The Effects of Private Stocks versus Public Stocks on Food Price Volatility

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Abstract:

This paper investigates the role of storage and its effects on price dynamics and volatility with an application to food markets. It investigates the differences between private stock and public stock as they affect the distribution of market price. Based on a reduced-form approach, the analysis relies on quantile autoregression (QAR) as a flexible representation of price dynamics. Applied to US wheat and corn markets, the paper documents how storage affects commodity price dynamics and price volatility. Stocks have statistically significant price effects but these effects vary in different parts of the price distribution (e.g., lower tail versus upper tail of the distribution). We find strong statistical evidence that private stock and public stock have different effects on price dynamics and price volatility (including variance, skewness and kurtosis). For wheat, increasing private stock shifts the price distribution to the left, while increasing public stock shifts the price distribution to the right. Studying the effects of storage on price dynamics, we uncover evidence of local dynamic instability in the upper tail of the price distribution. We evaluate how the private/public stock portfolio affects the odds of facing price crashes and spikes.

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Keywords: food price volatility, price dynamics, storage, private stock, public stock, quantile,

JEL: C1, E3, Q1

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1. Introduction

The economics of storage and its implications for intertemporal smoothing of commodity prices and consumption have been the subject of much research (e.g., Working, 1949; Gustafson, 1958; Williams and Wright, 1991; Deaton and Laroque, 1992, 1996; Chavas et al., 2014). When applied to food, grain stocks are crucial to support food security in the face of much agricultural production uncertainty. Such issues have been important since the beginning of civilization. This includes the history of ancient Egypt where failures of the Nile floods generated great famines and endangered Nile civilizations (e.g., Shaw, 2000; Marriner et al., 2012). And it is relevant today when weather shocks and climate change threaten to disrupt food production, with adverse impacts on food security around the world (e.g., Headey, 2011; Kalkuhl et al., 2016). When facing stochastic production, storage can smooth temporary gluts and reduce the adverse effects of future shortages. In market economies, storage behavior affects both consumption and prices over time. (e.g., Gustafson, 1958; Gardner, 1979; Williams and Wright, 1991; Deaton and Laroque, 1992, 1996; Benirschka and Binkley, 1995; Bobenrieth et al., 2013; Gouel, 2013a, 2013b; Knittel and Pindyck, 2016). Motivated by current concerns about food security, this paper takes a new look at the economics of storage, with an application to price dynamics in agricultural markets and its linkages to food price volatility.

Evaluating storage decision rules is complex for at least three reasons. First, while optimal stock decisions can be formulated as the solution of a dynamic programming problem, obtaining this solution is the subject of empirical difficulties. As argued by Ng and Ruge-Murcia (2000) and Cafiero et al. (2011), a number of inaccuracies can adversely affect the validity of results obtained from empirical implementations of the structural model proposed by Deaton and Laroque (1992, 1996). Second, markets are often subject to systematic changes in production and/or consumption that affect pricing patterns (e.g., Gouel and Legrand, 2017). This raises questions about how structural changes in markets can affect storage behavior over time. Third, storage decisions typically depend on expectations about the future (Muth, 1961). But at a given time period, not all market participants may have access to the same information. This stresses the role of information in the formulation of price expectations (Peterson and Tomek, 2005). Such issues have made the empirical evaluation of storage behavior somewhat difficult. Note that such difficulties are especially important in structural models, but are less significant in reduced-form models (as reduced-form models are less sensitive to structural misspecifications). On that basis, the analysis presented below relies on a reduced-form approach to the dynamics of market equilibrium for a storable commodity. In this context, we propose a flexible representation of price distribution and its dynamics, with explicit linkages with storage.

Another motivation for our paper is the policy shift associated with food storage during the last few decades. Before the 1990's, public stock policies (e.g. buffer stock programs) were important aspects of food policy around the world (including the US and Europe). This changed with the advent of the World Trade Organization (WTO) in 1995 with the attempt to reduce market distortions. Over the last 20 years, many countries have gradually abandoned public food stock policy. This shift is illustrated for the US in Figure 1. Figure 1 shows that public stocks were relatively large for wheat and corn in the 1980's, but they vanished after 2005 for corn and after 2010 for wheat. In other words, for wheat and corn, the late 1990's saw a massive shift away from public stock toward private stock. But the last 15 years have seen a large increase in food price volatility around the world (Headey and Fan, 2008; Chavas et al., 2014). This raises the question: is there any linkage between the recent increase in food price volatility and the reduction in global food stocks? There is evidence that low stocks have contributed to high food price volatility (e.g., Wright, 2011; von Braun and Torero, 2012). This has raised a new debate about whether there may be a need for strategic public food stock policy (von Braun and Torero, 2012; Deuss, 2014; Kalkuhl et al., 2016). As such, this paper investigates the role of storage and answers the following questions: 1/ what are the effects of storage on the distribution of market price? 2/ do private stock and public stock play different role in affecting commodity price dynamics and price volatility? And 3/ what is the role of stocks in price instability? And does this role differ between private stock and public stock?

This paper develops a method to investigate the role of storage and its effects on commodity price dynamics and price volatility. The investigation is presented under general supply-demand conditions, with a special focus on possible differences between public stock and private stock. Our approach relies on a reduced-form representation of price dynamics based on quantile autoregression (QAR) proposed by Koenker and Xiao (2006). QAR model provides a flexible representation to assess price volatility and the evolving distribution of price. Our quantile analysis is conditional on storage, providing a framework to investigate how storage affects price volatility. And it will allow us to evaluate differences between public stock and private stock as each affects the distribution of prices and price dynamics. Compared with traditional mean regression (e.g., using Least Squares), the QAR approach is more informative: it provides a flexible representation of how stocks affect the tails of the price distribution (including skewness and kurtosis). QAR also gives a basis to evaluate the dynamics of the price distribution. By allowing for nonlinear dynamics, it can document the presence of local dynamic instability and the effects of stocks (private and public) on price instability. As such, this paper makes new and important contributions to dynamic price analysis and to the economics of storage.

We apply the method to two US agricultural commodity markets, wheat and corn, over the period of 1980-2014. Our focus on US corn and wheat markets has two important motivations. First, there is no reliable data on world grain stocks, which prevents us from conducting the analysis at the world level.[1] But good data on both private and public stocks are available in the US corn and wheat markets. Second, as noted above, the 35-year sample period 1980-2014 covers significant changes in US agricultural policy, including a large policy shift away from public stockholding. This will allow us to study the effects of storage on commodity prices, with a special focus on the different roles played by public stock versus private stock. Our QAR model allows us to estimate the determinants of the price distribution. While this includes effects on mean price, it also covers factors affecting price volatility and the tails (both upper tail and lower tail) of the price distribution. Our empirical analysis gives several important results. First, we find that stocks have statistically significant price effects;

but these effects vary in different parts of the price distribution (e.g., upper tail versus lower tail of the distribution). Second, we investigate the relative effects of private stock and public stock on commodity prices. We find strong statistical evidence that private stock and public stock have different effects on price dynamics and price volatility (including variance, skewness and kurtosis). We document the nature of these differences. We find evidence that private stockholding increases the probability of price crashes (reflecting an unwillingness of private stockholders to buy when the price is low), while public stockholding increases the probability of price spikes (reflecting an unwillingness of public stockholders to sell when the price is high). Both effects reflect some limitations in the role of storage (either private or public) in smoothing price fluctuations over time. Third, we find differences between the wheat market and the corn market. For example, the wheat market has exhibited greater price volatility than the corn market. And we find the price effects of public stock to be much larger for wheat than for corn. Finally, we study the effects of stocks on price dynamics. We uncover evidence of local dynamic instability in the upper tail of the price distribution. In situations where stocks are positive, our empirical analysis indicates that, compared to public stockholding, private stockholding offers better options in reducing the odds of price spikes.

The rest of paper is organized as follows. Section 2 presents a model of the role of storage under general supply/demand conditions in a commodity market, for both aggregate stock and mixed private/public stocks. In Section 3, we investigate the implications of storage for price dynamics. Section 4 develops an econometric model of quantile autoregression, providing a flexible way to estimate the price distribution conditional on past prices and stocks.

Section 5 reports data used in an application to the US wheat and corn markets. The role of aggregate stock on price dynamics and price volatility is analyzed in Section 6. The different role of private and public stocks is further discussed in Section 7. Finally, Section 8 concludes.

2. Conceptual model

Consider the market for a storable commodity. At time t, let $Q_t \in \mathbb{R}_+$ be the quantity produced, $D_t \in \mathbb{R}_+$ be the quantity consumed, and $P_t \in \mathbb{R}_+$ be the market price. And let $S_t \in \mathbb{R}_+$ be the quantity stored at the end of period t. It follows that the market equilibrium condition at time t is given by

$$D_t = Q_t + S_{t-1} - S_t, \tag{1}$$

Equation (1) states that demand D_t is equal to supply Q_t plus the change in stock $(S_{t-1} - S_t)$. When $S_{t-1} - S_t > 0$, stock is declining and $(S_{t-1} - S_t)$ is the quantity that comes out of stock and becomes available for consumption at time t. Alternatively, when $S_{t-1} - S_t < 0$, stock is increasing and $(S_t - S_{t-1})$ is the quantity that is withdrawn from current consumption and becomes available for future consumption. In general, equation (1) shows that market equilibrium reflects supply, demand and inventory conditions.

First, consider the demand for the commodity. Conditional on price P_t, let $D_t(P_t, e_{D_t})$ denote aggregate demand at time t and e_{D_t} represents demand shocks. We assume that the demand function $D_t(P_t,\cdot)$ is downward sloping with $\frac{\partial D_t}{\partial P_t} < 0$. The elasticity of demand $ED_t \equiv \partial \ln(D_t) / \partial \ln(P_t)$ affects the response of price to shocks. For example, the effect of a supply shock on market price P_t is larger (smaller) when $|ED_t|$ is smaller (larger), i.e., when the demand is more inelastic (more elastic).

Second, consider the commodity production. Assume that production involves a multi-stage process and that supply decisions are given by the decision rule $Q_t(P_t, P_{t-1}, \dots, P_{t-m}, e_{Q_t})$, where e_{Qt} represents supply shocks (e.g., weather shocks affecting crop yield). The lagged prices $(P_{t-1}, \dots, P_{t-m})$ capture the effects of prices on production decisions at different stages of the production process. We assume that $\frac{\partial Q_t}{\partial P_t} \geq 0$, as higher price P_t tends to stimulate current supply. The decision rule $Q_t(P_t, P_{t-1}, \dots, P_{t-m}, e_{Q_t})$ allows for dynamic supply response as current and past prices affect production decisions over time.[2]

2.1. Aggregate stock

We start our analysis focusing on the case of aggregate stock S_t, assuming that the choice of S_t is made by an optimizing agent over time. Using backward induction, the behavior of the stock manager is represented by a dynamic programming problem where the choice of S_t is given by Bellman's equation

$$V_t(S_{t-1}) = max_{S_t \geq 0} \ \{B_t(S_{t-1} - S_t) - C_t(S_t) + \beta E_t[V_{t+1}(S_t)]\}, \tag{2}$$

where $V_t(S_{t-1})$ is the value function at time t, $\beta \in (0,1)$ is a discount factor, E_t is the expectation operator based on the information available to the stock manager at time t, $B_t(S_{t-1} - S_t) < \infty$ is the benefit of choosing S_t at time t, and $C_t(S_t)$ is the cost of stockholding at time t. In general, the benefit function depends on the change in stock $(S_{t-1} - S_t)$. When stock is purchased/sold on the market place, it also depends on current price P_t. Finally, equation (2) involves expectations about future market conditions. Conditional on the information available at time t, denote the expected future price at time $t + 1$ by P^e_{t+1}. As showed in Bertsekas and Shreve (1996), Bellman's equation (2) provides a general representation of storage decisions. We write the solution to (2) as the stock decision rule $S_t(S_{t-1}, P_t, P_{t+1}^e)$.[3] This makes it clear that both current price P_t and expected future price P_{t+1}^e affect storage decisions. As proposed by Muth (1961), price expectations come from rational expectations. The determination of price expectations is discussed in section 3 below.

