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FIRMS UNDER TAX ASYMMETRY: PRICE UNCERTAINTY AND HEDGING

by

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Abstract: We study the optimal decisions regarding production and hedging of a competitive firm under price uncertainty. The firm faces asymmetric tax (i.e., profits and losses are taxed at different rates) and has access to futures markets. The main findings are: (a) Risk neutral firms will engage in hedging in order to lower the expected tax; moreover, their output increases as a result of such hedging activity. (b) For risk-averse firms, under asymmetric tax, the optimal output level is independent of the tax rates, the price distribution and its attitude towards risk. (c) The optimal hedging policy differs significantly compared with the symmetric tax case.

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1. Introduction

The engagement of firms operating under price uncertainty in hedging and its effect on production decisions has attracted a great attention of economists for many years (see, for example, Holthausen (1979), Feder, Just and Schmitz (1980), Eldor and Zilcha (1987)). Another strand of the literature studies the impact of taxation on firm's production decisions in the presence of uncertainty (see, for example Sandmo (1971)). This work integrates these two strands of the literature and studies the impact of taxation on production and hedging behavior of a competitive firm facing price uncertainty with access to a futures market to the commodity produced by the firm.

Examining the corporate tax codes of many countries we see a clear asymmetry in taxation between profits and losses (see for example: Corporate Taxes: A Worldwide Summary; 1996, Price Waterhouse). Whenever "carry forward" applies to losses, which is the case in most countries, we still have asymmetry between positive and negative profits for the following reasons: (a) In most countries, as can be seen from Table 1, there is no "carry back" of losses. Thus a firm which loses at a certain period cannot deduct the losses (or part of the losses) from taxes paid in earlier years. (b) When only carry-forward of losses applies there is a loss of interest and opportunities due to the delay in the (uncertain) future compensation. (c) When losses "carry back" is applicable (for example, in the U.S. and the U.K.) it is limited in time and it is not applicable for young firms. Table 1 presents a relevant part of the corporate tax codes in various countries, emphasizing the asymmetry between profits and losses and the progressivity of corporate taxes in some countries.

Since the corporate tax code exhibits asymmetry between profits and losses, we shall concentrate on this case. We claim that under tax asymmetry, hedging has a significant effect on firm's optimal production level and market value. This is an additional argument in support of hedging to those studied in the literature to explain the widespread phenomena of hedging behavior by corporations (see, e.g., Mian (1996)). The main argument mentioned

TABLE 1

Country	Corporate tax	Loss carry-forward	Loss carry-back
Argentine	30%	5 years	none
Australia	36%	7 years	none
Austria	34% (+minimum)	7 years	none
Belgium	39%	unlimited	none
Brazil	15% (+surcharge)	up to 30% of loss	none
Chile	15%	yes	none
Cyprus	20% (+surcharge)	none	none
Denmark	34%	5 years	none
Finland	28%	10 years	none
France	33%	5 years	3 years
India	40% (+ surcharge)	8 years	none
Indonesia	progressive , 30%	5 years	none
Israel	36%	unlimited	none
Japan	28% ; 37.5% for large	5 years	1 year
Korea	16%, 28% for large	5 years	none
Kuwait	progressive -55%	unlimited	none
Luxemburg	20%, 30%, 33%	5 years	none
Mexico	34%	10 years	none
Morroco	35%	4 years	none
The Netherlands	35%	unlimited	3 years
Norway	28%	10 years	2 years
Saudi Arabia	progressive, 45%	none	none
United Kingdom	progressive , 33%	unlimited	3 years
United States	progressive, 35%	15 years	3 years
Venezuela	15%, 22%, 34%	3 years	none

in the literature regarding hedging and taxation is that hedging results in lower average taxes for the firm and hence raises its value (see, Smith and Stulz (1985), Froot, Scharfstein and Stein (1993)). However this claim has not been established analytically in a specific model. Other arguments are related to the "informational effect" of hedging where it is used by the management to signal this way to the shareholders about its performance (see DeMarzo and Duffie (1995)), or as a result of managerial risk aversion (see Smith and Stulz (1985)).

We consider here competitive firms facing price uncertainty and asymmetric (and symmetric) taxation. We begin the paper with a "benchmark case" where tax is at a fixed percentage and applies to both profits and losses (i.e., subsidy in the latter case). This case will be called the 'symmetric tax' case. For this case we derive a 'separation property' when forward/futures markets for this good exist: Optimal production level does not depend on the tax rate, as well as the price distribution and attitude towards risk of the firm. The behavior of optimal hedging policies as a function of the tax rate is studied as well for this case.

