Political Support and Tax Compliance: A Social Interaction Approach

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Abstract

People may express their political opinion by adopting different measures of civil disobedience. Tax compliance is an example of an economic decision that may be affected by anti-goverment sentiment. We consider a model in which political opinion as well as tax compliance decisions are both formed as part of a social interaction process in which individuals interact, exchange ideas and observe behavior. Tax compliance is affected by the level of government support and political opinion may be affected by government's auditing policy. The government's role is to set a social spending program which is viewed differently by rich and poor individuals. The paper focuses on the interdependence between tax compliance, government's social policies and political support, embedding this interdependence in a dynamic social interaction process.

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1 Introduction

The focus of political economics is on the relationship between government's policies and the probability of reelection. Governments may modify their policies in order to increase the probability of being reelected. While participating is an election is the obvious mechanism

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for individuals to express government support, there are other civil and daily actions that are affected by political opinion. People, for example, may express their political opinion by demonstrating or by adopting various measures of civil disobedience.[1] Legal disobedience may be viewed by some people as a legitimate way of expressing their anti-government sentiment[2]. But legal disobedience may evoke penalties. People may resent some of these penalties in particular when they are caught and punished. So while legal disobedience may be affected by political views, being punished for such a disobedience may affect government support. The paper focuses on tax compliance as the most obvious civil duty that might be affected by political views. Individuals may be more inclined to evade taxation when they oppose the ruling regime then when they support it.

Individuals form their political opinion by observing the government's policies but also by discussing these policies with their friends and peers. People like to talk about politics, they like to express their political opinion, argue, debate, demonstrate and express their political support or opposition. Political opinion is formed by the objective assessment of government policies but also by a social interaction process in which individuals with different opinions express their views and try to convince others regarding their political preference. Such a process is an important part of shaping political opinion. Social interaction allows for information exchange but also facilitates a mechanism of influencing other individuals' opinion and behavior. Glaeser, Sacerdote and Scheinkman (1996) discuss the importance of social interaction in determining criminal behavior (and tax evasion clearly falls under the definition of criminal behavior). On the other hand Huckfeldt, Beck, Dalton and Levine (1995) argued that social interaction plays an important role in the formation of political opinion.

There is extensive literature on tax compliance (for surveys see Andreoni, Erard and Feinstein (1998), Cowell (1990), Slemrod (2007) and Schneider and Enste (2000)). The literature focuses on the strategic interaction between taxpayers and tax authorities. The taxpayer makes his tax report decision based on the gains from misreporting his income, the probability of being caught and the expected penalty. The tax authority determines the tax rate and the auditing probability. Larger tax rates encourage tax evasion and a larger auditing (and penalties) discourage such a behavior.[3] Our analysis is focused differently

[1] Cearly, in different countries there are different traditions. For example, casual observation indicates that in Italy there are many more demonstrations and strikes than in the US.

[2] An interesting annectodal evidence is that in Israel some of the Arabic and the Druze population even do not obey some of the traffic rules (like using safety belts) as part of showing their attitude towards the current regime.

[3] See also Allingham and Sandmo (1972), Reinganum and Wilde (1986), Sanchez and Sobel (1993), Erard and Feinstein (1994) and the extensive reference list in the survey of Andreoni, Erard and Feinstein (1998).

since we emphasize the interdependence between tax compliance, government policy and political opinion embedding this interdependence in a dynamic social interaction process.

We consider a model in which individuals are heterogenous with respect to their earning and government support. Income is subject to taxation but individuals may misreport their income. The government may audit and punish those individuals. Our key assumption is that political opinions may affect the individuals' tax compliance decision. We assume for simplicity that only individuals who politically oppose the government would consider misreporting their income while government's supporters report their income properly as a manifestation of their political support.[4]

The government's role, in our model, is to collect and spend the taxes. The main government's decision is the portion of the budget (tax revenues) to be spent on social redistributive activities. Social spending affects government's support directly. Rich individuals oppose social spending that imposes wealth distribution while poor individuals support it.

We assume that government support is not fixed and may change over time. The dynamics of government support is governed by a social interaction process by which individuals meet and interact every period in small groups. In these meetings people observe the type of the individuals they meet, they discuss and express their political views and may exchange other relevant information. It is this social interaction, together with government's policies, that governs the formation of government support. The last component that affects government support is our assumption that individuals do not like to be penalized, even if they think that the penalty is justified. We assume therefore that an individual who was punished for tax evasion will continue to oppose the government and cannot be persuaded to change his/her views. We assume that the tax compliance decision is also formed and affected by the social interaction process. For example meeting individuals who were audited and punished for tax evasion reduces the probability of such a behavior in future periods.[5]

Our assumed social interaction process implies that both social spending and auditing policies affect government support and tax compliance. Penalizing an individual for tax evasion may convince this individual to report his income correctly. But at the same time penalizing individuals increases the portion of opposition in the population. These individuals socially interact with other individuals and may convince them to become government opposition. Having a large portion of the population who oppose the government may induce more tax evasion in future periods.

[4]This assumption captures the intuition that individuals who oppose the government would be more inclined to misreport their income.

[5]See Lipatov (2008) for a model of tax evasion in which there is a social interaction process by which individuals learn about the government's auditing policy.

The paper focuses on the effect of the social spending policy, the auditing probability, and the proportion of rich individuals in the population, on government's support, tax revenues and tax evasion. Given the complexity of the social interaction process we do not derive our results analytically but conduct a numerical analysis in which we find the stable steady state for different sets of parameters and investigate the effects of different policies and parameters on the steady states.

Our analysis indicates that the effect of social spending on government's popularity is not monotonic. A higher level of social spending may result in more opposition even among the poor population despite the fact that poor individuals prefer higher social spending. Tax revenues are also indirectly affected by social spending as its level affects government support among the rich which affects their tax compliance decisions. Since government's support is not monotonic with social spending the same pattern holds for tax revenues as well. We show that it is possible to increase social spending, a policy which is unpopular with rich individuals, and yet achieve lower levels of tax evasion and higher tax revenues. The interdependence between social spending, political popularity and tax revenues is however sensitive to the population composition.

Our assumptions imply that increasing the auditing probability leads to larger opposition. Consequently the effect of auditing on tax collection is not necessarily positive. Despite the positive direct effect, a higher auditing probability implies that more individuals become government's opposition which leads to more tax evasion and lower tax revenues. We indeed show that at low levels of auditing probability an increase of this probability yields a large increase of opposition among the rich which yields more tax evasion and lower tax revenues.

2 The Model

Consider a society of measure 1 in which individuals need to report their income which is subject to taxation. The government monitors the individuals' income reports and penalizes whenever tax evasion is exposed. Individuals in this society may have different political opinions. Some may support the government while others may oppose it. Political opinion may be affected by the policies adopted by the government. We focus on the government's social spending policy which is denoted by α, and defined by the proportion of the tax revenues that are spent on social issues. We assume that poor individuals support large social spending while rich individuals oppose it.

Our main assumption is that tax compliance is not just an economic decision - balancing the benefits of misreporting income with the expected penalty of being caught for tax evasion - but it may also be affected by individuals' political opinions. We assume that forming political opinion as well as tax compliance decisions are both determined as part of a social interaction process in which individuals interact, exchange ideas, observe each others' political opinion and disclose information regarding their tax report history.

Individuals: There are two types of individuals; poor individuals who earn L and rich individuals who earn H; such that $H > L$. There is a fixed proportion, denoted by γ, of rich individuals. Each individual must report his/her income and pay the appropriate taxes. The poor truthfully report L while rich individuals have a choice; they either report their income H or they may try to lower their tax payment by misreporting their income, i.e., reporting L instead of H.

Individuals may also differ with respect to government support. We assume, for convenience, that there are only two categories; S - individuals who support the government and O - individuals opposing it.[6] The shares of opposition in the rich and poor populations in period t are denoted as o_t^r and o_t^p respectively (similarly the share of supporters among rich and poor individuals is $s_t^r = 1 - o_t^r$ and $s_t^p = 1 - o_t^p$ respectively). The overall government support at period t is given by $s_t = \gamma s_t^r + (1-\gamma)s_t^p$.

We assume that government support induces tax compliance and assume, for convenience, that government supporters always report their income correctly. Thus only rich individuals who oppose the government consider tax evasion as an option.

Consequently, in the beginning of every period t there are 6 different types of individuals (omitting index t for convenience):

(i) P^O - a poor individual who opposes the government,

(ii) P^S - a poor individual who supports the government,

(iii) R^S - a rich individual who supports the government and therefore reports his earnings accurately,

(iv) R_H^O - a rich individual who opposes the government but in period $t-1$ he honestly reports his income,

(v) R_C^O - a rich individual who opposes the government, and in period $t-1$ he cheated on his tax report and has been caught by the government monitors,

(vi) R_N^O - a rich individual who opposes the government, and in period $t-1$ he cheated on his tax report but was not caught by the government.

The proportion of rich individuals who misreport their income in period $t-1$ is given by $q_{t-1} = (R_{C,t}^O + R_{N,t}^O)/(R_{C,t}^O + R_{N,t}^O + R_{H,t}^O)$ and it is endogenously determined.

[6] In a more general framework one may introduce different levels of government support - which all may affect the individuals tax report decision.

Social Interaction: We consider a dynamic social interaction process by which individuals are randomly matched into pairs every period.[7] The matched individuals observe each other's type and may discuss their political inclination. The outcome of such an interaction is a possible transition of individuals' opinions regarding government support and their tax compliance decisions.

We consider a two-stage social interaction process. In the first stage, political opinions are formed and shaped while in the second stage individuals make their tax compliance decisions given their new updated opinions. We assume a Markov process in which only the current type of the matched players affects their decisions and the formation of their opinions.