2.2. Public stock versus private stock

Equation (2) represents the choice of aggregate stock S_t which provides general insights on the role of storage and its effects on prices and price dynamics. As discussed in the introduction, we are also interested in distinguishing between two types of stocks: private stock Sr_t and public stock Su_t, which satisfy $S_t = Sr_t + Su_t$. This distinction is of interest when decisions differ between private stock Sr_t and public stock Su_t.

In the case of private stockholding, the benefit function in (2) takes the form $B_t(Sr_{t-1} - Sr_t) = [P_t \cdot (Sr_{t-1} - Sr_t)]$, measuring the revenue obtained from private stockholding at time t. Let $P_{r,t+1}^e$ denote price expectation by private stock manager. Then from (2), the decision rule for Sr_t is $Sr_t\left(Sr_{t-1}, P_t, P_{r,t+1}^e\right)$: it is the optimal private stock that maximizes the expected present value of profit from storage activities. In this context, private stock holding is motivated by profit generated from anticipated price fluctuations (e.g., Gustafson, 1958; Williams and Wright, 1991; Deaton and Laroque, 1992, 1996). Expected price increases (decreases) provide incentives (disincentives) for private storage. Under this scenario, private stock would contribute to stabilizing the market by buying (thus putting upward pressure on price) when the price is low and selling in the following period (thus putting downward pressure on price) when the price is high. Note that the non-negativity of stock $S_t \geq 0$ implies that storage effects are necessarily nonlinear: storage can prevent price increases only when lagged stock S_{t-1} is positive (Gustafson, 1958; Williams and Wright, 1991; Deaton and Laroque, 1992, 1996).

What if stock is managed by a public institution? The behavior of public stock managers may differ from private stock managers for at least three reasons: 1/ the public stock managers may have a different assessment of benefit $B_t(Sr_{t-1} - Sr_t)$ in (2); 2/ they may discount the future at a lower rate; and 3/ their access to information may vary, implying that they exhibit different price expectation. In such cases, the decision rules of private versus public stock managers would differ. For instance, public stock could be chosen according to a price band $[P_L, P_M]$ supporting the following decision rule: buy and increase public stock when $P_t < P_L$; sell and decrease public stock when $P_t > P_M$; and do nothing when $P_t \in [P_L, P_M]$ (Newbery and Stiglitz, 1981, p. 408-410). Below, we consider $Su_t\left(Su_{t-1}, P_t, P^e_{u,t+1}\right)$ as a general decision rule for public stock, where $P^e_{u,t+1}$ denotes price expectation by the public stock manager.

Given $S_t = Sr_t + Su_t$, it follows that the decision rule for aggregate stock is $S_t\left(Sr_{t-1}, Su_{t-1}, P_t, P^e_{r,t+1}, P^e_{u,t+1}\right) = Sr_t\left(Sr_{t-1}, P_t, P^e_{r,t+1}\right) + Su_t\left(Su_{t-1}, P_t, P^e_{u,t+1}\right)$. Again, current price P_t affects storage. So does future price expectations $(P^e_{r,t+1}, P^e_{u,t+1})$. The determination of price expectations is discussed next.

3. Price expectation and price dynamics

Next, we investigate the implications of storage for price dynamics. As discussed above, we consider two scenarios: 1/ the case of aggregate stock where the associated decision rule is $S_t(S_{t-1}, P_t, P_{t+1}^e)$; and 2/ the case of mixed private/public stock where the aggregate stock decision rule is $S_t\big(Sr_{t-1}, Su_{t-1}, P_t, P_{r,t+1}^e, P_{u,t+1}^e\big)$.

First, consider the case of the aggregate stock decision rule $S_t(S_{t-1}, P_t, P_{t+1}^e)$. In this context, combining equations (1) and (2) gives the market equilibrium condition for the commodity at time t

$$D_t\big(P_t, e_{D_t}\big) = Q_t\big(P_t, P_{t-1}, \dots, P_{t-m}, e_{Q_t}\big) + S_{t-1} - S_t(S_{t-1}, P_t, P_{t+1}^e), \tag{3a}$$

which has for solution the market equilibrium price

$$P_t = P_t{}'(P_{t-1}, \dots, P_{t-m}, P_{t+1}^e, S_{t-1}, e_t), \tag{3b}$$

where $e_t = \big(e_{D_t}, e_{Q_t}\big)$ are stochastic shocks assumed to have a given probability distribution. Equation (3b) can be used to evaluate price expectation. Following Muth (1961), under rational expectation, the expected price P_{t+1}^e is given by $P_{t+1}^e = E_t[P_{t+1}'(P_t, \dots, P_{t-m+1}, P_{t+2}^e, S_t, e_{t+1})]$, where E_t is the expectation operator about the shocks e_{t+1} based on the information available at time t. Using backward induction, solving this equation along with $S_t(S_{t-1}, P_t, P_{t+1}^e)$ gives the rational expected price $P_{t+1}^e(P_t, \dots, P_{t-m+1}, S_{t-1})$. Substituting this expression into equation (3b) and solving for P_t gives the market equilibrium price

$$P_t = P_t(P_{t-1}, \dots, P_{t-m}, S_{t-1}, e_t), \tag{4a}$$

Equation (4a) is an m-th order stochastic difference equation representing the dynamics of market prices under general conditions. It is a reduced-form equation that gives a valid representation of the net effects of past prices $(P_{t-1}, \dots, P_{t-m})$ and lagged stock S_{t-1} on current price. Equation (4a) will provide the basis for the empirical analysis presented in section 6 below.

Second, consider the case of the stock decision rule $S_t(Sr_{t-1}, Su_{t-1}, P_t, P^e_{r,t+1}, P^e_{u,t+1})$ where private stock management possibly differs from public stock management. In this context, after replacing S_{t-1} by (Sr_{t-1}, Su_{t-1}), note that the above analysis presented in (3a)-(3b) still applies. The market equilibrium price in (3b) then becomes $P_t = P^{"}_{t+1}(P_{t-1}, \dots, P_{t-m}, P^e_{r,t+1}, P^e_{u,t+1}, Sr_{t-1}, Su_{t-1}, e_t)$. Following Muth (1961), under rational expectation, the expected price becomes $P^e_{k,t+1} = E_{kt}[P^{"}_{t+1}(P_t, \dots, P_{t-m+1}, P^e_{r,t+1}, P^e_{u,t+1}, Sr_{t-1}, Su_{t-1}, e_{t+1})]$, where E_{kt} is the expectation operator based on the information available at time t to agent $k \in \{r, u\}$. This shows that the private/public stock mix can affect price expectations. As in (4a), using these price expectations gives the market equilibrium price

$$P_t = P_t(P_{t-1}, \dots, P_{t-m}, Sr_{t-1}, Su_{t-1}, e_t), \tag{4b}$$

Equation (4b) is a reduced-form representation of price dynamics in the presence of both private stock and public stock. Given $S_t = Sr_t + Su_t$, equations (4a) and (4b) become identical when private stock and public stock are perfect substitutes. In this case, we would have $\frac{\partial P_t}{\partial Sr_{t-1}} = \frac{\partial P_t}{\partial Su_{t-1}}$. Alternatively, in situations where $\frac{\partial P_t}{\partial Sr_{t-1}} \neq \frac{\partial P}{\partial Su_{t-1}}$, then private stock and public stock would be imperfect substitutes, corresponding to situations where private stock and public stock are managed differently and have different impacts on pricing. Differences between $\frac{\partial P_t}{\partial Sr_{t-1}}$ and $\frac{\partial P_t}{\partial Su_{t-1}}$ can come from two sources: 1/ the private decision rule $Sr_t\left(Sr_{t-1}, P_t, P^e_{r,t+1}\right)$ and the public decision rule $Su_t\left(Su_{t-1}, P_t, P^e_{u,t+1}\right)$ differ (due to differences in perceived storage benefit and/or in time discounting); and 2/ price expectations differ (as $P^e_{r,t+1} \neq P^e_{u,t+1}$). Thus, testing the null hypothesis that $\frac{\partial P_t}{\partial Sr_{t-1}} = \frac{\partial P_t}{\partial Su_{t-1}}$ provides a basis to investigate whether the effects of private and public stocks differ. And if this null hypothesis is rejected, then equation (4b) can provide useful information on how public stock differs from private stock as far as pricing in concerned. These issues will be investigated empirically in section 7 below.

With the understanding that (4a) is a special case of (4b) (when private stock and public stocks are perfect substitutes), our discussion proceeds in the context of equation (4b). Note that equation (4b) can be alternatively written as the first-order difference equation

$$w_t \equiv \begin{bmatrix} P_t \\ \vdots \\ P_{t-m+1} \end{bmatrix} = \begin{bmatrix} f_t(P_{t-1}, \dots, P_{t-m}, Sr_{t-1}, Su_{t-1}, e_t) \\ \vdots \\ P_{t-m+1} \end{bmatrix}$$

$$\equiv H_t(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t) \tag{5}$$

where $w_t \in \mathbb{R}^m_+$. Equation (5) can be used to characterize the nature of price dynamics. Under differentiability, let $DH_t(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t) = \partial H_t(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t)/\partial w_{t-1}$ be a $(m \times m)$ matrix. Denote the dominant characteristic roots of $DH_t(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t)$ by $\lambda_1(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t, t)$. This dominant root provides useful information on dynamics. In general, $|\lambda_1(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t, t)|$ reflects the speed of dynamic adjustments in the neighborhood of point $(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t, t)$. From equation (5), price dynamics is locally stable if the dominant root satisfies $|\lambda_1(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t, t)| < 1$; and it is locally unstable if $|\lambda_1(w_{t-1}, Sr_{t-1}, Su_{t-1}, e_t, t)| > 1$.[4]

Given (4b) or (5), define the conditional distribution function $F(c \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}, t) = Prob[P_t \leq c \ | \ P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}, t] = Prob[f_t(P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}\ , e_t) \leq c \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1},\ t]$. The associated conditional quantile function is defined as the inverse function $q(\tau \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1},\ t) \equiv inf_c \ \ \{c\colon F(c \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1},\ t) \geq \tau\}$ where τ is the τ^{th} quantile, $\tau \in (0, 1)$. When $\tau = 0.5$, this includes as special case the conditional median $q(0.5 \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}, t)$. In the rest of the paper, we will make extensive use of the quantile function $q(\tau \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}, t)$ in the analysis of the dynamics of P_t.

