Measurement Error in Prospect Theory Field Elicitations

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Field Elicitations in Applied Economics

- It has become increasingly popular to include prospect theory parameter estimates in analyses of agricultural and environmental decision making.
  - Demand for crop insurance
  - Pesticide use
  - Technological adoption
  - Forest harvesting decisions
  - Food waste
- Generally, subjects participate in a prospect theory lab experiment in the field to generate parameter estimates (following Tanaka, Camerer and Nguyen, AER 2010), which are then analyzed for correlations with other covariates and/or included in a regression model predicting economic behavior.

Outline

- In this project we...
  - Demonstrate how the field elicitations work, by mapping the parameter space into choice space.
  - Consider the effect of "choice errors" by subjects on the resulting parameter estimates.
  - Design and run experiments, and present some preliminary results.

Tanaka, Camerer and Nguyen (AER, 2010)

Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam.

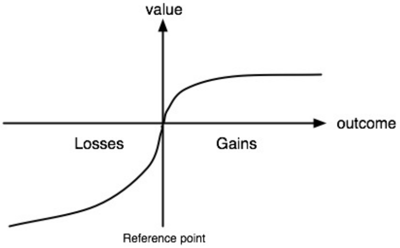
- 500+ Google Scholar citations.
- Elicitation uses incentivized menus of real-money gambles.
- Simple elicitation of 3-parameter CPT model.
- 181 subjects in rural Vietnam.
- Replications include Liu 2013; Liu and Huang 2013; Paul, Weinthal, Bellemare and Jeuland 2016; Sullivan, Uchida, Sproul and Xu 2018; etc.

TCN (2010) Background

- Innovation: Three simple lottery menus to fit a three-parameter CPT model.
- Clever parsing of parameter space into a grid.
- Parameters are easy to interpret.
- Calculations can be done rapidly using their lookup table.

Value function: $v = (x - r)^\sigma$ for gains, $v = -\lambda(r - x)^\sigma$ for losses.

Probability weights (Prelec, 1998): $w = \exp(-(-\ln p)^\alpha)$.



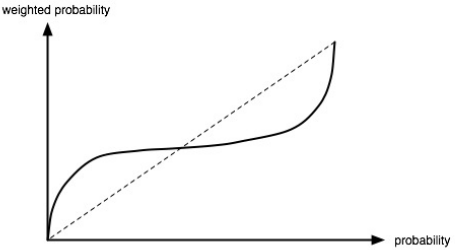


TABLE 2—THREE SERIES OF PAIRWISE LOTTERY CHOICES (*in 1,000 dong*)

Option A		Option B		Expected payoff difference (A–B)
Series 1				
Balls 1–3	Balls 4–10	Ball 1	Balls 2–10	
40	10	68	5	7.7
40	10	75	5	7.0
40	10	83	5	6.0
40	10	93	5	5.2
40	10	106	5	3.9
40	10	125	5	2.0
40	10	150	5	−0.5
40	10	185	5	−4.0
40	10	220	5	−7.5
40	10	300	5	−15.5
40	10	400	5	−25.5
40	10	600	5	−45.5
40	10	1,000	5	−85.5
40	10	1,700	5	−155.5
Series 2				
Balls 1–9	Ball 10	Balls 1–7	Balls 8–10	
40	30	54	5	−0.3
40	30	56	5	−1.7
40	30	58	5	−3.1
40	30	60	5	−4.5
40	30	62	5	−5.9
40	30	65	5	−8.0
40	30	68	5	−10.1
40	30	72	5	−12.9
40	30	77	5	−16.4
40	30	83	5	−20.6
40	30	90	5	−25.5
40	30	100	5	−32.5
40	30	110	5	−39.5
40	30	130	5	−53.5
Series 3				
Balls 1–5	Balls 6–10	Balls 1–5	Balls 6–10	
25	−4	30	−21	6.0
4	−4	30	−21	−4.5
1	−4	30	−21	−6.0
1	−4	30	−16	−8.5
1	−8	30	−16	−10.5
1	−8	30	−14	−11.5
1	−8	30	−11	−13.0