When tax is asymmetric (i.e., profits are taxed at a higher rate than losses) we consider first *risk-neutral* firms. Unlike the symmetric tax case, the optimal behavior of risk-neutral firms includes hedging in the futures/forward markets (although we assume unbiased futures market) which results in lower expected taxes. Moreover, production by risk-neutral firms is enhanced by the introduction of hedging tools and expected taxes are lower. For risk-averse firms we obtain that (a) the separation property holds under asymmetric taxation, (b) in the abscence of hedging tools the optimal production level of the firm under asymmetric tax differs from that of the symmetric tax case. Moreover, its behavior with respect to changes in the tax rate differs significantly. (c) compared with the symmetric tax case the optimal hedging policy of risk averse firms differs as a result of tax asymmetry in a way that reduces the variability of the profits.

The paper is organized as follows. In Section 2 we consider a 'benchmark case': competitive firms facing both price uncertainty and symmetric taxation in the presence of futures market. The impact of asymmetric taxation on production level and hedging policies is analyzed in Section 3, for risk neutral firms. Section 4 presents the case of a risk averse competitive firm facing asymmetric taxation. Section 5 contains a discussion and some concluding remarks.

2. The Benchmark Case: Tax Symmetry with Futures Markets

Consider a price-taking firm which produces a commodity under random price $\tilde{P}$. The firm determines its output Q at time 0 where production is completed at time 1, the spot price $\tilde{P}$ is realized and transactions take place. We assume that the firm's technology of production gives rise to a cost function $C(Q)$ which satisfies: $C(0) = 0$, $C'(Q) > 0$ and $C''(Q) \geqq 0$. In this secion we take the firm to be risk-averse with a von-Neumann Morgenstern utility function defined on profits $U(\pi)$, where $U(0) = 0$, $U' > 0$ and $U'' < 0$. Thus the firm maximizes expected utility of net profits. We also assume in this section that the same tax rate t applies to profits as well as to losses, i.e., the firm's net profits are $(1-t)\tilde{\pi}$. This is the case considered by Sandmo (1971). However, Sandmo does not allow the firm to engage in risk-sharing activities such as futures markets, as we do here.

The firm chooses its production level Q and its hedging level X in a way that maximizes its expected utility of net profits, i.e.,

$$\underset{Q,X}{Max}\; E\, U\left[(1-t)\tilde{\pi}(Q,X)\right] \text{ s.t.}$$
$$\tilde{\pi}(Q,X) = \tilde{P}Q + (P_f - \tilde{P})X - C(Q).$$

Necessary and sufficient conditions for optimum (Q^*, X_s^*) are:

$$(2.1) \qquad E\left\{\left[\tilde{P} - C'(Q^*)\right] U'\left[(1-t)\tilde{\pi}(Q^*, X_s^*)\right]\right\} = 0.$$

$$E\left\{(P_f - \tilde{P})U'\left[(1-t)\tilde{\pi}(Q^*, X_s^*)\right]\right\} = 0. \tag{2.2}$$

It follows from equations (2.1) and (2.2) that under symmetric tax the separation property holds.

Proposition 2.1. ***Separation with symmetric tax:*** *When the tax rate is symmetric the separation theorem holds, i.e., in the presence of futures market the firm's output is determined by: $C'(Q^*) = P_f$. In particular Q^* is independent of the tax rate.*

Now let us show that the optimal hedging behavior obtained in the no-tax case can be generalized to the symmetric tax case as well. Denote by X_s^* the optimal hedging policy in the *symmetric* tax case. Namely, from (2.2) we derive,

$$E(P_f - \tilde{P})EU'\left[(1-t)\tilde{\pi}(Q^*, X_s^*)\right] = Cov\left(\tilde{P}, U'\left((1-t)\tilde{\pi}\right)\right).$$

Therefore, one obtains:

$$X_s^* \mathrel{\substack{<\\=\\>}} Q^* \Leftrightarrow P_f \mathrel{\substack{<\\=\\>}} E\tilde{P}. \tag{2.3}$$

Note however that even though Q^* is independent of the attitude towards risk and the distribution of the random price (with futures markets), the optimal hedging policy X_s^* depends on both as well as on the tax rate t. Let us consider now the behavior of X_s^* as a function of t. Let $R_r(\pi)$ be the *relative* measure of risk aversion for this firm.