Relying on the conditional quantile function $q(\tau \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1},\ t)$, we focus our attention on the case where the conditional quantile function takes the form $q(\tau \ |P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}, t) = X(P_{t-1}, \dots, P_{t-m};\ Sr_{t-1}, Su_{t-1}, t)\ \alpha_\tau, \tau \in (0, 1)$, where $X(\cdot)$ is a $(1 \times K)$ vector and $\alpha_\tau \in \mathbb{R}^K$ is a $(K \times 1)$ vector of parameters. This restricts the analysis to situations where conditional quantiles are linear in the parameters α_r. Importantly, this specification allows the parameters α_τ to vary across quantiles, thus providing a flexible representation of the underlying distribution function. This flexibility extends to the effects of previous stock S_{t-1} on price volatility. In addition, the functions $X(P_{t-1}, \dots, P_{t-m}; Sr_{t-1}, Su_{t-1}, t)$ can possibly be nonlinear, thus allowing for the presence of nonlinear dynamics.

4. Econometric model

Below, for the τ-th quantile, following Koenker and Xiao (2006), we consider a quantile autoregressive QAR(m) model of the form

$$q(\tau \ |P_{t-1}, \dots, P_{t-m}; Sr_{t-1}, Su_{t-1}, t) = \alpha_{0,\tau}(Sr_{t-1}, Su_{t-1}, t) +$$

$$\sum_{j-1}^{m} \alpha_{i,\tau}(Sr_{t-1}, Su_{t-1}) \ P_{t-j} \tag{6}$$

$\tau \in (0, 1)$. When $\alpha_{j,\tau}(Sr_{t-1}, Su_{t-1}) = \alpha_j$, $j = 1, \dots, n$, the QAR specification (6) reduces to a standard autoregressive (AR(m)) model where the autoregression parameters $(\alpha_1, \dots, \alpha_n)$ are treated as constants. While this AR(m) specification still allows previous stock S_{t-1} to shift the intercept, it would restrict the autoregression parameters $(\alpha_1, \dots, \alpha_n)$ to be constant, i.e., not to change with (Sr_{t-1}, Su_{t-1}) or across quantiles.[5] In our quantile model, allowing the intercept $\alpha_{0,\tau}(Sr_{t-1}, Su_{t-1}, t)$ to vary across quantiles $\tau \in (0, 1)$ provides a flexible representation of the price distribution (including its moments: mean, variance, skewness and kurtosis). Perhaps more importantly, allowing the autoregression parameters $\alpha_{j,\tau}(Sr_{t-1}, Su_{t-1})$ to vary across quantiles can capture flexible dynamics for any moment of the price distribution (including mean, variance and skewness, kurtosis). Finally, allowing the stock variable (Sr_{t-1}, Su_{t-1}) to affect both the intercept $\alpha_{0,\tau}(Sr_{t-1}, Su_{t-1}, t)$ and the autoregression parameters $\alpha_{j,\tau}(Sr_{t-1}, Su_{t-1})$, the QAR(m) model in (6) gives a flexible representation of the effects of storage on the dynamics of the price distribution. The usefulness of this flexible approach is illustrated in our empirical analysis below.[6]

With $q(\tau \mid \cdot) = X(\cdot)\ \alpha_\tau, \tau \in (0,1)$, consider a sample of z observations on (P, X). Denote the l^{th} observation by (P_l, X_l), $l \in Z \equiv \{1, \ldots, z\}$. For a given quantile $\tau \in (0,1)$ and following Koenker (2005), the quantile regression estimate of α_τ is

$$\hat{\alpha}_\tau \in argmin_\alpha\ \{\sum_{l \in Z}\ \rho_\tau(P_l - X_l\ \alpha)\}, \tag{7}$$

where $\rho_\tau(w) =\ w\ [\tau\ - I(w < 0)]$ and $I(\cdot)$ is the indicator function. As discussed in Koenker (2005), the quantile estimator $\hat{\alpha}_\tau$ in (7) is a minimum distance estimator that can be obtained by solving simple linear programming problems. Under some regularity conditions, the estimator $\hat{\alpha}_\tau$ has desirable statistical properties, including consistency and asymptotically normality (Koenker, 2005). The usefulness of the quantile approach in the analysis of price dynamics and price volatility is illustrated in an application to food prices next.

5. Data

The analysis is applied to US agricultural markets over the period of 1980-2014. We focus on two key markets: the wheat market and the corn market. The sample data start in 1980, covering a period of US policy reform when public stock programs were gradually abandoned. This sample period allows us not only to study the effects of storage on commodity price but also to investigate the different functions of public versus private stock.

Monthly wheat and corn prices are collected from International Monetary Fund (IMF) primary commodity prices database. Wheat price refers to price of the No.1 hard red winter wheat in Kansas City; the corn price refers to the FOB price of the No.2 yellow corn in Gulf of Mexico. Agricultural commodity prices being highly correlated across locations, we treat such prices as representative of market conditions and explore their linkages with stocks.

Figure 1 shows the price trajectories of wheat and corn in the US market from 1980 to 2014, and Table 1 reports the summary statistics. During the sample period, wheat and corn prices followed similar trajectories, both showing a price spike around 1996 and several large price booms and busts after 2008.

The storage data are collected from the World Agricultural Supply and Demand Estimate (WASDE) database developed by the US Department of Agriculture (USDA). The WASDE database reports monthly ending stock (including total, private and public stock) and total use estimates in the US wheat and corn markets.[7] We calculated the stock-to-use ratio measured as the level of total (private, public) stock as a percentage of total use[8]. As illustrated in Figure 1, the wheat and corn total stock levels decreased over time. The total stock-to-use ratios for wheat and corn were respectively as high as 190 percent and 90 percent in the 1980's, and then decreased to about 60 percent and 20 percent after the 1990's. Note that, throughout the sample period, the total stock level for wheat was always much higher than for corn. As showed in Figure 1, high prices tend to occur when the stock level is low and vice versa, a common finding in previous studies (e.g., Williams and Wright, 1991; Cafiero et al., 2011). Our empirical analysis below will examine in details how storage affects market prices.

In the 1980's, public stock played a major role in US agricultural market, accounting for more than 50 percent of its total stock (see Figure 1). However, after 1990, US government policy started to rely less on public stock and eventually abandoned it in the mid 2000's. Such policy changes were motivated by arguments that public stock programs were distorting markets and were costly to the taxpayers. Yet, eliminating strategic food reserves has remained an issue to the extent that it may contribute to high food insecurity (e.g., Von Braun and Torero, 2012). In our empirical research reported below, we study the substitution possibilities between private and public stock, with a special focus on their effects on the lower tail (low price scenario) and on the upper tail (high price scenario) of the price distribution.

6. Analysis of storage effects

In this section, we examine the effects of aggregate stock on price dynamics and price volatility. The analysis is to be interpreted as a step toward a more refined investigation that distinguishes between private stock and public stock (as presented in section 7 below). While much research has examined the economics of storage (e.g., Gustafson, 1958; Wright and Williams, 1982a,b; Williams and Wright, 1991; Pindyck, 1994; Mitra and Boussard, 2012), the effects of storage on different parts of the price distribution are less well understood. The QAR model discussed above provides good basis for such an investigation. With a focus on aggregate stock, the analysis corresponds conceptually to equation (4a) and empirically to equation (6) (with (Sr_{t-1}, Su_{t-1}) replaced by S_{t-1}), applied to US wheat price and US corn price.

We start with preliminary estimates of autoregressive (AR(m)) processes representing underlying price dynamics. Table 2 reports estimated AR(m) models applied to wheat price and corn price. The AR(m) processes involves lagged prices up to m periods, m = 1, 2, 3, 4. The models also include lagged stock S_{t-1}, seasonality factors represented by quarterly dummies (Q_1, Q_2, Q_3) and two time trends capturing structural changes: T_1 as a general time trend, and T_2 as a time trend starting in 2005 and reflecting the effects of changes in US biofuel policy on the corn market (Wright, 2014; Carter et al., 2016). The model specification also includes interaction effects between stock S_{t-1} and lagged prices, allowing stock to affect price dynamics. And it includes the square of lagged stock S_{t-1}^2 reflecting possible nonlinear stock effects. As showed in table 2, the AR models have good explanatory power: the R-square varies between 0.952 and 0.975. A number of lagged prices have coefficients that are statistically significant, documenting the importance of price dynamics. Also, some of the coefficients associated with lagged stock S_{t-1} are statistically significant, providing evidence that stock affects prices. Finally, table 2 shows the time trend T_2 and seasonality have significant effects on corn price.

Table 2 reports the Bayesian Information Criterion (BIC) applied to AR(m) models for different lags m. According to the BIC criterion, an AR(2) process provides the best representation of dynamics for both wheat price and corn price. This indicates that lagged-one and lagged-two prices capture the relevant dynamics in these markets. On that basis, we proceed with our analysis with models including two lags.

Next, we estimate a quantile autoregression QAR(2) model (6) with two lags. The results are reported in Tables 3-4 for wheat price and corn price, respectively, for selected quantiles $\tau = (0.1, 0.3, 0.5, 0.7, 0.9)$.[9] Again, the results show that lagged-one and lagged-two prices are often statistically significant, documenting the importance of dynamics. Also, Table 3-4 report that lagged stock S_{t-1} can affect prices, although the effects vary across quantiles. We formally test whether the parameter estimates vary across quantiles. This is reported in table 5. For both wheat and corn, the test results find strong evidence that the parameters vary across quantiles, with a p-value less than 0.001. This indicates that price dynamics vary in different parts of the price distribution. Table 5 also reports two additional tests: 1/ testing that seasonality matters; and 2/ testing that stock S_{t-1} affects prices. The test results present strong statistical evidence that seasonality is important and that stock affect prices. Interestingly, the evidence that stock matters is weaker in the upper quantile (e.g., $\tau = 0.9$). For wheat, the stock effect is not statistically significant around the mean or median. But for both wheat and corn, such effects are highly significant in the lower tail of the price distribution (e.g., $\tau = 0.3$). This documents that stock effects are important; but they vary in different parts of the price distribution.

To evaluate these effects, we re-estimated the QAR(2) model (6) for all quantiles for both wheat and corn. The estimated quantile functions (and their associated distribution functions) were simulated under alternative conditions. Figure 2 reports the estimated distribution functions for wheat price and corn price for selected year. Defining relative quantiles as predicted quantiles divided by the median quantile, Figure 3 shows the evolution of relative quantiles over time, illustrating temporal changes in the price distribution and the recent increase in price volatility for both wheat and corn prices.

Next, we simulated the price distributions under alternative stock scenarios. We consider three scenarios: low stock, medium stock and high stock, corresponding respectively to the 0.2, 0.5 and 0.8 quantile of the stock distribution from the sample data. All scenarios are evaluated at the point corresponding to January 2000.[10] The impacts of different stock levels and the price distributions are reported in Figure 4 for wheat and corn. Figure 4 shows that increasing stock S_{t-1} tends to shift the price distribution to the left for wheat and corn. This is intuitive: having larger initial stock increases the quantity currently available, putting downward pressure on prices. Figure 4 shows that these effects are moderate. Also, Figure 4 shows that such effects tend to be larger in the lower tail of the price distribution; and they tend to larger for corn than for wheat.