TCN (2010): Parameter Lookup Tables

σ	Switching question in Series 1														
Series 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Never
1	1.50	1.40	1.35	1.25	1.15	1.10	1.00	0.95	0.90	0.85	0.80	0.75	0.65	0.55	0.50
2	1.40	1.30	1.25	1.15	1.10	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.55	0.50
3	1.30	1.20	1.15	1.10	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.55	0.50	0.45
4	1.20	1.15	1.05	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.50	0.45	0.40
5	1.15	1.05	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.40	0.35
6	1.05	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35
7	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
8	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
9	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20
10	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.20
11	0.80	0.70	0.65	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.15
12	0.75	0.65	0.60	0.55	0.50	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.20	0.15	0.10
13	0.65	0.60	0.55	0.50	0.45	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.15	0.10	0.10
14	0.60	0.55	0.50	0.45	0.40	0.35	0.35	0.30	0.25	0.20	0.15	0.10	0.10	0.10	0.05
Never	0.50	0.45	0.40	0.40	0.35	0.30	0.30	0.25	0.20	0.15	0.10	0.10	0.05	0.05	0.05

α	Switching question in Series 1														
Series 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Never
1	0.60	0.75	0.75	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.40	1.45
2	0.60	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.35	1.40
3	0.55	0.60	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30
4	0.50	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25
5	0.45	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20
6	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
7	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10
8	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05
9	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
10	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
11	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
12	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
13	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
14	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
Never	0.05	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.45	0.55	0.55	0.65	0.60

$$\alpha \approx 0.7 - 0.05 \cdot (c_1 - c_2)$$

$$\sigma \approx 1.4 - 0.05 \cdot (c_1 + c_2)$$

$$\lambda = (x_{B1}^{\sigma} - x_{A1}^{\sigma}) \,/\, ((-x_{B2})^{\sigma} - (-x_{A2})^{\sigma})$$

TCN (2010): Choice Space and Parameter Space

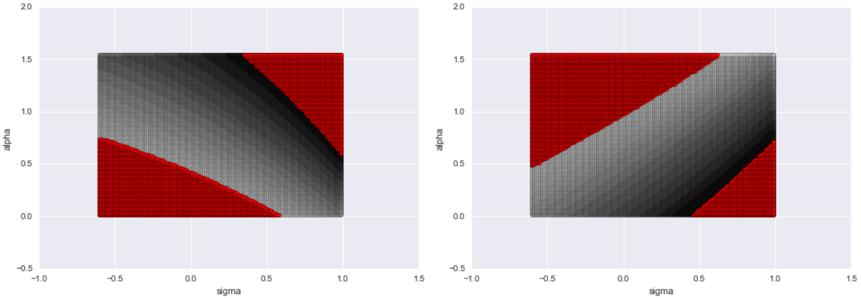


Figure: TCN Menu 1 and Menu 2, visualized

What About Choice Errors?

- Substantial evidence of inconsistent choices or 'choice errors'.
- Most common modeling approach is Normal errors on certainty equivalents.
  - e.g., with standard deviation parameter, ξ_i, as in Bruhin, Fehr-Duda and Epper (Econometrica 2010).
  - alternately, Luce errors (Holt and Laury 2002, 2005) drawing choice probabilities from utility.
- The main strength of the TCN approach (simplicity) ends up being its greatest weakness.
  - Exact identification increases measurement error in the presence of choice errors.
  - Leads to attenuation bias if parameter estimates used as regressors:
    - ★ $\hat{\beta}$ and t-statistics biased towards zero.

Measurement Error

Classical "errors-in-variables" model of measurement error: $y = x\beta + \epsilon$

What if we only know $\hat{x} = x + u$?

- Leads to attenuation bias in coefficient estimates:

$$\text{plim}\hat{\beta} = \lambda\beta = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}\beta < \beta.$$

- ...and in t-statistics:

$$\frac{\text{plim}\, t}{\sqrt{n}} = \sqrt{\lambda}\frac{\beta}{\sqrt{s^2 + (1-\lambda)\beta^2}} < \frac{\beta}{s}.$$

(these are the simple univariate versions only)

Proof of Concept: Experimental Design

- We conducted an experiment as proof of concept, containing multiple TCN style menus.
- To show attenuation bias, we compare regression predictions:
  $f(c_1, c_2, c_4, c_5) \rightarrow c_7$ vs. $f(c_1, c_2) \rightarrow c_7$.
- c_1, c_2 are TCN menu choices, while c_4, c_5, c_7 are choices in new menus.
  (c_3, c_6, etc. reserved for mixed menus)

Proof of Concept: Menu Detail

Menu 4

- **Option A**
  - p = 0.2, X = 10, Y = 2.5
- **Option B**
  - p = 0.1
  - Xs = [19.25, 22.25, 25.5, 28.5, 33, 37, 44.5, 55, 66, 78, 91, 105, 105, 105]
  - Ys = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1.25, 1.5]

