regional coverage, the model attempts to capture cross-commodity effects within each region as well as the relationship between the grain-oriented economies of the developing countries and the livestock-oriented economies of the developed countries.

The quantified description of the world's grain-oilseed-livestock economy are expressed in mathematical equation form by commodity, by region, and according to economic function. The model consists of about 1000 simultaneous linear equations in the same number of endogenous variables. The endogenous part of the model specifies interaction of production, consumption, trade and prices of grains, oilseeds and livestock products. The exogenous variables included in the model are the usual demand shifters such as population and income growth rates, and the time trend variable as a proxy for changes in consumer preference; and the supply shifters such as technological trend and cost of physical inputs. The model is solved simultaneously for given levels of exogenous variables. The mathematical relationships of the model can be grouped into nine major components: (1) demand block for livestock, (2) supply block for livestock, (3) demand block for feed, (4) demand block for food, (5) supply block for crops, (6) linkages within regions, (7) regional balance equations, (8) price equations linking regions and (9) world equilibrium equations for each commodity.

Within each regional model, the interrelationships between multiple product output supply and demand are represented by direct and cross price elasticities of supply and demand. The derived demand for feed includes physical input-output coefficients relating it to the supply of livestock products as well as direct and cross price elasticities of feed demand. The elasticity matrix for the model is obtained from existing studies and judgement of experts. Because of the size of the GOL model, the coefficients of the model equations are mainly synthesized from the selected elasticities and base period data rather than statistically estimated. The procedure of deriving the synthesized coefficients is such that, assuming that the observed base period price-quantity data in all markets in all countries/regions are in equilibrium, the coefficients for the model equations are then synthesized from the direct and cross elasticities of supply and demand by multiplying the elasticities by the ratio of the base period quantity to the base period price of each commodity. The value of the constants is then determined by the base period data and the computed coefficients.

The 1969-71 data of production, consumption and trade quantities, and demand, supply and trade prices are the data base for the GOL model. Main data sources are from the foreign agricultural statistics.4/ These data are aggregate measures at the national or regional level under the assumption that individual producers and consumers can be aggregated into homogenous groups. These data are used to derive the coefficients for the model equations. The data base for the model has not been updated since its completion in 1974. It is widely felt that the model needs to be rebased, and 1975-77 is the most recent 3-year for which complete data are available. Also, this period could be considered more or less normal in the world grains, oilseeds and livestock markets, with minor exceptions (61). So, we have decided to rebase the model to 1975-77 from 1969-71.

4/ These data are the computerized data base obtained from the Foreign Agricultural Service, USDA. The Foregin Agricultural Service compiled these data from various sources such as the U.N. Food and Agriculture Organization, Organization for Economic Cooperation and Development, individual countries, etc.

III. Issues Concerning Modeling a Multiple Output Supply System

The major methodological problem within the crop supply and factor demand part of the GOL model is the simulation of a multiple output production system. Several problems can be identified.

First, because the reason for simultaneously treating the multiple commodities in an agricultural trade model is to capture the interrelationships and feedback effects among commodities, it is essential to have adequate estimates of all relevant cross price effects in the model. This is particularly important in the grains, oilseeds and livestock sectors on which the GOL model focuses, where important interactions and substitution possibilities exist on both the supply and demand sides of the market. As Thompson points out, omission of relevant variables can lead to biased estimates of the own price term, 5/ and if significant cross-price effects among commodities exist, simulation of the effects of policy changes with the model can lead to erroneous conclusions (60). This further emphasizes the importance of including the relevant simultaneities among subsectors in the supply and demand components of the multiple-commodity model.

Second, most existing multiple commodity agricultural trade models lack detail on the agricultural factor markets. Because factor market adjustments have important effects on the cost structure and thus the supply schedules in each country, knowledge of factor market adjustments are important as we move toward general equilibrium models (60). They are also critical to policy analysis because decisions on trade policy questions as well as on domestic agricultural policy issues are often made via factor market adjustments.

5/ This is primarily the problems of model specification and parameter estimation. But the result of the estimation (bias) has implications for model simulation.

The third problem is the impact of technological change on the shape of the product transformation surface. The shape of the production possibility surface changes with technological change, and this in turn changes the matrix of own and cross-price elasticities of supply. Moreover, in a study of supply response in Australian agriculture, Vincent, Dixon and Powell (64) point out that the estimation of product transformation relationships in terms of relative prices may fail to capture the rate of adoption or adjustment path of technological change towards new production possibility frontiers. In order to account for the impact of technological change on the shape of the product transformation surface, the incorporation of a suitably shaped trend curve (for example, the logistic curve) into the product supply functions can allow for this type of adjustment process.

The fourth issue concerns resource capacities and the allocation of productive resources among crop alternatives. Consistency of resource demand and availability should be ensured in a multiple product production system. This is particularly important for long-run projections where major substitutions among commodities and substantial changes of the resource capacities can take place. Consistency as defined here assumes that projected outputs of the various commodities can be realized with available resources.

Given the above concerns, specification of a comprehensive multiple crop supply and factor demand component of a multiple-commodity agricultural trade model needs to carefully consider interactions and substitutions among commodities, linkage of product and factor markets, effects of technological change on the shape of production transformation surface, and in turn price responsiveness and consistency of resource availability. The objective of this paper is thus to develop an improved conceptual framework for specifying the multiple crop supply and input demand relationships in the GOL agricultural trade model. The following section will review the theoretical consideration for a multiple output production system.

IV. Theoretical Foundations

(1) Static Theory for a Multiple Output Production System:

(A) Duality Theory:

Assuming the multiple output production sector confronts downward sloping output demand schedules and upward sloping factor supply schedules, and its sole goal is profit maximization, the general multiple product, multiple input production decision model for a competitive industry is characterized by Diewert (12) as:

[1] $$\text{Max.} \quad \pi = P'Y - W'X$$
$$\text{s.t.} \quad (Y;X) \varepsilon T$$

where

π = profit function,
Y = vector of outputs, $Y_1, Y_2, \ldots, Y_m$,
X = vector of n nonnegative inputs, $X_1, X_2, \ldots, X_n$,
T = the set of feasible inputs and outputs (production possibilities set),
P = vector of prices, $P_1, P_2, \ldots, P_m$, and
W = vector of input prices, $W_1, W_2, \ldots, W_n$.

Using the concept of transformation, equation [1] describes the set of efficient input-output combinations. Conditions on the set T are defined as: (1) T must be a closed, nonempty, and convex set, (2) T has characteristics of diminishing marginal rates of transformation of output for inputs (i.e., decreasing returns to scale), increasing marginal rates of transformation of outputs for outputs, and diminishing marginal rates of substitution of inputs for inputs, (3) boundedness from above, 6/ and (4) free disposal.

6/ This condition implies that the set of efficient input-output combinations is constrained by resource availability (the adding-up condition).

Under these conditions, Diewert has shown an equivalence between the set of production possibilities, a transformation function and a profit function.

The set of efficient input-output combinations, T, can be described symmetrically as the set of (Y;X) which satisfy the equation $t^*(Y;X)=0$, where t^* is the transformation function for one output, for example, Y_1. This means that one output Y_1 can be singled out, and the efficient set can be described by $Y_1 = t(Y_2, Y_3, \ldots, Y_m, X)$, where the transformation function t tells us what the maximum production of Y_1 is, given the vector of inputs X, and the vector of other outputs $Y^0 = (Y_2, Y_3, \ldots, Y_m)$. 7/ Thus, the outcome of equation [1] is equivalent to revenue maximization subject to the transformation function for Y_1, i.e.