Table 6 reports the summary statistics of price distributions presented in Figure 4 under different levels of aggregate stock. For both wheat and corn, storage is found to decrease mean prices, shifting the price distributions to the left. Again, a higher aggregate stock tends to put downward pressure on the market price. The impacts of storage on variance, skewness and kurtosis are more complex, with distinct differences between the wheat market and the corn market. For wheat, increasing stock first increases variance and kurtosis up to a point and then starts to decease them under a high stock level. Note that both skewness and kurtosis are positive and statistically significant for wheat. For corn, kurtosis is positive and statistically significant.[11] These results indicate that prices depart from normality, with strong evidence of thick tails. The finding of positive skewness for wheat price is consistent with previous studies documenting that price distributions are often asymmetric, with a higher probability of facing price increases than price decreases (Gustafson, 1958; Williams and Wright, 1991; Deaton and Laroque.1992, 1996; Cafiero et al., 2011). Table 6 shows that skewness decreases with higher stock (especially for wheat). We also find that increasing storage tends to increase both the variance and kurtosis of corn price.

Table 6 reports that storage can have different effects across the two commodities, wheat and corn. We expect the demand for wheat to be more price-inelastic than the demand for corn (as wheat is used to feed people while corn is used in large part as animal feed). This indicates that supply shocks would have larger impacts on wheat price than corn price. This argument helps explain why wheat exhibits greater price volatility than corn (including larger variance and skewness in Table 6). In addition, as showed in Figure 1, the stock-to-use ratio for wheat has been much higher than for corn. This helps explain why a higher stock tends to reduce skewness more for wheat than for corn.

Finally, we investigated price dynamics by evaluating the dominant root $|\lambda_1|$ associated with our estimated model. The results are reported in Figure 5 for wheat and corn prices for different stock levels. Figure 5 shows the root $|\lambda_1|$ is less than 1 around the median and in the lower tail of the price distribution, but it is greater than 1 in the upper tail. Recall that $|\lambda_1|$ < 1 (> 1) corresponds to local stability (local instability). Thus, for both wheat and corn, Figure 5 shows evidence of local price instability in the upper tail of the price distribution. In addition, Figure 5 shows that higher stock S_{t-1} tends to reduce $|\lambda_1|$ in the upper tail of the distribution. This is one of our key results: stock has important effects on price dynamics especially in the presence of price spikes (when prices are in the upper tail of the price distribution). Indeed, Figure 5 shows that higher stock reduces the local instability of price dynamics in the presence of price spikes. This issue will be revisited below when we distinguish between private and public stocks.

7. Analysis of private stock versus public stock

In this section, we investigate the separate effects of private and public stock on price dynamics and price volatility. The analysis presented below now distinguishes between the private and public stock, corresponding conceptually to equation (4b) and empirically to equation (6). Investigating whether private stock and public stock have similar effects on commodity prices has been investigated in previous research (e.g., Newbery and Stiglitz, 1981; Wright and Williams, 1982b; Williams and Wrights, 1991). As we document below, such effects often take place in the tails of the price distribution. This is a context where standard regression analysis (e.g., using least squares) is not particularly informative, while our QAR approach can be very useful.

As shown in Figure 1, US public stock of wheat and corn has become zero toward the end of our sample period. This occurred around 2010 for wheat and 2005 for corn. This is problematic for our analysis of public stock effects. Indeed, when a variable ceases to vary, the associated data points in the sample no longer provides information to estimate its impact. Our analysis of public stock effects should be based on sample information where public stock varies. On that basis, the empirical investigation presented in this section does not include observations in the later part of the sample (when public stock is always zero). Our estimation of joint public/private stock effects is thus limited to the period 1980-2010 for wheat and 1980-2005 for corn.

Table 7 and Table 8 report the econometric results for wheat and corn prices, respectively. Consistent with the results discussed above, lagged prices and some of the coefficients associated with private stock Sr_{t-1} and public stock Su_{t-1} are statistically significant, indicating the importance of price dynamics and stock. To quantify the different effects of private and public stock, we conducted a series of hypothesis tests (HT) about whether the coefficients associated with private stock Sr_{t-1} and public stock Su_{t-1} are statistically equal. As discussed above, the equality of these coefficients corresponds to testing whether private stock and public stock are perfect substitutes. Table 9 reports the test results.

For both wheat and corn, Table 9 shows strong evidence that the effects of private stock and public stock have statistically different effects on price. For wheat, the coefficients of private stock Sr_{t-1} and public stock Su_{t-1} differ significantly across quantiles. For instance, as shown in Table 7 for the 0.1 quantile, the coefficients of private stock and public stock in the wheat price equation are -52.321 and -24.100, respectively. They are each significant at 1 percent level; and they are statistically different from each other (from test HT1 in table 9). And for corn, the coefficients of Sr_{t-1}^2 and Su_{t-1}^2 are statistically different for most quantiles. For example, Table 8 shows, in the 0.3 quantile, the coefficients of ${Sr_{t-1}}^2$ and ${Su_{t-1}}^2$ are 64.8 and 27.8. Each coefficient is statistically significant at 5 percent level, and they are statistically different from each other (from test HT2 in Table 9). For both wheat and corn, the joint test (called HT5 in Table 9) shows that private stock and public stock have different impacts on the price distribution for most quantiles. This is one of our key findings. Private stock and public stock are not perfect substitutes: they each have different impacts on price volatility, including effects on both the lower tail and upper tail of the price distribution. Importantly, such results would not be obtained from standard regression results (e.g., Table 9 reports that, under Least Squares, HT5 shows no statistical differences between private and public stocks). In other words, our QAR approach shows that the differences between private and public stock effects come in large part from impacts on the tails of the price distributions. The exact nature of these differences is discussed next.

We evaluate the effects of private stock and public stock by simulating the price distributions under different stock scenarios. We consider three scenarios for private stock Sr_{t-1} and public stock Su_{t-1}: low, medium and high, again corresponding respectively to the 0.2, 0.5 and 0.8 quantile of the stock distribution from the sample data. All scenarios are evaluated at the point corresponding to January 2000.

Figure 6 plots the simulated distribution functions of wheat price and corn price under different private stock levels. We find that increasing private stock tends to shift the whole price distribution (including median and both tails) to the left. This result is obtained for both wheat and corn. It means that private stockholding has a uniformly negative impact on price across all quantiles. This is a key difference with the public stock effects in Figure 7, which documents that public stock shifts the price distribution to the right. Recall (from Section 6) that the negative impact of stock on price was also obtained at the aggregate level. This indicates that the price effects of stocks are dominated by the role of private stock. Again, our analysis shows that higher private stock increases current available supply and puts downward pressure on prices. Finding that such effects apply across quantiles indicate that private stockholding reduces the probability of price spikes. To the extent that decreasing price spikes is desirable, our investigation shows that private stockholding can help attain this objective.

But the leftward shift in the lower tail of the distribution indicates that private stock increases exposure to low prices. It means that private stockholding does not help reduce large price declines. Why is it that private stock managers do not take advantage of low prices to increase their stocks? This may reflect that private stockholders exhibit a high discount rate, indicating that they generate limited prospects to smooth price fluctuations over time.

Figure 7 plots the simulated distribution functions of wheat price and corn price under different public stock levels. In contrast with the effect of private stock shown in Figure 6, increasing public stock tends to shift the lower tail of the price distributions to the right for both wheat and corn. This indicates that public stock policies tend to reduce the probability of facing low prices. This may reflect that buffer stock policies take advantage of low prices to increase public stock. This may also reflect that public stock programs are often put in place to support commodity prices and farm income when those are low. However, in this case, increasing public stock level shifts the upper tail to the right as well. This is a scenario where large public stock increases the probability of price spikes. It can be seen as an example where public stockholding is not contributing to stabilizing the market. To the extent that stabilizing commodity markets is an objective of public stock programs, our analysis indicates that the high levels of public wheat stock observed in the 1980's were excessive.

Table 10 presents summary statistics of the simulated price distributions under different private and public stock levels (corresponding to Figure 6 and Figure 7). First, Table 10 reports strong statistical evidence of positive skewness and kurtosis for wheat price and corn price. This documents that price distributions are right-skewed and with presence of thick tails.

Second, for wheat, Table 10 shows that a higher private stock decreases the mean, skewness and kurtosis of price and increases its variance. Note that the price effects of public stock are in opposite directions: public stock increases the mean, skewness and kurtosis of price and decreases its variance. This is another illustration that private and public stock have very different impacts on price volatility. Also, note that the contribution of public wheat stock to increasing the probability of facing price spikes comes from the increase in skewness (which works against the decrease in variance). This stresses the importance of going beyond a simple mean-variance analysis. Third, for corn, Table 10 shows that the price effects of private and public stock are often similar to wheat, but of much smaller magnitude. This reflects a relatively more important role of storage in the wheat market than in the corn market.

Finally, we investigated price dynamics and evaluated the dominant root $|\lambda_1|$ associated with our estimated model. The results are reported in Figure 8 for different private stock levels and in Figure 9 for different public stock levels, respectively. From the two figures, we find that the private stock has little effect on $|\lambda_1|$, while higher public stock tends to reduce $|\lambda_1|$ in the upper tail of the distribution. This finding applies to both wheat and corn, indicating that private and public stocks have different effects on price dynamics.

To further evaluate the effects of private vs. public stock, we calculated the dominant roots $|\lambda_1|$ based on 5 scenarios involving different proportions of private and public stock, holding aggregate stock constant.[12] The scenarios are: 1/ all private stock; 2/ 75% private stock, 25% public stock; 3/ 50% private stock, 50% public stock; 4/ 25% private stock, 75% public stock; 5/ all public stock. Figure 10 plots the modulus of the dominant root for the dynamics of wheat price and corn price under these 5 scenarios. First, noting that $|\lambda_1| > 1$ corresponds to local instability, Figure 10 indicates the presence of local instability in the upper tail of the price distribution. Second, it shows that higher proportion of public stock tends to reduce the dominant root $|\lambda_1|$ in the upper tail of the price distribution for both wheat and corn. This means that public stockholding tends to lower local instability in the presence of price spikes. Alternatively, private stockholding increases local instability under price spikes. A region of local instability means a dynamic escape away from that region. This gives us another of our key results: compared to public stockholding, private stockholding is associated with a faster dynamic escape from price spikes. It indicates that, when faced with high prices, public stock managers may be more reluctant to reduce their stock than private stock managers. Consistent with what we found in Figure 6 and Figure 7, it means that private stock would perform better than public stock in avoiding price spikes. To the extent that avoiding price spikes is seen as desirable, our analysis shows that private stockholding can offer good options in reducing the odds of facing high prices (as long as private stocks remain positive). And as discussed above, our results indicate that public stockholding does not necessarily decrease the odds of price spikes. This result must be seen as a significant challenge for public stock managers.

8. Conclusion

This paper has presented an economic analysis of the effects of storage on commodity price dynamics and price volatility. The investigation applies under general supply-demand conditions, with a special focus on possible differences between public stock and private stock. The analysis relies on quantile autoregression (QAR) as a flexible representation of dynamics in the price distribution. Applied to US wheat and corn markets over the period 1980-2014, our analysis documented the role of storage and its effects on prices.