Proposition 2.2. *The behavior of the optimal hedging policy $X_s^*(t)$ as a function of the (symmetric) tax rate t depends on the monotonicity of $R_r(\pi)$ as follows:*

$$\frac{\partial X_s^*(t)}{\partial t}\begin{cases} <0 \;\; if \;\; R_r \;\; is\ increasing \\ =0 \;\; if \;\; R_r \; = constant \\ >0 \;\; if \;\; R_r \;\; is\ decreasing \end{cases} \tag{2.4}$$

This result, concerning the behavior of the optimal hedging policy in relation to the monotonicity of the **relative** measure of risk aversion, is reminiscent of the result obtained by Sandmo (1971) relating the *optimal output* (with respect to the tax rate t) to this measure (of course, in the absence of hedging markets). Sandmo has shown that when no futures markets exist, the optimal output $Q^*(t)$ **increases** in t if $R_r(\pi)$ is increasing and **decreases** in t if $R_r(\pi)$ is decreasing. In the presence of futures markets Q^* is **independent** of the tax rate t (and the utility function as well!) while the behavior of $X_s^*(t)$ is **reversed** as shown in (2.4).

The economic rational underlying proposition 2.2 is the following. Increasing the tax rate t lowers the expected profits on the one hand, but also lowers its variance; thus the resulting hedging policy depends on the attitude towards risk of this firm. Increasing relative risk aversion implies more aversion to risk in the firm's profits on the one hand, but higher t results in lower variance as well, hence in this case the exposure to risk, via $Q^* - X_s^*$, increases. For constant relative risk aversion the two opposing effects are balanced, hence $Q^* - X_s^*$ remains unchanged as t increases.

Proof. Differentiating equation (2.2) with respect to t we obtain that

$$\frac{\partial X_s^*}{\partial t} = \frac{E\{(P_f - \tilde{P})\tilde{\pi}U''\left[(1-t)\tilde{\pi}\right]\}}{(1-t)E\{(P_f - \tilde{P})^2 U''\left[(1-t)\tilde{\pi}\right]\}} \tag{2.5}$$

Let us find the sign of the numerator since the denominator is negative. Assume first that $R_r(\pi)$ is increasing, then

$$R_r\left[(1-t)\pi\right] > R_r(\bar{\pi}), \;\; \text{for} \;\; P > P_f \;\; (\text{or } \pi > \bar{\pi})$$

where $\bar{\pi} = (1-t)[P_f Q^* - C(Q^*)]$. Therefore,

$$-\pi U''\left[(1-t)\pi\right] > R_r(\bar{\pi})U'\left((1-t)\pi\right) \quad \text{for} \quad \pi > \bar{\pi} \text{ (or } P > P_f).$$

Thus we have,

$$(2.6) \qquad \pi(P_f - P)U''\left[(1-t)\pi\right] > R_r(\bar{\pi})(P_f - P)U'\left[(1-t)\pi\right] \quad \text{for all } P > P_f.$$

However, (2.6) holds also for all $P < P_f$ as well. Therefore,

$$E(P_f - \tilde{P})U''\left[(1-t)\pi\right] > R_r(\bar{\pi})E(P_f - \tilde{P})U'\left[(1-t)\bar{\pi}\right] = 0$$

which implies that $\dfrac{\partial X_s^*}{\partial t} < 0$. The other two cases are proved similarly. ■

In the next section we analyze the tax asymmetry case for a risk-neutral firm facing price uncertainty.

3. Risk Neutrality and Tax Asymmetry

Throughout the rest of this work we assume that firms pay tax at rate t_1 whenever the profit is positive; However, when a firm suffers losses it is compensated by the tax authorities, but at a **lower rate** t_2, namely, $t_1 > t_2$. To simplify the analysis and the notations we shall assume that $t_1 = t > 0$ while $t_2 = 0$. Our results remain unchanged as long as $0 \leqq t_2 < t_1$.This assumption represents reality in many countries.

We now take the random price $\tilde{P}$ to assume values in $[\underline{P}, \bar{P}]$, $0 < \underline{P} < \bar{P} < \infty$. Denote by $f(P)$ the probability density function corresponding to $\tilde{P}$.

We shall demonstrate now that risk-neutral firms will engage in hedging under asymmetric taxation. Such an activity is redundant in our framework under a symmetric tax system. Namely, the existence of unbiased futures market will not vary the **expected** profits if $(1-t)\tilde{\pi}$ is the net profit in all states of nature.

