Who Cares about Unemployment Insurance?

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 Abstract
How is the optimal level of unemployment insurance affected when accounting for skill differences? We analyze this question using a general equilibrium model that has a number of key elements: (i) a search and matching friction in the labor market; (ii) workers who have the ability to save and cannot perfectly insure idiosyncratic risks; and (iii) ex-ante heterogeneity in unemployment risk and labor income. Considering a proportional tax and replacement rate UI system, our model suggests an optimal replacement rate of 32%, while a model without ex-ante heterogeneity calls for a much lower replacement rate (12%). We show that both dimensions of heterogeneity are responsible for these results. Specifically, we argue that income differences induce an incentive to redistribute consumption across skill groups. However, given the UI system, such redistribution is feasible only when there are differences in unemployment risk.

1 Introduction

Unemployment Insurance (UI) is a central policy for insuring workers against unemployment risk. Providing insurance in the form of UI may also be costly, as it may alter workers labor supply and/or firms labor demand incentives. The optimal choice of UI has been studied extensively in frameworks where the government trades-off the insurance against incentives of workers and firms.[1]

Most existing studies consider workers with the same ex-ante characteristics, who then differ in their ex-post employment histories.[2] Thus, the analysis abstracts from a potentially important and relevant source of heterogeneity – skill differences. In US data, it is well documented that workers with lower levels of education typically experience a higher unemployment rate, and have a lower labor income. For instance, the average unemployment rate of a college graduate is 2.8%, while that of a high school dropout is 9.4%. The wage "college premium" for males in US data is around 80% (Krueger, Perri, Pistaferri, and Violante (2010)). Motivated by these observations, we study whether and how the inclusion of ex-ante heterogeneity along these dimensions affects the choice of optimal replacement rate.

To analyze this question, in section 2 we construct a general equilibrium model that has a number of key elements. First, unemployment in the model is endogenously determined by a search and matching friction as in Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP). Second, as in Bewley (undated), Huggett (1993), and Aiyagari (1994) (BHA), workers have access to savings in the form of aggregate assets, and are subject to an ad hoc borrowing constraint. This implies that workers cannot perfectly insure their idiosyncratic income and unemployment risks, and that in equilibrium there is heterogeneity in asset holdings. Finally, we assume that the population is divided into groups that permanently differ in their productivity levels and unemployment rates.

The search and matching friction places the incentive cost of UI on firms labor demand. Specifically, in the DMP framework wages are determined by Nash bargaining, hence higher levels of UI improve workers outside options, increase wages, and depress firms' incentives to maintain vacancies. Therefore, more generous UI benefits implies a higher unemployment rate, and the optimal replacement rate should be zero. The fact that workers cannot insure their idiosyncratic risk implies ex-post heterogeneity in asset holdings, and consumption. Absent any costs of reallocation,

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[1]See for example Hansen and Imrohoroglu (1992) and Hopenhayn and Nicolini (1997) for workers labor market incentives, and Fredriksson and Holmlund (2001) and Krusell, Mukoyama, and Şahin (2010) for analyzing the incentives through vacancy posting.

[2]A notable exception is Pallage and Zimmerman (2001).

models in the BHA framework call for high levels of UI benefits, in order to equalize consumption among workers. A recent study by Krusell, Mukoyama, and Şahin (2010) (henceforth, KMS) runs the "horse race" between these two competing effects. In the quantitative analysis, KMS find that the forces of the DMP model dominate, and therefore the optimal replacement rate should be low, at about 12%.

Our consideration of ex-ante heterogeneity among workers relies on three sources that generate endogenous differences in unemployment and income across groups. In particular, we consider workers who permanently differ in the level of productivity, the separation rate, and the cost of recruiting. These parameters have a direct effect on the cost and the surplus generated from a match, which in turn determine the wage and firms' job creation decisions. The latter is crucial in characterizing workers' job finding rates that together with the separation rates determine the equilibrium level of unemployment rate for each group. In section 3 we describe the details of our calibration of these parameters, relying on US labor market data.

In section 4 we study the quantitative implications of our model. The main result is that the optimal replacement rate is 32%. This is in contrast to the model without exogenous heterogeneity that calls for a much lower replacement rate at around 12%. To learn which of the sources of exogenous heterogeneity is responsible for this difference, we investigate two alternative calibrations of the model. First, we consider a calibration with separation rates as the only source of heterogeneity. This yields differences in unemployment levels but essentially no income differences. Then we consider a calibration with productivity as the only source of heterogeneity, and keep unemployment constant across groups. In this case there are income differences that are similar to our baseline model. The somewhat surprising finding is that both of these alternative calibrations support a rather low optimal replacement rate (12%-14%) that is very close to the ex-ante homogeneity case. Therefore, it appears that the high replacement rate in our baseline model stems from heterogeneity in both income and unemployment.

We interpret this set of results relying on UI as a mechanism for redistribution. In a model where redistribution can be achieved without cost, a utilitarian policy maker would choose equal consumption for all workers. Moreover, the more dispersed consumption is (across workers), the larger should be the benefits from redistribution. Therefore, we use measures of consumption dispersion within groups and for the economy as a whole for the various calibrations. In all calibrations we find a relatively small dispersion within groups, even when the replacement rate is low (10%). This suggests that the combination of self-insurance and low UI results in small dispersion, and there is not much to be gained from more redistribution, given the cost. We also document

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low dispersion for the aggregate economy for the calibration with no productivity differences. This suggests that when income differences are small, so are the consumption differences and the incentive to redistribute.