Opposition/Support Decision:

In the beginning of period t, individuals have a political opinion that was shaped in period $t-1$. As part of their social interaction individuals discuss their political opinion, try to convince their friends and acquaintances regarding their political views but may also be convinced by others to change their views. We follow a probabilistic transition rule that is affected by government social policy, the individuals' type and the type of individuals they are matched with.

The underlying process of persuasion implies that if two individuals who have the same political opinion are matched then they are not going to change their opinions. We also assume that individuals' wealth affects their government support only via the different views that rich and poor individuals have regarding social spending. In Table 1 below we specify our assumption regarding the transition probabilities in any matching depending on the types that are matched.[8,9]

We assume transition probabilities with the following properties:

(i) when both individuals have the same opinion, then social interaction does not affect their opinion.

(ii) when two different types are matched, only one of them will switch his opinion. That is, we do not allow the government supporter to convince his matched partner to

[7] We assume social interaction of pairs in order to simplify the population dynamics that we investigate. One can clearly extend our model to a more complex social interaction but this would be at a cost of considerably complicating the analysis.

[8] Note that there are eight possible types of mixed matches depending on the different charecteristics of individuals (opposition, support), (rich, poor), (cheat, comply), (caught, not caught).

[9] In the table we use for convinience the terminolgy $\Pr(SS|\cdot,\cdot)$ for having the two players becoming government supporters as a result of the social interaction. Since a change of opinion happens only when individuals have a different opinion we assume for such couples that the first individual is type O while the second is type S and therefore we do not have a possible transition to SO which implies that both individuals change their opinion.

switch to support the government and at the same time he himself is convinced to switch to the opposition.

(iii) individuals do not like government penalties and thus if a (rich) individual was caught for tax evasion and was punished he would not be convinced to become a government supporter.

(iv) poor individuals like redistribution, whereas rich individuals are averse to it. Thus a higher level of social spending, α, increases the probability that a rich individual would oppose the government and increase the probability that a poor individual would support the government.

match	$\Pr(OO\|\cdot,\cdot)$	$\Pr(SS\|\cdot,\cdot)$	$\Pr(OS\|\cdot,\cdot)$
$R^O R^O$	1	$-$	$-$
$P^O P^O$	1	$-$	$-$
$R^O P^O$	1	$-$	$-$
$R^O_C R^S$	$\alpha\beta$	0	$1-\alpha\beta$
$R^O_C P^S$	$(1-\alpha)\beta$	0	$1-(1-\alpha)\beta$
$R^O_N R^S$	$\alpha\beta$	$(1-\alpha)\beta$	$1-\beta$
$R^O_N P^S$	$(1-\alpha)\beta$	$(1-\alpha)\beta$	$1-2\beta(1-\alpha)$
$R^O_H R^S$	$\alpha\beta$	$(1-\alpha)\beta$	$1-\beta$
$R^O_H P^S$	$(1-\alpha)\beta$	$(1-\alpha)\beta$	$1-2\beta(1-\alpha)$
$P^O R^S$	$\alpha\beta$	$\alpha\beta$	$1-2\alpha\beta$
$P^O P^S$	$(1-\alpha)\beta$	$\alpha\beta$	$1-\beta$
$R^S R^S$	$-$	1	$-$
$P^S P^S$	$-$	1	$-$
$R^S P^S$	$-$	1	$-$

Table 1: The transition probabilities of government support

When an individual who supports the government is matched with an individual who opposes the government, the probability that the government supporter will switch to opposition is $(1-\alpha)\beta$ if he is rich and does not like social spending and $\alpha\beta$ if he is poor and supports social spending. The parameter β ; $\beta < \frac{1}{2}$, is the inclination to switch. A small β implies that it is more difficult to convince individuals to change their political opinion and there is more inertia in such opinions while a high β implies that switching political opinion is more frequent.

Tax Report Decision:

One way to model tax compliance decisions is to assume that rational individuals who are familiar with the governments auditing policy and the penalty for tax evasion make their tax compliance decisions by balancing the gains from tax evasion and the expected punishment. We choose to model tax compliance decision in a different way by emphasizing two aspects: the relationship between political views and the tax report decision and the effect of social interaction on the tax compliance decision. That is, the decision by an individual whether to misreport his/her income depends on his own (recent) experience and on the experience of the individuals he is matched with. Individuals that were caught for tax evasion and those whose matched partners were caught and punished for tax evasion will be less likely to do so themselves. We do not explicitly introduce the penalty for tax evasion into our analysis but it may clearly affect the parameters that we do introduce.

We assume that political opinion affects the individuals' tax compliance decision but the political opinion of his/her matched partner has no effect on tax reporting decision. For simplicity we assume that government supporters always report their income correctly and only individuals who oppose the government consider tax evasion.

We let $Q(x, y)$ be the probability that type x individual will misreport his income when he is matched with a type y individual. Since we assume that government supporters do not consider tax evasion we get that $Q\left(R^S, y\right) = 0$ for every possible type y. The three types that may consider tax evasion are the rich individuals who oppose the government, R_C^O, R_H^O, and R_N^O.[10] We assume that type R_C^O, who was caught in the previous period for tax evasion, would report his income correctly independently of whom he was matched with. Type R_N^O will continue to misreport his income unless he is matched with type R_C^O who was caught and punished. Given the bad experience of his matched partner he is going to misreport income only with probability $1 - \eta$. Type R_H^O, who was honest in the previous period, continues to be honest if he is matched with type R_C^O but otherwise there is a positive probability, η, that he would switch and try to misreport his income. Our assumption regarding tax compliance is summarized in Table 2 which specifies the probabilities of misreporting income of each type (columns) as a function of the individual he is matched with:

	R_C^O	R_N^O	R_H^O
R_C^O	0	$1 - \eta$	0
R_N^O	0	1	η
R_H^O	0	1	η
R^S, P	0	1	η

Table 2: The probabilities of non-compliance.

Note that in our formulation the tax compliance decision is entirely based on population dynamics and not on the expected benefit of tax evasion. We assume that the auditing

[10] We also need to specify the tax decision of a rich opposition that was a supporter in the previous period. But given that this individual (as a government supporter) reported his income correctly in the previous period, we assume that he behaves in the same manner as the honest opposition R_H^O.

probability p and the penalty are exogenously given. Clearly these variables may affect the individuals' tax compliance decision. In our setting the probability of auditing indirectly affects the tax compliance decision via the population dynamics. As p goes up there are relatively more type R_C^O individuals and thus the probability of type R_N^O to be matched with type R_C^O is higher. Such a match results in a lower probability of tax evasion. We may also capture such considerations by assuming that the probability of experimenting with tax evasion, $\eta(p, D)$, is a deceasing function of the probability of auditing and the penalty of misreporting.

Government: The government in our model collects taxes and determines α - the share of social spending out of the tax revenues. We assume that the government does not have any ideology regarding the proper social spending levels and it chooses α in order to maximize its objective function. We assume that the government commits to a specific α at the outset of the game and that the tax policy which includes tax rate, the probability of auditing and the penalty for tax evasion are all exogenously given.

We assume that the government fully internalizes the social interaction process and can calculate its resultant steady state. We let $s(\alpha)$ and $q(\alpha)$ be the steady state shares of government's supporters among the entire population and the percentage of non-compliant among rich individuals respectively as functions of the government's policy α.

The objective of the government is to have the highest possible budget (tax revenues) but on the other hand it wishes to maximize the political support it gets. When all the people correctly report their income, tax revenues are maximized and we denote this tax potential as T. However, given that the proportion of rich people who misreport their income and are not caught for tax evasion is $q(\alpha)(1-p)$, the government's tax revenue in steady state is given by

$$T(\alpha) = T(1 - q(\alpha)(1 - p)). \tag{1}$$

We let $T^*(\alpha)$ be the proportion of the tax potential that the government manages to capture, i.e., $T^*(\alpha) = T(\alpha)/T$. We assume that the government's objective function is additive in the two terms and is given by:

$$G \equiv \theta T^*(\alpha) + (1 - \theta)s(\alpha), \tag{2}$$

where θ is a parameter reflecting the relative importance of government support and tax revenues.

3 Population Dynamics

The transition probabilities provided by Tables 1 and 2 define a Markovian population dynamic. In each period we have a distribution of types that socially interact and transform according to the transition matrix defined by the two tables. A steady state of this population dynamics would be a distribution of types and tax compliance decisions that remain fixed over time given the assumed transition matrix. The state of the population is fully characterized by the triple (o_t^r, o_t^p, q_t) which defines the percentage of government opposition among the rich in period t, the percentage of government opposition among the poor in period t and the probability that at period t a rich individual who opposes the government will be engaged in tax evasion. That is, (o_t^r, o_t^p, q_t) uniquely defines the distribution of all the types of individuals. A steady state is characterized by (o^r, o^p, q) which remains unchanged over time.