The optimization problem solved by the risk-neutral firm can be written as follows:

$$\text{(3.1)} \qquad \underset{Q,X}{Max}\,[(1-t)E\;Max\,(0,\tilde{\pi}(Q,X)) + E\;Min\,((0,\tilde{\pi}\,(Q,X))]$$
$$= E\tilde{\pi}(Q^*,X^*) - t\;E\;Max\,(0,\tilde{\pi}(Q^*,X^*))\,.$$

A mean-preserving squeeze in the distribution of profits will reduce the expected tax payment. Thus any hedging operation that achieves such a change in the distribution of profits is desirable. Write,

$$\tilde{\pi}(Q,X) = \tilde{P}(Q-X) + P_f X - C(Q), \quad \text{hence}$$
$$E\tilde{\pi} = (Q-X)E\tilde{P} + P_f X - C(Q)$$

Since $E\tilde{\pi}(Q,X)$, when $P_f = E\tilde{P}$ does not depend on X it is maximized at:

$$\text{(3.2)} \qquad C'(Q^*) = P_f.$$

This condition is basically the separation property for risk-neutral firms under unbiasedness assumption. In addition, when $P_f = E\tilde{P}$ hedging does not vary the mean $P_f Q^* - C(Q^*)$, hence by (3.1) the hedging is aimed at

$$\text{(3.3)} \qquad \underset{X}{Min}\,\{E\;Max(0,\tilde{\pi}(Q^*,X)) \mid E\tilde{\pi} = P_f Q^* - C(Q^*)\}\,.$$

This minimum is achieved when $\tilde{\pi}(Q^*,X) \geqq 0$ in probability 1 for the following reason. Any X which satisfies $\Pr\{\tilde{\pi}(Q^*,X) < 0\} > 0$ will increase the profits for some states of nature since the expected value is fixed. Negative profits do not affect the function being minimized at (3.3), while the higher positive profits increases its value. Thus, at the optimum, i.e., minimum of $E\max(0, \pi(Q^*, X))$, we obtain that $\tilde{\pi}(Q^*, X^*) \geqq 0$ in probability 1. However, this implies that there is no unique hedging level X^*, which is optimal. To show this claim let us rewrite this minimization problem as follows:

$$\min_X \int_{P^*(Q^*,X)}^{P^{**}(Q^*,X)} \left[\tilde{P}(Q^* - X) + P_f X - C(Q^*)\right] f(p)dp \tag{3.4}$$

where $P^*(Q^*, X)$ is the minimal price for which $\pi(Q^*, X) = 0$ and $P^{**}(Q^*, X)$ is the highest price for which $\pi(Q^*, X) = 0$. The first-order condition for (3.4) is (noting that profits are 0 at **both** limits of the integral):

$$\int_{P^*(Q^*,X^*)}^{P^{**}(Q^*,X^*)} \left[-\tilde{P} + P_f\right] f(p)dp = 0. \tag{3.5}$$

However, condition (3.5) holds if $P^*(Q^*, X^*) = \underline{P}$ and $P^{**}(Q^*, X^*) = \bar{P}$. Namely, if $\tilde{\pi}(Q^*, X^*) \geqq 0$ in probability 1. Consequently, the optimal hedging X^* is not unique, since $\underline{P} < P_f < \bar{P}$, X^* must lie in the interval:

$$X^* \in [X^*_{\min}, X^*_{\max}]$$

where

$$X^*_{\min} = \frac{C(Q^*) - \underline{P}\, Q^*}{P_f - \underline{P}} \quad \text{and} \quad X^*_{\max} = \frac{\bar{P}Q^* - C(Q^*)}{\bar{P} - P_f}. \tag{3.6}$$

where Q^* is given by (3.2). This demonstrates that a risk-neutral firm operating in an **unbiased** futures market will engage in hedging even though the optimal hedge level may vary in some interval (not containing 0). Moreover, it is easy to verify that $X^*_{\min} < Q^* < X^*_{\max}$; thus full hedging is one possibility at the optimum.

Proposition 3.1. *Let the futures market be unbiased. A risk-neutral firm will engage in hedging, its optimal hedge is nonunique and it is any X^* in $[X^*_{\min}, X^*_{\max}]$ given by (3.6), and $X^*_{\min} < Q^* < X^*_{\max}$.*

As we have indicated earlier it is optimal for this firm to eliminate all states of nature with negative profits, where the benefits from losses are nonexistent. Thus, either X^* lower than $X^*_{\min}$ or X^* larger than $X^*_{\max}$ are not optimal for this firm, while for any X^* in this interval we have $\tilde{\pi}(Q^*, X^*) \geqq 0$.