[2] $$\text{Max. } \pi = P'Y - W'X$$
$$\text{s.t. } Y_1 = t(Y_2, \ldots, Y_m; X)$$

The production possibilities set, T, which corresponds to the transformation function t for output Y_1, is $T = [(Y_1, Y^0; X): Y_1 \leq t(Y^0;X),\ Y_1 \geq 0,\ Y^0 \geq 0,\ X \geq 0]$. This means that given the same bundle of inputs X to produce multiple outputs Y_1 and $Y^0=(Y_2, Y_3, \ldots, Y_m)$, the corresponding production possibility frontier for the sector is T, the efficient input-output combinations of Y_1, Y^0 and X.

Another equivalent parameterization of the industry's technology can also be obtained by means of the profit function. Given a vector of output prices P, and a vector of inputs prices W, with all prices positive, and a production possibility set T, then the profit function is defined by $\pi(P;W) = P'Y - W'X$. For the given vector of prices (P,W), the producer is assumed to choose a feasible production plan $(Y;X) \varepsilon T$ which maximizes

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7/ The vector of other outputs $Y^0=(Y_2, Y_3, \ldots, Y_m)$ are defined as a finite, nonnegative vector, $Y^0 \geq 0$.

profit. From this, the profit function can be used to generate a production possibility set, $T = [(Y;X):P'Y - W'X \leq \pi(P;W)$ for every $(P,W) > (0,0)$ and $(Y;X) \geq (0,0)]$. This relationship establishes a duality between transformation function and profit function. The input demand and output supply functions are derivable from the concept of duality.

If the profit function is differentiable, the input demand and output supply functions can be generated from the profit function by straight forward differentiation. 8/ The derivation of output supply and input demand functions can be shown as follows. If a profit function $\pi(P;W)$ is differentiable with respect to output and input prices at the point $(P^*,W^*) > 0$, then we can obtain $\frac{\partial\pi(P^*;W^*)}{\partial P_i} = Y_i(P^*,W^*)$, for $i = 1, 2, \ldots, m$, and $\frac{\partial\pi(P^*;W^*)}{\partial W_j} = X_j(P^*;W^*)$, for $j = 1, 2, \ldots, n$. This leads to an explicit formulation of the input demand and output supply functions which can be estimated by econometric techniques. Furthermore, it can be shown that the supply function for output (Y) and demand function for input (X) both are homogeneous of degree zero in all prices (33, 48 and 52).

The above discussion is applicable to "long-run" profit maximization when all output and factor prices are exogenous to the sector. Duality theory is also applicable to the situation when additional exogenous variables or additional information is incorporated in the analysis. That is, the dual relationships between quantities and prices are unaffected by the existence of additional variables which affect the production technology. Because of the flexibility of duality theory, exogenous variables such as an index of technological change, policy variables, and fixed inputs can be explicitly incorporated into the dual structure. The incorporation of an index of technological changes is essential to the construction of an operational model of industry demand and supply. Incorporating fixed inputs in the model is primarily applicable to "short-run" profit maximization or cost minimization.

8/ This result is known as Hotelling's Lemma (24).

Letting tc represent the "state of technology", and letting Z represent a K-dimensional vector of fixed inputs, and R represent the vector of prices of fixed inputs, $R = (R_1, R_2, \ldots, R_k)$, the production possibility set can be written as:

$T = [(Y;X,Z,tc):P'Y - W'X - R'Z \leq \pi_1(P;W,R)$ for every $(P,W,R) > 0$ and $(Y,X,Z) \geq 0]$, where $\pi_1(P;W,R) = \text{Max. } \pi(P,W,R)$ is defined as a "variable" profit function by Samuelson (54) and Gorman (18). Hotelling's Lemma (24) is equally applicable to the profit function to generate the profit maximizing derived input demand and output supply functions.

(B) Primal-Dual Envelope Scheme:

Another methodology for deriving the comparative static input demand and output supply functions is to apply the primal-dual envelope scheme of Samuelson (53) and Silberberg (57). As indicated by Silberberg, all comparative static and symmetry conditions can be derived from the positive semidefiniteness restrictions on the primal-dual Lagrangean function.

Consider a profit maximizing firm with multiple product $q_i = \Psi i(X_1, X_2, \ldots, X_n)$ which purchases its inputs X_j, $j=1,\ldots,n$ at constant unit factor prices W_j, and which sells its products q_i, $i=1,\ldots m$ at constant output prices P_i, respectively. Hence, the firm maximizes its (direct) profit function $\pi = Pq - WX$. Under these assumptions, a primal-dual problem can be formulated as follows:

[3] Find the saddle point of $L^* = \pi^* - \pi$

where

π = Pq-WX (direct profit function), q=F(X),
π^* = indirect profit function in which optimal input quanitities are solved and inserted in the profit function,
L^* = Lagrangean function.

The primal-dual scheme indicates the following marginal conditions which describe the optimum $X=X^*$, $q=q^*$:

[4] $\frac{\partial L^*}{\partial X_j} = -\pi_{xj} = 0$, where $\pi_{xj} = \frac{\partial \pi}{\partial X_j}$

[5] $\frac{\partial L^*}{\partial P_i} = \pi^*_{Pi} - \pi_{Pi} = 0$, where $\pi P_i = \frac{\partial \pi}{\partial P_i} = q_i^*$ for all i

so [5'] $\pi P_i^* = q_i^*$ for all i (the output supply functions)

[6] $\frac{\partial L^*}{\partial W_j} = \pi W_j^* - \pi_{Wj} = 0$, in which $\pi_{Wj} = \frac{\partial \pi}{\partial W_j} = -X_j^*$

and [6'] $\pi W_j^* = -X_j^*$ for all j (the input demand functions)

Equations [5'] and [6'] are the output supply and input demand functions respectively, identically derivable from the concepts of duality and transformation functions discussed in previous pages. Silberberg has further shown that the matrix of cross-partials ($L^*_{\alpha\beta}$) of the Lagrangean function L* must be positive semidefinite (concavity condition) (57). From the positive semidefiniteness of $L^*_{\alpha\beta}$ the following comparative static and symmetric conditions can be derived: (a) The positive semidefiniteness of $L^*_{\alpha\beta}$ implies that the diagonal elements of $L^*_{\alpha\beta}$ are nonnegative, hence

$$\frac{\partial q_i^*}{\partial P_i} \geq 0 \text{ for all i, and } \frac{\partial X_j^*}{\partial W_j} \leq 0 \text{ for all j}$$

i.e., the supply curve are not downward sloping and the factor demand curves are not upward sloping.

(b) By the symmetry of $L^*_{\alpha\beta}$, the following conditions will also hold:

$$\frac{\partial X_j^*}{\partial P_i} = \frac{\partial q_i^*}{\partial W_j}, \text{ and } \frac{\partial X_k^*}{\partial W_j} = \frac{\partial X_j^*}{\partial W_k} \text{ for all j, k}$$

$$\frac{\partial q_i^*}{\partial P_h} = \frac{\partial q_h^*}{\partial P_i} \text{ for all i, h}$$

(c) The output supply and input demand functions are homogenous of degree zero (46). Homogeneity implies:

$$\sum_i P_i \frac{\partial q_i^*}{\partial P_i} + \sum_j W_j \frac{\partial q_i^*}{\partial W_j} = 0 \qquad \text{for the supply function}$$

or

$$\sum_i P_i \frac{\partial X_j^*}{\partial P_i} + \sum_j W_j \frac{\partial X_j^*}{\partial W_j} = 0 \qquad \text{for the demand function}$$

(2) Dynamics of Production:

The static structure outlined in the foregoing pages implicitly assumes that the response of decision makers to changes in prices is instantaneous, i.e. changes in prices may change the choice of output and input mix simultaneously and without any time lags or dynamics. However, in the agricultural production process, we must recognize that the biological lag in supply response and uncertainty in prices and weather conditions have important impacts on factor allocation and resource mix. Lags in adoption and diffusion of technology also have impacts on factor mix and factor demand. 9/ Assuming the farmers' sole goal is profit maximization, based on expected prices (often represented by lagged prices) and other determining factors (such as government farm programs), the farmers' production decision is to choose total fertilizer and machinery purchases simultaneously with enterprise (crop) combination. Since prices received and yields (because of weather uncertainty) cannot be foreseen perfectly at the time the factor allocation decisions are made, expected values for both of these variables must be projected or assumed. The comparative static theory reviewed above assumes price certainty but is equally applicable to the case of expected profit maximization in deriving the "optimal" output supply and input demand (57).

