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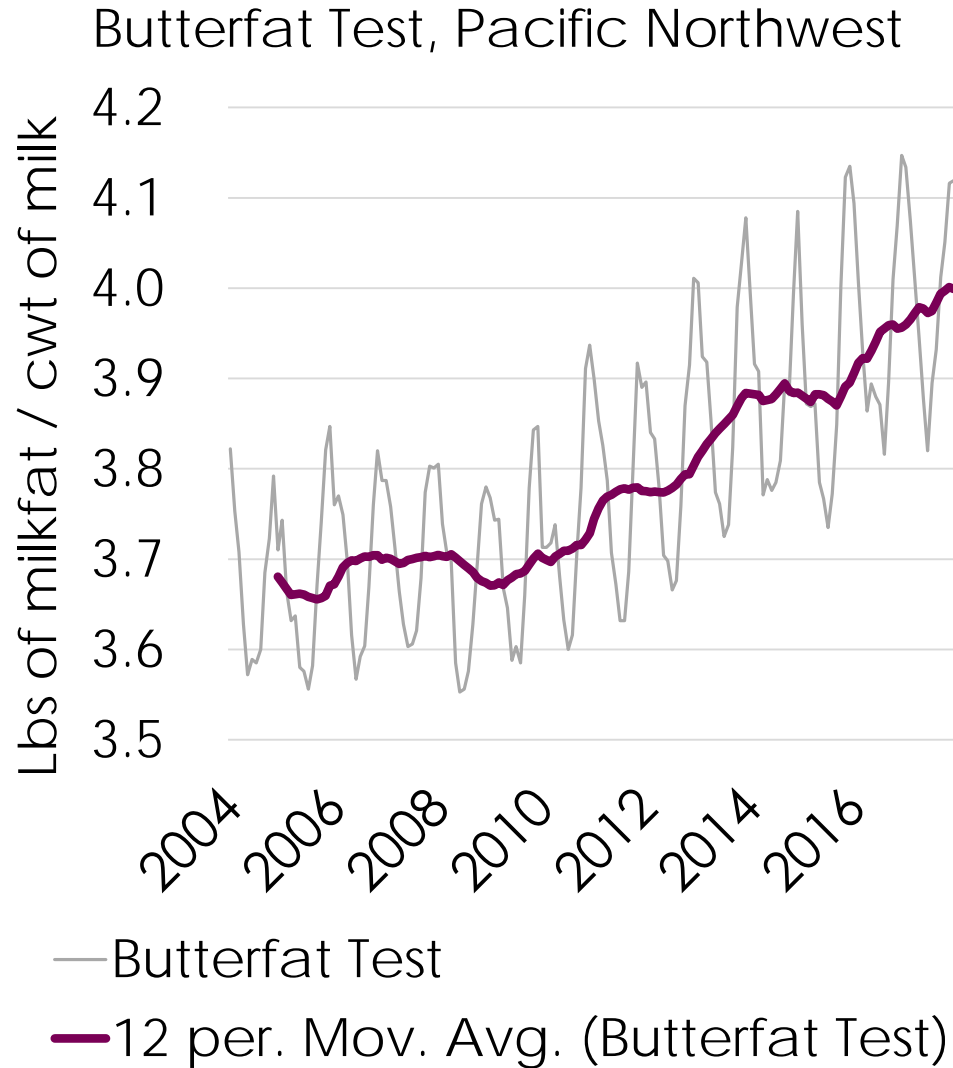
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Pricing Exotic Basket Options for Milk Using Arbitrage-based Dynamic Conditional Correlation