Once we consider the calibrations that involve substantial income differences across groups, there is a much larger dispersion of consumption. However, we show that absent differences in unemployment rates, and given that the only policy tool is a UI system that is based on proportional taxes and benefits, it is impossible to achieve any redistribution across groups. Therefore we conclude that the (realistic) case with both income and unemployment heterogeneity is the one where the policy maker has a relatively strong incentive to redistribute, and the ability to do so.

Our paper is related to the large body of literature on UI, and most closely related to a number of previous studies. KMS consider a model with imperfect self-insurance and a search and matching friction. We use their results as benchmark for comparison with ours, and stay close to their calibration where possible to facilitate comparison. Pallage and Zimmermann (2001) consider the question of optimal UI with ex-ante heterogeneity in skills. Their model and analysis are different from ours on two accounts. First, in their model UI affects workers' incentives through moral hazard. In addition, their choice of UI is based on political, or voting considerations, rather than welfare. Their results indicate that it is the voting that matters for the determination of optimal UI, regardless of whether workers are ex-ante homogenous or heterogeneous. Finally, Mukoyama and Şahin (2006) consider the welfare consequences of business cycle fluctuations once skill differences are taken under consideration.

2 The model

The model presented in this section consists of a number of central building blocks. First, workers belong to groups that permanently differ in their productivity and unemployment risk. Second, heterogeneity within groups arises from individual workers' asset accumulation decisions. Third, there exists a search and matching friction in the labor market that results in positive unemployment in equilibrium. Fourth, a typical neoclassical production function determines the level of output produced by each type of workers, and we assume that these outputs are perfect substitutes. Finally, a government can choose a replacement rate for unemployment insurance, and tax workers in order to keep a balanced budget. The model is in discrete time and there is no aggregate risk.

2.1 Sources of Heterogeneity

We assume N types of workers, where the measure of type $i \in \{1, ..., N\}$ is ϕ_i, and we normalize the sum of ϕ_i to be 1. The fraction of workers in each group is constant and exogenous. Members of separate groups differ along three dimensions: (i) the productivity level z_i; (ii) the separation rate σ_i; and (iii) the cost of posting a vacancy for a worker of type i, denoted by ξ_i. These sources of heterogeneity generate income and unemployment differences between groups.

Workers can hold risk-free assets to partially insure themselves against unemployment, in the tradition of BHA models. Asset holdings create another dimension of heterogeneity, both within and across groups. Therefore, in equilibrium, there exists a non-degenerate distribution asset holding.

2.2 Matching

We assume that a worker's type is known, and that the labor market is segmented by types $i \in \{1, ..., N\}$. Therefore, in order to recruit workers of type i, firms must maintain type-specific vacancies v_i. Type i workers can only apply to "type i jobs". There is no on-the-job search. Type i vacant jobs (v_i) and type i unemployed workers (u_i) are randomly matched according to a constant returns to scale matching technology. Each matching function, $M(u_i, v_i)$, represents the number of type i matches in a period. The probability of filling a vacant job, λ_i^f , thus equals $M(u_i, v_i)/v_i = M(u_i/v_i, 1) = M(1/\theta_i, 1)$, where $\theta_i = v_i/u_i$ is the vacancy–unemployment ratio of type i. Similarly, the job finding probability of an unemployed worker of type i, λ_i^w, equals $M(u_i, v_i)/u_i = (v_i/u_i) M(u_i/v_i, 1) = \theta_i \lambda_i^f$. A type i match separates with constant and exogenous probability σ_i in each period. Finally, we maintain the conventional assumption that matches that are formed in the current period become productive in the next period. As such, the evolution of the number of type i unemployed workers, u_i, can be described by

$$u_i' = (1 - \lambda_i^w)u_i + \sigma_i(1 - u_i)$$

where a next period variable is denoted by a prime ($'$).

2.3 Unemployment Insurance and Taxes

In our baseline model, we consider an unemployment insurance policy that involves a single replacement rate h for all unemployed workers.[3] By assuming a replacement rate policy, rather than a fixed-level transfer, the provided insurance is not a trivial redistribution between high and low-income workers. We assume that UI is financed by a proportional tax τ on labor earnings - wages for the employed, and unemployment benefits for the unemployed. The government sets τ in order to keep a balanced budget.

2.4 Production and Asset Structure

Producers in the economy rent capital (k) to produce output with their matched worker using a standard neoclassical production function $z_i f(k)$ where z_i is a type specific productivity and $f' > 0$ and $f'' < 0$. Since the capital market is frictionless there is one single rental per unit of capital, which in turn implies that the marginal product of capital must be equal across producers. Hence, all producers of type i will employ the same level of capital k_i. We assume that all types produce identical final output, thus total production is $\sum_{i=1}^{N} n_i z_i f(k_i)$, where $n_i = \phi_i (1 - u_i)$ is the total number of workers of type i.

Workers have access to two types of assets: capital that they rent to the firm for a rate r, and equity x – a claim to aggregate profits.[4] Workers cannot hold claims on individual jobs. Therefore, they cannot insure the idiosyncratic employment risk that they face.

Let δ be the capital depreciation rate, p be the price of equity, and d be the dividend. Imposing a no arbitrage condition implies that the return on capital must equal the return on equity. Therefore the equity price is

$$p = \frac{d + p}{1 + r - \delta} \tag{1}$$

Since both capital and equity markets are complete, the worker is indifferent with respect to

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[3] For computational reasons, we model the actual unemployment benefit for an unemployed worker of type i as a constant replacement rate h times the average wage earned by an employed worker of type i, $\bar{w}_i$. We use the average wage in order to avoid the need to keep track of individual workers' histories. In addition, the wage functions below suggest that there is relatively little variation in wages *within* group i, and we verify in all numerical exercises that within each group the lowest wage is higher than the unemployment benefit.