Menu 5

- **Option A**
  - p = 0.8, X = 6, Y = 3
- **Option B**
  - p = 0.7, Y = 1
  - Xs = [6.1, 6.3, 6.6, 7, 7.5, 8, 8.6, 9.2, 9.8, 10.5, 11.2, 11.9, 12.7, 13.5]

Proof of Concept: Menus

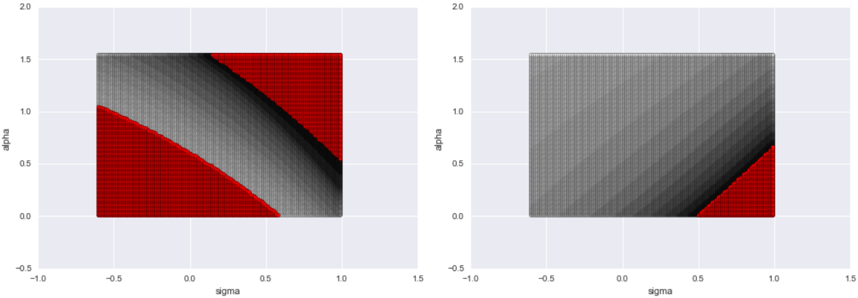
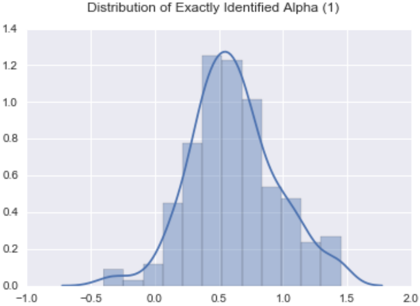
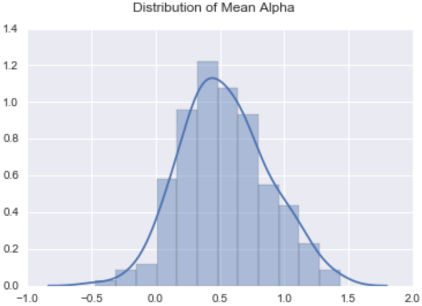


Figure: Our first menus 4 and 5, visualized

Proof of Concept: Alpha Parameter

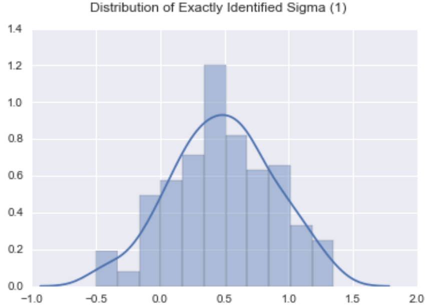
- Parameter estimates for alpha (probability weighting), exactly identified (TCN) vs. averaged.

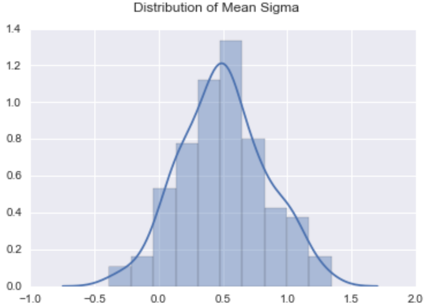




Proof of Concept: Sigma Parameter

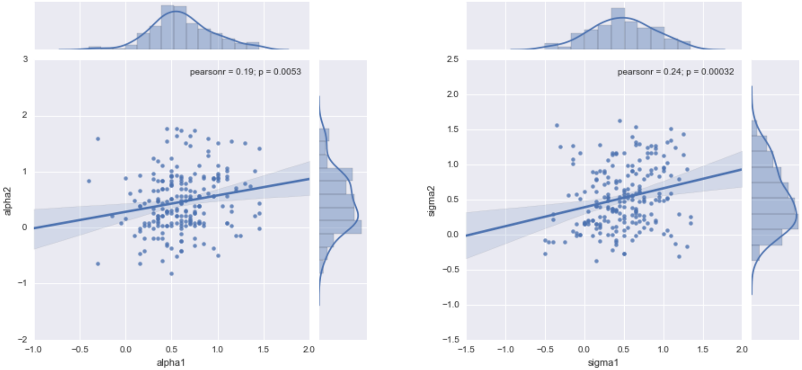
- Parameter estimates for sigma (value function curvature), exactly identified (TCN) vs. averaged.





Evidence of Choice Errors

- Below plots compare exactly identified estimates of α and σ, within subjects.
- We observe evidence of choice errors, in terms of substantial variation of estimates within subjects.



