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SPECIALIZATION AND TRADE: THE PERSPECTIVE OF CLUB AND LOCAL PUBLIC GOOD THEORIES

by

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ABSTRACT

This paper suggests a unified framework for explaining the potential Pareto superiority of a heterogeneous club (jurisdiction) structure over a homogeneous one when there are multiple private goods. The superiority of the heterogeneous structure has been observed by Wilson (1987) in the context of an economy with local public goods and by Gilles and Scotchmer (1996, 1997) in the context of an economy with clubs. In the unified framework suggested in this paper, the two models are combined and reduced to a simple optimization problem. The reduced model shows that the advantage of a heterogeneous structure consists of the **gains from trade** between differentiated jurisdictions; the disadvantage being the loss from **inefficient jurisdiction size**. Thus, the club and local public good theories provide a new rationale for specialization and **trade among identical individuals**. The unified framework is also used in showing that earlier results regarding the possibility of decentralizing the optimal allocation in the case of one private good are equally applicable to the case of multiple private goods.

1 Introduction

A standard explanation for the very existence of trade is that **agents differ from one another in their endowments and preferences** and, therefore, their marginal valuations of goods vary. As a consequence, the exchange of goods among the different individuals is beneficial to all the parties involved. It was, therefore, surprising when Wilson (1987), in the context of local public good theory, and Gilles and Scotchmer (1996, 1997), in the context of club theory showed that trade among (ex ante) identical individuals may also be advantageous. Both studies demonstrated the potential advantage of forming a heterogenous group (communities/clubs) system vis-à-vis a homogenous one and thus realizing the gains from trade. They deviated from standard local public good/club models by accounting for multiple private goods rather than only one private good. However, these studies differed from each other by emphasizing different facets of the issue; consequently, they yielded different implications. More importantly, in both studies the fundamental trade-off involved was not sufficiently clarified.

The objectives of the present paper are twofold:

1. To synthesize Wilson's (1987) local public good model with Gilles and Scotchmer's (1996, 1997) club model by accounting for both production and an impure collective good; and to use the integrated model for analyzing the fundamental trade-off involved in evaluating the advantages versus the disadvantages of heterogenous vis-à-vis homogenous group (community/club) structure.

2. To use the integrated model for evaluating whether Wilson's (1987) results are sufficiently robust to withstand the impurity of the collective good.

The standard club-LPG theories (see Berglas (1976), Stiglitz (1977)), and Scotchmer and Wooders (1987)) showed that, with identical populations and **one private good**, it is optimal to form **identical groups, each of optimal size,** for performing collective activities (production of goods and public goods consumption). The **optimal size** of each such group depends on the advantage –associated with sharing the public good cost on the one hand– and the disadvantage of congestion in consumption (club specification) or in production (LPG specification) on the other hand. Of course, with only one private good, there is no point in trade among identical groups and, therefore, each should be autarkic.

Wilson (1987) extends Stiglitz' model by allowing for the production of multiple private goods. Specifically, he assumes that there are identical agents who live in a large number of geographically isolated communities, each extending over one unit of land. Each individual lives and works in one community, deriving utility from **two private goods** and one **pure local public good**. Each of the private goods is produced by land and labor, according to a linear homogenous production function. The labor/land intensity for each of the two private goods differs. The local public good is produced by one of the private goods. The private goods are costlessly transportable, as is migration of individuals among the communities. However, commuting between communities is prohibitively costly. With this extension, Wilson (1987) shows that heterogenous community structure is optimal: Sorting themselves out in communities of different size, identical individuals differentiate themselves from one another ex-post, thus allowing gains from trade as if they were ex-ante different. However, assuming a pure local public good, or only mild impurity, Wilson (1987) ignores the possibility that the gains from trade may not be large enough to justify the formation of a heterogenous community structure with full specialization. Thus, the fundamental trade-off involved in the choice between a homogenous versus heterogenous community structure is not sufficiently highlighted.