The effect of hedging markets on production is given by

Proposition 3.2. *For a risk-neutral firm:*

(a) Under symmetric taxation introducing futures market has no effect on optimal output and expected profits.

(b) Under asymmetric tax introducing futures market results in higher production level and higher expected profits.

Proof. In the symmetric tax case the output is given by

$C'(\bar{Q}) = E\tilde{P}$. With futures market it is also $C'(\bar{Q}) = P_f = E\tilde{P}$.

Consider the asymmetric taxation case in the absence of futures market. The firm produces according to:

$$\underset{Q}{Max}\ \{E\tilde{\pi}(Q) - t\ E\max(0, \tilde{\pi}(Q))\}$$

where $\tilde{\pi}(Q) = \tilde{P}Q - C(Q)$. The maximand can be rewritten as:

$$E\tilde{P}Q - C(Q) - t\int_{C(Q)/Q}^{\bar{P}} [\tilde{P}Q - C(Q)]f(p)dp.$$

The first-order condition is:

$$E\tilde{P} - C'(Q^*) - t\int_{P_0}^{\bar{P}} [\tilde{P} - C'(Q^*)] f(p)dp = 0$$

where $P_0 = C(Q^*)/Q^* > \underline{P}$. This can be rewritten as

$$C'(Q^*) = \frac{1}{1-t}\left[E\tilde{P} - t\int_{P_0}^{\bar{P}} \tilde{P} f(p)dp\right].$$

Since the density of $\tilde{P}$ on $[\underline{P}, P_0)$ is positive and $E\tilde{P} < \int_{P_0}^{\bar{P}} P f(p)dp$ we obtain:

$$C'(Q^*) < \frac{1}{1-t}\left[E\tilde{P}(1-t)\right] = E\tilde{P}.$$

On the other hand, when futures markets exist and $P_f = E\tilde{P}$ the optimal output $\hat{Q}$ is given by $C'(\hat{Q}) = P_f$ which implies that $C'(Q^*) < C'(\hat{Q})$, i.e., $Q^* < \hat{Q}$. Moreover, when hedging markets exist the optimal outputs under symmetric and asymmetric taxation coincide. The fact that the risk-neutral firm hedges when tax is asymmetric demonstrates that its expected profits are higher. ■

Now let us consider the effect of a mean-preserving spread (MPS) in the distribution of $\tilde{P}$ on the optimal hedging of risk-neutral firm. Let us assume only cases where $\underline{P}$ declines, $\bar{P}$ increases while $E\tilde{P}$ remains unchanged.

Proposition 3.3. *A Mean-preserving spread in the distribution of the random price results in a contraction of the optimal hedge interval* $[X^*_{\min}, X^*_{\max}]$, *i.e.,* $X^*_{\min} \uparrow$ *while* $X^*_{\max} \downarrow$.

Proof. Consider a MPS where $\underline{P}$ declines and $\bar{P}$ increases. It is easy to verify from (3.6) that $X^*_{\min}$ is a decreasing function of $\underline{P}$ (since P_f remains unchanged) and $X^*_{\max}$ is a decreasing function of $\bar{P}$. Therefore, $X^*_{\min} \uparrow$ and $X^*_{\max} \downarrow$. ■

Let us consider the following progressive tax: When profits are positive the tax rate is t for profits in the range $0 \leqq \pi \leqq \bar{\pi}$ and it increases to $t_1 > t$ if $\pi > \bar{\pi}$. Moreover, let us assume that $\pi^* = P_f Q^* - C(Q^*) \leqq \bar{\pi}$ (i.e., profits when $X^* = Q^*$ are at the lower tax bracket). Also assume that the probability that profits without hedging are higher than $\bar{\pi}$ is positive, i.e., $\Pr\left\{\tilde{P}Q^* - C(Q^*) > \bar{\pi}\right\} > 0$. Now let us show that under a progressive tax system hedging can be used to avoid the higher tax bracket.

Proposition 3.4. *Under the above assumptions if additional higher tax bracket $t_1 > t$ is introduced, then the optimal hedging interval shrinks to*

$$[X^{**}_{\min}, X^{**}_{\max}], \text{ where } X^{**}_{\min} \geqq X^*_{\min} \text{ and } X^{**}_{\max} \leqq X^*_{\max},$$

such that the highest tax rate t_1 is not applicable to the firm after the hedging.