(3) Government Policies and Other Distortions:

There are two aspects of government policies: (1) government policies may impose constraints, e.g., on land area, and (2) government policies may affect or change the supply inducing price (i.e., it might become the government's announced minimum price instead of expected market price).

9/ The length of adoption lags varies with different technologies. In agriculture, the length of adoption lags has ranged from 3 years for DDT to 53 years for cotton pickers (37). In studying the diffusion of hybrid corn, Griliches indicates that Alabama took 8 years to increase its adoption rate from 20 to 80 percent (followed an S-shaped growth curve) (20).

The effects of government policies on multiple-product supply response may be many fold. For example, government acreage control policies place restrictions on land use, and thus lead to changes in the mix of outputs or inputs. The implications may be illustrated by a concept based on a generalization of Hick's measure of the bias in technological change (65). Just as technological change may shift the production possibility surface in many different ways, changes in the level of any restricted or fixed input will shift the production possibility surface which traces optimal combinations of variable inputs and outputs. In either case, changes in technology or effects of government policies may shift production possibility curves and lead to changes in the choice of outputs and input mix. Following Hick's concept (65), one can measure the impact of policy-imposed input restrictions in terms of the biases introduced in resource allocation and relative changes in the product mix.

The effects of policy distortion on factor markets can be illustrated in three cases as discussed by Floyd (15). Assuming that the sector confronts downward sloping product demand and upward sloping factor supply schedules, and its goal is profit maximization, under a government acreage control policy, there is no limit on the level of output producers can produce. Rather, there is a limitation on the amount of production they will find profitable. Since the amount of land that can be used is limited, any given quantity of output must be produced with more non-land inputs. The producer is forced to produce each level of output with a non-optimum combination of resources, 10/ consequently at a higher cost than would be the case in the absence of the acreage control program.

10/ Non-optimum means when comparing the resource combination under the case of policy distortions to a solution based on the free market case.

Under a policy of restricting sales, output can be reduced by any method producers choose. Assuming a profit maximizing goal, producers will attempt to produce the new output at minimum cost. The demands for the resources used in agriculture will decline. Those inputs with the most elastic supplies will experience the relatively greatest decline in quantity; those with the least elastic supplies will have the relatively greatest price declines. Under the price support program without output controls, gross farm income will rise and the demand for all factors will increase accordingly. One would expect that the prices of the factors with the least elastic supplies would rise most. For the case of labor and land, since labor is more elastic in supply than land in most countries, an increase in product prices will result in increasing the quantity of labor more than the quantity of land, but land values would rise proportionately more than farm wage rates. Floyd also argues that with respect to the relationship between capital, labor and land, for any given supply elasticity of labor, the larger the elasticity of supply of capital, the greater the effect of a change in product prices on the price of labor, and by a similar relation, on the price of land as well.

Therefore, in modeling a multiple crop supply system, it is important to include multiple factor demand and multiple product supply functions. When the relevant price elasticities are included in the model, then the cross product and cross factor effects by exogenous factors such as policy distortions can be determined.

(4) Summary:

From the above discussion of the theoretical consideration for deriving the multiple factor demand and output supply function and the complexities of the agricultural production (i.e., dynamics, policy distortions and technological change), in specifying a multiple product supply system, we need to carefully consider interactions and substitutions among commodities and factors, effects of lagged prices and government policy distortions. Under the profit maximizing assumption, an implicit function for the production technology can be specified as g(X) = Z = f(Q), where Z = an index of production capacity, X = vector of inputs, $X_1,\ldots,X_j,\ldots X_n$, Q = vector of outputs, $Q_1,\ldots,Q_i,\ldots Qm$. Given output prices P_i, i = 1,...m and input prices W_j, j=1,...n, by duality theory or the primal-dual envelope scheme, the output supply and input demand functions can be explicitly specified as $Q_i = f(P_1,\ldots P_m,W_1,\ldots W_n)$ and $X_j = g(P_1,\ldots P_m,W_1,\ldots W_n)$ respectively.

The output supply and input demand functions are homogenous of degree zero in all prices. The homogeneity of output supply function, $Q_i = f(P_1,\ldots P_i,\ldots P_m,W_i,\ldots W_n)$ implies that:

$$\frac{\partial Q_i}{\partial P_1} P_1 + \ldots + \frac{\partial Q_i}{\partial P_i} P_i + \ldots \frac{\partial Q_i}{\partial P_m} P_m + \frac{\partial Q_i}{\partial W_1} W_1 + \ldots + \frac{\partial Q_i}{\partial W_n} W_n = 0 \quad i=1,\ldots,m.$$

these results may be alternatively expressed in elasticity form as:

$$\frac{\partial Q_i}{\partial P_1} \frac{P_1}{Q_i} + \ldots + \frac{\partial Q_i}{\partial P_i} \frac{P_i}{Q_i} + \ldots + \frac{\partial Q_i}{\partial P_m} \frac{P_m}{Q_i} + \sum_j \frac{\partial Q_i}{\partial W_j} \frac{W_i}{Q_i} = 0$$

The sum of all of the price elasticities equals to zero. This is a classical result of production theory: in the short-run, homogeneity implies that the supply elasticity with respect to product price is determined by the cross price elasticities with respect to other outputs and the production elasticities with respect to input prices. The homogeneity of input demand function can also be shown in a similar way. Furthermore, the lagged prices and exogenous variables such as technological trend, government policy variables can be explicitly incorporated into the equation specification and the theoretical restrictions can equally be applied.

V. Review of Empirical Studies

The literature related to modeling multiple output production systems can be divided into three main categories: (1) aggregate multiple-product supply system, (2) sectoral programming models and (3) models of single commodity (partial) supply response and marketed surplus of a subsistence crop. A few relevant empirical studies are reviewed in this section.

(1) Aggregate Multiple-Product Supply System:

In empirical analysis of multiple-product production functions, the standard neoclassical models such as the agricultural sector models by Floyd (15), Gisser (17), Rosine and Helmberger (51) are in the class of aggregate production function approaches (i.e., all outputs and inputs are aggregated together). The model structure in these studies mainly consists of system of equations for production, output demand, input supply and input demand. These studies were used to evaluate the major exogenous impacts (such as technological change, exogenous price changes and farm programs) on the farm sector. Farm output are aggregated into a single homogeneous product.

Mundlak has pointed out several problems with aggregation in the study of multiple-product production functions (40). He investigates the problem of aggregating outputs of multiple products by using product price weights to obtain an aggregate or composite output. The limitation of using product price weights to obtain an aggregate measure is that when relative product prices change, composite output is not a single valued function of a given bundle of inputs. Thus, it is possible to have a situation where an increase in total quantity of inputs (when accompanied by an appropriate change in relative product prices) results in an increase, no change, or a decrease in composite output. The magnitude of the error introduced depends upon the shape of the production possibilities curve as well as on the extent of the change in relative product prices.

In order to avoid the problem of product aggregation, Mundlak and Razin develop procedures for deriving multi-stage multiple product production functions (41). The production process is formulated as F(a) = G(v), where a and v are vectors of products and factors, respectively. F(a) is further specified as a multi-stage constant elasticity of substitution (CES) function, and estimated by using the profit maximizing first-order conditions and duality relations. The restrictions imposed by the model include the selection of an appropriate functional form for the product transformation curve and separability conditions, which state that the marginal rate of transformation of a pair of products within a given group is independent of other products within or outside that particular group. The model was used in an attempt to construct the product component of multi-product production function for Israeli agriculture for the period 1954-1968. However, the results did not show high substitution within some groups.