[4] One interpretation of the claim to aggregate profits is that each worker holds a portfolio of all firms. We could alternatively allow consumers to hold specific claims to each type of firm. Since in such a setting all firms would have the same return on their equity, consumers would be indifferent with respect to the composition of those types of equity.

the composition of the two assets. We denote the total assets of the worker by a, which is equal to:

$$a \equiv (1 + r - \delta) \, k + (p + d)x.$$

2.5 Workers

In this subsection we use the structure described thus far to specify the worker optimization problem and her optimal policy functions.

Let $\tilde{W}_i(w, a)$ denote the value function of a worker of type i, who is employed, earns a wage w, and owns a assets. Let $U_i(a)$ denote the value function of a worker of type i who is unemployed and owns a assets. Workers move between employment and unemployment according to the (endogenous) job finding rate (λ_i^w), and the (exogenous) job separation rate (σ_i). Workers take both probabilities parametrically.

Workers period utility is represented by an increasing and strictly concave function $u(c)$, and they discount future streams of utility by a discount factor $\beta \in (0, 1)$.

Workers derive utility from consumption (c) only, and there is no disutility from labor or home production. Hence the only flow benefit for an unemployed worker is UI. In addition, search effort does not entail any cost. Therefore, all unemployed workers actively seek employment.

2.5.1 Employed workers

An employed worker enters a period with some level of assets (a), and earns the period wage (w) net of the tax rate τ. The employed worker allocates her available resources between consumption and accumulation of assets for the next period, in order to maximize the discounted value of lifetime utility. In addition, we impose an ad-hoc borrowing constraint of $\underline{a}$, and we denote the inverse of the gross real interest by $q \equiv \frac{1}{1+r-\delta}$. The employed worker's problem is

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\begin{aligned}
\widetilde{W_i}(w, a) &= \max_{c, a'} u(c) + \beta \left[\sigma_i U_i(a') + (1 - \sigma_i) W_i(a') \right] \quad\quad (2) \\
\text{s.t.} \quad &: \\
& c + qa' = a + w(1 - \tau) \\
& a' \geq \underline{a}
\end{aligned}
$$

We denote the implied decision rule for a' from (2) by $\widetilde{\psi}_i^e(w, a)$. The worker's wage, which is determined by Nash equilibrium as explained below, is a function of the worker's assets and is

denoted by $w = \omega_i(a)$. With this notation, we can formally define $W_i(a)$ and $\psi_i^e(a)$ as:

$$W_i(a) \equiv \widetilde{W}_i(\omega_i(a), a)$$
$$\psi_i^e(a) \equiv \widetilde{\psi}_i^e(\omega_i(a), a).$$

2.5.2 Unemployed workers

An unemployed worker enters a period with some level of assets (a), and receives unemployment benefits ($h\overline{w}_i$) net of the tax rate τ, where h is the economy-wide replacement rate described above and $\overline{w}_i$ is the average income of type i worker. The unemployed worker allocates her available resources between consumption and accumulation of assets (capital) for the next period, in order to maximize the discounted value of lifetime utility. The unemployed worker's problem is

$$U_i(a) = \max_{a'} u(c) + \beta\left[(1 - \lambda_i^w)U_i(a') + \lambda_i^w W_i(a')\right] \tag{3}$$
$$\text{s.t.} \quad :$$
$$c + qa' = a + h\overline{w}_i$$
$$a' \geq \underline{a}$$

We denote the implied decision rule for a' from (3) by $\psi_i^u(a)$.

2.6 Firms

Firms create jobs, rent capital from workers, and produce. The firm maximizes the discounted present value of the profits of shareholders. To create a job, a firm first posts a vacancy of a certain type i. There is a flow cost of posting a vacancy of type i, denoted by ξ_i. The value of posting a vacancy of type i, V_i, is

$$V_i = -\xi_i + q\left[(1 - \lambda_i^f)V + \lambda_i^f \int J_i(\psi_i^u(a))\frac{f_i^u(a)}{u_i}da\right], \tag{4}$$

where $V = \max\{V_1, V_2, ...V_N, 0\}$ because firms are free to choose between any of the N types of vacancies, or not opening a vacancy at all.

The firm discounts the future at the rate q, which is the market rate and the marginal rate of substitution of workers with positive holdings of firm's equity. $J_i(a)$ is the value of a job filled by a worker of type i whose asset level is a. Because the matching process is random, the firm can

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be matched with any worker of type i in the current period unemployment pool. An unemployed worker with current asset level a will have asset level $\psi_i^u(a)$ next period. The density function of the unemployed workers of type i over a is $f_i^u(a)/u_i$. In equilibrium, firms will post new vacancies until $V_i = 0$.

The value of a filled job, given the wage w, is

$$\tilde{J}_i(w, a) = \max_{k_i} z_i f(k_i) - rk_i - w + q\left[\sigma_i V_i + (1 - \sigma_i)J_i(\tilde{\psi}_i^e(w, a))\right] \tag{5}$$

Note that $\tilde{J}_i$ depends on a because with probability $1 - \sigma_i$ the firm continues to be matched with the same worker, whose next period asset level depends on a. $\tilde{J}_i$ and J_i are related by

$$J_i(a) \equiv \tilde{J}_i(\omega_i(a), a) \tag{6}$$

The firm's first-order condition implies that $r = z_i f'(k_i)$. In equilibrium, the period profit is equal to $\pi_i(a) = z_i f(k_i) - rk_i - \omega_i(a)$.