Prediction Menu Detail

Menu 7A

- **Option A**
  - p = 0.9, X = 6, Y = 0
- **Option B**
  - p = 0.6
  - Xs = [6, 6.6, 7.5, 8.6, 9.8, 11.5, 14, 16.25, 18, 20, 23, 23, 23, 23]
  - Ys = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 3, 4]

Menu 7B

- **Option A**
  - p = 0.3, X = 8, Y = 3
- **Option B**
  - p = 0.1
  - Xs = [24.25, 27, 30, 34.25, 40.75, 49, 58, 68.75, 80, 93, 93, 93, 93, 93]
  - Ys = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.75, 1, 1.4, 1.8]

Prediction menus (oops!)

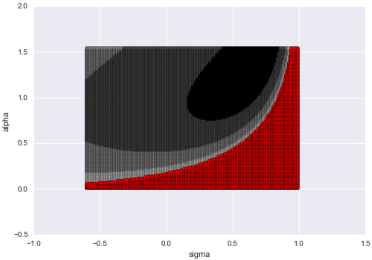


Figure: Our menu 7A, accidental "black hole".

Prediction menus

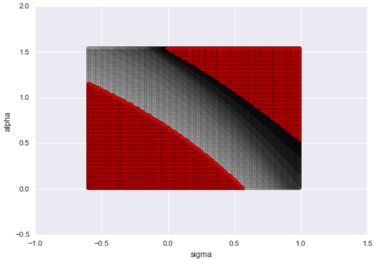


Figure: Our menu 7B, prediction test

Evidence of Attenuation Bias

- We compare the out of sample predictive power of a single exactly identified parameter estimate (α, σ from TCN menus 1 and 2) with the mean value of our two estimates (those, averaged with our menus 4 and 5).
- We regress both α and σ on the observed switching point for an additional TCN-style menu (our menu 7).
- As expected, we observe:
  - Variation in noise, across menu pairs
  - Convergence to theoretical values, when averaging
  - Increasing t-statistics, when averaging

OLS Regression Results

Theoretical Values

Intercept	-7.211133
alpha	12.440129
sigma	14.814084

OLS Regression Results (n=217)
Dependent Variable: s7

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	4.0853	0.821	4.979	0.000	2.468	5.703
alpha12	1.7387	0.976	1.782	0.076	-0.184	3.662
sigma12	2.8630	0.772	3.707	0.000	1.341	4.385

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2.5139	0.779	3.227	0.001	0.978	4.049
alpha45	3.6196	0.765	4.730	0.000	2.111	5.128
sigma45	4.7297	0.792	5.972	0.000	3.169	6.291

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.1206	0.952	1.176	0.241	-0.757	2.998
alpha1245	4.4759	1.080	4.145	0.000	2.348	6.604
sigma1245	6.3406	0.974	6.511	0.000	4.421	8.260

Second Experiment Menu Detail

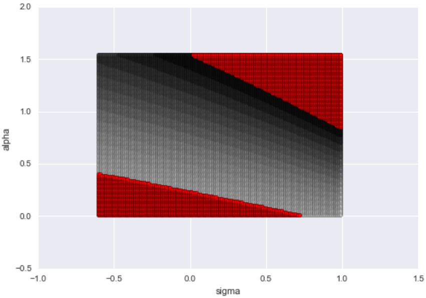
Menu 4

- **Option A**
  - p = 0.4, X = 55, Y = 36
- **Option B**
  - p = 0.1, Y = 28
  - Xs = [79, 84, 90, 98, 108, 120, 135, 154, 177, 210, 255, 325, 430, 630]

Menu 5

- **Option A**
  - p = 0.8, X = 40, Y = 26
- **Option B**
  - p = 0.6, Y = 6
  - Xs = [55, 56.5, 58, 60, 62, 65, 68, 72, 77, 83, 90, 100, 115, 135]

Second Experiment Menus



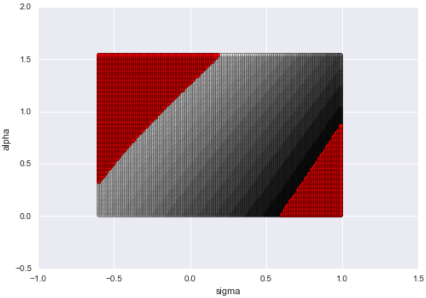


Figure: Our menus 4 and 5, second experiment

Second Prediction Menu Detail

Menu 7

- **Option A**
  - p = 0.4, X = 110, 63
- **Option B**
  - p = 0.4, Y = 32
  - Xs = [144, 147, 150, 154, 158, 163, 168, 177, 188, 202, 218, 238, 265, 300]