By deriving the constant elasticity of transformation (CET) production possibility sets for a two-product case, Powell and Gruen have shown that the CET model is useful in the analysis of multiple product supply systems (49). A linear supply system for wheat, coarse grains and wool was estimated by generalized least squares on aggregate Australian data. They estimated a complete set of cross price elasticities of supply for three agricultural products.

Vincent, Dixon and Powell (64) have further developed a theory of input demand and product supply relationships which constitutes an improvement over the CET specification adopted by Powell and Gruen (49). They have formulated the production system as $Z = f(X_1,\ldots.,X_m)$ and $g(Y_1,\ldots.,Y_n) = Z$, where Z is a scalar index defining generalized capacity to produce, X_j are the total levels of factors which are inputs into the multi-product production activity, and Y_i are the bundle of multiple products. The capacity index Z serves to locate the product transformation schedule. The functional form for f and g were postulated as the constant ratio of elasticities of substitution, homothetic (CRESH) and constant ratio of elasticities of transformation, homothetic (CRETH). The CRESH/CRETH behavioral equations were derived by maximizing total farm gross margins subject to product transformation and input substitution constraints. In the empirical application, they have estimated the product supply system (CRETH) for the three regions comprising the cereals-livestock complex of Australian agriculture for the period 1952/53 to 1973/74. Due to the lack of sufficient independent variation in inputs and input prices over the data sample, the input substitution relationships (CRESH) were not estimated.

Clements has developed an aggregate multi-product supply system analogue to consumer demand (10). In his model, the firm chooses its production mix and hires factor inputs to maximize profit subject to a production technology represented by a multiple-output production function. The output supply and input demand equations are parameterized in terms of proportional changes in resource shares and thus resemble equations in the Rotterdam model of consumer demand (59). An aggregate multi-product supply model for three output groups—exportables, importables, and nontraded goods, is empirically estimated. All the parameters of the model are estimated simultaneously by full information maximum likelihood method, and the validity of the restrictions (such as homogeneity, symmetry and adding-up conditions) implied by the theoretical model are tested by likelihood ratio tests.

Frohberg, et. al. have developed a multiple product nonlinear production model of the European Community as part of the Food and Agriculture program at the International Institute for Applied System Analysis (16). A unique feature of their method is that because there are no data on allocation of inputs among outputs, they simultaneously optimize the allocation of input and estimate the associated multiple-product production function. Under the profit maximization assumption, the model can be described as: Max. PY, subject to $V^i = \bar{V}$, where P is the vector of expected product price, Y is the output vector, $Y = f(V^1, \ldots, V^n)$, V^i is the vector of inputs used in producing product i and $\bar{V}$ is the vector of total available inputs. Assuming the constraints of the model form a convex set and that the model has a unique solution, then the supply function is derived as a function of the parameters P,V and α, i.e., $Y = Y(P,V,\alpha)$, where α is the vector of coefficients of the function $f(V; \ldots, V^n)$. A nonlinear optimization routine is used in estimating the model.

The empirical studies of multiple product production system reviewed above generally assume that a given production possibility frontier is defined for the sector's underlying production technology. The selection of an appropriate functional form for the production possibility frontier is important to the estimation of theoretically plausible multi-product supply system (41 and 49). Among the class of production possibility frontiers, the transcendental production possibility frontier, or more simply, the translog production frontier--formulated by Christensen, Jorgenson and Lau (9)--is the most desirable on grounds of flexibility (43). 11/

The translog function can provide a second-order approximation to a general production function. Kmenta has considered it, in particular, as an approximation to the CES functional form (28). Properties of the translog production function are discussed further in Christensen, Jorgenson and Lau (9). Based on the translog production frontier, they have empirically estimated

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11/ The class of flexible functional forms is defined by the property that these forms can provide a second order approximation to an arbitrary differentiable function.

an econometric model of production using time series data for the U.S. private domestic economy for the period 1929-1969. In a study to test the specification of aggregate production functions within the context of Taiwan's developing agriculture, Shih, Hushak, and Rask have compared the properties of the Cobb-Douglas and translog production functions (55). They have shown that through the use of the translog function, many restrictive properties of the Cobb-Douglas function can be relaxed without causing computational difficulties and that the translog function performs better on economic and statistical grounds than the Cobb-Douglas function. The translog function has also been used to measure technical change bias with many factors of production by Binswanger (5) and to test the existence of a consistent aggregate index of labor inputs by Berndt and Christensen (4).

(2) Sectoral Programming Models:

For economy-wide sectoral modeling (within the context of mathematical programming), a linear programming formulation, using a production possibility set represented by a set of production activities and product demand and factor supply schedules has recently been used widely, especially by the World Bank (Mexico, Portugal, Ivory Coast, etc.). A continuous production function can be represented as closely as desired through the use of linear segments. Assuming producers' sole goal is profit maximization and the production sector confronts downward-sloping output demand and upward-sloping input supply functions, linear programming models can be used to derive multiple output supply response, input demand, factor substitution and product transformation frontiers (38). Within a sectoral programming model, the allocation of resources among a set of production activities (such as land allocation among crop alternatives) is done in a profit maximization manner. The production possibilities set is represented by the set of production activities and the sector is constrained to stay on (move around) the production possibility surface for a given technology and available resources.

In the empirical applications, the CHAC model of Mexican crop production is a linear programming type of agricultural sector model (13). Lattimore and Thompson have also applied the sectoral programming approach to specify a multi-sectoral general equilibrium trade model for Canada with emphasis on the agricultural sector and agricultural trade (32). The model is a linear programming type with an input-output matrix in the tableau to provide the transactions linkages between sectors. The model structure explicitly includes price responsive commodity demand functions and factor supply functions so that all commodity prices and most factor prices are determined endogenously along with the corresponding market clearing quantities in the product and factor markets. The model was used to estimate the domestic effects of changes in trade policy.

The Center for Agricultural and Rural Development at Iowa State University under the direction of Earl O. Heady has developed a series of linear programming models for analysis of national production possibilities and policies (39). These models included land, water, soil loss, demand, crop, livestock and transportation sectors. These models were capable of simulating crop production, input demand, consumer demand and international trade activities under alternative scenarios.

The Takayama-Judge type spatial-temporal equilibrium quadratic programming approach has also been applied to agricultural sector models (58). Pandey and Takayama have developed a temporal equilibrium model of India's food grains economy (44). Using quradratic programming approach, they have studied the alternative price and output allocation pattern of rice and wheat under alternative policy assumptions. Pant and Takayama have attempted to explore the potential usefulness of several agricultural planning models in planning India's agricultural sector (45). They have applied Leontief input-output, linear programming and Takayama-Judge type spatial equilibrium models to study the structural relationships in India's agricultural sector.

McCarl and Spreen have discussed wide ranging applications of the mathematical programming approach for modeling supply response (38). A principal feature of the mathematical programming sector model structure is its ability to capture changes in the economic environment. Modification may be incorporated through specification of new activities, new constraints (such as restrictions on resource availability), modification of factor supply product demand schedules (imposing price ceilings, increased taxes or introducing imports). The basic validation tests of the mathematical programming models which have been used involved validating the model solution to correspond to a known solution or validating the activity levels and the shadow prices. As to the test of accuracy of prediction, Shumway and Chang (54) conclude that the linear programming technique is a feasible method for estimation of the direct and cross elasticities of supply with approximately the same degree of accuracy as a single equation econometric models. However, there are some drawbacks with mathematical programming sector models. The aggregation problems introduced by aggregating individual producers into homogenous groups and regional grouping in spatial specification are concerns of researchers (38). The development of a detailed sector programming model can be costly. The data requirements are extensive and time and resource requirements can be overwhelming.