Ignoring the production side, the model constructed by Gilles and Scotchmer (1996, 1997) emphasizes the crowding effect on consumption. Each of their agents is endowed with two private goods and her utility depends on the consumption of the two private goods and the size of the group (club) to which he belongs. In some of the examples used to illustrate the model, the supply of the collective good (club good) is suppressed and thus does not appear representative of standard club models. In fact, however, this simple model may represent a reduced form of a standard club model with an impure public good, extended to include two private goods, where the cost saving dominates the crowding effect for small club size and the crowding effect dominates the cost saving for large club size. In contrast to Wilson (1987), however, this model implies that the heterogenous group structure may sometimes, but not necessarily always, be superior to the homogenous one. However, because Gilles and Scotchmer's (1996, 1997) analysis focuses on the implications of the multiple private goods in economies with **heterogenous individuals,** it blurs the main surprising result, i.e., that even ex ante **identical individuals may gain from trading their initial endowments**.

The present paper suggests a unified framework for illuminating the fundamental reason for forming a heterogenous group (communities and clubs) structure by **identical individuals,** a structure which leads to specialization and trade. In this unified framework, the two models, those of Wilson (1987) and Gilles and Scotchmer (1996, 1997), are integrated by accounting simultaneously for the effect of production, a key element in Wilson (1987), and for the effect of an impure collective good (club good/ local public good), a key element in Gilles and Scotchmer (1996, 1997). The unified model is then reduced to a simple optimization problem with two variables: the relative prices of the private goods and the consumption-production group size. This reduced form sheds new light on the sources for the advantages/disadvantages of a heterogenous structure with specialization and trade vis-a-vis a homogenous structure of autarkic groups.

The unified framework in its reduced form is also helpful in highlighting the issue of decentralization of the optimal allocation. Wilson (1987) asserted that the decentralization result that applies to one private good can be directly applied to the case of multiple private goods as well. Gilles and Scotchmer (1996, 1997) are more skeptical, emphasizing that decentralizing the optimal allocation in the case of multiple private goods requires a replication of optimal economies rather than a replication of optimal clubs, as in the case of one private good. In fact, however, the analysis in this paper shows that the difficulty in decentralizing the optimal allocation in the case of multiple private goods is the same as in the case of one private good: the notorious integer problem. Despite that similarity, however, ruling out the integer problem in the case of multiple private goods is a more restrictive requirement than in the case of only one private good.

Section 2 presents a general formulation of an economy with an impure local public good. The model is then reduced to a two-dimensional choice problem with the advantages and disadvantages of a heterogenous versus a homogenous structure of communities explained in terms of the dilemma of choosing between gains from trade, on the one hand, and gains from efficient group size, on the other. Several examples representing both Wilson's (1987) and Gilles and Scotchmer's (1996, 1997) specifications, are provided and used to illustrate the different outcomes the integrated model can yield. In Section 3, I discuss decentralization of the optimal allocation and characterize the supporting price systems. Section 4 summarizes the results and adds some final comments.

2 The Model

2.1 The Setup

I here discuss a generalized version of a local public good model à la Stiglitz (1977), with a crowding effect on the utility and on the provision cost of the collective good. There are

N individuals in the economy who are identical in terms of both preferences and initial endowments.[1] The welfare of an individual living in one of these communities, say i, is represented by a well-behaved utility function, $u(x^i, y^i, z^i, n^i)$, where x^i and y^i are the private consumptions of tradable goods X and Y, z^i is a congested local public good, and n^i is the community's population size. z^i is local in the sense that only residents of community i, where it is provided, can benefit from its provision; it is congested in the sense that the benefit from and the cost of its provision change with the community's population size. Excessive crowding is intolerable, that is, as $n^i \to \infty$, $\partial u(\cdot)/\partial n^i \to -\infty$ and x^i, y^i, and z^i are essential.

There is a large number, $\overline{M}$, of potential site, each extending over one unit of land, for locating communities. Each of the N individuals in the economy lives, works (supplies one unit of labor), and enjoys the local public good in one community, say i, out of M $\left(< \overline{M}\right)$ communities ($i = 1, ..., M$).

Each of the two private goods, X and Y, is produced by land and labor according to a distinct linear homogenous production function of land and labor, $f(n^{xi}, l^{xi})$ and $g(n^{yi}, l^{yi})$, respectively, where l^{xi}, n^{xi}, l^{yi}, and n^{yi} are the land and labor input used in producing x^i and y^i. The production functions $f(\cdot)$ and $g(\cdot)$ are distinct in the sense that, for the same input combination, the slope of the isoquants of the two commodities differs for each. X is used for either direct consumption or production of the local public good z^i according

[1] I refrain from the more general case of a heterogeneous population with differentiated tastes and initial endowments, as in Gilles and Scotchmer (1996, 1997), in order to highlight the specific implications of multiple private goods vis-à-vis one private good for the advantage of specialization and trade. Extending the model to a heterogeneous population may blur the surprising result regarding the effect of multiple private goods, that is: trading even among identical individuals may be advantageous (gains from trade among heterogeneous individuals is never surprising).

to the cost function $c(z^i, n^i)$.[2] Excessive crowding is intolerable in this respect, too: as $n^i \to \infty$, $c(z^i, n^i)/n^i \to \infty$.