The dividend is paid out as the sum of profit minus the total vacancy cost:

$$d = \sum_i \left[\int \pi_i(a) f_i^e(a) da - \xi_i v_i\right] \tag{7}$$

where $f_i^e(a)$ is the population of the matched workers of type i with wealth level a.

2.7   Wage determination

The wage is determined by generalized Nash bargaining. We assume that firms cannot commit to wages, so that wages are set period by period. The Nash bargaining solution solves the problem:

$$\max_w \left(\tilde{W}_i(w, a) - U_i(a)\right)^\gamma \left(\tilde{J}_i(w, a) - V_i\right)^{1-\gamma} \tag{8}$$

$\gamma \in (0, 1)$ represents the bargaining power of the worker of any type. We denote the type i solution $w = \omega_i(a)$; the dependence of w on a stems from $\tilde{W}_i(w, a) - U_i(a)$ depending on a.

2.8 Computation

In order to maximize her utility, the worker needs to know the entire wage function, $\omega_i(a)$. Therefore the algorithm we use aims at finding a *functional* fixed-point.

1. Start with an initial guess for $\omega_i(a)$, r, θ_i and τ.

2. Given the current guess for θ_i, compute the probability of finding a job λ_i^w for each type and the associated unemployment level u_i.

3. Solve the workers' dynamic programming problem for each type of worker and for each level of assets. This gives both the value function and the capital accumulation path.

4. Given the employee's capital accumulation path and the wage associated with her next period's assets, calculate the firm's value function for each type of employee and for each asset level. This does not require the asset distribution, which is calculated in the next step.

5. Based on the optimal saving decisions of workers and the transitions probabilities between employment and unemployment, calculate the stationary distribution of assets for employed and unemployed workers of each type. Calculate the aggregate stationary distribution of workers across asset holdings given the weights ϕ_i of each group and the measures of employed and unemployed workers within each group. This gives the total capital stock.

6. Update of the guess for $\{\omega_i(a), r, \theta_i, \tau\}$ as follows.

   - Given the value functions of workers in step 3 and firms in step 4 perform Nash bargaining, which delivers an update for $\omega_i(a)$.

   - Use the total capital stock from step 5, u_i from step 2 and the first-order condition of each type of firm to compute k_i and r.

   - Use the firm's value and the distribution of assets over unemployed to calculate the expected value for the firm from a match. Given the vacancy cost and the value of a match we update θ_i such that the value of a vacancy is zero. Note that we do not force θ_i to be the same across types.

   - Given u_i, $w_i(a)$ and the stationary distribution of workers across asset holdings of each type in step 5, update the tax rate so that the budget is balanced.

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3 Calibration

In this section we describe the calibration of the model that is used for the numerical analysis. We first describe a set of standard parameters that are kept constant across groups, as well as across numerical exercises. To facilitate comparability with existing results, we maintain the calibration of KMS whenever possible. We then describe in detail the calibration of parameters that govern the heterogeneity in the model.

One period is set to be 6 weeks. The production function is $F(k) = k^\alpha$. We choose $\alpha = 0.3$, $\delta = 0.01$ and $\beta = 0.995$ using the following calibration targets: a capital share of 0.3, an investment–output ratio of 0.2 and annual real rate of return on capital of 4.5%. The borrowing constraint $\underline{a}$ is set at 0. We use the utility function $u(c) = \log(c)$.

We assume a Cobb-Douglas matching function for all types: $M(u_i, v_i) = \chi u_i^\eta v_i^{1-\eta}$, so that

$$\lambda_i^w = \theta_i \lambda_i^f = \chi \theta_i^{1-\eta}$$

We follow Shimer (2005), and KMS by setting the matching functions elasticity $\eta = 0.72$, and the worker's bargaining power $\gamma = \eta$.[5] Finally, in our benchmark calibration we set the replacement rate $h = 0.4$, i.e. a 40% replacement rate.

3.1 Type Specific Parameters

For the analysis in this paper, we identify skill with levels of education. Specifically, using data from the Current Population Survey (CPS), we divide the labor force into four groups: "Less than high-school", "High-school graduates", "Some college" and "Bachelor's degree and over", denoted $\{1, 2, 3, 4\}$, respectively. We use data on labor force participants who are older than 25, reflecting the assumption that most people have made their education level choice by that age.

The top row of Table 1 describe the share of each group (ϕ_i), obtained by averaging over the share of each group in the labor force between 1992 and 2012.

Workers with different education levels differ substantially in terms of earnings and unemployment. The calibration of parameters governing productivity levels (z_i), separation rates (σ_i), and the cost of recruiting (ξ_i) enable us to generate such differences using our model.

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[5]In a textbook DMP model, setting $\gamma = \theta$ guarantees that the allocation is constrained efficient, as this calibration satisfies the Hosios condition. It is important to stress that satisfying this condition in our model does not guarantee efficiency. As Davila, Hong, Krusell, and Rios-Rull (2012) show, BHA models are not generally efficient because of externalities involved in the accumulation of capital.

To capture differences in earnings, we calculate "education premiums" using CPS data.[6] Specifically, we calculate the premium for each group as the ratio of median wage for this group divided by the median wage of group 1. Averaging over all periods, the education premiums for groups 2, 3, and 4 are 1.32, 1.51, and 2.04 respectively.