First, we found that the price effects of storage are statistically significant but they vary in different parts of the price distribution. Stocks tend to have stronger impacts in the tails of the price distribution. This stresses that studying the effects of storage just on mean prices is too narrow (as it would fail to capture the effects of the storage on price volatility). Second, we found that increasing aggregate stock tends to shift the price distribution to the left for wheat and corn. This is intuitive: having larger initial stock increases the quantity currently available, putting downward pressure on prices. Third, we investigated the relative effects of private stock and public stock on commodity prices. We found strong statistical evidence that private stock and public stock have different effects on price dynamics and price volatility (including variance, skewness and kurtosis). In other words, private and public stock are not perfect substitutes. Our analysis presented evidence that private stockholding increases the probability of price crashes (reflecting an unwillingness of private stockholders to buy when the price is low), while public stockholding increases the probability of price spikes (reflecting an unwillingness of public stockholders to sell when the price is high). If reducing price volatility means avoiding both price crashes and price hikes, this indicates some limitations in the role of storage (either private or public) in smoothing price fluctuations over time. Fourth, we found many differences of storage effects across markets. The wheat market exhibits greater price volatility than the corn market. This probably reflects a more inelastic demand for food than for feed. Public stock effects were estimated to be much larger for wheat than for corn. Our investigation also indicated that large public stock of wheat (e.g., as observed in the 1980's) did not reduce the odds of facing high wheat prices. Fifth, we studied the effects of storage on price dynamics. We uncovered evidence of local dynamic instability in the upper tail of the price distribution. We also found that the private/public stock portfolio affects this instability. In situations where stocks are positive, our analysis indicated that, compared to public stockholding, private stockholding offered better options in reducing the odds of price spikes. To the extent that avoiding price spikes is seen as desirable, this finding seems to be a significant challenge for public stock managers.

The analysis presented in this paper could be expanded in several directions. First, our empirical findings are specific to the US wheat and corn markets. There is a need to present applications to other commodity markets. Second, given the different effects of private and public stocks found in this paper, the linkages between storage and economic welfare need further investigations. Third, it would be useful to expand the analysis of storage in a multi-commodity framework (e.g., following Nishimura and Stachurski, 2009). Finally, the issue of designing policies that could help improve food security remains an important topic for future research.

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Table 1: Summary statistics for price and storage data, 1980-2014.

Commodity	Variable	Statistics			
		Mean	S.D.	Max	Min
Wheat	Total stock (S_t)	0.733	0.401	1.823	0.218
	Private stock (Sr_t)	0.460	0.165	0.859	0.105
	Public stock (Su_t)	0.273	0.373	1.377	0
	Market price (P_t)	165.806	55.958	403.810	88.550
Corn	Total stock (S_t)	0.239	0.184	0.890	0.050
	Private stock (Sr_t)	0.170	0.103	0.558	0.020
	Public stock (Su_t)	0.069	0.124	0.541	0
	Market price (P_t)	135.778	58.013	332.950	65.350

Note: Wheat and corn prices are collected from IMF database (unit: US dollar per metric ton). Wheat and corn stock data are collected from WASDE database, USDA. The stock-to-use ratios are measured as the level of total (private, public) stock as a percentage of total use.

Table 2: Parameter estimates of selected AR processes

Variable	Parameter Estimates							
	Wheat				Corn			
	AR(1)	AR(2)	AR(3)	AR(4)	AR(1)	AR(2)	AR(3)	AR(4)
Intercept	14.575**	16.113**	14.316**	14.463**	14.152***	14.936***	15.887***	17.185***
	(7.100)	(7.004)	(7.117)	(7.185)	(4.928)	(4.176)	(4.187)	(4.210)
P_{t-1}	0.944***	1.173***	1.218***	1.218***	0.970***	1.194***	1.160***	1.144***
	(0.033)	(0.123)	(0.127)	(0.127)	(0.021)	(0.077)	(0.079)	(0.079)
P_{t-2}		-0.250*	-0.476**	-0.479**		-0.237***	-0.071	-0.079
		(0.129)	(0.202)	(0.204)		(0.076)	(0.123)	(0.124)
P_{t-3}			0.195	0.213			-0.141*	-0.009
			(0.135)	(0.207)			(0.079)	(0.124)
P_{t-4}				-0.016				-0.119
				(0.137)				(0.080)
S_{t-1}	-22.015	-23.814*	-21.597	-21.850	-15.443	-13.100	-13.754	-14.609
	(13.680)	(13.407)	(13.497)	(13.577)	(20.078)	(19.503)	(19.467)	(19.399)
S_{t-1}*P_{t-1}	0.035*	0.055	0.008	0.005	-0.184	-0.131	-0.075	-0.047
	(0.051)	(0.209)	(0.214)	(0.215)	(0.113)	(0.383)	(0.402)	(0.401)
S_{t-1}*P_{t-2}		0.004	0.245	0.259		-0.050	-0.346	-0.388
		(0.216)	(0.340)	(0.344)		(0.374)	(0.625)	(0.638)
S_{t-1}*P_{t-3}			-0.211	-0.272			0.251	0.248
			(0.227)	(0.348)			(0.394)	(0.635)
S_{t-1}*P_{t-4}				0.051				0.027
				(0.278)				(0.397)
S_{t-1}^2	8.000	7.521*	7.635*	7.592*	25.910*	23.129	22.221	22.023
	(4.440)	(4.315)	(4.311)	(4.328)	(14.432)	(14.101)	(14.082)	(14.067)
T_1	0.108	0.148*	0.132	0.130	-0.105	-0.122	-0.132*	-0.145*
	(0.091)	(0.088)	(0.089)	(0.089)	(0.080)	(0.078)	(0.078)	(0.078)
T_2					0.621	0.934**	1.108***	1.308***
					(0.381)	(0.376)	(0.383)	(0.392)
Q_1	-2.780	-2.331	-2.165	-2.181	-1.994	-1.718	-1.733	-1.985
	(1.734)	(1.689)	(1.698)	(1.704)	(1.309)	(1.272)	(1.271)	(1.271)
Q_2	0.181	0.599	0.812	0.743	-5.511***	-4.589***	-4.522***	-4.587***
	(1.724)	(1.683)	(1.722)	(1.740)	(1.300)	(1.286)	(1.289)	(1.284)
Q_3	-0.067	-0.006	0.232	0.214	-2.584**	-2.030	-1.851	-1.651
	(1.724)	(1.676)	(1.682)	(1.690)	(1.301)	(1.268)	(1.287)	(1.305)
R-square	0.9526	0.9555	0.9558	0.9558	0.9750	0.9766	0.9768	0.9771
BIC	3318.3	3304.1	3312.8	3324.7	3087.4	3072.9	3080.2	3086.8

NOTE. — Standard errors are in parentheses below the corresponding parameter estimates. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 3: Quantile regression estimates of the wheat price for selected quantiles, 1980-2014.

Variable	LS	Quantile regression				
		τ = 0.1	τ = 0.3	τ = 0.5	τ = 0.7	τ = 0.9
Intercept	11.748*	25.729***	13.262***	6.670	4.322	-10.888
	(7.106)	(9.281)	(4.130)	(5.016)	(6.523)	(9.222)
P_{t-1}	1.215***	1.030***	1.294***	1.248***	1.235***	1.612***
	(0.133)	(0.188)	(0.093)	(0.114)	(0.174)	(0.233)
P_{t-2}	-0.277**	-0.182	-0.376***	-0.273**	-0.235	-0.498**
	(0.134)	(0.192)	(0.084)	(0.106)	(0.176)	(0.229)
S_{t-1}	-14.862	-29.051**	-10.411**	-5.453	-5.044	1.461
	(13.287)	(11.499)	(5.235)	(6.819)	(8.842)	(10.716)
S_{t-1}*P_{t-1}	-0.019	0.067	-0.152	-0.019	0.095	-0.328
	(0.219)	(0.244)	(0.110)	(0.148)	(0.232)	(0.226)
S_{t-1}*P_{t-2}	0.053	0.020	0.176*	0.014	-0.091	0.292
	(0.222)	(0.244)	(0.101)	(0.136)	(0.234)	(0.228)
S_{t-1}^2	4.921	7.411*	3.494**	2.759	1.599	1.707
	(4.179)	(4.165)	(1.643)	(1.823)	(2.932)	(3.062)
T	0.145*	-0.157	-0.036	0.029	0.111	0.361***
	(0.088)	(0.100)	(0.058)	(0.074)	(0.088)	(0.127)
Q_1	-0.184	-1.914**	-1.493*	-0.962	-1.022	-0.986
	(1.681)	(0.914)	(0.815)	(0.849)	(1.177)	(1.044)
Q_2	-2.160	-5.892***	-2.388***	-1.872	-1.098	-0.427
	(1.678)	(1.964)	(0.560)	(1.145)	(1.208)	(2.090)
Q_3	0.552	-2.501***	-1.932*	-0.151	1.709	3.427*
	(1.678)	(1.185)	(1.119)	(1.142)	(1.439)	(2.043)

NOTE. — Standard errors are in parentheses below the corresponding parameter estimates. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 4: Quantile regression estimates of the corn price for selected quantiles, 1980-2014.

Variable	LS	Quantile regression				
		τ = 0.1	τ = 0.3	τ = 0.5	τ = 0.7	τ = 0.9
Intercept	10.643***	11.260**	15.552***	0.695***	5.374	2.197
	(3.888)	(5.414)	(3.581)	(3.034)	(4.356)	(7.397)
P_{t-1}	1.204***	1.168***	1.163***	1.175***	1.368***	1.562***
	(0.077)	(0.122)	(0.095)	(0.083)	(0.113)	(0.197)
P_{t-2}	-0.238***	-0.241*	-0.251**	-0.223***	-0.368***	-0.491**
	(0.077)	(0.133)	(0.098)	(0.080)	(0.103)	(0.190)
S_{t-1}	-7.548	8.506	-22.046**	-9.444	-10.228	10.831
	(17.797)	(23.009)	(11.359)	(9.900)	(10.405)	(19.801)
$S_{t-1}*P_{t-1}$	-0.158	-0.328	-0.046	0.077	-0.346	-1.297*
	(0.380)	(0.542)	(0.407)	(0.231)	(0.400)	(0.770)
$S_{t-1}*P_{t-2}$	-0.026	0.052	-0.001	-0.154	0.272	1.012
	(0.377)	(0.578)	(0.400)	(0.220)	(0.370)	(0.750)
S_{t-1}^2	19.187	5.924	20.122***	12.331**	16.066***	16.906
	(13.264)	(13.013)	(7.361)	(6.247)	(5.842)	(11.300)
T_1	-0.098	-0.215***	-0.179***	-0.070	0.006	0.028
	(0.076)	(0.053)	(0.046)	(0.042)	(0.051)	(0.094)
T_2	0.795**	0.715	1.1**39	0.730**	0.612	1.042
	(0.369)	(0.641)	(0.445)	(0.219)	(0.537)	(0.903)
Q_1	1.859	0.652	0.802	1.697**	0.745	-1.339
	(1.269)	(0.520)	(0.704)	(0.588)	(0.790)	(0.929)
Q_2	0.277	-0.653	-0.915	-0.188	-0.694	-2.259**
	(1.269)	(0.482)	(0.740)	(0.544)	(0.877)	(1.142)
Q_3	-2.457*	-6.046***	-3.511***	-1.254	0.387	-1.382
	(1.271)	(1.241)	(1.001)	(1.072)	(1.201)	(1.442)

NOTE. — Standard errors are in parentheses below the corresponding parameter estimates. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 5: Hypothesis testing for quantile effects, seasonality and storage effects: a comparison between Least Squares and Quantile Regression.