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April 6, 2017













UNIVERSITY OF MINNESOTA
Driven to DiscoverSM

Motivation: Insurance for High-solids Milk

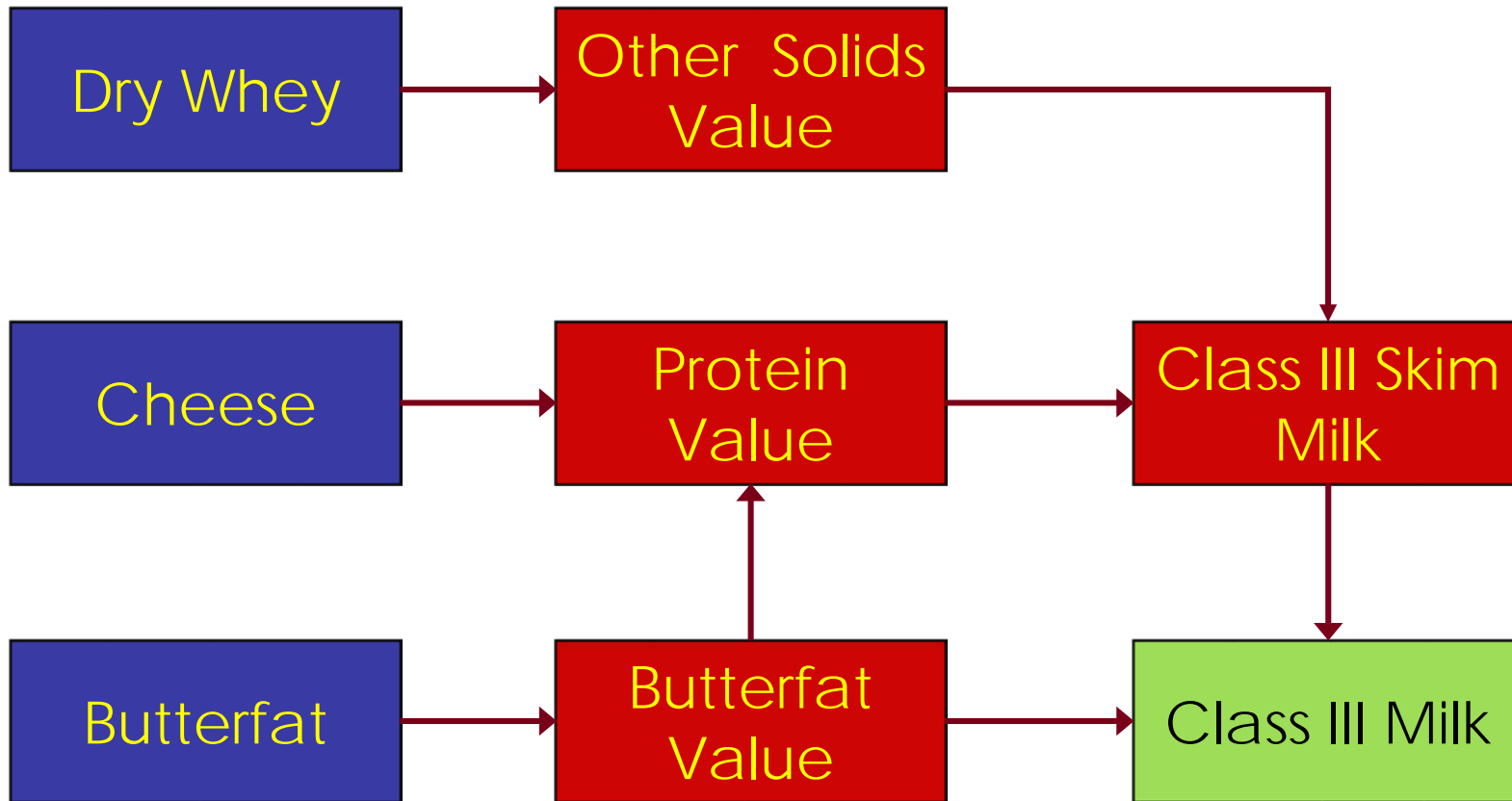


Dairy at CME

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| Product | Code | Contract | | Last | Change | Chart | Open | High | Low | Globex Vol |
|-----------------------------|-------|----------|---|---------|--------|---|---------|---------|---------|------------|
| Class III Milk Futures | DCK8 | MAY 2018 |  | 14.71 | +0.13 |  | 14.57 | 14.76 | 14.55 | 316 |
| Class IV Milk Futures | GDKM8 | JUN 2018 |  | - | - |  | - | - | - | 0 |
| Non-fat Dry Milk Futures | GNFM8 | JUN 2018 |  | 75.900 | -0.900 |  | 76.400 | 76.875 | 75.750 | 27 |
| Dry Whey Futures | DYN8 | JUL 2018 |  | 29.000 | -0.075 |  | 28.825 | 29.250 | 28.825 | 5 |
| Cash-settled Butter Futures | CBM8 | JUN 2018 |  | 234.000 | +1.500 |  | 233.250 | 234.175 | 233.250 | 33 |
| Cash-Settled Cheese Futures | CSCN8 | JUL 2018 |  | 1.664 | +0.007 |  | 1.660 | 1.672 | 1.657 | 37 |

Class III Milk Price



Class III Milk Price – a benchmark price for milk with 3.5% butterfat, 3.1% protein in skim milk, 5.9% other solids.