In our model the wage is tightly linked to the productivity level. We calibrate the productivity level z_i in order to match the education premium, normalizing the weighted average productivity to be 1.0. These values are presented in the second row of table 1.

According to CPS data, the average unemployment rates for groups 1-4 equal 9.4%, 5.7%, 4.7%, and 2.8%, respectively.[7] The immediate observation is that low skilled workers are more likely to be unemployed. We interpret the differences in unemployment rates as workers facing different unemployment risk.

Similar to a standard DMP model, we use the (group-wise) labor evolution equation to express the steady state unemployment rate for each group:

$$u_i = \frac{\sigma_i}{\sigma_i + \lambda_i^w}$$

This term suggests that differences in unemployment risk across groups can stem from different separation rates, different job finding rates, or both. In order to determine the appropriate weights we use the data on unemployment flows described by Elsby, Hobijn, and Şahin (2010), who study the flows into and out of unemployment by education groups that correspond to our classification.[8] The flows data suggest that differences in the average unemployment to employment (UE) transition rates are minor. On the other hand, differences in the average employment to unemployment (EU) transition rates are substantial - the average EU transition rate for the lowest skill group is about four times as high as the rate of the highest skill group.

Motivated by the findings regarding the transition rate, we calibrate the model such that in the baseline calibration, the entire difference in unemployment rates between groups is due to the variation in the job separation rate. Specifically, we target a single equilibrium job finding rate of 0.45 per month for all groups, which translates into a job finding rate of $1 - (1 - 0.45)^{1.5} = 0.6$, and compute the implied separation rates such that the unemployment rate per group is consistent with CPS data. We then rescale the separation rates σ_i to reflect the six weeks period, and to

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[6]We use data on earnings by educational attainment for both men and women 25 years and older, in 1979, 2002.

[7]CPS monthly unemployment rates by educational attainment, 25 years and older, 1992:01-2012:07.

[8]Elsby, Hobijn, and Şahin (2010), figure 8

Table 1: Calibrated parameters

	Less than High-school	High-school	Some college	Bachelor's degree and over
Population share (ϕ_i)	0.10	0.31	0.28	0.31
Productivity (z_i)	0.730	0.888	0.978	1.220
Separation rate (σ_i)	0.0973	0.0596	0.0479	0.0271
Vacancy cost (ξ_i)	0.3115	0.4405	0.5166	0.7370

be consistent with an average separation rate of 0.05, as in KMS.[9] The resulting values of σ_i are described in the third row of Table 1.

We use the set of zero profit conditions - one for each type of vacancies - to calibrate the recruiting cost parameters ξ_i as follows. In order to keep the job finding rate constant across groups, we target the values of θ_i to equal 1 for all groups, and set the matching efficiency parameter χ to equal 0.6. Given the productivity levels z_i, and the separation rates, the zero profit conditions can be used to calculate ξ_i such that $\theta_i = 1 \; \forall i$. The resulting values of ξ_i are described in the fourth row of Table 1.

There is some empirical evidence supporting the result of vacancy costs that increase with education. Dolfin (2006) finds that the number of hours required for recruiting, searching, and interviewing workers depends on their education. Assuming that the skill of the workers engaged in recruiting is independent of the worker recruited, Dolfin (2006) finds that the cost of recruiting a high-school graduate is 50% higher than the cost of recruiting a lower skill worker, and that the cost of recruiting a worker with more than high-school education is 170% higher than the cost of recruiting a worker with less than high-school education. Barron, Berger, and Black (1997) report findings based on a variety of data sources, all clearly suggest that the cost of recruiting a worker is increasing in the worker's level of education.

[9]Once again we follow the calibration targets in KMS to verify that our results are not affected by setting different targets. Also note that the resulting aggregate unemployment rate is higher than the corresponding figure in the data.
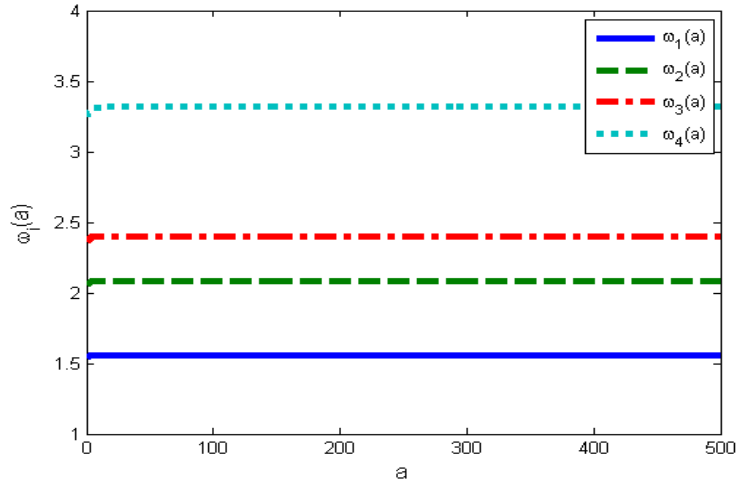
Figure 1: The wage functions

4 Results

In this section we use the calibrated model to study the role of unemployment insurance in the economy by computing the optimal replacement rate. We do this by calculating the model equilibrium at the benchmark calibration with a 40% replacement rate, as well as for alternative economies with different replacement rates. First, we characterize the behavior of the macroeconomic aggregates. Then we turn to a welfare analysis that is instrumental for the choice of the optimal replacement rate. The results are presented as gains or losses relative to the 40% replacement rate benchmark.