Testing items	Estimate method		Wheat P-value	Corn P-value
Same coefficients across quantiles	QR		0.000***	0.000***
Seasonality	LS		0.390	0.010***
	QR	τ=0.1	0.005***	0.000***
		τ=0.3	0.001***	0.000***
		τ=0.5	0.340	0.002***
		τ=0.7	0.262	0.165
		τ=0.9	0.130	0.182
Storage effects	LS		0.813	0.011
	QR	τ=0.1	0.143	0.003***
		τ=0.3	0.048**	0.000***
		τ=0.5	0.538	0.001***
		τ=0.7	0.916	0.001***
		τ=0.9	0.602	0.015**

NOTE.— Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 6: Summary of simulated price distributions under different aggregate stock levels.

Simulated item	Scenario	Wheat Mean	Wheat Variance	Wheat Skewness	Wheat Kurtosis	Corn Mean	Corn Variance	Corn Skewness	Corn Kurtosis
Total stock	Low stock	167.415	80.073	0.542*** (0.000)	0.512*** (0.002)	136.037	42.398	-0.058 (0.478)	0.926*** (0.000)
	Medium stock	166.794	84.688	0.480*** (0.000)	0.535*** (0.001)	134.998	43.785	-0.071 (0.380)	1.058*** (0.000)
	High stock	166.277	81.746	0.435*** (0.000)	0.511*** (0.002)	133.535	46.342	-0.106 (0.190)	1.257*** (0.000)

NOTE. — 1. P-values are in parentheses below the corresponding skewness and kurtosis.

2. The kurtosis in this table refers to "excess kurtosis" with the value 0 for the normal distribution.

3. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 7: Quantile regression estimates of the wheat price for selected quantiles: private stock vs. public stock, 1980-2010.

Variable	LS	Quantile regression				
		τ = 0.1	τ = 0.3	τ = 0.5	τ = 0.7	τ = 0.9
Intercept	4.830	29.493***	22.384***	7.058	13.933	-13.251
	(9.921)	(8.920)	(8.313)	(9.300)	(10.218)	(15.513)
P_{t-1}	1.192***	0.959***	1.156***	1.373***	1.313***	1.319***
	(0.162)	(0.182)	(0.206)	(0.224)	(0.200)	(0.214)
P_{t-2}	-0.218	-0.096	-0.296	-0.404*	-0.354*	-0.225
	(0.170)	(0.210)	(0.192)	(0.215)	(0.209)	(0.227)
Sr_{t-1}	-37.527*	-52.321***	-40.550**	-33.624*	-50.451***	-17.300
	(22.523)	(17.241)	(19.363)	(19.902)	(18.030)	(30.514)
Su_{t-1}	11.195	-24.100***	-1.414	8.063	6.296	33.523**
	(12.091)	(9.127)	(4.283)	(4.919)	(7.383)	(13.203)
${Sr_{t-1}}^2$	37.749**	66.438***	27.542**	33.009***	31.567***	-1.381
	(17.856)	(17.024)	(13.264)	(11.182)	(10.247)	(18.883)
${Su_{t-1}}^2$	-10.985	-14.022***	-6.306***	-5.719*	-3.431	-2.542
	(7.650)	(4.590)	(2.233)	(3.281)	(4.766)	(8.836)
$Sr_{t-1} * P_{t-1}$	-0.048	0.079	-0.240	-0.347	-0.078	0.082
	(0.308)	(0.306)	(0.441)	(0.474)	(0.382)	(0.564)
$Sr_{t-1} * P_{t-2}$	-0.028	-0.247	0.251	0.310	0.171	-0.027
	(0.323)	(0.375)	(0.401)	(0.449)	(0.392)	(0.591)
$Su_{t-1} * P_{t-1}$	0.010	0.060	0.065	0.055	0.120	-0.373*
	(0.278)	(0.165)	(0.084)	(0.103)	(0.171)	(0.199)
$Su_{t-1} * P_{t-2}$	0.059	0.236	0.042	-0.029	-0.127	0.209
	(0.277)	(0.163)	(0.078)	(0.104)	(0.172)	(0.248)
T_1	0.594***	0.007	0.306***	0.362***	0.276	0.834***
	(0.201)	(0.175)	(0.113)	(0.135)	(0.180)	(0.315)
Q_1	-0.707	-1.924*	-2.162***	-0.046	-0.671	-3.971*
	(1.707)	(1.094)	(0.702)	(0.716)	(0.963)	(2.292)
Q_2	-2.399	-4.433**	-3.177***	-1.924**	-1.324	-2.554
	(1.713)	(2.191)	(0.617)	(0.837)	(1.397)	(2.117)
Q_3	0.066	-2.668**	-2.919***	0.400	2.319*	1.717
	(1.723)	(1.055)	(0.819)	(1.206)	(1.212)	(1.958)

NOTE.— Standard errors are in parentheses below the corresponding parameter estimates. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 8: Quantile regression estimates of the corn price for selected quantiles: private stock vs. public stock, 1980-2005.

Variable	LS	Quantile regression				
		τ = 0.1	τ = 0.3	τ = 0.5	τ = 0.7	τ = 0.9
Intercept	14.627***	7.109	17.408***	15.788***	2.890	5.998
	(4.353)	(6.794)	(3.305)	(4.710)	(4.171)	(6.303)
P_{t-1}	1.329***	1.344***	1.251***	1.261***	1.408***	1.553***
	(0.105)	(0.161)	(0.078)	(0.116)	(0.082)	(0.154)
P_{t-2}	-0.399***	-0.395***	-0.344***	-0.346***	-0.375***	-0.499***
	(0.104)	(0.141)	(0.075)	(0.111)	(0.077)	(0.147)
Sr_{t-1}	-29.030	31.741	-34.808***	-13.913	-2.142	-14.689
	(22.512)	(29.157)	(10.363)	(18.238)	(16.418)	(34.462)
Su_{t-1}	11.069	22.253	-20.674*	-20.389	1.093	44.486**
	(19.336)	(26.046)	(11.615)	(16.435)	(17.794)	(21.831)
${Sr_{t-1}}^2$	59.604	36.201	64.795***	49.254***	49.974***	50.886
	(22.773)	(28.922)	(14.400)	(18.668)	(12.161)	(32.946)
${Su_{t-1}}^2$	-7.105	-52.912	27.880**	18.742	25.547	21.537
	(23.386)	(32.225)	(14.154)	(15.576)	(17.660)	(37.697)
$Sr_{t-1} * P_{t-1}$	-0.354	-1.256	-0.240	-0.354	-0.669*	-0.859
	(0.571)	(0.819)	(0.268)	(0.442)	(0.373)	(0.845)
$Sr_{t-1} * P_{t-2}$	0.240	0.673	0.191	0.174	0.398	0.659
	(0.567)	(0.785)	(0.231)	(0.398)	(0.360)	(0.789)
$Su_{t-1} * P_{t-1}$	-0.603	-0.713	-0.236	0.063	-0.226	-1.983***
	(0.399)	(0.693)	(0.219)	(0.431)	(0.881)	(0.450)
$Su_{t-1} * P_{t-2}$	0.433	0.524	0.215	-0.033	0.036	1.435***
	(0.395)	(0.658)	(0.205)	(0.422)	(0.843)	(0.525)
T_1	-0.071	-0.124	-0.180**	-0.130**	-0.023	-0.005
	(0.081)	(0.081)	(0.076)	(0.064)	(0.067)	(0.100)
Q_1	0.635	1.369**	0.531	1.122	0.839	-1.820*
	(0.888)	(0.692)	(0.679)	(0.728)	(0.612)	(0.985)
Q_2	-0.207	-0.148	-0.066	-0.465	-0.209	-3.043**
	(0.884)	(0.504)	(0.609)	(0.659)	(0.840)	(1.519)
Q_3	-2.299	-4.875***	-2.613***	-1.472	-0.002	-0.013
	(0.884)	(0.989)	(0.850)	(0.985)	(1.013)	(1.257)

NOTE.— Standard errors are in parentheses below the corresponding parameter estimates. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 9: Hypothesis testing for the equality of effects of public stock and private stock.

(wheat)

Method		HT1	HT2	HT3	HT4	HT5
LS		0.056*	0.023**	0.920	0.669	0.030
QR	0.1	0.010***	0.071*	0.880	0.890	0.000***
	0.2	0.002***	0.003***	0.685	0.538	0.000***
	0.3	0.021**	0.027**	0.816	0.720	0.000***
	0.4	0.013**	0.011**	0.791	0.883	0.000***
	0.5	0.036**	0.006***	0.300	0.436	0.000***
	0.6	0.008***	0.008***	0.501	0.482	0.000***
	0.7	0.000***	0.001***	0.782	0.419	0.000***
	0.8	0.000***	0.044**	0.636	0.492	0.001***
	0.9	0.056**	0.193	0.009***	0.010***	0.001***

(Corn)

Method		HT1	HT2	HT3	HT4	HT5
LS		0.231	0.012	0.747	0.800	0.172
QR	0.1	0.702	0.001***	0.755	0.934	0.000***
	0.2	0.969	0.048**	0.339	0.479	0.007***
	0.3	0.479	0.013**	0.561	0.327	0.004***
	0.4	0.595	0.107	0.991	0.962	0.051*
	0.5	0.696	0.045**	0.240	0.565	0.013**
	0.6	0.723	0.023**	0.130	0.194	0.015**
	0.7	0.931	0.163	0.649	0.695	0.144
	0.8	0.392	0.024**	0.225	0.181	0.007***
	0.9	0.073*	0.069*	0.055*	0.164	0.001***

NOTE. — HT1: hypothesis testing that coefficients of STu_{t-1} and STr_{t-1} are statistically equal;
HT2: hypothesis testing that coefficients of ${STu_{t-1}}^2$ and ${STr_{t-1}}^2$ are statistically equal;
HT3: hypothesis testing that coefficients of $STu_{t-1} * P_{t-1}$ and $STr_{t-1} * P_{t-1}$ are statistically equal;
HT4: hypothesis testing that coefficients of $STu_{t-2} * P_{t-1}$ and $STr_{t-1} * P_{t-2}$ are statistically equal;
HT5: all the hypotheses above are jointly true.
Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Table 10: Summary of simulated price distributions under different public/private stock levels.