Proof. Write $t_1 = t + \Delta$, $\Delta > 0$. By the same argument used to derive (3.3) in this case the optimization implies:

$$\underset{X}{Min}\ \{t\ E\ \max(0, \tilde{\pi}(Q^*, X)) + \Delta\ E\ \max[0, \tilde{\pi}(Q^*, X) - \bar{\pi}]\}.$$

Thus an optimal hedge X^* will guarantee that $\Pr ob\{\tilde{\pi}(Q^*, X^*) - \bar{\pi} > 0\} = 0$.
In this case we obtain that the following constraints should hold:
$0 \leqq \bar{P}(Q^* - X) + P_f X - C(Q^*) \leqq \bar{\pi}.$
$0 \leqq \underline{P}\,(Q^* - X) + P_f X - C(Q^*) \leqq \bar{\pi}.$
Hence,

$$X^{**}_{\min} = \max\left(X^*_{\max} - \frac{\bar{\pi}}{\bar{P} - P_f},\ X^*_{\min}\right)$$

$$X^{**}_{\max} = \min\left(\frac{\bar{\pi}}{P_f - \underline{P}} - X^{*}_{\min},\ X^{*}_{\max}\right)$$

which proves our claim. ■

In the next section we discuss the production and hedging policies of a a risk-averse competitive firm under price uncertainty and tax asymmetry.

4. Competitive Firm with Tax Asymmetry

4.1. No Futures Markets

Consider now the behavior of risk averse competitive firm under asymmetric taxation, in the absence of risk-sharing markets. The firm chooses its production Q in a way that maximizes its expected utility of the **net** profit $\tilde{\pi}_n$, i.e., the gross profits $\tilde{\pi}(Q) = \tilde{P}Q - C(Q)$ minus the tax,

(4.1)
$$\max_Q \ EU\left[\tilde{\pi}_n\right] \ \ s.t.$$

$$\tilde{\pi}_n(Q) \ = \ \begin{cases} (1-t)[\tilde{P}Q - C(Q)] & \text{if } \ \tilde{\pi}(Q) \geqq 0 \\ \tilde{P}Q - C(Q) \ \text{ if } \ \tilde{\pi}(Q) < 0. \end{cases}$$

In the case of symmetric tax i.e., when $\tilde{\pi}_n(Q) = (1-t)\tilde{\pi}(Q)$, (see Sandmo (1971)), the optimal output depends on the tax rate t. Clearly, the same phenomena occurs here; however, some results vary due to the tax asymmetry. For example, Sandmo has shown that when the measure of **relative risk aversion is constant** the optimal Q^* does not depend on t. This is not the case under asymmetric tax as the following example demonstrates:

Example: Let $U(\pi) = 3\pi^{1/3}$, $C(Q) = 0.5Q^2$. The random price $\tilde{P}$ assumes two values, either

10 or 4 with equal probabilities. The optimal production level Q satisfies the equation:

$$\left(\frac{10-0.5Q}{4-0.5Q}\right)^2\left(\frac{10-Q}{Q-4}\right)^{-3}=1-t$$

Thus the optimum Q^* which solves this equation satisfies $4<Q^*<10$ and it depends on the tax rate t (imposed on positive profits only) although R_r is constant.

4.2. Futures Markets: The Role of Hedging

Now assume that the above risk-averse firm, facing price uncertainty, has access to a forward market (at date 0 for deliveries at date 1, when production is completed) with forward/futures p

rice P_f. We assume in the sequel that given $\tilde{P}$, P_f and the tax at rate t (on positive profits only) the firm produces and participates at the forward market as well. Let us assume for simplicity throughout this section that $\underline{P}=0$.

The firm chooses its production level Q and its hedging level X in a way that maximizes $\tilde{\pi}(Q,X)=\tilde{P}Q+(P_f-\tilde{P})X-C(Q)$.

$$Max_{Q,X}EU\left[\tilde{\pi}_n\right] \tag{4.2}$$

where,

$$\tilde{\pi}_n=\begin{cases}(1-t)[\tilde{P}Q+(P_f-\tilde{P})X-C(Q)] & \text{if } \tilde{\pi}(Q,X)\geqq 0\\ \tilde{P}Q+(P_f-\tilde{P})X-C(Q) & \text{if } \tilde{\pi}(Q,X)<0.\end{cases}$$

We shall add the following assumption to simplify our proofs in this section.

Assumption: The density function $f(p)$ of the random price is a continuous function on $[\underline{P},\bar{P}]$.