A **feasible allocation** with equal treatment of individuals inside a community is one where the entire population is accommodated in M $(< \overline{M})$ communities and the material balance is satisfied. That is, a feasible allocation is a set $(x^i, y^i, z^i, n^i, l^{xi}, n^{xi}, M; i = 1, ..., M)$ satisfying:

$$N - \sum_{i=1}^{M} n^i = 0,$$

$$\sum_{i=1}^{M} [n^i x^i + c(z^i, n^i) - f(n^{xi}, l^{xi})] = 0,$$

$$\sum_{i=1}^{M} [n^i y^i - g(n - n^{xi}, 1 - l^{xi})] = 0.$$

A **feasible equal-treatment allocation** is a feasible allocation in which the utility is equalized across communities, i.e.,

$$U - u(x^i, y^i, z^i, n^i) = 0; \quad i = 1, ..., M.$$

An **optimal allocation** is a feasible equal-treatment allocation which maximizes U.

[2]The simplifying assumption that X is used for both direct consumption and for the provision of the public good allows us to reduce our three-good model to the two-good model of Gilles and Scotchmer (1996, 1997), thus facilitating the exposition considerably. The main result, i.e., that under some conditions heterogenous group structure is optimal and under others it is not, is not affected by this simplifying assumption, though the details of the conditions may be more complex.

In elaborating on the optimal allocation, I **assume** that the optimal community system includes either one type, a structure referred to as homogenous, or two distinct types, say a and b, a structure referred to as heterogenous.[3] This assumption allows us to formulate the optimal allocation as a set $(U, x^i, y^i, z^i, n^i, n^{xi}, l^{xi}, m^i; i = a, b)$ which satisfies:

$$\max_{x^a, y^a, z^a, n^a, n^{xa}, l^{xa}, m^a, x^b, y^b, z^b, n^b, n^{xb}, l^{xb}, m^b} U$$

subject to

(1)
$$U - u(x^a, y^a, z^a, n^a) = 0,$$

(2)
$$U - u\left(x^b, y^b, z^b, n^b\right) = 0,$$

(3)
$$m^a\left[n^a x^a + c(z^a, n^a) - f(n^{xa}, l^{xa})\right] + m^b\left[n^b x^b + c\left(z^b, n^b\right) - f\left(n^{xb}, l^{xb}\right)\right] = 0,$$

(4)
$$m^a\left[n^a y^a - g(n^a - n^{xa}, 1 - l^{xa})\right] + m^b\left[n^b y^b - g\left(n^b - n^{xb}, 1 - l^{xb}\right)\right] = 0,$$

(5)
$$N - \left(n^a m^a + n^b m^b\right) = 0,$$

$$x^a, y^a, z^a, n^a, n^{xa}, l^{xa}, m^a, x^b, y^b, z^b, n^b, n^{xb}, l^{xb}, m^b \geq 0.$$

[3] I ignore herein the possibility that there are more than two types of jurisdictions in the optimal system.

Equations (1) and (2) represent the equal-treatment requirement; (3) and (4) represent the material balance regarding X and Y, respectively; and (5) guarantees accommodation of every individual in some community.

The solution may imply that either only one type of community is optimal or, alternatively, that two distinct types are optimal. If community i is optimal, the following first-order conditions hold, where the subscripts x, y, z, n, and l denote a partial derivative of the relevant function with respect to the subscript:[4]

$$(6) \qquad n^i \frac{u_{z^i}}{u_{x^i}} = c_{z^i},$$

$$(7) \qquad m^i \left(x^i + c(z^i, n^i)/n^i - f\left(n^{xi}, l^{xi}\right)/n^i\right) + \left(y^i - g\left(n^i - n^{xi}, 1 - l^{xi}\right)/n^i\right)p - \mu = 0,$$