Pricing Insurance Products

General Template

1. Define the asset at risk
2. Assume structure of marginal distributions
3. Fit marginal distributions
4. Assume structure of copula
5. Fit copula
6. Simulate expected losses

Example: Livestock Gross Margin for Dairy Cattle

1. Milk – (Corn + Soymeal)
2. Lognormal
3. Futures & Option-implied volatility
4. Normal
5. Historical futures shocks
6. 5000 draws of milk-feed margin

Pricing Insurance Products: Best Practices & Challenges

- Rely on forward-looking measures as much as feasible
 - Using futures for expected prices, assuming zero risk premium
 - Using option-based implied volatility to fit marginal distributions
- Use historical data if must
 - Correlation between price shocks
 - Yields
 - Correlation between yield shocks and price shocks
- But we understand that historical data may be outdated
 - GARCH-style modelling of conditional variance
 - Practitioners base pricing on 30-day historical volatility
- **Challenge: can we reverse-engineer market-based, forward-looking copula?**

Pricing Insurance Products: High-solids Milk

$$\begin{aligned} \text{Expected Price} = & \text{ Butterfat Test} \times \text{ Butterfat Price} \\ & + \text{ Protein Test} \times \text{ Protein Price} \\ & + \text{ Other Solids Test} \times \text{ Other Solids Price} \end{aligned}$$