Surprisingly, the 'Separation theorem' remains valid in the asymmetric tax case as well. Denote by (Q^*, X^*) the optimum for Problem (4.2).

Proposition 4.1. *Consider the above competitive firm operating under asymmetric tax. If futures markets are available, then its optimal output Q^* is given by:*

$$C'(Q^*) = P_f$$

Namely, Q^* is independent of its attitude towards risk, its belief about the price distribution function and the tax rates.

Proof. To verify that first-order conditions are sufficient for the optimum let us note that the maximand in Problem (4.2) is strictly concave in Q and X. To see this, let us define

$$r(X) = \begin{cases} (1-t)X & \text{if } X \geqq 0 \\ X & \text{if } X < 0 \end{cases} .$$

$r(X)$ is a concave function and the maximand in (4.2) can be rewritten as,

$$\underset{Q,X}{Max}\; E\, U\left[r(\tilde{\pi}(Q,X))\right] \tag{4.3}$$

Since $\tilde{\pi}$ is linear in Q and X, r is concave while U is strictly concave, we see that the maximand is strictly concave in (Q, X).

The maximand in (4.3) is not differentiable since $r(X)$ is not differentiable at $X = 0$. This implies that at the optimum (Q^*, X^*) we have the following first-order conditions: (We denote by $f'_-(x)$ and $f'_+(x)$ the left-hand side and right-hand side derivatives):

$$E\left\{r'_-(\tilde{\pi}(Q^*, X^*))\left[\tilde{P} - C^*(Q^*)\right] U'\left[r(\tilde{\pi}(Q^*, X^*))\right]\right\} \geqq 0 \tag{4.4}$$

$$E\left\{r'_+(\tilde{\pi}(Q^*, X^*))\left[\tilde{P} - C'(Q^*)\right] U'\left[r(\tilde{\pi}(Q^*, X^*))\right]\right\} \leqq 0 \tag{4.5}$$

$$E\left\{r'_-\left(\tilde{\pi}(Q^*,X^*)\right)\left[P_f-\tilde{P}\right]U'\left[r\left(\tilde{\pi}(Q^*,X^*)\right)\right]\right\} \geqq 0 \tag{4.6}$$

$$E\left\{r'_+\left(\tilde{\pi}(Q^*,X^*)\right)[P_f-\tilde{P}]U'\left[r\left(\tilde{\pi}(Q^*,X^*)\right)\right]\right\} \leqq 0 \tag{4.7}$$

The strict concavity of the maximand in (4.3) implies that there is a unique optimum (Q^*, X^*) to this optimization. Now let us define,

$$A = \{P \mid P(Q^* - X^*) + P_f X^* - C(Q^*) \neq 0\}.$$

By our assumptions, the probability of the event A, given Q^*, X^*, is 1. Since $r(X)$ is differentiable everywhere except at $X = 0$, we obtain from conditions (4.4)-(4.7) that,

$$E_A\left\{r'\left(\tilde{\pi}(Q^*,X^*)\right)\left[\tilde{P}-C'(Q^*)\right]U'\left[r\left(\tilde{\pi}(Q^*,X^*)\right)\right]\right\} = 0. \tag{4.8}$$

$$E_A\left\{r'\left(\tilde{\pi}(Q^*,X^*)\right)\left[P_f-\tilde{P}\right]U'\left[r\left(\tilde{\pi}(Q^*,X^*)\right)\right]\right\} = 0 \tag{4.9}$$

where the expectations are on the event A, where Pr $A = 1$. Now, from equations (4.8) and (4.9) we obtain that:

$$[P_f - C'(Q^*)]\, E_A r'\left(\tilde{\pi}(Q^*,X^*)\right) U'\left[r\left(\tilde{\pi}(Q^*,X^*)\right)\right] = 0 \tag{4.10}$$

which is possible only if $C'(Q^*) = P_f$. ∎

Let us consider now the hedging behavior of the firm in our case. Although we introduced tax asymmetry we claim that the following results hold:

Proposition 4.2. *Given the firm's optimal decision (Q^*, X^*), then the hedging policy X^* satisfies:*

(a) If $P_f < E\tilde{P}$ then $X^* < Q^*$.

(b) If $P_f = E\tilde{P}$ then $X^* = Q^*$, i.e., full hedging in an unbiased futures market.

(c) If $P_f > E\tilde{P}$ then $X^* > Q^*$.