(3) Partial Equilibrium Single Commodity Approach:

In crop production, because of the biological nature of the production process, the time lags involved between planting and harvesting, the influence of weather on crop yields and uncertainty in prices, farmers' production decisions can be thought to occur in stages. Farmers allocate the available land area among crops and decide on the total amount of current inputs (such as labor, capital and fertilizer). These current inputs are then allocated to various crops to maximize expected gross revenue minus variable costs. Because the nature of crop production and farmers' production decision process, researchers generally attempt to separate total crop production into acreage and yield components in modeling crop supply. Most of the studies of acreage response and yield functions of individual commodities are single equation models using ordinary least squares (OLS) estimation. Systems of simultaneous equations are only occasionally estimated. The following sections focus on literature related to partial single commodity supply response models (mainly acreage response) and yield equations.

(A) Acreage Response Models:

In studies of partial equilibrium, single commodity acreage supply response, it is usually assumed that producers attempt to maximize expected profit and therefore allocate the available land area among crops on the basis of expected relative prices. The "desired" long-run (equilibrium) acreage to be planted to crop i can be postulated as a function of lagged price of crop i and the lagged prices of alternative crops which compete for the same inputs (42).

[7] $$A^{*}_{i,t} = f(P^{i}t-1,\ P^{c}t-1)$$

where

$A^{*}_{i,t}$ = desired acreage of crop i,
P^{i}_{t-1} = lagged price of crop i,
P^{c}_{t-1} = lagged price(s) of competing crop(s).

Lagged prices serve here as proxies for expected prices because current year prices are only known with certainty after harvest, but not when the planting decision is made. A Koyck-Nerlove distributed lag framework (29, 42) has frequently been applied in acreage response analysis under the assumption that only some fraction, γ, of the difference between actual and desired acreage is reduced in any one year:

[8] $$A_{i,t} - A_{i,t-1} = \gamma(A^{*}_{i,t} - A_{i,t-1}),\ 0 < \gamma < 1$$

Substituting equation [3] into the above equation, we get:

[9] $$A_{i,t} = \gamma \cdot f(P_{i,t-1}, P_{i,t-1}) + (1-\gamma)A_{i,t-1}$$

Direct estimation (often in logarithms) of equation [9] is usually accomplished by means of ordinary least squares. The estimated price coefficients refer to short-run elasticity coefficients while the coefficients for the "desired" (long-run equilibrium) acreage function may be derived from the estimated coefficients on prices and lagged area ($A_{i,t-1}$). The adjustment coefficient, γ, determines the relationship among the short-run elasticities and the long-run (full equilibrium) elasticities and can be obtained by subtracting the statistically determined coefficient on $A_{i,t-1}$ from one.

The Nerlovian model, hypothesizing producer reactions in terms of price expectations and/or partial area (or production) adjustments, has been adopted, modified and even extensively revised by numerous authors in examining supply response. For example, Askari and Cummings have made an extensive survey of the literature related to the price responsiveness of agricultural supply with the Nerlovian model and have compiled estimates of supply elasticities by crop and by region around the world (2). While the geometrical distributed lag formulation in the partial adjustment (Koyck-Nerlove type) model is the most commonly used method in acreage response studies, other types of lag structure such as Pascal or polynomial distributed lags have also been applied to supply response studies (7 and 34).

In research on crop acreage supply response for the U.S., Houck, et.al. (26), recognizing the importance of the multi-crop environment (several crop alternatives compete for same resources) and the impacts of changing government programs on crop acreage. They have specified the acreage supply response as a function of the current and past market prices for both own and major competing crops, and inputs which are identified as basic supply-inducing factors. The policy variables considered influencing supply factors are devised to account for the effects of changes in government programs on crop acreage. The estimation of these acreage supply response equations is accomplished by means of OLS regression.

Penn, in his study of econometric policy models of commodity supply response, recognizes the importance of the competition for crop land among major crops (47). He points out that the use of OLS to estimate each crop acreage supply response function separately is not appropriate when decisions on acreage allocation are made simultaneously. Zellner's seemingly unrelated regression (Joint Generalized Least Squares) was applied to estimate a system of acreage response functions simultaneously (67).

Colman (11), in a study of Prairie grain and oilseed acreage response in Canada has used simultaneous and constrained estimation methods for estimating the acreage supply response equations. Of key concern in his study is that the total cultivatable acreage to be allocated to different crops is limited. Two alternative systems have been tested to deal with this problem. One alternative is called the naive system of acreage response. Five crops, wheat, barley, oats, rapeseed and flax are considered for the study. He estimates an aggregate acreage equation for grains and oilseeds which is the sum of the five crop acreages. Any one of the five individual acreages, such as for oats, is then treated as a residual equal to aggregate acreage minus sum of the four other individual acreages.

Another alternative suggested by Colman is the application of the multinomial logit model to the allocation of a total acreage between alternative crops (11). The functional form determining each individual acreage is assumed to be: $A_i = e^{fi}$, fi is an unspecified function of prices and other variables, i=1,...,m and the share of any acreage in the total can be expressed as: $W_i = \frac{A_i}{\sum_{j=1} A_j} = \frac{e^{f_i}}{\sum_{j=1} e^{f_j}}$ where i,j=1,...,m while the ratio and share to one common share B, can be expressed as: $A_i/A_B = e^{f_i}/e^{f_B}$. Taking logarithms, this equation can be restated as $\log (A_i/A_B) = f_i - f_B$, which is the estimating form of the equation.

Only m-1 equations need to be estimated since $\log (A_B/A_B) = 0$, while the estimate of the m-1 equations of the system, the shares of the m alternative crops are estimated as $(A_i/\sum_{j=1}^{m} A_j) = (A_i/A_B)/\sum_{j=1}^{m}(A_j/A_B)$, where i,j=1,2,...,B,...,m, and $(A_i/A_B)=1$ when i=B. It follows that the sum of the estimated shares always equals 1 and that the allocation of alternative crop acreages will be constrained to any given level of total crop land availability. Furthermore, the estimated share of each acreage depends upon the econometrically projected estimates of all m-1 ratios of A_i/A_B and thus depends upon all explanatory variables appearing in the m-1 equation system. Hence the model provides estimates of a full matrix of own and cross-price effects. The Zellner 3-stage-least-squares estimator for seemingly unrelated regressions can be applied to estimate the structure of the equations.

Parikh and Narayana, in a case study of India's agriculture, have estimated acreage supply response for all major crops grown in the country (46). Based on growing seasons and soil types, an overall substitution pattern among the crops at the all-India level was developed. According to the pattern, crops were then classified into ten groups. The Nerlovian-type acreage response equations were estimated using expected revenue instead of expected price as a proxy for expected profits. A Box-Jenkins' auto regressive integrated moving average model was postulated for the independent estimation of crop-revenue expectation functions. These functions were later substituted into the acreage allocation and adoption scheme, and acreage-response functions were then estimated. An area-allocation scheme was formulated to ensure that the individual crop areas would add up to the exogenously specified total cropland area in India.

Researchers have also recognized the simultaneous nature of acreage, yield and production response to price changes. For example, Evans and Bell (14), (although not original to them), in a study of cotton acreage response, have used a system of two behavior equations (acreage and yield equations) and an identity (production=area*yield) to account for the interdependencies between acreage and yield responses.

Many of acreage supply response studies (such as Houck, et. al. (26), Evans and Bell ((14), and Lin (34)) have attempted to introduce measures of government policy instruments to account for the effects of changes in government programs on crop acreage response. They have, however, failed to recognize that these instruments are continuously related to acreage and other production decisions only over a limited range of time horizon. Parameters estimated from these acreage response equations using time series data were assumed to be stable over the entire time series involved, despite variation of government policies. In Weaver's recent work on acreage supply response, he specifies the U.S. wheat acreage response during various post-war policy regimes by presenting a theory of choice under discontinuous policy (66). An extension of Tobin's maximum likelihood method is introduced to generate consistent estimates of the model.