The dynamics of government support:

We first construct the transition probabilities of government opposition, $o_{t+1}^r(o_t^r, o_t^p, q_t)$, and $o_{t+1}^p(o_t^r, o_t^p, q_t)$ i.e., the percentage of government opposition among rich individuals in period $t+1$ as a function of the distribution of types at period t. Collecting the terms summarized in Tables 1 and A1, we obtain the following expressions for the transition probabilities:[11]

$$\begin{aligned} o_{t+1}^r = {} & \gamma\left(o_t^r\right)^2 + (1-\gamma)\, o_t^r o_t^p + \\ & \gamma o_t^r \left(1-o_t^r\right)\left(pq_t\left(1+\alpha\beta\right) + \left(1-pq_t\right)\left(2\alpha\beta - \beta + 1\right)\right) + \\ & (1-\gamma)\, o_t^r \left(1-o_t^p\right)\left(1 - \left(1-pq_t\right)\left(1-\alpha\right)\beta\right) + (1-\gamma)\, o_t^p \left(1-o_t^r\right)\alpha\beta, \end{aligned} \tag{3}$$

$$\begin{aligned} o_{t+1}^p = {} & (1-\gamma)\left(o_t^p\right)^2 + \gamma o_t^r o_t^p + \gamma o_t^r \left(1-o_t^p\right)\left(1-\alpha\right)\beta + \\ & \gamma o_t^p \left(1-o_t^r\right)\left(1-\alpha\beta\right) + (1-\gamma)\, o_t^p \left(1-o_t^p\right)\left(1-2\alpha\beta+\beta\right). \end{aligned} \tag{4}$$

The dynamics of Tax Compliance decision:

In every period t, q_t describes the probability that a rich individual who opposes the government would misreport his/her income. This probability is an outcome of the social interaction process. We can use Tables 1 and 2 to define the transition probability $q_{t+1}(o_t^r, o_t^p, q_t)$ (see Appendix 3 for details):

[11] See Appendix 2 for details.

$$
\begin{aligned}
q_{t+1}(o_t^r, o_t^p, q_t) \;=\;& \gamma o_t^r \left(q_t + \eta - q_t\eta - pq_t + p^2 q_t^2 \eta - pq_t\eta\right) + \\
& (1-\gamma)\, o_t^p \left((1-q_t)\,\eta + (1-p)\,q_t\right) + \\
& (1-p)\, q_t \gamma\, (1-o_t^r)\,(1-\beta\,(1-(1+\eta)\,\alpha)) + \\
& \gamma\,(1-q_t)\,(1-o_t^r)\,(2\alpha\beta + 1 - \beta)\,\eta + (1-\gamma)\,\frac{o_t^p}{o_t^r}\,(1-o_t^r)\,\alpha\beta\eta + \\
& ((1-q_t)\,\eta + (1-p)\,q_t)\,(1-\beta+\alpha\beta)\,(1-o_t^p)\,(1-\gamma)
\end{aligned}
\tag{5}
$$

Steady state: A steady state is a triple (o^r, o^p, q) that remains unchanged under our Markovian transition probabilities defined by (3)-(5).

A **stable steady state** is (o_t^r, o_t^p, q_t) which is a steady state and satisfies the following stability condition: if there is a population of types which is in a ε neighborhood of the steady state then the population dynamics given by (3),(4) and (5) will converge back to (o_t^r, o_t^p, q_t).

The steady state conditions imply that full support and full opposition (with corresponding compliance levels) are always a steady state - but not necessarily a stable one. We refer to such steady states as corner solutions and steady states for which $o^r \in (0,1)\,, o^p \in (0,1)$ will be referred to as interior steady states.

Proposition 1 *(i) There always exists a unique stable steady state (o^r, o^p, q), which is the solution to the system of equations (3)-(5). (ii) There are α_1 and α_2; $\alpha_2 \geq \alpha_1$, such that a (unique) interior stable steady state exists only when either $\alpha < \alpha_1$ or when $\alpha > \alpha_2$.*

Proof. See Appendix 4. ■

An interior stable steady state implies that there is a mixed population; some are government supporters while others are government opposition. An interior steady state exists for either small or high levels of social spending. For these levels the government policy plays an important role in the formation of political opinion. Rich individuals resent high levels of α while poor individuals resent low levels of α. In the middle range the resentment is weaker and the social interaction process may result in a boundary steady state in which all individuals have the same political opinion.

4 Results

Our analysis focuses on the effect of the social spending policy, α , and the proportion of rich individuals in the population, γ, on government's support and on tax evasion. Given the complexity of the social interaction process we do not derive our results analytically and we conduct a numerical analysis in which we find the stable steady state for different sets of parameters and investigate the effects of different policies and parameters on the steady states that we get.

Parameters' Values: We determine the following parameters' values for our numerical analysis:

- p - The auditing probability will be $0.1 - 0.2$, i.e., between $10\% - 20\%$ of the tax reports are monitored.
- γ - The percentage of rich individuals in the society will be between $30\% - 50\%$. For most cases we will assume 40%.
- β - $0.2 - 0.4$
- η - $0.1 - 0.2$

Our benchmark case is with the parameter values $\{p = 0.2, \gamma = 0.4, \beta = 0.3, \eta = 0.1\}$. Proposition 1 tells us that for this benchmark case the necessary and sufficient conditions for a stable interior steady state are that $\alpha \in \{[0, 0.375) \cup (0.525, 1]\}$. That is, for the middle level of social spending $\alpha \in [0.375, 0.525]$ there is only a boundary stable steady state.

4.1 The level of social spending α:

In an environment without social interaction the effect of social spending, α, on government popularity is relatively simple. Poor individuals like a higher α while rich individuals dislike it. Thus any increase of α increases government support among the poor and decreases it among the rich. In our model there are two additional effects. First, there is a population dynamics by which people may convince one another regarding their political opinion. The second effect is through the interdependence between political opinion and tax compliance. A higher α implies that there are more rich individuals that oppose the government and therefore it would imply a larger pool of rich individuals who misreport their income. When there are more R_C^O type individuals then the opposition decision becomes reinforced as this type of individual remains in opposition regardless of whom he/she is matched with.

In Figure 1 we present our baseline case. We vary the social spending α between 0 and 1 and calculate the stable steady state for each level of α. Figure 1_a depicts the level of opposition from the poor population as a function of α; Figure 1_b depicts the opposition level of rich individuals and Figure 1_c depicts the total level of government opposition (which is the weighted sum of the two types of individuals) and the total amount of tax revenues collected by the government which are given by $TC = (1 - q + pq)\, o^r + 1 - o^r = 1 - q\,(1 - p)\, o^r$. Figure 1_d provides information on the level of tax evasion and depicts the percentage of actual cheaters, q, among the rich-opposition population.

We first examine the two extreme cases of $\alpha = 0$ and $\alpha = 1$. When $\alpha = 0$, the unique stable steady state is a complete division of government support; poor individuals oppose the government while all the rich individuals support it. In this case, the government maximizes its tax revenues as all the rich people support the government and correctly report their income. When $\alpha = 1$ (the government uses all its tax revenues on social spending) the only stable steady state is such that all the rich individuals oppose the government while the poor individuals support it.

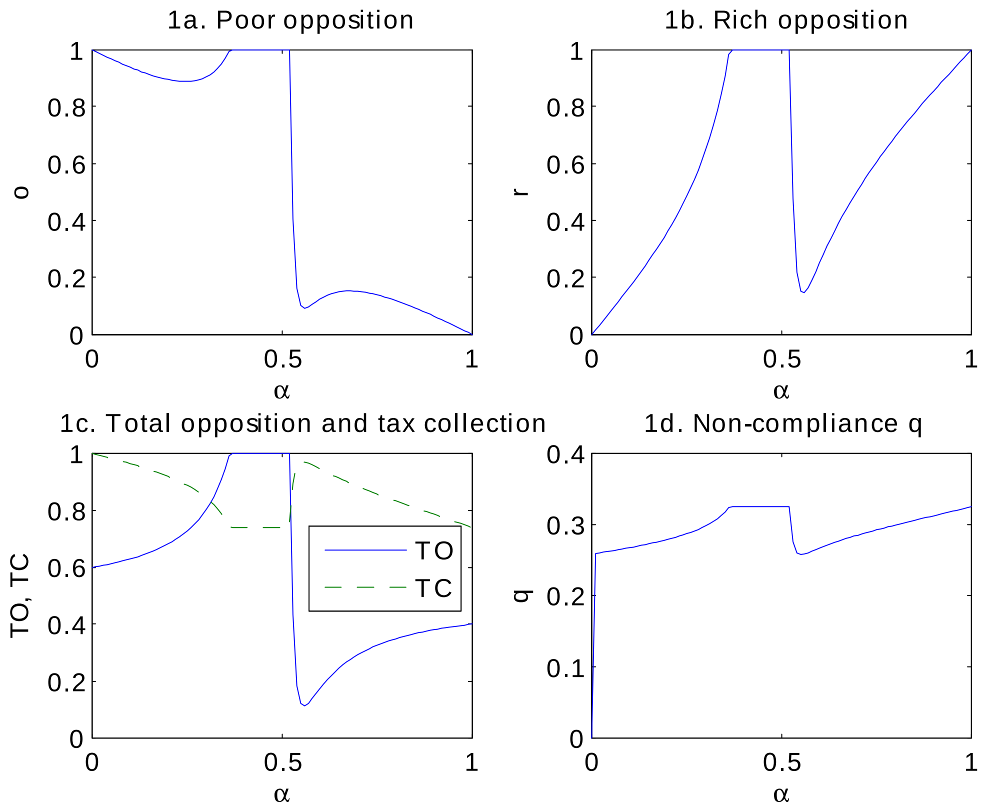

Figure 1, $\gamma = 0.4$.

The effect of α on government's support is not monotonic. When $\alpha = 0$, all the poor individuals oppose the government and all the rich individuals support it. Raising α at the beginning yields the intuitive effect - poor individuals increase their support while rich individuals reduce their support. But government support among the poor is not monotonically increasing in α. At this benchmark case 60% of individuals are poor and prefer a higher α, yet a further increase of α does not always imply a higher government's popularity among the poor (see Figure 1_a). If we look at the overall government support from the entire population we see that initially it is indeed decreasing with α but then a further increase in α induces a large increase in overall popularity (or a reduction in the opposition level). Government's popularity is maximized in our benchmark case around $\alpha = 0.57$. A further increase of α beyond this level would imply an increase of the level of opposition. Tax revenues are maximized in this benchmark case at $\alpha = 0$. When there are no social spending all the rich individuals are government supporters and by assumption government supporters report their income correctly. Interestingly, when $\alpha \approx 0.57$ tax revenues are not far from their maxium level.