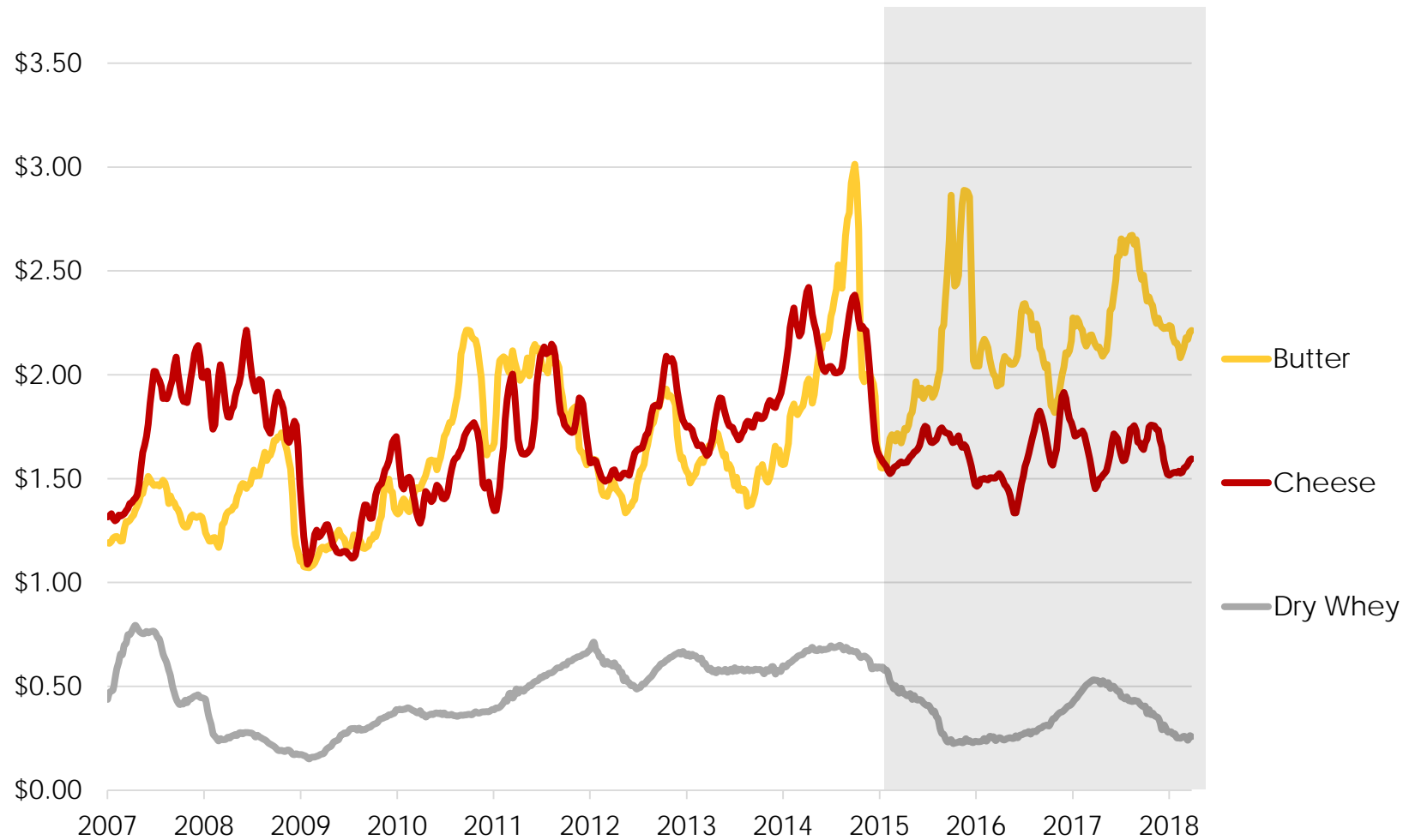
Measuring Risk

- Use implied volatility from butter, cheese and dry whey options

Measuring correlation among prices

- Historical correlations?

Structural Changes in Dairy Markets Post 2015



Historical Correlation of Futures Shocks

| 3-Months Out | 2007-2014 | 2015-2018 |
|---------------------|------------------|------------------|
| Butter-Cheese | 0.53 | 0.05 |
| Butter-Whey | 0.32 | -0.18 |
| Cheese-Whey | 0.22 | 0.17 |

| 6-Months Out | 2007-2014 | 2015-2018 |
|---------------------|------------------|------------------|
| Butter-Cheese | 0.60 | -0.29 |
| Butter-Whey | 0.34 | -0.38 |
| Cheese-Whey | 0.13 | 0.41 |

| 8-Months Out | 2007-2014 | 2015-2018 |
|---------------------|------------------|------------------|
| Butter-Cheese | 0.61 | -0.20 |
| Butter-Whey | 0.37 | -0.37 |
| Cheese-Whey | 0.20 | 0.44 |

Non-arbitrage Relationship

$$\begin{aligned} \textit{Class III} &= 0.42379 \times \textit{Butter} \\ &+ 9.63933 \times \textit{Cheese} \\ &+ 5.86430 \times \textit{Dry Whey} \\ &- 3.17100 \end{aligned}$$

Identity Relationship Implication for Variance

$$m = -3.17100 + 0.42379 \times b + 9.63933 \times c + 5.86430 \times w$$

$$m = \alpha + \beta \times b + \gamma \times c + \delta \times w$$

$$\begin{aligned} \text{Var}(m) &= \text{Var}(\alpha + \beta \times b + \gamma \times c + \delta \times w) \\ &= \beta^2 \text{Var}(b) + \gamma^2 \text{Var}(c) + \delta^2 \text{Var}(w) \\ &\quad + 2 \times \text{Cov}(\beta \times b, \gamma \times c) \\ &\quad + 2 \times \text{Cov}(\beta \times b, \delta \times w) \\ &\quad + 2 \times \text{Cov}(\gamma \times c, \delta \times w) \end{aligned}$$

Identity Relationship Implication for Variance

$$\begin{aligned} \text{Var}(m) &= \beta^2 \text{Var}(b) + \gamma^2 \text{Var}(c) + \delta^2 \text{Var}(w) \\ &\quad + 2 \times \rho_{b,c} \beta \times \gamma \times \sqrt{\text{Var}(b) \text{Var}(c)} \\ &\quad + 2 \times \rho_{b,w} \beta \times \delta \times \sqrt{\text{Var}(b) \text{Var}(w)} \\ &\quad + 2 \times \rho_{c,w} \gamma \times \delta \times \sqrt{\text{Var}(c) \text{Var}(w)} \end{aligned}$$

Identity Relationship Implication for Variance

$$\text{Var}(b) = \left[e^{\sigma_b^2} - 1 \right] e^{(2\mu_b + \sigma_b^2)}$$

$$\sigma_b^2 = IV_b^2 \tau$$

$$\mu_b = \ln(b) - 0.5 \times \sigma_b^2$$

Identity Relationship Implication for Variance

$$\begin{aligned} \text{Var}(m) &= \beta^2 \text{Var}(b) + \gamma^2 \text{Var}(c) + \delta^2 \text{Var}(w) \\ &+ 2 \times \rho_{b,c} \beta \times \gamma \times \sqrt{\text{Var}(b) \text{Var}(c)} \\ &+ 2 \times \rho_{b,w} \beta \times \delta \times \sqrt{\text{Var}(b) \text{Var}(w)} \\ &+ 2 \times \rho_{c,w} \gamma \times \delta \times \sqrt{\text{Var}(c) \text{Var}(w)} \end{aligned}$$