Comparing the hedging policy under symmetric tax with the asymmetric tax case, i.e., $X_s^*(t)$ vs. $X^*(t)$, we come to the following conclusions. Whenever $X^*(t) < \frac{C(Q^*)}{P_f}$ the sign of $\frac{\partial X^*(t)}{\partial t}$ differs from that of $\frac{\partial X_s^*(t)}{\partial t}$ for the case where $R_r(\pi)$ is constant or decreasing. On the other hand if $X^* \geqq \frac{C(Q^*)}{P_f}$ the sign of $\frac{\partial X^*}{\partial t}$ is determined by the monotonicity of $R_r(\pi)$, in the same way as in the symmetric tax case. For example, consider the case of constant relative risk aversion. In this case $X_s^*(t)$ does not vary with the tax rate, while for certain levels of P_f $X^*(t)$ will decrease with t.

Proof. Let us use equation (4.9) to obtain:

$$
\begin{aligned}
(4.11) \qquad & E_A\left[P_f - \tilde{P}\right] E_A\left\{r'\left(\tilde{\pi}(Q^*, X^*)\right) U'\left[r\left(\tilde{\pi}(Q^*, X^*)\right)\right]\right\} + \\
& Cov_A\left(P_f - \tilde{P},\ r'\left(\tilde{\pi}(Q^*, X^*)\right) U'\left[r\left(\tilde{\pi}(Q^*, X^*)\right)\right]\right) = 0.
\end{aligned}
$$

But $r'(X) = 1$ for $X < 0$ while $r'(X) = 1 - t$ for $X > 0$, thus as we vary the price P we shall **not change** the monotonicity of $r\left[\pi(Q^*, X^*)\right]$ in P : it is increasing in P if $Q^* - X^* > 0$, it is decreasing in P if $Q^* - X^* < 0$ and it is constant if $Q^* = X^*$. Since the event A has probability 1 we derive from (4.11) that if $P_f - E\tilde{P} < 0$ then the $Cov_A(\cdot,\cdot) > 0$. This implies that $U'\left[r\left(\tilde{\pi}(Q^*, X^*)\right)\right]$ is decreasing in $\tilde{P}$, i.e., $\tilde{\pi}(Q^*, X^*)$ is increasing in $\tilde{P}$. Thus $Q^* - X^* > 0$. Similarly, $P_f - E\tilde{P} = 0$ implies that $\tilde{\pi}(Q^*, X^*) =$ constant, hence $Q^* = X^*$. ■

5. Concluding Remarks

The evidence regarding the hedging behavior of firms is very limited in most countries. Particularly, such information is hard to gather in countries where tax asymmetry is strong. It is claimed by Nance, Smith and Smithson (1993) using a sample of 169 firms in the US, where tax asymmetry is very weak, that 104 firms use some hedging tools. The authors claim that an important reason for hedging is the reduction in the expected taxes. Another study by Mian (1996) uses data from financial reports of firms to the Stock Exchange in the US. It demonstrates that firms classified as "hedgers", according to their 1992 annual reports, constitute a significant portion of the sample considered in this study, and that hedging activities exhibit economies of scale. Moreover, the conjecture that tax consideration plays an important role in this hedging activity cannot be rejected. This sample includes mainly large firms that are already well diversified in their activities and profitable; thus the asymmetry in taxation is a weak motive in this case. Gøczy, Minton and Schrand (1996) show that firms might use currency derivatives to reduce cash flow variations. Moreover, firms with extensive foreign exchange-rate exposure and economies of scale in hedging also use currency derivatives.

We consider our argument regarding the role of hedging when taxes on profits and "compensation" for losses are asymmetric as another important reason for this observed evidence about hedging. The literature dealing with this topic (see, e.g., Smith and Stulz (1985)) did not specify explicitly the reasons for the lower expected taxes (and hence higher value of the firm) and this work, which demonstrates this phenomena in the tax asymmetry case, closes this gap, since corporate tax codes treat profits and losses differently worldwide.

In economies with tax carry-back, a firm that has accumulated significant profits will behave as in the symmetric taxation case which has been analyzed in Section 2. Some of the results attained for risk-neutral firms, such as the impact of tax progressivity on optimal hedging, can be extended to risk-averse firms as well.

Our framework is suitable for exporting firms under exchange rate uncertainty as well (see, for example, Ethier (1973), Katz and Paroush (1979)). In this case the price volatility results from the random exchange rate. One can obtain from our analysis the impact of hedging under asymmetric taxation on international trade. For example, in the asymmetric tax case for both risk-neutral and risk-averse firms, introducing hedging tools results in higher output, and hence higher international trade.

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