Observation 1 (Case 1: Majority are poor): In our benchmark setting: (i) Social spending affects government's popularity in a non-monotonic way. This property holds for both the poor and the rich populations. (ii) Overall government support is maximized when social spending is in the middle range, i.e., at $\alpha \approx 0.57$ - but a small decrease of social spending implies a large reduction of the government's popularity. (iii) Tax collection is affected by the level of social spending; it is maximized at $\alpha = 0$ but an increase of social spending does not always reduce tax collection. Tax revenues reaches almost the same maximum level also at $\alpha \approx 0.57$ but increasing social spending beyond that level implies an decrease in overall popularity, lower tax revenues and more tax evasion.

Our assumptions regarding the population dynamic process play an important role in deriving the above results. An increase of α increases government support among the poor who convince some of the rich individuals to support the government despite the fact that rich people object to high α. This support also affects tax revenues as individuals who support the government do not engage in tax evasion. Note also that although government popularity is maximized at $\alpha = 0.57$ a small reduction of α implies a sharp decline in government popularity. This decline is a nice example of the effect of the underlying social interaction process.

We considered the above benchmark case assuming that the majority of people were poor (i.e., $\gamma = 0.4$). The balance between poor and rich plays an important role in our analysis as it affects the distribution of types each individual is matched with. We now change this assumption and assume that 60% of the population is rich i.e., $\gamma = 0.6$. The effect of changing the population composite is beyond the direct effect on each group as having groups of different sizes affects the underlying social interaction process. In figure 2 below we present the steady state government's popularity, tax revenues and tax evasion as a function of α when $\gamma = 0.6$.

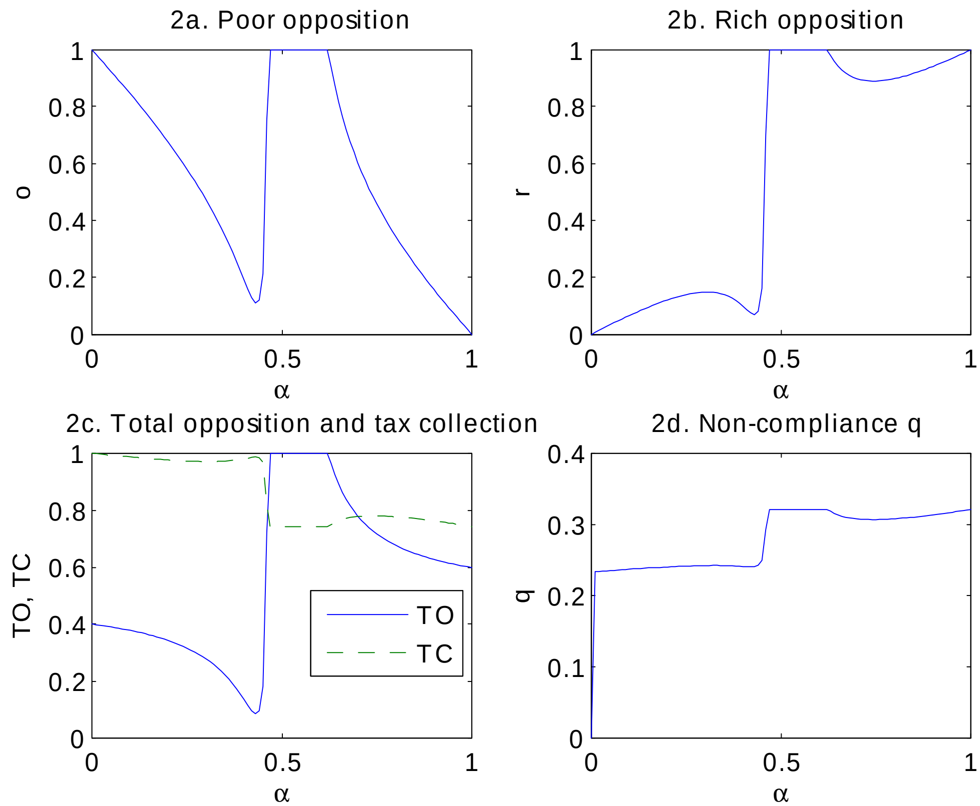

Figure 2, $\gamma = 0.6$

Comparing Figures 1 and 2 tells an interesting story about the role of social interaction. Figures $1_a, 1_b$ and $2_a, 2_b$ regarding government's opposition describe a similar pattern. For example the support of the rich individuals when $\gamma = 0.6$ is a mirror picture of the support of poor individuals when $\gamma = 0.4$.

Observation 2 (Case 2: Majority are rich): (i) For $\alpha = 0$ all the poor individuals oppose the government. Increasing α in the range of $\alpha < 0.4$ dramatically increases government support among the poor. When $\alpha = 0.4$ more than 80% of the poor are government supporters. (ii) A further increase of α from $\alpha = 0.4$ to $\alpha = 0.6$ induce a dramatic reduction in government support among the poor (from 80% to 0%) even though the poor support larger social spending. (iii) For low levels of α, tax collection is not affected much by α but increasing α beyond 0.4 implies a 20% drop of tax collection.

It is interesting to compare Figure 1_a and Figure 2_a; when the poor were the majority (Figure 1) and the government policy was $\alpha = 0.4$ then all of the poor population oppose the government. However when the poor are the minority (Figure 2) at the same level of government spending $\alpha = 0.4$ most of the poor population support the government. We can thus conclude that whenever political opinion is determined by a social interaction process government support of each segment of the society crucially depend on the ratio of rich and poor in the population.

4.2 The effect of η - the tendency to misreport income.

The parameter η captures the tendency to misreport income. Specifically, a R_H^O type (a rich individual who opposes the government and reports his income correctly) is going to experiment with cheating with probability η unless he is matched with an individual who was caught and penalized for cheating (an R_C^O type).[12] Having $\eta = 0$ implies that an individual who did not cheat is not going to change his behavior regardless of whom he is matched with. At the same time an individual who cheated without being caught will continue to cheat as long as he remains in opposition. However, this individual will eventually meet government supporters and there is always a positive probability that he will be convinced to become one. Being a government supporter implies an honest income report. Consequently, whenever $\eta = 0$ there is no cheating in the steady state.

[12] On the other hand, whenever an individual who cheated without being caught is matched with an individual who was caught for cheating he becomes more careful and cheats in the next period with probability $1 - \eta$.

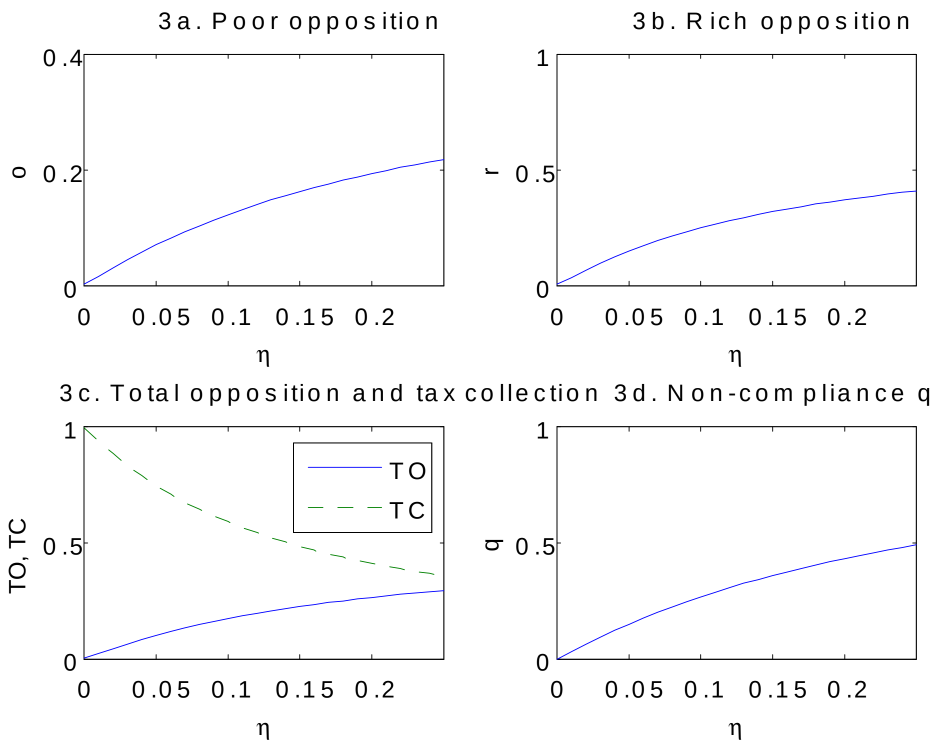

Figure 3.

Observation 3: An increase of η (tendency towards tax evasion) implies a higher opposition level for both rich and poor individuals, lower tax collection and higher tax evasion.

Observation 3 illustrates the interdependence between tax evasion and government support which is the focus of our model. The parameter η does not affect the poor individuals directly and therefore should not affect their behavior or opinion. But our assumptions regarding social interaction and the relationship between political opinion and tax evasion introduce a link between the political opinion of the poor and the rate of tax evasion among the rich. Increasing η implies that in the new steady state there is more experimentation with tax evasion and more individuals are caught and penalized for tax evasion. Having more individuals of type R_C^O affects the tax reporting dynamics but also the level of government support. An individual of type R_C^O remain in opposition regardless of the types that he/she is matched with. Thus an increase in η implies more opposition among the poor and rich population despite the fact that this parameter has no direct effect on political opinion as it determines only experimentation with respect to tax compliance.