(B) Crop Yield Models:

In the context of crop yield modeling, the analysis of per acre (or hectare) crop yields over time usually employ weather, an indicator of technologial trend, and sometimes product and input prices as explanatory variables. Weather and price relationships are used to capture short-run environmental and economic influences, respectively, which affect yields. Technological trends are designed to capture long-run yield increases resulting from producers' investment into new technology. Lyons and Thompson have developed a conceptual framework for analyzing cross-country differences in corn yields (35). In their conceptual model, a separable two-stage production function with four inputs is assumed:

$$Q = F[f(T,K_T),\ g(L,K_L)] \qquad [10]$$

where

Q = physical output,
T = the area of land in production,
L = the flow of labor services,
K_T = land-saving forms of capital (embodying biological and chemical technology)
K_L = labor-saving forms of capital (embodying mechanical technology).
$f(T,K_T)$ = the biological subfunction, and
$g(L,K_L)$ = the husbandry subfunction.

From their discussion of this conceptual model, Lyons and Thompson show that crop yields are closely related to soils, climate, technology and management ability (or farmer education). Factor-product price ratios also have significant effects on crop yields. Assuming that producers allocate resources in a profit maximizing manner, producers would produce where the value of the marginal product of each factor is equal to its price in all uses. This suggests that changes in product or input price affect input use, and consequently alter crop yields. So, assuming profit maximizing behavior, crop yield can be estimated as a function of the product/factor price ratio and a vector of factors (such as technology, climate and soils) which shift the response surface among geographical regions and through time. The yield function can be expressed as: $Y_i = G(\frac{P_i}{P_n}, Z)$, where Y_i is the yield of the ith crop, P_i is the farmgate price of crop i, P_n is the farm purchase price of input such as nitrogen fertilizer, and Z is a vector of factors. Lyons and Thompson used the above model to conduct a cross-country study to determine the effects of distortions in relative prices on corn productivity and exports. Other studies such as Guise

(22), Hee (23), Krishna (30), Houck and Gallagher (25) also have attempted to measure the price responsiveness of crop yields.

In measuring technological trend in crop yields, several functional forms such as linear, log-linear, semilog, log-inverse, logistic or Spillman functions have been used for crop yield research. These functions are of the following forms:

linear: $Y = a+b \cdot t$

log linear: $\ln Y = a+b \cdot \ln t$

semilog: $\ln Y = a+b \cdot t$
or: $Y = a+b \cdot \ln t$

log inverse: $\ln Y = a-b/t$

logistic: $Y = \frac{C}{1+e^{-(a+bt)}}$ ($a>0$, $b>0$, $C>0$, as $t \to \infty$, $Y \to C$)

Spillman: $Y = C-ab^t$

where Y = average yield,
t = time
C = maximum (ceiling) yield (generally based on biological information or linear trend yield estimate),
a,b = coefficients to be estimated by regression.

In addition to linear, log linear and semilog yield functions in time, researchers often include other explanatory variables such as last year's yield, relative product factor prices and weather.

Because, it approaches a maximum, e^a, which can be set at the biological maximum or some other "reasonable" maximum, the log inverse function seems most attractive for representing trend yield curves.

Logistic trends are not commonly used, but they can represent a large number of growth curves (36). C is the ceiling or maximum yield, and b the proportionality factor in the growth rate. The growth rate at any time is given by $\frac{dYt}{dt} = b \cdot \frac{Yt(C-Yt)}{C}$. The parameters that are of interest are the ceiling C and the proportionate rate of growth b. In empirical application, Griliches (19) applied the logistic function to estimate the values of b and C for a number of corn-growing regions in the U.S. and tried to explain the differences in these parameters in terms of some economic variables.

(C) Marketable Surplus Supply Response of the Subsistence Crop:

The theoretical framework for a multiple-output production system developed previously implicitly assumes that all factors and products can be valued, i.e., that value can come from a market determination of a price or from some kind of implied shadow price determining process. However, in developing countries, a major part of agriculture comprises semi-commercial family farms in a multi-crop environment (1). The family farm engages in subsistence production in that it retains some part of its output for household consumption and markets some surplus. The household and the farm are the two interdependent, fundamental units of these family farms or agricultural households. For these households, food is a wage good and farm profits from producing crop alternatives are a component of total household income and hence a determinant of household consumption behavior. In studying the price responsiveness of peasant family farms in developing agriculture, the distinction between total production and marketed surplus is very important. 12/ The elasticity of marketed surplus is important in the design of agricultural price policy in developing countries because the policy may affect the magnitude of the rural domestic food surpluses which are available to the urban area and upon which the rate of economic development may be partially dependent. Because of the importance of policy implications of elasticity of marketed surplus to agricultural planning in developing countries, the supply response of the marketed surplus of subsistence crops is a relevant focus on modeling the quantity in multiple crop supply systems in LDC's.

12/ The short-run price elasticity of the marketed surplus of a subsistence crop is a mixture of the direct price and indirect income effects, and the long-run elasticity of marketed surplus involves supply (production response) as well as marketing response.

To estimate the elasticity of marketed agricultural surplus in LDC's, several methods of estimating the response of marketings to changes in product prices (the elasticity of marketed surplus) have been developed. Estimation of the price elasticity of the marketed surplus of a subsistence crop can be obtained by first estimating the response of total output to a change in price (own-price elasticity of supply) and then estimating the relationship between quantities marketed and produced (the elasticity of sales with respect to output) using either time series or cross-section data (3, 30, and 63). The price elasticity of marketed surplus is then estimated by multiplying the elasticity of sales with respect to output by the price elasticity of supply. Toquero, Duff, Anden-Lacsina and Hayami (63) have also attempted to measure the relevant elasticities by a model of allocating output of a subsistence crop between home consumption and market sale as follows:

[11] $Q = C + M$

[12] $M = f(P,Q)$

[13] $C = g(P,Q) = Q - f(P,Q)$

where

Q = the output of a subsistence crop,
C = quantity consumed by the producer's household,
M = quantity sold in the market, and
P = product price (deflated by an index of price paid by farmers for nonconsumption goods).

The model states that the output of a subsistence crop (Q) is allocated between the quantity consumed by the producer's household (C) and the quantity sold in the market (M). Assuming that the farm income is generated solely from production of a subsistence crop, the producer will attempt to allocate output between home consumption and sales in a manner to maximize the utility of the household. The marketable surplus supply function can be expressed as a function of product price and quantity produced, and the demand for home consumption is derived as output minus marketable surplus.

The partial price elasticity of supply for the marketable surplus at a given output is defined as $\alpha_P = \frac{\partial f}{\partial P}\frac{P}{M}$. The elasticity of market supply with respect to output is defined as $\alpha_Q = \frac{\partial f}{\partial Q}\frac{Q}{M}$ and the price elasticity of output with respect to price is defined as $e = \frac{\partial Q}{\partial P}\frac{P}{Q}$. Then the total price elasticity of supply for the marketable surplus is obtained as $\alpha = \alpha_P + \alpha_Q \cdot e$. If the "barter component" of marketed surplus and farmer's carryover stocks are important for a subsistence crop, then the output should also include these components and the elasticity of marketed surplus will be derived in a slightly different fashion from the above (8).

For multi-crop economy, Ahn, Singh and Squire (1) develop a model to integrate production and consumption decisions within the context of farm-household theory (assuming the farm-household attempts to maximize farm profit and household utility). The model can solve for labor supply, household consumption, marketable surplus of farm output and resource use (including family labor) simultaneously.

VI. Specification of Crop Supply in Present GOL

The above review of theoretical foundations and empirical studies of multiple product supply systems are applicable to modeling multiple output production system in a single region as well as in multiple region cases such as the GOL agricultural trade model. Based on the theoretical and empirical considerations discussed previously, this section examines the structure of the GOL crop supply component.