Simulated item	Scenario	Wheat				Corn			
		Mean	Variance	Skewness	Kurtosis	Mean	Variance	Skewness	Kurtosis
Private stock	Low stock	158.158	69.648	1.180*** (0.000)	1.109*** (0.000)	110.472	21.601	1.090*** (0.000)	0.889*** (0.000)
	Medium stock	155.793	75.114	1.090*** (0.000)	0.785*** (0.000)	109.312	22.368	1.260*** (0.000)	1.534*** (0.000)
	High stock	154.725	77.426	1.027*** (0.000)	0.689*** (0.000)	108.327	22.076	1.342*** (0.000)	1.881*** (0.000)
Public stock	Low stock	152.685	81.976	0.756*** (0.000)	0.358** (0.028)	109.641	20.382	0.357*** (0.000)	-0.136 (0.464)
	Medium stock	153.230	80.275	0.806*** (0.000)	0.379** (0.020)	109.474	20.609	0.627*** (0.000)	0.174 (0.347)
	High stock	158.988	74.498	1.445*** (0.000)	1.559*** (0.000)	109.195	21.434	1.048*** (0.000)	1.002*** (0.000)

NOTE. — 1. P-values are in parentheses below the corresponding skewness and kurtosis.

2. The kurtosis in this table refers to "excess kurtosis" with the value 0 for the normal distribution.

3. Asterisks indicate the significance level: * at the 10 percent level, ** at the 5 percent level, and *** at the 1 percent level.

Figure 1: The US stock-to-use ratio (total, private and public) and market price, 1980-2014

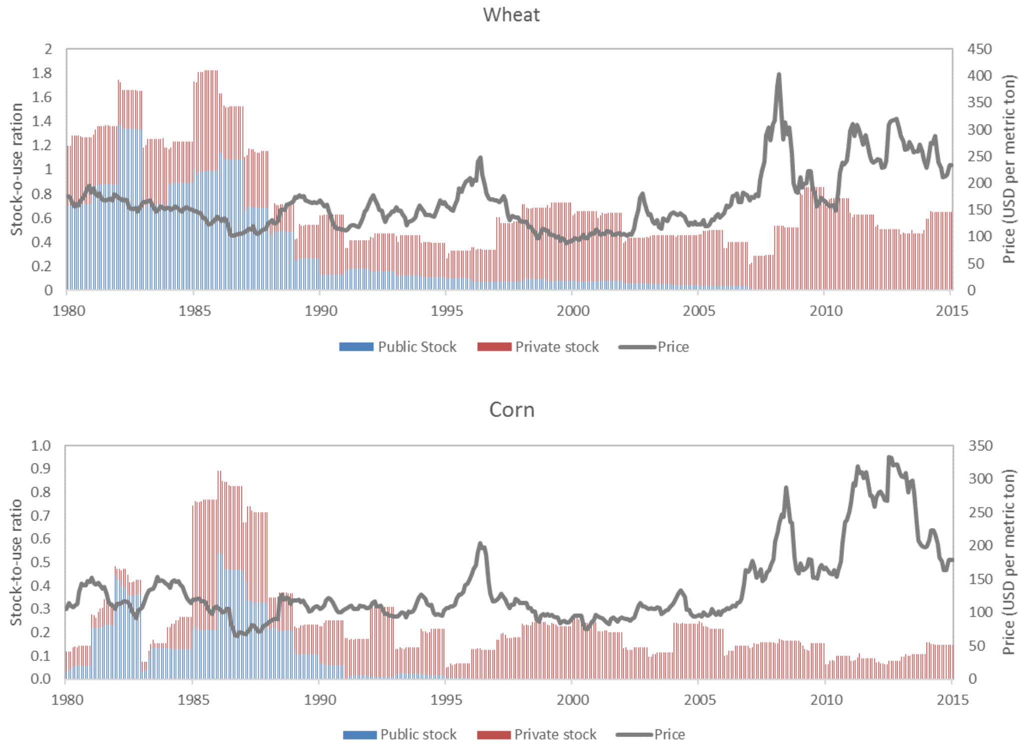


Figure 2: Estimated distribution of the wheat price and corn price in 1990, 2000 and 2010.

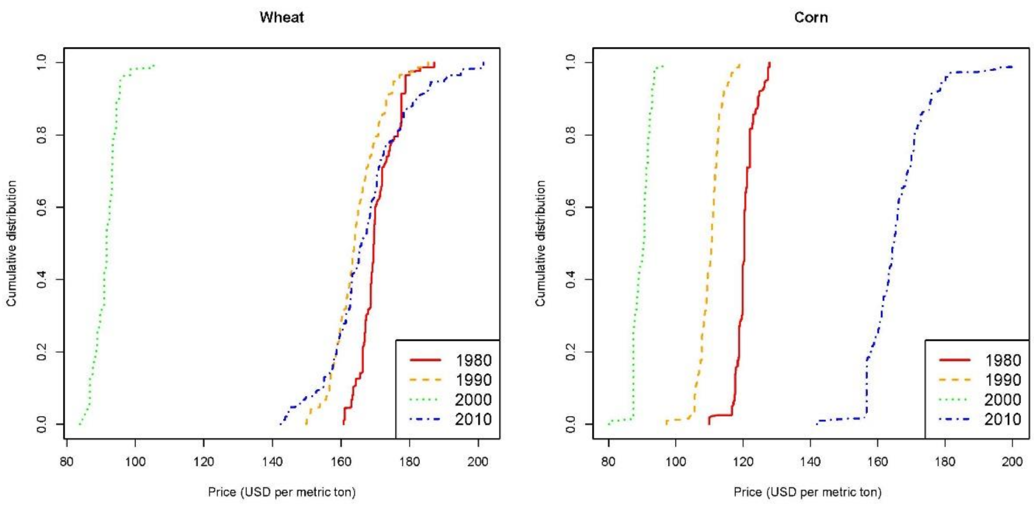


Figure 3: Estimates of relative quantiles for the distribution of wheat price and corn price (relative to the median)

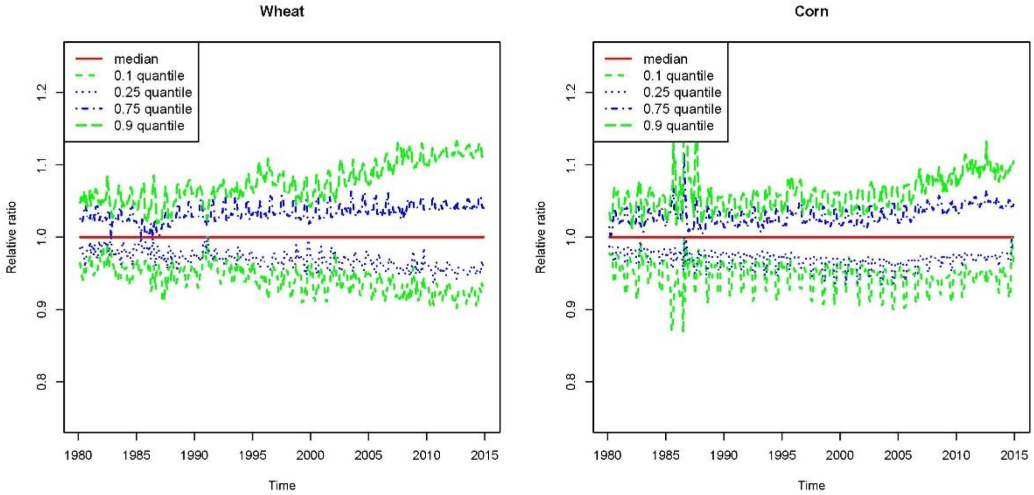


Figure 4: Simulated distribution functions of wheat price and corn price under different aggregate stock levels

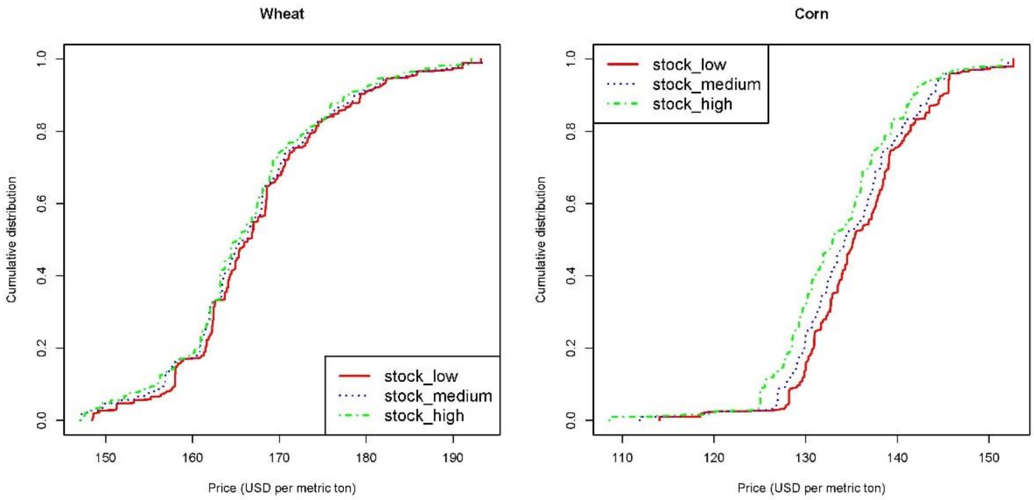


Figure 5: Modulus of the dominant root for the dynamics of wheat price and corn price under different aggregate stock levels

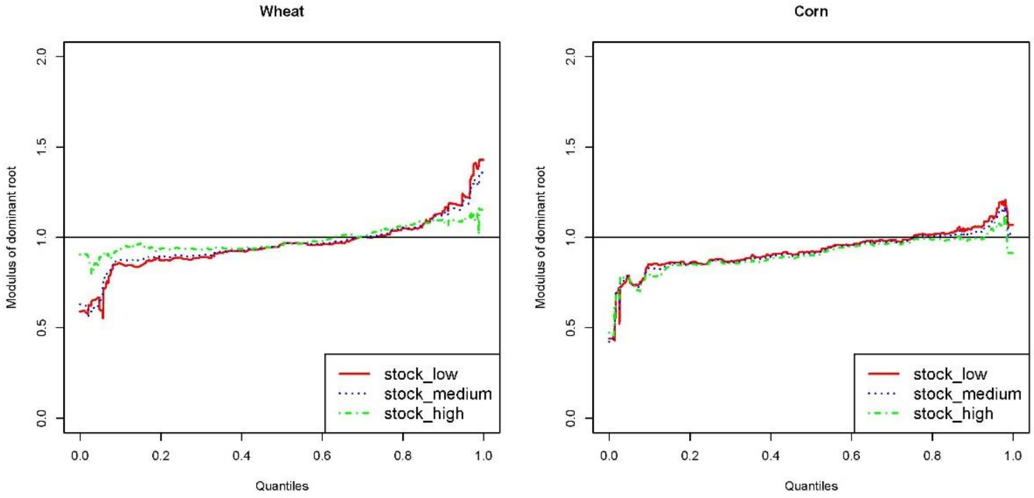


Figure 6: Simulated distribution functions of wheat price and corn price under different private stock levels