4.3 The effect of auditing probability

The literature on tax evasion focuses on the direct effect of auditing - a higher auditing probability reduces the incentives for tax evasion. Our analysis focuses on the indirect effect of the auditing probability.[13] A higher auditing probability implies that a higher percentage of individuals who misreport their income is being caught. This higher percentage has two effects. The first effect is with respect to tax compliance: individuals who have been caught will not cheat on their taxes in the coming period. Moreover, these individuals socially interact with other individuals and reduce the probability that those individuals will tax evade in the coming period. The second effect is on individuals' political opinion: people who were caught for tax evasion remain government opposition in the next period without even considering supporting the government.

In Figure 4 we present the effect of government auditing on government support, on tax collection and tax evasion. We assume for this calculation the benchmark parameters for which $\gamma = 0.4$ and $\alpha = 0.6$.

[13] As was previously discussed we can incorporate the direct effect of p by assuming that the tendency to misreport income, denoted by η, is a function of the probability of monitoring p.

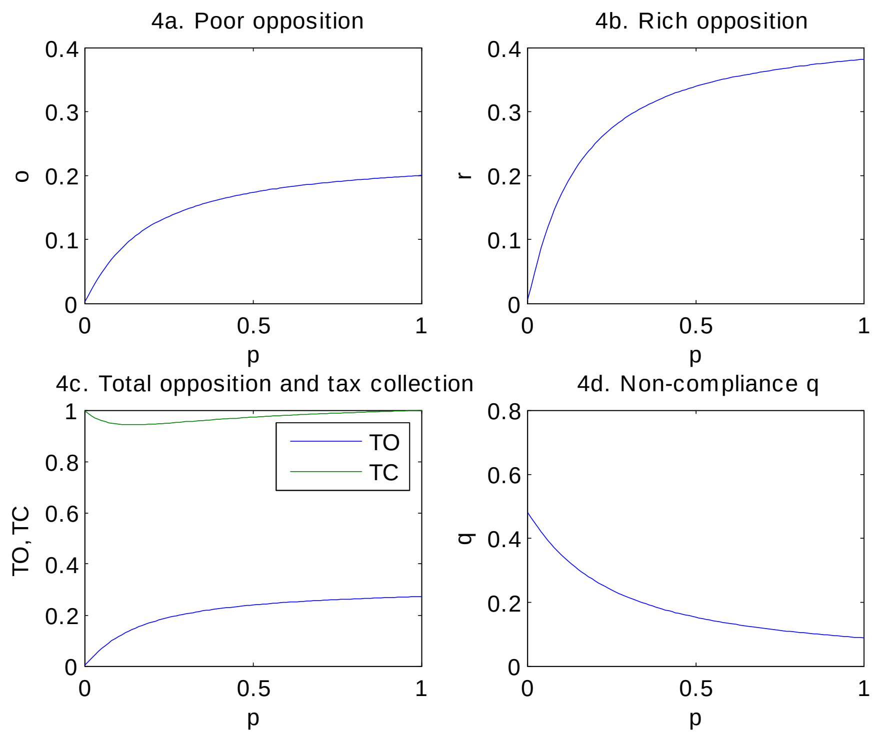

Figure 4, $\alpha = 0.6$

Observation 4: (i) The levels of opposition among the poor and among the rich are both increasing with p. (ii) The effect of the auditing probability p on total tax collection is not monotonic. At low levels of p total tax revenues goes down with p. At higher levels, as expected, tax revenues goes up with p. (iii) Tax evasion goes down with p.

Note that poor individuals are not affected directly by the auditing probability. Yet our analysis indicates that a higher p implies that more of them become government opposition. The reason for this effect is the assumed social interaction process. Poor individuals' political opinion is affected by rich individuals who they are matched with. Since the high p implies that more rich individuals become government opposition, the social dynamics implies that such views would be more common also among poor individuals.

Part (ii) of Observation 4 indicates that there is an interesting non-monotonicity of total tax revenues with respect to the auditing probability. The direct effect of a higher p is an increase in tax collection. A higher auditing probability implies that less rich individuals are able to get away with tax evasion and at the new steady state more of them report their income correctly. But there is an indirect effect as well. At lower levels of p an increase of p triggers a large increase of opposition level among the rich (see Figure 4*b*). More opposition among the rich implies a larger number of rich individuals who consider the option of tax evasion which results in lower tax collection. At a higher level of p it is the direct effect that dominates and a higher p implies more tax revenues.

Figure 4*d* indicates that non-compliance goes down with the auditing probability. Note however that we do not assume any direct effect of auditing on the individuals' decision to misreport income. The negative effect is due to the social interaction process that we assume. We assume that an individual who has been caught for tax evasion refrains from tax evasion in the coming period. Moreover if an individual who considers misreporting income is matched with an individual who had been caught and punished for doing it, then it would reduce the probability that he would evade paying taxes in the next period.

5 Government's Optimal Social Spending Policy

Consider now the government's optimal social spending policy, α. We assume that the government fully internalizes the social interaction process and can predict the properies of its steady state; $T^*(\alpha)$ and $s(\alpha)$, where is the percentage of tax potential that is collected as a function of α and $s(\alpha)$ is the level of government support at the steady state as a function of α. The government's objective function is given by (2).

Figure 5*a* describes the optimal α as a function of γ and θ. The figure is derived in the following way: Given the parameter γ (the percentage of rich individuals in the population) we calculate the steady state associated with each possible value of α. We then find the α that gives the highest possible value to the government objective function with the appropriate θ.

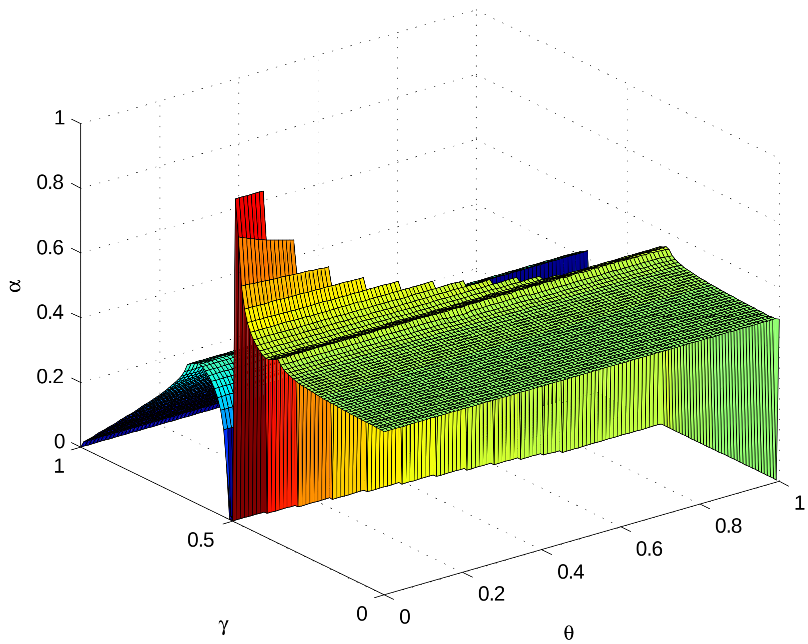

Figure 5a. Optimal α as a function of γ and θ.

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From Figure $5a$ we can see that for low levels of γ the optimal α does not vary much with θ. However when the population is (almost) equally divided between rich and poor the optimal α is highly sensitive to changes of γ and θ. In order to have a better understanding of these effects lets look at the optimal α as a function of the composition of the population γ keeping θ constant at $\theta = 0.5$.

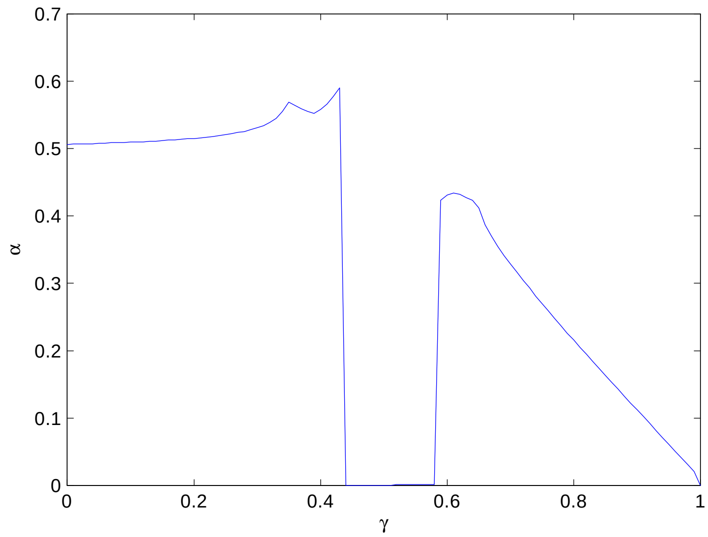

Figure 5b. Optimal α as a function of γ (with $\theta = 0.5$).

As Figure 5b indicates, the optimal α does not change much with γ whenever $\gamma < 0.42$. But when the population mix of rich and poor individuals becomes relatively balanced the optimal α becomes 0. When γ goes up to 0.6 the optimal α goes up again. Thus figure 5b describes a very special pattern of the optimal social spending level which is highly sensitive to the composition of rich and poor in the society. When 40% of the population is rich the optimal share of social spending is around 60%, when the population is equally divided between rich and poor the optimal social spending becomes zero and when the percentage of rich individuals increases to 60% the optimal social spending goes up to 40%. In order to understand this result let us examine Figure $A1$ in appendix 5 which describes the case of $\gamma = 0.5$.

From figure $A1c$ we can see that the overall government support is maximized at $\alpha = 0$ or $\alpha = 1$ while tax revenues are maximized at $\alpha = 0$. Thus regardless of the weights the government will put on its two objectives, $T^*(\alpha)$ and $s(\alpha)$, the optimal policy would be $\alpha = 0$. We can compare Figure 1 ($\gamma = 0.4$) and Figure $A1$ ($\gamma = 0.5$) and see that in Figure 1 the government support was maximized at $\alpha = 0.55$ and thus for this case we would have an interior solution where the government needs to balance its two objectives.