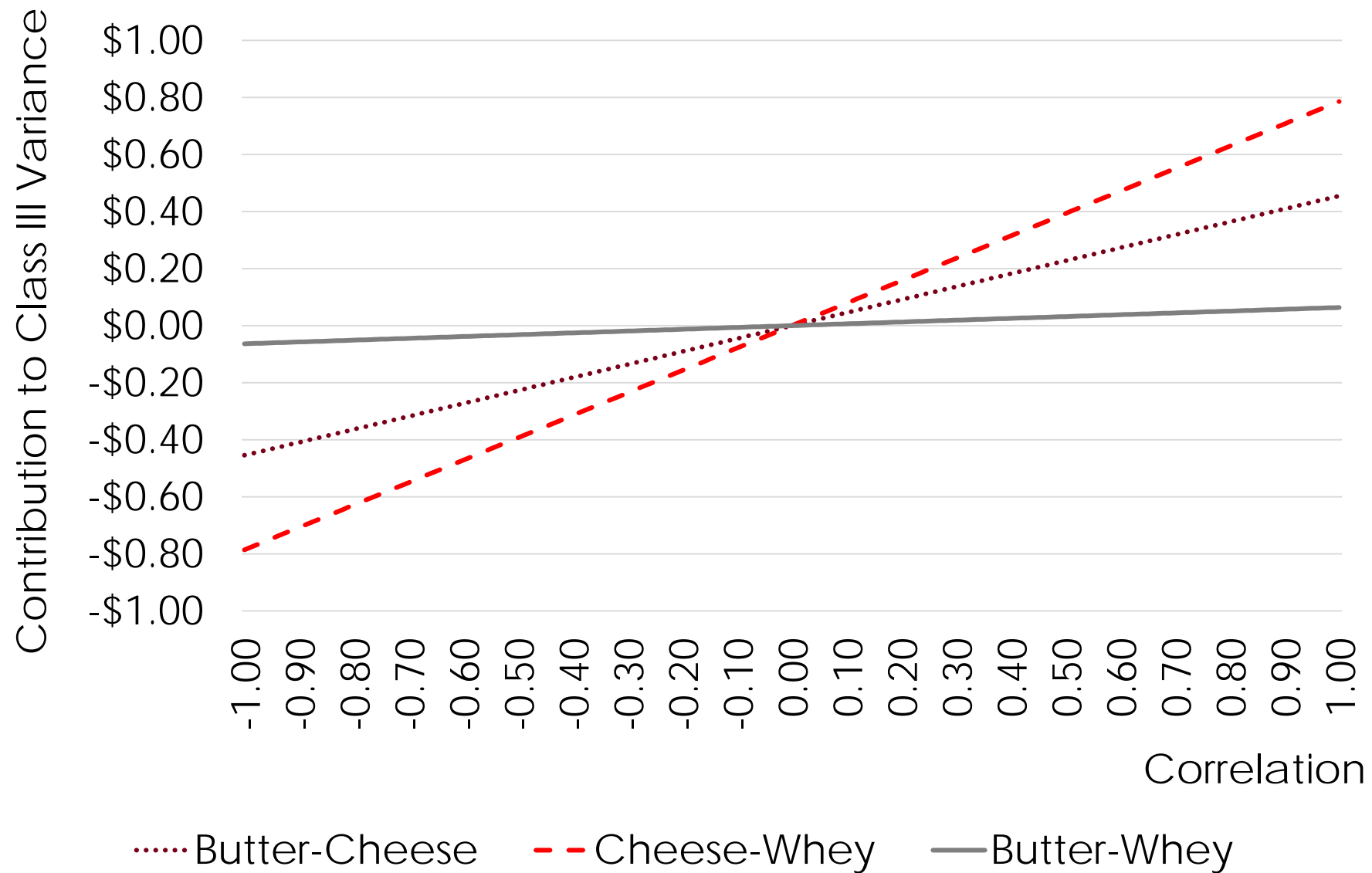
Case Study with Prices for Dec 2018 on April 5, 2018

| Commodity | Variance | Contribution to Class III Variance | | | |
|-------------------------------|----------|------------------------------------|-------------------|--------------------|---------------|
| | | Zero Corr. | Hist.Corr '07-'14 | Hist.Corr. '15-'18 | Implied Corr. |
| Butter | 0.10 | 0.02 | 0.02 | 0.02 | 0.02 |
| Cheese | 0.03 | 2.81 | 2.81 | 2.81 | 2.81 |
| Dry Whey | 0.00 | 0.06 | 0.06 | 0.06 | 0.06 |
| Butter-Cheese | | 0.00 | | | |
| Butter-Whey | | 0.00 | | | |
| Cheese-Whey | | 0.00 | | | |
| Calculated Class III Variance | | 2.88 | | | |
| Actual Class III Variance | 2.76 | 2.76 | 2.76 | 2.76 | 2.76 |
| Difference | | +0.12 | | | |