Figure 1 describes the wage as a function of the asset for each of the four groups for the benchmark. The wage dispersion is driven by changes in productivity. Note that the wage is fairly inelastic with respect to assets. The high job-finding probability implies that from a relatively low level of assets all workers have a high ability to insure themselves. Therefore, differences in assets, except at very low levels, have a very small effect on the wage.[10]

The relatively low variability of wages within education groups also enables us to simplify and characterize UI as a replacement rate times the average wage for that group. The small decline in wages for low level of assets implies that it is very unlikely that UI benefits are higher than a worker's wage. Indeed, throughout our analysis, we verify that this is never the case.

Figure 2 shows how key variables in the economy are affected by the replacement rate. The

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[10]Krusell, Mukoyama, and Şahin (2010) and Bils, Chang, and Kim (2011) present similar wage functions in similar frameworks.
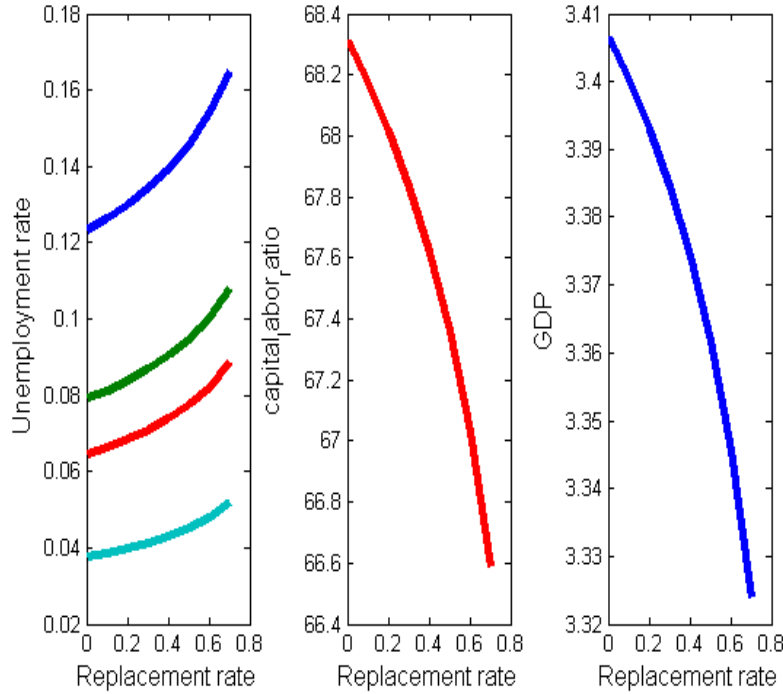
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Figure 2: The effect of the replacement rate on unemployment, capital-labor ratio, and GDP.

left panel shows the group-wise unemployment rate. As expected, increasing the replacement rate increases unemployment for all groups. This is due to a standard mechanism in DMP models: higher unemployment benefits increase the outside option of workers, and depress firms' incentives to maintain vacancies. With a lower level of labor input, capital is less productive, and therefore firms demand less capital per worker. Higher replacement rate also implies a weaker incentive for workers to accumulate assets. As a result, a higher replacement rate is associated with lower levels of capital, and trivially, lower levels of GDP.[11] Taken together, this set of results verifies straightforward intuition – when judged by aggregate variables, unemployment benefits are costly.

To study the choice of an "optimal" replacement rate, we adopt the welfare criterion used in KMS, and is similar to Pallage and Zimmermann (2001). First, we calculate the stationary competitive equilibrium for economies that differ only in their replacement rate.[12] We then move every individual, with her labor status and asset holdings, from the benchmark economy to each of

[11]From a quantitative standpoint, moving from the benchmark 40% replacement rate to 10% replacement rate increases GDP by about 0.8 percentage points.

[12]An initial set of parameters is calibrated at the 40% benchmark according to the description in the calibration section. These parameters are kept constant for all economies.

the alternative economies. The welfare gain (or loss) for each individual from such a transition is characterized by λ, defined by:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log \left((1 + \lambda) c_t \right) \right] = E_0 \left[\sum_{t=0}^{\infty} \beta^t \log \left(\widetilde{c}_t \right) \right]$$

where c_t is the consumption under the benchmark case ($h = 0.4$) and $\widetilde{c}_t$ is the consumption under a particular experiment.

Finally, for each of the alternative replacement rates, we compute the sum of individual gains from adopting the particular replacement rate relative to the 40% benchmark. Alternatives that involve a positive aggregate gain are considered preferable over the benchmark. The "optimal" choice is the alternative that has the highest aggregate gain.[13]

Before describing the results, we stress two issues regarding the welfare criterion. First, while admittedly we do not consider the entire transition path, we view this welfare criterion as preferable over summing steady state welfare levels for the different economies. The main reason is that a plain steady state comparison would miss the potentially important short run consequences of the need to adjust savings while transitioning to a new steady state. Second, note that the optimal choice is determined by the total gain. Our choice is consistent with the analysis in KMS, but departs from the analysis in Pallage and Zimmermann (2001), who consider voting patterns, and therefore emphasize the fraction of population that gains from a change in UI policy.

Panel A of Table 2 presents the results of our calibrated model. The main result is visible in the second column: when we consider deviations from the 40% benchmark, the highest aggregate welfare gain is at a replacement rate of 32%. While lower than the benchmark, this optimal replacement rate is substantially higher than the 12% replacement rate that is chosen in an economy with only one group of workers.[14] Other results in this table are suggestive with respect to the source of the difference between the one group and multiple groups models. Columns 3 and 4 show that generally the unemployed lose from reducing the replacement rate, while the employed mostly gain. Moreover, taking a closer look at poor individuals, whose borrowing constraint is binding, we observe (columns 5,6) that this group of the population always lose from less generous UI. Finally, we also note that had the replacement rate been subject to a vote, the 32% replacement rate is backed by a majority of the population.