Observation 5: The optimal social spending is not an inceasing function of the proportion of poor indiiduals. In particular, in a society with a relatively similar number of poor and rich the optimal social spending is lower than in a society with a majority of rich individuals or with a majority of poor individuals.

6 Concluding Comments

Political opinion can be expressed in different ways. In some countries people express their support or opposition mainly on election day. In other societies government support or resentment may affect daily life. People strike, demonstrate or violate different laws as a way of expressing their political opinion. This paper focuses on tax evasion as a way of expressing opposition to a regime and on the interdependence between political opinion and legal obedience. Tax evasion is a convenient example as it allows us to use wealth level as a source of heterogeneity among individuals which is not directly related to government's support. But tax evasion is just an example of such an interdependence. Different types of laws and regulations can be affected by political opinion and at the same time shape them. Giving that political opinion and law obedience are both typically subject to social interaction implies an interesting interdependence that should be taken into account when searching for optimal government policies.

The social interaction process assumed in this paper is charecterized by no segragation. Poor individuals interact with rich individuals and the probability of matching between them depends solely on the relative size of the two groups. An interesting extension of our paper is having a segregation parameters that govern the social interaction process. High segregation level implies that rich individuals are most likely to interact with rich individuals and poor individuals are more likely to interact with poor individuals. Since segregation affects the social interaction process it establishes an interesting link between segregation level, government support, tax compliance and optimal social spending.

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8 Appendix

8.1 Appendix 1 - population shares

match	share
$R^O R^O$	$(\gamma o_t^r)^2$
$P^O P^O$	$((1-\gamma) o_t^p)^2$
$R^O P^O$	$2\gamma(1-\gamma) o_t^r o_t^p$
$R_C^O R^S$	$2pq_t\gamma^2 o_t^r (1-o_t^r)$
$R_C^O P^S$	$2pq_t\gamma(1-\gamma) o_t^r (1-o_t^p)$
$R_N^O R^S$	$2(1-p) q_t\gamma^2 o_t^r (1-o_t^r)$
$R_N^O P^S$	$2(1-p) q_t\gamma(1-\gamma) o_t^r (1-o_t^p)$
$R_H^O R^S$	$2\gamma^2 (1-q_t) o_t^r (1-o_t^r)$
$R_H^O P^S$	$2\gamma(1-\gamma)(1-q_t) o_t^r (1-o_t^p)$
$P^O R^S$	$2\gamma(1-\gamma) o_t^p (1-o_t^r)$
$P^O P^S$	$2(1-\gamma)^2 o_t^p (1-o_t^p)$
$R^S R^S$	$(\gamma(1-o_t^r))^2$
$P^S P^S$	$((1-\gamma)(1-o_t^p))^2$
$R^S P^S$	$2\gamma(1-\gamma)(1-o_t^r)(1-o_t^p)$

Table A1: Population shares of matches in support decisions.

8.2 Appendix 2

Collecting the terms from appendix 1 and multiplying them with corresponding probability for a rich individual to switch ($\Pr(O|S) = \Pr(OO|\cdot,\cdot)$) to or to stay in opposition ($\Pr(O|O) = 1 - \Pr(SS|\cdot,\cdot)$), we get

$$\begin{aligned} \gamma o_{t+1}^r = {} & (\gamma o_t^r)^2 + \gamma(1-\gamma) o_t^r o_t^p + \\ & pq_t\gamma^2 o_t^r (1-o_t^r)(1+\alpha\beta) + pq_t\gamma(1-\gamma) o_t^r (1-o_t^p) + \\ & (1-p) q_t\gamma^2 o_t^r (1-o_t^r)(1-(1-\alpha)\beta+\alpha\beta) + \\ & (1-p) q_t\gamma(1-\gamma) o_t^r (1-o_t^p)(1-(1-\alpha)\beta) + \\ & \gamma^2(1-q_t) o_t^r (1-o_t^r)(1-(1-\alpha)\beta+\alpha\beta) + \\ & \gamma(1-\gamma)(1-q_t) o_t^r (1-o_t^p)(1-(1-\alpha)\beta) + \gamma(1-\gamma) o_t^p (1-o_t^r)\alpha\beta + \\ & (\gamma(1-o_t^r))^2 * 0 + ((1-\gamma)(1-o_t^p))^2 * 0 + \gamma(1-\gamma)(1-o_t^r)(1-o_t^p) * 0. \end{aligned} \quad (6)$$

Collecting terms again and dividing by $\gamma \neq 0$, we arrive at (3).

Similarly, for the poor individuals and $\gamma \neq 1$ we first arrive at

$$
\begin{aligned}
o^p_{t+} &= (1-\gamma)\, o^{p2}_t + \gamma o^r_t o^p_t + \\
& pq_t \gamma o^r_t \left(1-o^p_t\right)(1-\alpha)\beta + (1-p)\, q_t \gamma o^r_t \left(1-o^p_t\right)(1-\alpha)\beta + \\
& \gamma (1-q_t)\, o^r_t \left(1-o^p_t\right)(1-\alpha)\beta + \gamma o^p_t \left(1-o^r_t\right)(1-\alpha\beta) + (1-\gamma)\, o^p_t \left(1-o^p_t\right)(1-\alpha\beta + (1-
\end{aligned}
$$

that then simplifies to (4).

8.3 Appendix 3

The derivation of compliance dynamics is a bit more involved, as there is no rematching between the support and compliance decision, so the latter takes place given a match described by the support decision. Thus, every term in the support equation of the rich (6) must be multiplied by the corresponding probability to comply that in turn depends on the history of compliance and auditing. In general then, dividing table values by $\gamma o^r_t \neq 0$, we have

$$
\begin{aligned}
q_{t+1} &= \gamma o^r_{t+1} \left((1-q)\, pq \left(Q\left(R^O_H R^O_C\right) + Q\left(R^O_C R^O_H\right)\right) + (1-q)(1-p)\, q \left(Q\left(R^O_H R^O_N\right) + Q\left(R^O_N R^O_H\right)\right)\right. \\
& \gamma o^r_{t+1} \left((1-q)^2 Q\left(R^O_H R^O_H\right) + p^2 q^2 Q\left(R^O_C R^O_C\right) + (1-p)^2 q^2_t Q\left(R^O_N R^O_N\right)\right) + \\
& \gamma o^r_{t+1} p (1-p)\, q^2 \left(Q\left(R^O_N R^O_C\right) + Q\left(R^O_C R^O_N\right)\right) + \\
& (1-\gamma)\, o^p_{t+1} \left((1-q)\, Q\left(R^O_H P^O\right) + pqQ\left(R^O_C P^O\right) + (1-p)\, qQ\left(R^O_N P^O\right)\right) + \\
& pq_t \gamma \left(1-o^r_{t+1}\right)\left(\alpha\beta \left(Q\left(R^O_C R^O_-\right) + Q\left(R^O_- R^O_C\right)\right) + (1-\alpha\beta)\, Q\left(R^O_C R^S\right)\right) + \\
& pq_t (1-\gamma)\left(1-o^p_{t+1}\right)\left((1-\alpha)\,\beta Q\left(R^O_C P^O\right) + (1-(1-\alpha)\,\beta)\, Q\left(R^O_C P^S\right)\right) + \\
& (1-p)\, q_t \gamma \left(1-o^r_{t+1}\right)(1-\alpha\beta + \beta(2\alpha-1))\, Q\left(R^O_N R^S\right) + \\
& (1-p)\, q_t (1-\gamma)\left(1-o^p_{t+1}\right)\left((1-\alpha)\,\beta Q\left(R^O_N P^O\right) + (1-(1-\alpha)\,\beta)\, Q\left(R^O_N P^S\right)\right) + \\
& \gamma (1-q_t)\left(1-o^r_{t+1}\right)\left(\alpha\beta \left(Q\left(R^O_H R^O_-\right) + Q\left(R^O_- R^O_H\right)\right) + (1-\beta)\, Q\left(R^O_H R^S\right)\right) + \\
& (1-\gamma)(1-q_t)\left(1-o^p_{t+1}\right)\left((1-\alpha)(1-\beta)\, Q\left(R^O_H P^O\right) + \alpha Q\left(R^O_H P^S\right)\right) + \\
& (1-\gamma)\, \frac{o^p_{t+1}}{o^r_{t+1}} \left(1-o^r_{t+1}\right) \alpha\beta Q\left(R^O_- P^O\right).
\end{aligned}
$$

Using the compliance probabilities $Q\,()$, we arrive then at

$$
\begin{aligned}
q_{t+1} &= \gamma o^r_{t+1} \left((1-q_t)^2 \eta + (1-p)^2 q^2_t + (1-q_t)(1-p)\, q_t (1+\eta) + p(1-p)\, q^2_t (1-\eta)\right) \\
& (1-\gamma)\, o^p_{t+1} \left((1-q_t)\,\eta + (1-p)\, q_t\right) + (1-p)\, q_t \gamma \left(1-o^r_{t+1}\right)(\beta(1+\eta)\,\alpha + 1 - \beta) + \\
& (1-p)\, q_t (1-\gamma)\left(1-o^p_{t+1}\right)((1-\alpha)\,\beta + 1 - 2(1-\alpha)\,\beta) + \gamma(1-q_t)\left(1-o^r_{t+1}\right)(2\alpha\beta\eta + (\\
& (1-\gamma)(1-q_t)\left(1-o^p_{t+1}\right)\eta\,((1-\alpha)\,\beta + 1 - 2\beta(1-\alpha)) + (1-\gamma)\, \frac{o^p_{t+1}}{o^r_{t+1}} \left(1-o^r_{t+1}\right)\alpha\beta\eta,
\end{aligned}
$$

that further simplifies to (5)

8.4 Appendix 4: Proof of Proposition 1

We start with part (i): Our steady state system is described by the three quadratic equations. The solution to the rich support (3) can then be generally written as

$$\begin{aligned} o^r &= \frac{A+G+C \pm \sqrt{(A+G+C)^2 - 4AC}}{2A}, \\ A &: = \gamma\left(1-2\alpha - pq\left(1-\alpha\right)\right), \\ C &: = \left(1-\gamma\right) o^p \alpha, \\ G &: = \left(1-\gamma\right)\left(1-o^p\right)\left(1-\alpha\right)\left(1-pq\right). \end{aligned}$$

for $\beta \neq 0$.