The supply block for both grains and oilseeds in a typical GOL region includes area equations for total crop area and individual crops, and a production equation to represent yield for each crop.

The total crop area equation for each region is generally:

[14] $$HAT = A_0 + \sum_{i=1}^{m} b_i P_i + a_1 ZI, \quad i = 1, 2, \ldots, m$$

The area equation for each individual crop i is generally:

[15] $$HA_i = A_{i0} + b_i P_i + \sum_{j=1}^{m-1} b_j P_j + b_{m+1} HAT$$

The production equation for each individual crop i is generally:

[16] $$QS_i = A_{si0} + b_{si1} HA_i + b_{si2} P_i + a_{si1} T + a_{si2} ZI$$

where HAT = total crop area supplied in a region,
i, j = indexes for the major crops in a region, $i=1, \ldots, m$, $j=1, \ldots, m$,
P_i = domestic price of crop i in each region,
P_j = domestic price of crop j, crop j is the competing crop for crop i,
HA_i = area planted for individual crop i,
QS_i = production of individual crop i,
ZI = index of cost of physical inputs in a region (exogenous variable),
T = time trend variable (exogenous variable).

In these crop supply equations, all relationships are synthesized (61). Total crop area is specified as a function of the prices of major crops in the region (this equation is assumed to be a land supply equation). Individual crops compete for total crop area based on historical shares and relative crop prices. Production is a function of individual crop area, own price, technological trend and an index of the cost of physical inputs. Yield equations are not included in the model. There is an implied yield equation in this system, however (dividing equation 16 by equation 15 will generate the implied yield equation).

As discussed in section II, the usual framework for the structural specification of the multiple crop supply system is one of profit maximization subject to technological constraints within the context of comparative static theory for a multiple product supply system. From the concept of production transformation, consistency of resource demand and availability should be ensured in a multiple crop supply system. That is, in order to assure that the sector stays on (or moves around) its production transformation surface, the feasible input-output combinations can only be realized with available resources. The structure of the supply equations in the present GOL does not conform to this condition, however.

The area equations primarily consist of some price variables and other explanatory variables (such as index of cost of physical inputs). This makes them very inappropriate in the long-run projections since consistency between land allocation among crops and outputs is not guaranteed. When the GOL model was originally built, there was an attempt to impose the restrictions such as the homogeneity and the adding-up conditons on parameters of crop supply equations (50). These restrictions were incorporated in the supply equations by using supply price elasticities which summed close to zero and by basing each individual crop area equation on its historical share in total crop land. However, some of the important competitors for land with grains and oilseeds (for example cotton) were not included in the original crop supply system. Because some important crops were excluded from the crop supply system, the model specification omits relevant variables (mainly relevant cross price effects) in the supply equations. The model may not realistically describe substitution effects among crops and resources and the homogeneity restrictions may not be satisfied. 13/ Also, even though it is assumed that total crop area is allocated among major crops (such as wheat, coarse grains and soybeans in the U.S.) on the basis of historical shares, the adding-up condition is not guaranteed during model solution.

Since all relationships in the GOL model are synthesized, many researchers have concerned about the quality of the model's empirical content. Main criticisms concern substitution among crops, omission of relevant variables,

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13/ The homogeneity of degree zero in prices of supply functions holds in general only in a partial equilibrium framework where all relevant prices are included (6).

and the functional forms used in the model. Other shortcomings of the current GOL supply structure are discussed below.

First, the GOL model consists of a system of linear equations, for which there is no guarantee against negative values in the model solution. The linear function implies the constant slope coefficients (elasticities may change through time) and can only be expected to perform well in the neighborhood of the base period observation point if the true function is non-linear. Movements far away from these observed points along a linear schedule can and often do lead to unreasonable solutions (61). Because of the linear specification, non-linearities among endogenous variables, such as production=yield*area, are not allowed. This restriction was imposed since the solution algorithm used to solve the GOL system could only handle linear equations. As a consequence, no yield equations are explicitly specified in the present GOL, although equation [16] implicitly serves this purpose.

Second, although individual crop areas were assumed to sum to total area, this restriction was not explicitly imposed on the system of supply equations. Also, no maximum total crop land area restriction was imposed for any region and no consistent acreage allocation among crops was assured.

Third, there are no policy variables included in the supply equations to reflect the effects of government farm policies on crop supply response. From both theoretical and empirical grounds, the effects of government policies have important implications on multiple crop supply response, also for future use of the model for policy analysis.

Fourth, the GOL model generally lacks linkage between product and factor markets (except area allocation and through the use of index of cost of physical inputs 14/). It is important to include factor markets (such as for labor, capital and current inputs) into multiple commodity trade model because the factor market adjustments have important effects on cost structure and thus the supply schedules in each country.

Fifth, in developing countries, when there is a large amount of self consumption by farmers, the distinction between the responsiveness of total production and of marketed surplus to price changes for a subsistence crop becomes important. The response of marketable surplus to changes in product prices (the elasticity of marketed surplus) has important policy implications to agricultural planning in LDC's. In the present GOL model, only the response of total output to changes in product prices (price elasticity of supply) was considered in the crop supply component. Ultimately, the specification of supply response in LDC's should consider the relationship between the total crop production, household consumption, marketable surplus of subsistence crops, and resource use simultaneously. 15/

Sixth, elasticity estimates in the model are outdated. The need to review and update these elasticities estimates can be illustrated by the large own price elasticities of supply of grains and oilseeds in the U.S. supply equations. These elasticities are in the range of 2.5-3.0, while those in all the other principal grain exporting countries are about 0.5 (50). As a result, the U.S., in response to any shock to the system, does most of the adjusting for the whole world market, i.e., always serves as "residual supplier" at unrealistically low prices, to the world market.

14/ The linkage exists in equation 14 and 16 through ZI variable (index of cost of physical inputs). In which individual factor market impacts affect ZI which affects the crop area and production.

15/ This may be done in a sector programming framework, such as the model developed by Ahn, Singh and Squire (1) to integrate production and consumption decisions within the farm-household theory.

VII. Planned Modifications in GOL Crop Supply System

Based on the theoretical and empirical studies of multiple product supply systems discussed previously and from the review of the present GOL crop supply component, planned modifications in the GOL crop supply system focus on the following. (1) Ensure consistency of resource requirement and availability by imposing a restriction on total cropland area. This will guarantee that projected outputs of the various crop alternatives can be realized with the available resources. (2) The model will be made to more realistically capture cross-price effects or substitution possibilities between alternative crops and resources. This is important in grains, oilseeds and livestock subsectors, where important interactions and substitution possibilities exist on both the supply and demand sides of the market. The required restrictions on parameters (such as homogeneity condition) should be met when possible. (3) Strong efforts will be made to include policy variables in crop supply or equilibrium equations to reflect the effects of government farm policies on crop supply response. From both theoretical and empirical standpoints, the effects of government policies have important implications on the choice of product and input mix. (4) The economic behavioral relationships of crop supply response will be better specified recognizing the simultaneous nature of area, yield and production response to price change. (5) Nonlinearities should be introduced into constant slope supply schedules to overcome the problem of negative solution values.

(1) Functional Forms:

In replacing the present GOL linear system of crop supply response, the following desirable features of the functional forms will provide selection guidelines: (1) functions should be defined only in positive quadrant; and (2) should approach a maximum. For crop yeild a biological maximum or some other "reasonable" maximum could be adjusted upward over time to account for technological changes. For land area supply, a maximum could be set at the limit of total arable land in a region).