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[13]We implement this by moving away from the 40% benchmark in increments of a 10%, and then a refinement of the grid until the optimal level is reached.

[14]For brevity we do not present this case here. The analysis of the one group model is identical to KMS.

Table 2: Welfare gain by type of worker

(1) h from 0.4 to	(2) Total (%)	(3) Unemployed (%)	(4) Employed (%)	(5) Poorest unemployed (%)	(6) Poorest employed (%)	(7) % gaining
Panel A: Benchmark calibration						
0.10	-0.067	-0.509	-0.031	-3.603	-2.708	44.13
0.20	-0.017	-0.314	0.008	-2.161	-1.712	56.759
0.30	0.011	-0.141	0.022	-1.017	-0.830	56.93
0.32	**0.011**	**-0.110**	**0.021**	**-0.805**	**-0.660**	**56.93**
0.40	0	0	0	0	0	0
Panel B: No heterogeneity in productivity						
0.10	0.118	-0.330	0.155	-3.390	-2.490	58.89
0.14	**0.119**	**-0.271**	**0.151**	**-2.781**	**-2.101**	**58.96**
0.20	0.112	-0.191	0.137	-2.014	-1.565	58.99
0.30	0.076	-0.077	0.089	-0.935	-0.747	59.00
0.40	0	0	0	0	0	0
Panel C: No heterogeneity in separation risk						
0.10	0.123	-0.026	0.135	-1.243	-0.317	92.29
0.12	**0.124**	**-0.014**	**0.135**	**-1.044**	**-0.263**	**92.29**
0.20	0.116	0.016	0.124	-0.558	-0.095	99.54
0.30	0.078	0.028	0.083	-0.195	-0.002	99.97
0.40	0	0	0	0	0	0

To better understand the mechanisms that yield this result, we turn to the breakdown of welfare gains and losses by education groups. The motivation is intuitive – in our model (as in the data), lower education level is associated with a higher likelihood to be unemployed, and a higher likelihood to be "poor" (in terms of the model). Table 3 presents the group-wise welfare gains associated with changing the replacement rate. The results coincide with the intuitive reasoning: members of groups 1 and 2 (the two lowest education groups) generally lose from reducing the replacement rate. The highest group generally gains from lowering the replacement rate. Members of group 3 appear to be gaining from lowering the replacement rate. However, this masks some members of the group who have lower levels of assets, and for whom UI is a beneficial source of insurance.

The analysis thus far suggests that the heterogeneity measures we introduced to the model are qualitatively and quantitatively important with respect to the determination of optimal UI. Based on our assumptions and calibration, these results may stem from income level differences, unemployment risk differences, or both. We believe that it is important to distinguish between the two sources. If income differences among groups are the source for the higher UI, then we can argue that UI is a way to achieve redistribution in the economy. If unemployment risk is the main source, then we can argue for an insurance role for UI.

To isolate the role of heterogeneity in unemployment risk, we consider a second calibration of the model with invariant productivity across groups. We maintain the difference in separation rates hence the difference between groups is the likelihood of being unemployed.[15]

The main result from this experiment is described in panel B of table 2 – the optimal replacement rate is 14%, substantially lower than the 32% in the baseline model. Panel B of Table 3 presents the breakdown of gains and losses by groups. While it is still the case that the lower groups lose from lowering the replacement rate, these loses are smaller in magnitude relative to the baseline case. Looking at the top group, the opposite is true – this group gains from a reduction of the replacement rate by more than the baseline model. Therefore, the economy as a whole prefers a lower replacement rate.

To isolate the role of heterogeneity in productivity, we consider a third calibration of the model with invariant separation rates across groups. We maintain the difference in productivity. Panel C of table 2 presents the results of that calibration. In this case, all workers in the economy face the same separation risk. The optimal replacement rate is 12% - once again substantially lower relative to the baseline model, and closer to the model with unemployment risk only.

Taking stock of these results, it appears that the relatively high replacement rate in our baseline

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[15]We recalibrate the model as described above and then repeat the experiments of varying the replacement rate.

Table 3: Welfare gain by group

(1) h from 0.4 to	(2) Total (%)	(3) Group 1 (%)	(4) Group 2 (%)	(5) Group 3 (%)	(6) Group 4 (%)
Panel A: Benchmark calibration					
0.10	-0.067	-1.832	-0.475	-0.015	0.826
0.20	-0.017	-1.210	-0.292	0.019	0.612
0.30	0.010	-0.600	-0.130	0.028	0.330
0.32	**0.011**	**-0.476**	**-0.100**	**0.025**	**0.267**
0.40	0	0	0	0	0
Panel B: No heterogeneity in productivity					
0.10	0.118	-1.693	-0.291	0.176	1.060
0.14	**0.119**	**-1.458**	**-0.238**	**0.169**	**0.939**
0.20	0.112	-1.111	-0.165	0.151	0.748
0.30	0.076	-0.544	-0.064	0.096	0.399
0.40	0	0	0	0	0
Panel C: No heterogeneity in separation risk					
0.10	0.123	0.119	0.122	0.123	0.124
0.12	**0.125**	**0.122**	**0.123**	**0.123**	**0.125**
0.20	0.116	0.113	0.115	0.116	0.117
0.30	0.078	0.077	0.078	0.078	0.079
0.40	0	0	0	0	0

Table 4: Gini coefficient by group ($h = 0.1$)

Group	Group 1	Group 2	Group 3	Group 4
Benchmark	0.070	0.053	0.046	0.030
Heterogeneity in productivity	0.051	0.045	0.044	0.041
Heterogeneity in separation risk	0.061	0.049	0.044	0.031

model is driven by a combination of heterogeneity in both productivity and unemployment risks, rather than one of them in isolation.