Second Prediction Menu

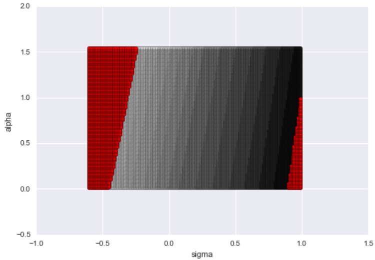


Figure: Our menu 7, second prediction test

Second Experiment Results

Theoretical Values

Intercept	5.235879
alpha	-1.123610
sigma	9.521997

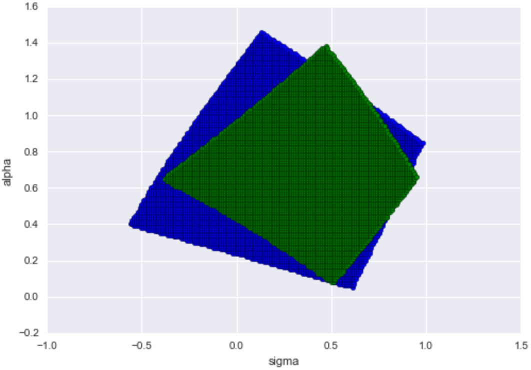
OLS Regression Results (n=74)
Dependent Variable: s7

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	5.9599	1.331	4.479	0.000	3.307	8.613
alpha12	-1.1004	1.731	-0.636	0.527	-4.553	2.352
sigma12	3.4921	1.290	2.708	0.008	0.921	6.063

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	5.8459	0.940	6.217	0.000	3.971	7.721
alpha45	0.2783	1.279	0.218	0.828	-2.272	2.828
sigma45	2.7279	1.134	2.406	0.019	0.467	4.988

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	5.3543	1.269	4.218	0.000	2.823	7.886
alpha1245	-0.9336	1.805	-0.517	0.607	-4.533	2.666
sigma1245	5.4459	1.512	3.601	0.001	2.430	8.461

Comparing Domains in Parameter Space



What We Learned

- All of above design difficulties can be worked out from theory.
  - $c_{im} = a_{im} + b^1_{im} \cdot \alpha_i + b^2_{im} \cdot \sigma_i + e_{im}$
- Large and approx. equal theoretical effect sizes are desirable.
- Closely matching parameter domains and choice-spacing, too.
- Challenge is to have apparent variety of menus within constraints (humans are not robots).

Loss Aversion is Hard

- Speaking of design constraints...

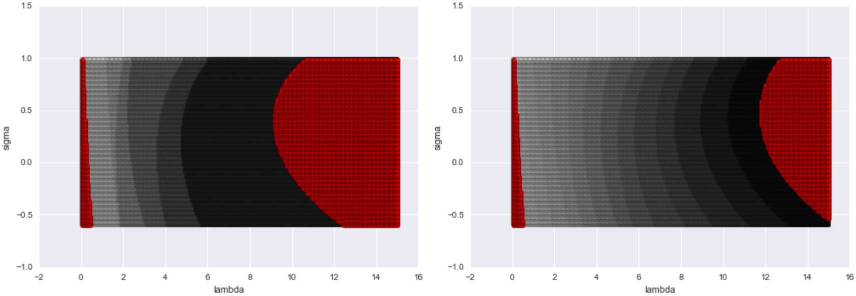


Figure: TCN Menu 3, vs. a cleaner version

Loss Aversion is Hard II

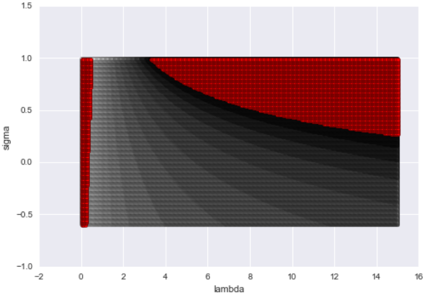


Figure: Other possible lambda menus

Conclusion

- Exactly identified behavioral parameter estimates do not account for choice errors.
- Resulting measurement error produces attenuation bias, which can result in Type 2 Errors (False Negatives).
- This potential bias may be solvable with the right (extra) menus.

- Future Work...
  - Calibration of theoretical effect sizes in future tests.
  - Assessment of non-linear effects on solved λ estimates.