Alternative functional forms such as log-linear, semilog, log-inverse, and logistic functions have been reviewed in previous sections on measuring technological trend in crop yields. These are also applicable to crop area supply response relationships. The log linear (double log) function implies a constant elasticity over all price ranges and has the practical advantage that the coefficient on the logarithm of price is equal to the price elasticity. In addition, solution values are always positive. However, the log linear form does not provide for a satiation (maximum) level and thus can be used only if area projections are far below a maximum limit. In order to overcome this a maximum limit can be set exogenously for the variable (for example, total cropland area) under study. Because of resource limitations on intermediate-run GOL revision activities, log-linear functional forms will be strongly considered for specifying crop supply equations. Other functional forms such as log-inverse, logistic functions may be used later, if log-linear functions do not work.

(2) A Multiple Crop System:

In the context of specifying a multiple-product supply system, it is usually assumed that producers allocate resources (available land and other inputs) in a profit maximizing manner. Based on expected prices and other determining factors (such as Government farm policies), producers allocate the available land area among crops and decide on the total amount of current inputs such as labor, capital and fertilizer simultaneously so as to maximize expected profit. Also, because of the simultaneous nature of acreage, yield and production response to price changes (14), in specifying crop supply equations, a system of behavioral equations for acreage and yield and an identity equation for production (=area*yield) is specified. In addition, consistency of crop land availability and its allocation among alternative crops will be ensured in the multiple crop supply system; that is, outputs of the various crop alternatives will be realized with the available cropland and other resources.

The improved multiple crop supply system for each region consist of the following equations: (1) a total cropland supply equation, (2) individual crop area response equation, (3) land market clearing condition, (4) a crop yield equation for each individual crop and (5) the production identity for each crop. The total productive cropland supply equation is specified as a function of expected average gross return of land (or use the expected rental price of land as a proxy variable), cost of production index and a cost of land development to account for the effects on increased land supply. To recognize the importance of the competition for cropland among crop alternatives (i.e., several crop alternatives compete for the same, limited resource - productive cropland), and the importance of changing government policies on crop area response, individual crop area allocation is specified as a function of total available productive cropland, prices of own and competing crops and inputs, and government farm policy variables in supply or equilibrium equations. A land market clearing condition is defined to ensure the consistency of cropland allocation and cropland availability. Crop yield is dependent upon relative prices of own crop and variable inputs (such as fertilizer), individual crop area, crop productivity growth trend and weather index. Production is then derived by multiplying yield by area. The tentative system of equations (in log-linear form) is specified as follows:

Total crop area equation for each region:

$$[17] \qquad HAT = A_{S1}\,(REV)^{b_{S1}}\,(ICP)^{b_{S2}}\,(CLD)^{b_{S3}}$$

Definition of average gross return of land (REV):

$$[18] \qquad REV = \left(\sum_{i=1}^{m} P_i \cdot QS_i\right) / \sum_{i=1}^{m} HA_i$$

Area equation for the individual crop i:

$$[19] \qquad HA_i = A_i\, HAT^{b_{i1}}\, P_F^{b_{i2}}\, P_i^{b_{i3}}\, GP^{b_{i4}} \prod_{\substack{j=1 \\ j\neq i}}^{m-1} P_j^{b_{i(j+4)}}$$

Land market clearing condition: 16/

$$[20] \qquad \sum_{i=1}^{m} HA_i = HAT \text{ or } \sum_{i=1}^{m} HA_i + HARS = HAT$$

Crop yield equation for each individual crop i:

$$[21] \qquad Y_i = A_{yi}(P_i/P_f)^{b_{i1}}(HA_i)^{b_{i2}}\, T^{b_{i3}}\, (WINDEX)^{b_{i4}}$$

Production identity for each individual crop i:

$$[22] \qquad QS_i = HA_i \cdot Y_i$$

where

REV = weighted average price received by farmers as a proxy of gross return to land,
ICP = cost of production (index),
CLD = cost of land development,
HAT = total crop land area in a region,
HA_i = individual crop area in each region, for crop i, i=1, 2, ...,m,
HARS = residual cropland use,
m = number of major crops produced in a region,
Y_i = average yield for crop i,
QS_i = production quantity for crop i,
P_i = producer price of crop i,
P_j = producer price of competing crop(s), $j \neq i$, j = 1, 2, ...,m-1
P_f = price of variable inputs, such as fertilizer
GP = government policy variable such as "effective" support price, diversion payment, etc.,
T = time trend,

16/ Assume all of the individual crop area will sum up to total cropland area. If this condition does not hold, then a variable for residual crop area (HARS) should be added.

WINDEX = weather index variable,
b's = elasticities used in different equations, and
A's = constant terms.

For the individual area equation, the theoretical restrictions such as the adding-up and homogeneity conditions are assumed. The adding-up condition implies that the allocation of productive cropland among various crop alternatives should be constrained by total available productive cropland in a region, and the sum of individual crop areas should be equal to total productive cropland (equation 16, the land market clearing condition implies this restriction). The homogeneity condition implies that in the short-run:

$$\sum_{i=1}^{m} \frac{\partial HA_i}{\partial P_i} = \sum_{i=1}^{m} \frac{\partial HA_i}{\partial P_j} = \sum_{i=1}^{m} \frac{\partial HA_i}{\partial P_f} = 0$$

and in the long-run:

$$\sum_{i=1}^{m} \frac{\partial HA_i}{\partial HAT} = 1, \text{ where } \sum_{i=1}^{m} HA_i = HAT$$

These relationships state that the individual area equation is homogeneous of degree zero in all prices (the sum of all of the price elasticities equals to zero). Uniform changes in all prices will not affect crop area, but the changes in crop area allocation is conditional on total productive cropland availability.

The log-linear form of land supply and crop yield functions has the undesirable property that no maximum limit of total arable land or no biological limit on crop yield. For simulation of the model, a maximum limit of total cropland area can be set to constraint the total cropland supply in a region. One alternative is to specify the cropland supply and crop yield equations in terms of log-inverse functions.

The present crop supply component of the GOL model emphasises grains (wheat, coarse grains and rice) and oilseeds (mainly soybeans). Other field crops including other annual crops (such as cotton), perennials, fallow and pasture are left out of the present commodity coverage. These other field crops are the important competitors for land with grains and oilseeds. Inclusion of other field crops will be considered in the longer-term modifications.

In the near term phase of model revision, the relevant own and cross-price elasticity estimates will be reviewed and updated and the base period for the model will be changed to 1975-77.

VIII. Concluding Remarks

The major modifications in the GOL model crop supply system to be undertaken over the next year or so primarily emphasize structural consistency to assure consistent acreage allocation among crops and to conform the theoretical restrictions of production theory to the crop supply system. Because of resource limitations and time constraints, the model parameters will continue to be synthesized from existing studies, analysis and expert opinion, rather than econometrically estimated. As data and resources permit, we will turn our attention to obtaining the best structural estimates possible, including consistent estimates of important cross-price effects to improve the quality of the empirical content of the model. Some other desirable model attributes such as inclusion of factor markets into multiple product production system, distinctions between the price responsiveness of total production and of marketed surplus for a subsistence crop in developing countries should be considered in the longer-term modeling activities.

In empirical estimation of the structural relationships for the multiple crop supply sector, recent development in estimation of CRESH/CRETH production system by Vincent, Dixon and Powell (61) can be applied to estimate the supply response for a set of multiple products and multiple inputs (if data are available). The empirical developments in general equilibrium analysis such as the work of Laitinen and Theil (31) on the supply and demand of the multi-product firm and Clements' aggregate multi-product supply model (10) can also be applied to estimate the supply responses for a set of multiple products. Other types of constrained estimation can be tried to ensure that output is constrained by resource availability such as allocating available land area among alternative crops. The multinomial logit approach applied by Colman (11) to estimate Prairie grain and oilseed acreage response in Canada can be used to simulate situations in which total land in production is allocated between crop alternatives.

Although budget and time limitations mean many of these more desireable model attributes will be developed only slowly over time, a U.S. model is being developed in which a strong attempt will be made to incorporate the most desireable set of model specifications and functional forms.

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