What may account for these results? Our interpretation relies on the redistributive role of the UI system in the model. Had there been no costs associated with redistribution a utilitarian policy maker would choose the same level of consumption plan to all workers. In our model we note three important features. First, using UI is costly, as illustrated by the higher unemployment, lower capital, and lower GDP in figure 2. Second, there is some heterogeneity within groups that may justify some degree of reallocation. Finally, our model introduces an exogenous heterogeneity between groups. This may add another incentive to redistribute resources in the economy.

To illustrate this in more detail, we first look at measures of consumption dispersion within groups and for the aggregate economy. The more unequal consumption is, the higher should be the benefit from reallocation of resources towards consumers who consume relatively little. Table 4 presents the Gini coefficient of consumption for the four groups in our model, for each specification of the model, and at a replacement rate of 10%.[16] Generally, all the coefficients are small, suggesting a relatively equal consumption within groups, and therefore a weak incentive for reallocation within the group. This result is fairly consistent with the results reported by KMS – a low replacement rate, combined with consumers ability to save implies a fairly low variability of consumption. This indicates that further increases of the replacement rate involve costs that outweigh the benefits from providing further insurance. Indeed, in our model, with further increases of UI, the group-wise Gini coefficients remain fairly stable.

Turning to the aggregate economy, we note that the measure of aggregate consumption dispersion is inclusive of both the within group and across group heterogeneity. Table 5 presents the Gini coefficient of consumption for the aggregate economy, using the three specifications of the model, and at replacement rates of 10%, 20%, 30%, and 40%. The first observation we highlight is the fact that in the model with no productivity differences, the Gini coefficients are much smaller,

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[16]Similar analysis using coefficients of variation yields the same results.

Table 5: Gini coefficient by replacement rate

Replacement rate	0.1	0.2	0.3	0.4
Benchmark	0.136	0.135	0.133	0.132
Heterogeneity in productivity	0.134	0.134	0.134	0.134
Heterogeneity in separation risk	0.044	0.044	0.044	0.044

and close in magnitude to the group-wise coefficients. This is not surprising. In this model workers productivity is identical, which implies small variation in labor income. Therefore, the main source of variation in consumption is the within group heterogeneity. Given the analysis above this is consistent with a weak incentive to do any further redistribution above the low replacement rate.

Once productivity differences are introduced, consumption in the economy becomes more unequal, and the Gini coefficients are roughly three times as large as the model with identical productivity. At a 10% replacement rate, this implies a much stronger incentive to redistribute consumption in the economy – an incentive that is mostly driven by across-group variability.

What is even more revealing about the mechanisms at work is the evolution of the Gini coefficient when replacement rates are higher. In the baseline model, higher replacement rates imply more equal consumption, as expected. The optimal choice that we described above can be viewed as trading off the (aggregate) utility gain from redistribution, and the costs described in figure 2. However, in the model without differences in unemployment risk, consumption inequality remains unchanged. This suggests that in our model, absent heterogeneity in unemployment risk, the economy loses the ability to reallocate resources across groups.

To show this more explicitly, we calculate the net transfers that a worker of group i receives on average (i.e. unemployment benefits received minus taxes paid by the group). This can be expressed as $\overline{w}_i [u_i h - e_i \tau]$, where $\overline{w}_i$ is the average wage of group i, and u_i and e_i are the unemployment and employment rates of group i. Since balanced budget imposes that the sum of net transfers is zero, it follows that when the unemployment rates are identical across groups, net transfers must be zero for all groups. This implies that if the only policy tool is UI and there are no differences in unemployment risk, the policy maker has no ability to redistribute across groups.

If the unemployment (and employment) rate are allowed to be type specific then net transfers in general differ from zero. More specifically, the group with the highest unemployment rate receives a positive net transfer and the group with the lowest unemployment rate faces a negative net trans-

fer.[17] Since in our benchmark calibration a higher unemployment rate is associated with a lower productivity, the redistribution implied by the UI system favors workers with low productivity, whose marginal utility is higher.

Using this observation, the economic forces that determine the optimal replacement rate in the three calibrations can be summarized as follows. When groups differ only by their productivity, redistribution can be effective because of the high inequality in the economy, yet it is impossible to use the UI as redistribution mechanism because the unemployment risk is the same across groups. Therefore, in this economy redistribution through UI is desirable yet infeasible. When groups differ only by their unemployment risk, this heterogeneity allows redistribution across groups, yet the benefit of redistribution is limited because of the low inequality in the economy. Therefore, in this economy redistribution through UI is feasible yet less desirable. Finally, when groups differ by both their productivity and their unemployment risk, as in the benchmark calibration (and in the data), redistribution is both desirable and feasible.

5   Conclusion

To be completed.

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[17]Other groups can have either a positive or a negative net transfer. However, when ordered according to the unemployment rate there is a threshold unemployment rate such that any group with a higher unemployment rate receives a positive net transfer and any group with a lower unemployment rate receives a negative net transfer.

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