Analogously, the solution to the poor support (4) can be written as

$$\begin{aligned} o^p &= \frac{a+h+c \pm \sqrt{(a+h+c)^2 - 4ac}}{2a}, \\ a &: = \left(1-\gamma\right)\left(2\alpha - 1\right), \\ c &: = \gamma o^r \left(1-\alpha\right), \\ h &: = \gamma\left(1-o^r\right)\alpha. \end{aligned}$$

Clearly, only one of the solutions to each of the two equations describes a necessary condition for a stable steady state (it is the smaller root if $A\left(a\right) > 0$, the larger root, if $A\left(a\right) < 0$, the only root otherwise). It can be shown that both relevant solutions belong to the unit interval (the proof is trivial, but lengthy, so we do not include it here).

Moreover, each of the relevant solutions may be represented as a function, $o^r\left(o^p\right)$ and $o^p\left(o^r\right)$ correspondingly. An intersection of these functions describes a stable steady state for given q^{ss} and parameters, if after a small deviation in o^p or o^r the system converges back to this intersection. Depending on α, there are following possibilities:

1) $\alpha < \frac{1-pq}{2-pq}$ $(A > 0, a < 0)$

$$\begin{aligned} o^r &= \frac{1}{2} + \frac{G+C}{2A} - \sqrt{\left(\frac{1}{2} + \frac{G+C}{2A}\right)^2 - \frac{C}{A}}, \\ o^p &= \frac{1}{2} + \frac{h+c}{2a} + \sqrt{\left(\frac{1}{2} + \frac{h+c}{2a}\right)^2 - \frac{c}{a}}; \end{aligned}$$

2) $\alpha > \frac{1}{2}$ $(A < 0, a > 0)$

$$\begin{aligned} o^r &= \frac{1}{2} + \frac{G+C}{2A} + \sqrt{\left(\frac{1}{2} + \frac{G+C}{2A}\right)^2 - \frac{C}{A}}, \\ o^p &= \frac{1}{2} + \frac{h+c}{2a} - \sqrt{\left(\frac{1}{2} + \frac{h+c}{2a}\right)^2 - \frac{c}{a}}; \end{aligned}$$

3) $\frac{1-pq}{2-pq} < \alpha < \frac{1}{2}(A < 0, a < 0)$

$$\begin{aligned} o^r &= \frac{1}{2} + \frac{G+C}{2A} + \sqrt{\left(\frac{1}{2} + \frac{G+C}{2A}\right)^2 - \frac{C}{A}}, \\ o^p &= \frac{1}{2} + \frac{h+c}{2a} + \sqrt{\left(\frac{1}{2} + \frac{h+c}{2a}\right)^2 - \frac{c}{a}}; \end{aligned}$$

And the special cases are

$$\begin{aligned} o^r &= \frac{C}{G+C}, \alpha = \frac{1-pq}{2-pq}; \\ o^p &= \frac{c}{c+h}, \alpha = \frac{1}{2}. \end{aligned}$$

Inspecting these expressions closely, one can establish that the intersection defining the stable steady state is unique. To show this, we first check the limiting expressions for each function. For the rich opposition we have with $\alpha < \frac{1-pq}{2-pq}$ $(A > 0)$:

$$\begin{aligned} \lim_{o^p \to 0} o^r &= \frac{1}{2} + \frac{G}{2A} - \sqrt{\left(\frac{1}{2} + \frac{G}{2A}\right)^2} = 0; \\ \lim_{o^p \to 1} o^r &= \frac{1}{2} + \frac{C}{2A} - \left|\frac{1}{2} - \frac{C}{2A}\right| = \frac{C}{A}, C < A, \\ &= 1, C > A. \end{aligned}$$

With $\alpha > \frac{1-pq}{2-pq}\,(A < 0)$:

$$\begin{aligned}
\lim_{o^p \to 0} o^r &= \frac{1}{2} + \frac{G}{2A} + \sqrt{\left(\frac{1}{2} + \frac{G}{2A}\right)^2} = 1 + \frac{G}{A}, 1 + \frac{G}{A} > 0, \\
&= 0, 1 + \frac{G}{A} < 0; \\
\lim_{o^p \to 1} o^r &= \frac{1}{2} + \frac{C}{2A} + \sqrt{\left(\frac{1}{2} - \frac{C}{2A}\right)^2} = 1.
\end{aligned}$$

The limiting expressions for the poor if $\alpha > \frac{1}{2}\,(a > 0)$

$$\begin{aligned}
\lim_{o^r \to 0} o^p &= \frac{1}{2} + \frac{h}{2a} - \sqrt{\left(\frac{1}{2} + \frac{h}{2a}\right)^2} = 0, \\
\lim_{o^r \to 1} o^p &= \frac{1}{2} + \frac{c}{2a} - \left|\frac{1}{2} - \frac{c}{2a}\right| = \frac{c}{a}, c < a; \\
&= 1, c > a.
\end{aligned}$$

If $\alpha < \frac{1}{2}\,(a < 0)$, we get

$$\begin{aligned}
\lim_{o^r \to 0} o^p &= \frac{1}{2} + \frac{h}{2a} + \left|\frac{1}{2} + \frac{h}{2a}\right| = 1 + \frac{h}{a}, 1 + \frac{h}{a} > 0; \\
&= 0, 1 + \frac{h}{a} < 0; \\
\lim_{o^r \to 1} o^p &= \frac{1}{2} + \frac{c}{2a} + \sqrt{\left(\frac{1}{2} - \frac{c}{2a}\right)^2} = 1.
\end{aligned}$$

>From the limiting expressions we see that each of the functions $o^r\,(o^p)$ and $o^p\,(o^r)$ has a unit interval as its domain; its range is contained in unit interval. We also know that these functions are parts of ellipses describing the whole set of solutions to the corresponding quadratic equations.

By the nature of ellipse, its upper part is concave and the lower is convex. So whenever our solution is the higher root, we know it is concave; when it is the lower root, it is convex. So, with $A\,(a) > 0$ we have a smaller root - a convex curve, in the opposite case a concave one. We must also remember that inverse of a convex function is a concave function.

We can see that for $\alpha \in \left[\frac{1-pq}{2-pq}, \frac{1}{2}\right]$ $(A < 0, a < 0)$, as $\lim_{o^r \to 1} o^p = 1$ and $\lim_{o^p \to 1} o^r = 1$, $(1, 1)$ is an intersection. As $\lim_{o^r \to 0} o^p \geq 0$ and $\lim_{o^p \to 0} o^r \geq 0$, $o^r\,(o^p)$ is concave and inverse of $o^p(o^r)$ is convex, the only other possible intersection is at $o^r = o^p = 0$. But this is an unstable steady state. To see that intuitively, one can plot the corresponding curves and check how the dynamic converges to $(1,1)$ for arbitrarily small deviation from $(0,0)$. Therefore, the unique stable steady state in this case is $(1,1)$.

For $\alpha < \frac{1-pq}{2-pq}$ $(A > 0, a < 0)$, $\lim_{o^r \to 1} o^p = 1$ and $\lim_{o^p \to 1} o^r \leq 1$; $\lim_{o^r \to 0} o^p \geq 0$ and $\lim_{o^p \to 0} o^r = 0$. There are at most 3 intersections; both curves are now convex. The stable steady state is again unique: it is an interior intersection, if it exists, and a corner intersection ($(0,0)$ or $(1,1)$) otherwise.

For $\alpha > \frac{1}{2}(A < 0, a > 0)$, $\lim_{o^r \to 1} o^p \leq 1$ and $\lim_{o^p \to 1} o^r = 1$; $\lim_{o^r \to 0} o^p = 0$ and $\lim_{o^p \to 0} o^r \geq 0$. Again, there are at most 3 intersections while both curves are concave. $(0,0)$ is always an intersection, but it is only a stable steady state, if no other intersection exists. Again, if an interior intersection exists, it describes a unique stable steady state.

To sum up, for any given parameter combination and q^{ss} a stable steady state (o^r, o^p) exists and is unique.

Finally, the compliance equation has the following coefficients:

$$\begin{aligned}
A_0 &= \gamma o^r p^2 \eta, \\
B_0 &= \gamma o^r (Q - p\eta) + QD + \gamma(1 - o^r)(Q(1 - \beta(1-\alpha)) - \alpha\beta\eta) - 1, \\
C_0 &= \eta\left((1 - o^r)\left(\gamma(2\alpha\beta + 1 - \beta) + (1-\gamma)\frac{o^p}{o^r}\alpha\beta\right) + D + \gamma o^r\right), \\
Q &:= 1 - p - \eta, \\
D &:= (1-\gamma)(1 - \beta(1 - \alpha - o^p + o^p\alpha)).
\end{aligned}$$

Since $A_0 \geq 0$, only the smaller solution characterizes the stable steady state. Thus, for each parameter combination we have at most one triple (o^r, o^p, q) that defines a stable steady state.

Proof of Proposition 1 (ii): From part (i) we know that each of the 3 equations characterizing the stable steady state always have a solution, but it is not always an interior solution. We have already seen that for $\alpha \in \left[\frac{1-pq}{2-pq}, \frac{1}{2}\right]$ no interior solution is possible, and a unique stable steady state in this case is $(1, 1, q^{ss})$.