Case Study with Prices for Dec 2018 on April 5, 2018

| Commodity | Variance | Contribution to Class III Variance | | | |
|-------------------------------|----------|------------------------------------|-------------------|--------------------|---------------|
| | | Zero Corr. | Hist.Corr '07-'14 | Hist.Corr. '15-'18 | Implied Corr. |
| Butter | 0.10 | 0.02 | 0.02 | 0.02 | 0.02 |
| Cheese | 0.03 | 2.81 | 2.81 | 2.81 | 2.81 |
| Dry Whey | 0.00 | 0.06 | 0.06 | 0.06 | 0.06 |
| Butter-Cheese | | 0.00 | 0.28 | -0.09 | |
| Butter-Whey | | 0.00 | 0.02 | -0.02 | |
| Cheese-Whey | | 0.00 | 0.16 | 0.35 | |
| Calculated Class III Variance | | 2.88 | 3.34 | 3.12 | |
| Actual Class III Variance | 2.76 | 2.76 | 2.76 | 2.76 | 2.76 |
| Difference | | +0.12 | +0.58 | +0.36 | |

Case Study with Prices for Dec 18 on April 5, 2018



Case Study with Prices for Dec 2018 on April 5, 2018

| Commodity | Variance | Contribution to Class III Variance | | | |
|-------------------------------|----------|------------------------------------|-------------------|--------------------|---------------|
| | | Zero Corr. | Hist.Corr '07-'14 | Hist.Corr. '15-'18 | Implied Corr. |
| Butter | 0.10 | 0.02 | 0.02 | 0.02 | 0.02 |
| Cheese | 0.03 | 2.81 | 2.81 | 2.81 | 2.81 |
| Dry Whey | 0.00 | 0.06 | 0.06 | 0.06 | 0.06 |
| Butter-Cheese | | 0.00 | 0.28 | -0.09 | -0.32 |
| Butter-Whey | | 0.00 | 0.02 | -0.02 | 0.01 |
| Cheese-Whey | | 0.00 | 0.16 | 0.35 | 0.19 |
| Calculated Class III Variance | | 2.88 | 3.34 | 3.12 | 2.76 |
| Actual Class III Variance | 2.76 | 2.76 | 2.76 | 2.76 | 2.76 |
| Difference | | +0.12 | +0.58 | +0.36 | 0.00 |

Historical vs. Implied Correlation

| 3-Months Out | 2007-2014 | 2015-2018 | Implied Corr |
|---------------|-----------|-----------|--------------|
| Butter-Cheese | 0.53 | 0.05 | -0.63 |
| Butter-Whey | 0.32 | -0.18 | 0.15 |
| Cheese-Whey | 0.22 | 0.17 | 0.21 |

| 6-Months Out | 2007-2014 | 2015-2018 | Implied Corr |
|---------------|-----------|-----------|--------------|
| Butter-Cheese | 0.60 | -0.29 | -0.64 |
| Butter-Whey | 0.34 | -0.38 | 0.12 |
| Cheese-Whey | 0.13 | 0.41 | 0.18 |

| 8-Months Out | 2007-2014 | 2015-2018 | Implied Corr |
|---------------|-----------|-----------|--------------|
| Butter-Cheese | 0.61 | -0.20 | -0.70 |
| Butter-Whey | 0.37 | -0.37 | 0.13 |
| Cheese-Whey | 0.20 | 0.44 | 0.24 |

Open Issues

- **Dealing with partial identification?**
 - The no-arbitrage only restricts the degrees of freedom of the space of three correlation coefficients. How to optimally adjust the correlation coefficients?
- **Ensuring implied correlation matrix is positive semi-definite.**
 - Derive implied correlations
 - Project onto space of positive semidefinite matrices
 - Repeat until convergence
- **Trust implied correlation or implied volatility?**
 - Implied Correlations may not pass the common-sense test. If implied butter-cheese correlation needed to match variances is -0.9 would you still trust the results?
 - An alternative conclusion may be that implied Class III volatility is too low relative to implied vol. of butter and cheese. Speculative positions?

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By Stik



Pricing Exotic Basket
Options for Milk using
Arbitrage-based Dynamic
Conditional Correlation

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