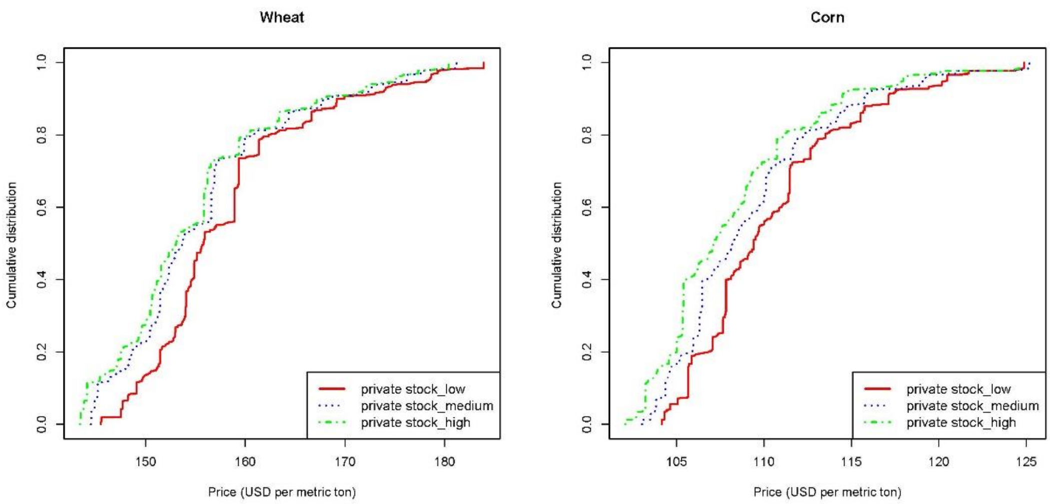


Figure 7: Simulated distribution functions of wheat price and corn price under different public stock levels

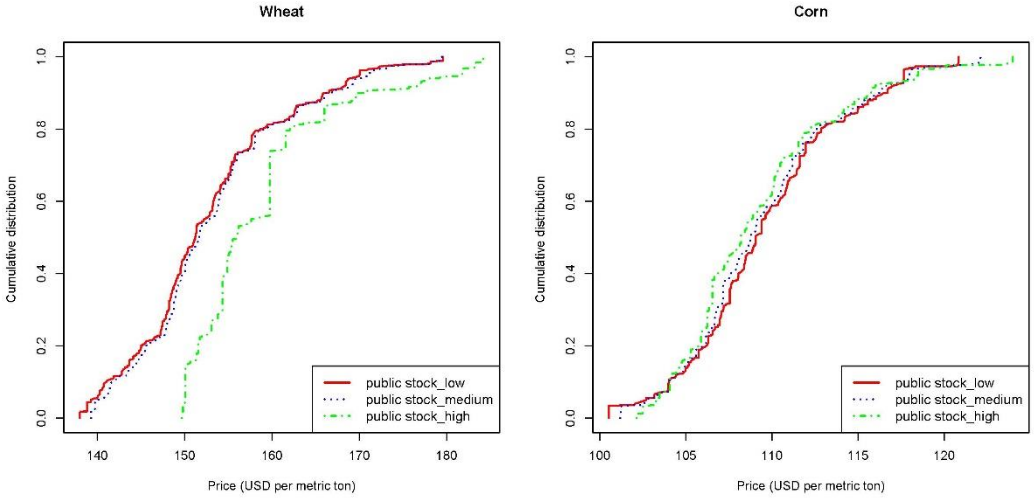


Figure 8: Modulus of the dominant root for the dynamics of wheat price and corn price under different private stock levels

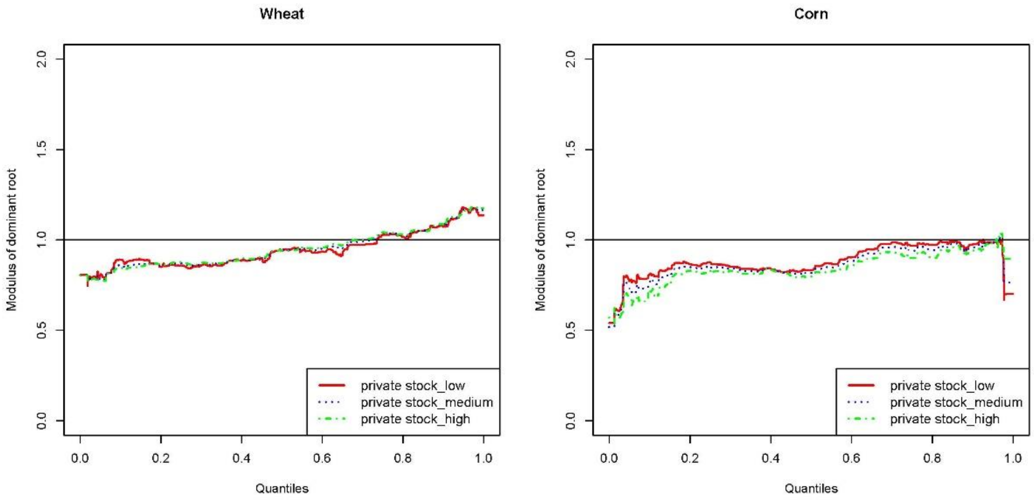


Figure 9: Modulus of the dominant root for the dynamics of wheat price and corn price under different public stock levels

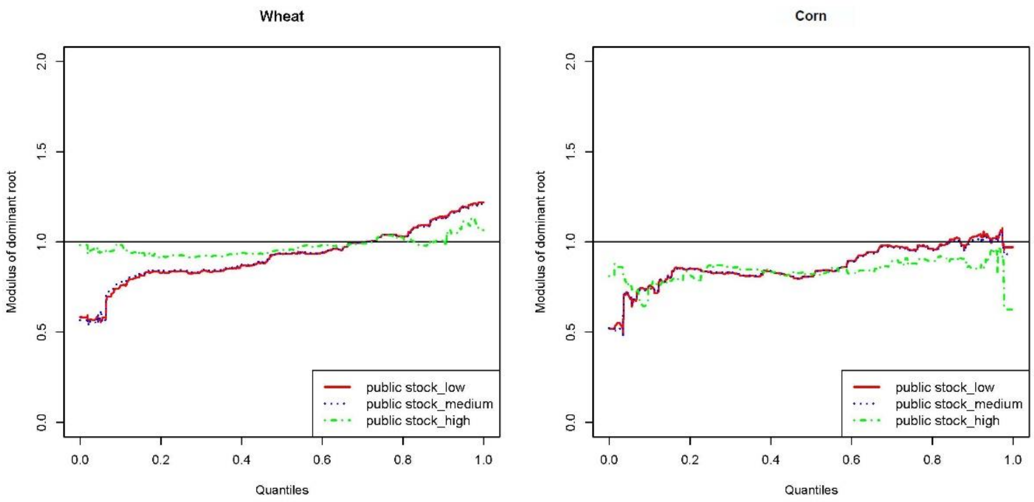
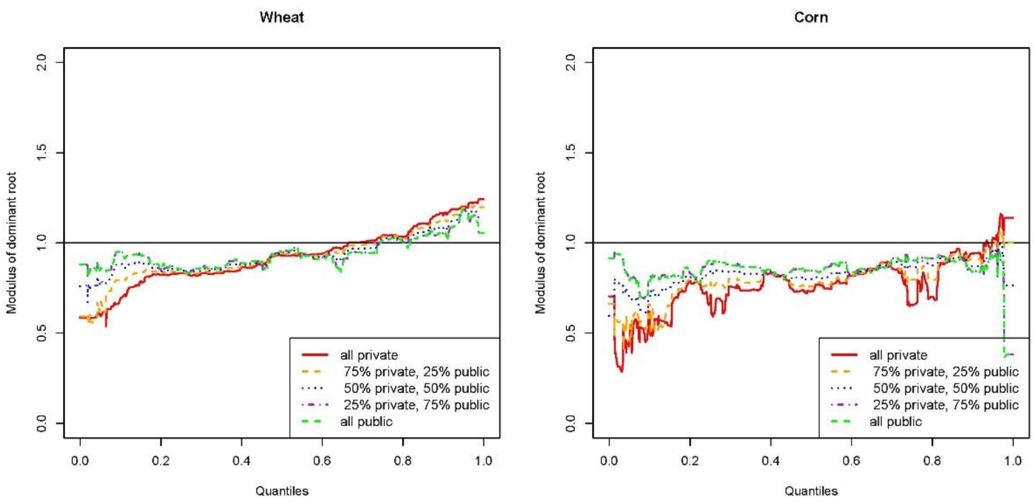


Figure 10: Modulus of the dominant root for the dynamics of wheat price and corn price under different private/public proportions



Footnotes

[1] For example, there is no data on grain stocks held in China.

[2] As discussed below, lagged prices could also capture the role of price expectation in production decisions over time.

[3] Note that the decision rule $S_t(S_{t-1}, P_t, P_{t+1}^e)$ associated with (2) holds under a finite horizon as well as under an infinite horizon (Bertsekas and Shrive, 1996). Importantly, it allows the storage decision rule to change over time. As such, it can capture the effects of seasonality and/or changes in market conditions.

[4] The case where $|\lambda_1(w_{t-1}, S_{t-1}, e_t, t)| = 1$ is a boundary threshold between local stability and local instability. In the special case where $|\lambda_1(\cdot)|$ is constant, this is the case of "unit root" dynamics that has received much attention in the econometric literature (e.g., Enders, 2014).

[5] Note that previous literature has explored various ways to generalize the standard AR(m) model. This includes model specifications allowing for dynamics in variance (e.g., the generalized autoregressive conditional heteroscedastic (GARCH) model proposed by Bollerslev (1986), Markov switching models (Hamilton, 1989)) and nonlinear dynamics (e.g., smooth transition autoregressive (STAR) model and threshold autoregression (TAR) model; see Tong (1990) and Van Dijk et al. (2002)). The QAR(m) in (6) is a flexible way to capture nonlinear dynamics of the price distribution.

[6] Another application of quantile autoregression (QAR) includes Chavas and Li (2017) who investigate the effects of price support policy on agricultural markets in China.

[7] As noted in the introduction, our analysis focuses on US grain stocks for two reasons. First, there is no reliable data on world grain stocks, which prevents us from presenting our analysis at the world level. Second, policy changes over the last two decades have moved the US away from relying on public grain stocks, making our focus on the effects of US private vs. public grain stocks of significant interest to both economists and policy makers. The WASDE stock data present stock estimates conditional on the month in which the estimate is made, reflecting the estimated stock level affected by market information available on a monthly basis.

[8] The WASDE grain stock data report farmer-owned reserve, Commodity Credit Corporation inventory (CCC inventory) and free stocks from 1980 to 1998, and report CCC inventory and free stocks after 1998. In this study, we consider farmer-owned reserve and CCC inventory as public stocks, and consider free stocks as private stocks.

[9] The standard errors reported in Table 4 were obtained from the asymptotic distribution of the quantile estimator (Koenker, 2005). We also evaluated bootstrapped standard errors and found that they gave similar results. Such a comment also applies to Tables 7 and 8.

[10] To check the robustness of our analysis, we considered alternative evaluation points. Our qualitative findings remained similar.

[11] Somewhat surprisingly, Table 6 shows that skewness of corn price is not statistically significant. This result is obtained under aggregate stock. Note that, Table 10 reports positive and significant skewness for corn price after we distinguish between private stock and public stock.

[12] In the reported analysis, the aggregate stock level (stock-to-use ratio) was set equal to be 50%. Changing the evaluation point gave similar results.