Inspecting the limiting expressions again (and plotting corresponding pictures), we can also see that sometimes the interior solution is sure regardless of the slope of our functions:

1) $A > 0, a < 0$ ("small alpha"). In this case, $C < A, 1 + \frac{h}{a} > 0$ give intersection at the interior. The conditions may be rewritten as

$$\frac{\alpha}{(1-p)\delta q + (1-q)(1-\alpha)} < \gamma < \frac{1-2\alpha}{1-\alpha}.$$

2) $A<0, a>0$ ("big alpha"). For large government spending, $c<a, 1+\frac{G}{A}>0$ give an interior solution. Rewritten, we have

$$\frac{1-\alpha}{\alpha}\frac{P}{(2-q-P)}<\gamma<\frac{1}{\left(q\beta\left(1-\delta\left(1-p\right)\right)/\left(1-\beta\right)+\left(1-\beta\right)\left(1-q\right)\right)\left(1-\alpha\right)/\left(2\alpha-1\right)+1}.$$

If these conditions are not satisfied, we may still have the interior solution under certain conditions on the slope of the two curves at the corners. We have then to compute the slope:

$A>0$:

$$\begin{aligned}
o^{r\prime} &= \frac{G'+C'}{2A}-\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)^2-\frac{C}{A}\right)^{-1/2}\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)\frac{G'+C'}{2A}-\frac{C'}{2A}\right),\\
o^{r\prime}\left(0\right) &= \frac{G'+C'}{2A}-\frac{G'+C'}{2A}+\frac{C'}{2A}\left(\frac{1}{2}+\frac{G}{2A}\right)^{-1}=\frac{C'}{2A}\left(\frac{1}{2}+\frac{G}{2A}\right)^{-1}=\frac{C'}{A+G}>0,\\
o^{r\prime}\left(1\right) &= \frac{C-A+G'}{A}\frac{C}{C-A}>0, C<A,\\
o^{r\prime}\left(1\right) &= \frac{G'+C}{2A}+\left(\frac{1}{2}-\frac{C}{2A}\right)^{-1}\left(\left(\frac{1}{2}+\frac{C}{2A}\right)\frac{G'+C}{2A}-\frac{C}{2A}\right)=\frac{G'}{A-C}, C>A.
\end{aligned}$$

$A<0$:

$$\begin{aligned}
o^{r\prime} &= \frac{G'+C'}{2A}+\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)^2-\frac{C}{A}\right)^{-1/2}\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)\frac{G'+C'}{2A}-\frac{C'}{2A}\right),\\
o^{r\prime}\left(1\right) &= \frac{G'+C}{2A}+\left(\frac{1}{2}-\frac{C}{2A}\right)^{-1}\left(\left(\frac{1}{2}+\frac{C}{2A}\right)\frac{G'+C}{2A}-\frac{C}{2A}\right),\\
&= \frac{G'}{A-C}>0;
\end{aligned}$$

$$\begin{aligned}
o^{r\prime}\left(0\right) &= \frac{G'+C'}{2A}+\left(\left(\frac{1}{2}+\frac{G}{2A}\right)^2\right)^{-1/2}\left(\left(\frac{1}{2}+\frac{G}{2A}\right)\frac{G'+C'}{2A}-\frac{C'}{2A}\right),\\
&= -\frac{1}{A}\frac{G}{A+G}\left(A+G-C'\right), 1+\frac{G}{A}>0,\\
&= \frac{C'}{A+G}, 1+\frac{G}{A}<0,\\
&= \frac{C'-G}{2A}, 1+\frac{G}{A}=0.
\end{aligned}$$

Note that $o^r\,(o^p)$ is monotonic.

Since problems for o^r and o^p are 'dual', it is enough to establish that $c(0) = 0, h(1) = 0, c(1) = c', h(0) = -h'$, and we can immediately write

for $a > 0$

$$\begin{aligned} o^{p\prime}(0) &= \frac{c'}{a+h}, \\ o^{p\prime}(1) &= \frac{c-a+h'}{a}\frac{c}{c-a}. \end{aligned}$$

for $a < 0$

$$\begin{aligned} o^{p\prime}(1) &= \frac{h'}{a-c} > 0, \\ o^{p\prime}(0) &= -\frac{a+h-c'}{a}\frac{h}{a+h}. \end{aligned}$$

Note that $o^p\,(o^r)$ is monotonic as well.

Once we know this, we can consider all the possible combinations of the parameters and formulate necessary and sufficient conditions for the existence of the interior stable steady state:

Necessary and sufficient conditions

1. $A > 0, a < 0:$

a) $C < A, 1 + h/a \leq 0, o^{r\prime}(0) = \frac{C'}{A+G} > o^{p\prime -1}(0) = -\frac{a+h}{a+h-c'}\frac{a}{h};$

b) $C > A, 1 + h/a > 0, o^{r\prime}(1) = \frac{C-A+G'}{A}\frac{C}{C-A} > o^{p\prime -1}(1) = \frac{a-c}{h'};$

c) $C < A, 1 + h/a > 0;$

d) $C > A, 1 + h/a < 0, o^{r\prime}(0) > o^{p\prime -1}(0), o^{r\prime}(1) > o^{p\prime -1}(1).$

2. $A < 0, a > 0:$

a) $1 + G/A > 0, c \geq a, o^{r\prime}(1) = \frac{G'}{A-C} > o^{p\prime -1}(1) = \frac{a}{c-a+h'}\frac{c-a}{c};$

b) $1 + G/A \leq 0, c < a, o^{r\prime}(0) = -\frac{1}{A}\frac{G}{A+G}(A + G - C') > o^{p\prime -1}(0) = \frac{a+h}{c'};$

c) $c < a, 1 + \frac{G}{A} > 0;$

d)$c < a, 1 + \frac{G}{A} > 0, o^{r\prime}(1) > o^{p\prime -1}(1), o^{r\prime}(0) > o^{p\prime -1}(0).$

Working out these conditions, we can get the following formulation:

1. $\alpha < \frac{P}{1+P}$, where $P := 1 - pq^{ss}$, *and*

a) $\alpha < \min\left\{\frac{\gamma P}{1+\gamma P}, \frac{1-\gamma}{2-\gamma}\right\}$

or b) $\frac{1-\gamma}{2-\gamma} < \alpha < \frac{\gamma P}{1+\gamma P}$

or c) $\frac{\gamma P}{1+\gamma P} < \alpha < \frac{1-\gamma}{2-\gamma}$ together with the following condition:

$$\frac{(1-\gamma)(1-\alpha)P}{\alpha - P\gamma + P\alpha\gamma} > \frac{\alpha\gamma - 2\alpha + 1}{\gamma\alpha}. \tag{7}$$

or d) $\alpha > \max\left\{\frac{\gamma P}{1+\gamma P}, \frac{1-\gamma}{2-\gamma}\right\}$ together with condition (7) and the following condition:

$$\frac{\gamma\alpha^2}{(1-\alpha)P - \gamma\alpha} > -(1-\gamma)(2\alpha - 1) - \gamma\alpha.$$

or 2. $\alpha > \frac{1}{2}$ *and*
a) $\frac{P}{P+\gamma} < \alpha < \frac{1}{2-\gamma}$
or b) $\alpha > \max\left\{\frac{1}{2-\gamma}, \frac{P}{P+\gamma}\right\}$
or c) $\alpha < \min\left\{\frac{1}{2-\gamma}, \frac{P}{P+\gamma}\right\}$ together with the following conditions:

$$\begin{aligned} \frac{-(1-\gamma)(1-\alpha)P}{P\gamma - \alpha - P\alpha\gamma} &> \frac{2\alpha - \alpha\gamma - 1}{-\gamma\alpha}, \\ \frac{(1-\gamma)\alpha}{P - P\alpha - \alpha\gamma} &> \frac{2\alpha + \gamma - \alpha\gamma - 1}{\gamma(1-\alpha)}. \end{aligned}$$

To sum up, we see that the interior stable steady state may exist only if $\alpha < \frac{P}{1+P} \leq \frac{1}{2}$ or if $\alpha > \frac{1}{2}$, and that is exactly the statement of the Proposition $1(ii)$.

8.5 Appendix 5

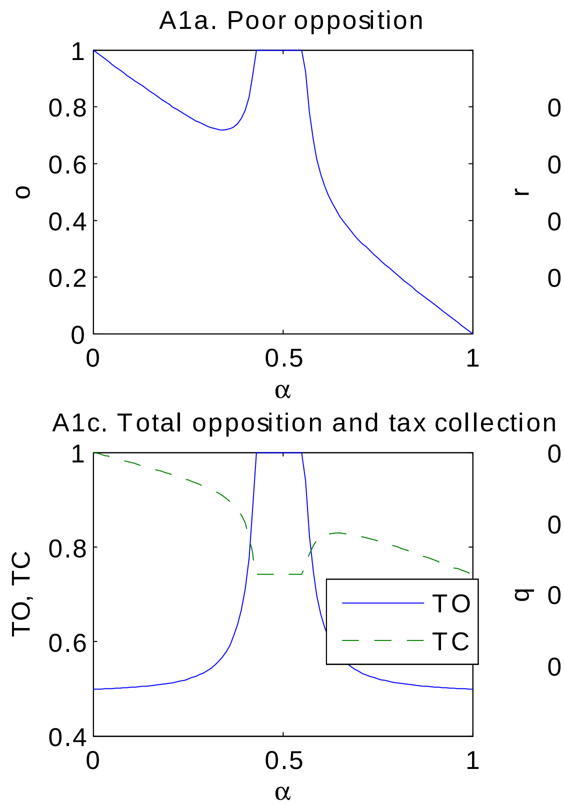

Figure A1, $\gamma = 0.5$.