$$(8) \qquad \left(x^i + y^i p + c_{n^i} - n^i \frac{u_{n^i}}{u_{x^i}}\right) - \left(g_{n^i}(\cdot)\, p + \mu\right) = 0,$$

$$(9) \qquad f_{n^i} = g_{n^i} p,$$

[4]The first-order conditions are derived from the appropriate Lagrangian, which can be written down in short-hand as follows:

$$\begin{aligned} \pounds &= U - \lambda_1 LHS(1) - \lambda_2 LHS(2) - \lambda_3 LHS(3) - \lambda_4 LHS(4) \\ &\quad - \lambda_5 LHS(5), \end{aligned}$$

where λ_i and $LHS(i)$ represent the Lagrange multiplier and the left-hand side of equation i, respectively.

$$f_{l^i} = g_{l^i} p, \tag{10}$$

$$p = \frac{u_{y^i}}{u_{x^i}}. \tag{11}$$

Using X as the numeraire, p and μ are the normalized Lagrange multipliers of equations (4) and (5), respectively.[5] p is the shadow price of y and μ is the shadow non-wage per-capita income.

Equation (6) illustrates the Samuelson rule for the present case.

Equation (7) implies that the net value of per-capita resources imported into community i is fully financed by the non-wage income.

Equation (8) implies that the per-capita expenditure required to finance the optimal consumption bundle is equal to per-capita income. The expenditure consists of the value of the consumption bundle, $x^i + y^i p$, and the user charge for the local public good. The latter comprises the marginal cost and the external effect. Per-capita income is composed of wages (the value of marginal product of labor), $g_{n^i} p$, plus the non-wage per-capita income, μ.

Multiplying the first part of (7) by n^i, summing over i, and using (3) and (5), it follows that μ vanishes. This implies that *optimality requires self-sufficiency of each community, that is, the value of the endowment of its members is equal to the value of resources used in the community, where Y is evaluated according to its shadow price as defined in* (11). The intuition behind this result is straightforward. If self-sufficiency is violated, it means that

[5] That is,

$$\begin{aligned} p &= \lambda_4/\lambda_3, \\ \mu &= \lambda_5/\lambda_3 \end{aligned}$$

(see the preceding footnote).

resources are being transferred from one community to another. However, in such a case, it would be possible to establish only communities which, at the outset, transfer resources to the other communities.. Each community would, then, have more resources than required for achieving the original utility, thus allowing it to achieve higher utility than in the original allocation.

This observation is also reflected in the Henry George theorem (see, for example, Serck-Hanssen (1969), Stiglitz (1977), Arnott (1979), Berglas and Pines (1981), and Starrett (1988)) which follows from (7)-(11), and the linear homogeneity of $f(\cdot)$ and $g(\cdot)$:

$$c(z^i, n^i) = n^i \left(c_{n^i} - n^i \frac{u_{n^i}}{u_{x^i}} \right) + g_{l^i} p. \tag{12}$$

Equation (12) says that the total cost of providing the local public good in community i is equal to the total revenue from the warranted user charge (that is, the marginal social cost of crowding, which comprises the direct marginal cost, c_{n^i}, plus the external effect, $-n^i u_{n^i}/u_{x^i}$) plus the aggregate land rent in the community.[6]

Now, consider a more restricted case where each community is constrained to be autarkic, i.e., the consumption of **each private good** in the community equals its production (rather than their aggregate value). I **assume** that, under this constraint, the optimal allocation is unique. Furthermore, when defining $U(n)$ as the maximum utility in an autarkic community with population n, $U(n)$ has an inverted "u" shape: The more n deviates from its optimal level, in either direction, the lower is the achievable utility.[7]

Given these assumptions on the characteristics of the autarkic solution, two cases for the overall solution of the model can be distinguished:

[6] $g_{n^i} p$ represents the unit rent in i (the value of the marginal product of land in producing Y). Since the supply of land equals one unit, it also represents the aggregate rent in jurisdiction i.

[7] $U(n)$ need not, however, be concave.

Case I: The optimal solution yields one type of community. Formally, either $m^a m^b = 0$, or $m^a m^b > 0$ but $(x^a, y^a, z^a, n^a) = (x^b, y^b, z^b, n^b)$. Of course, in such an optimal system, the community must be an autarky.

Case II: There are two types of communities differing in their population size and probably in their other attributes. This solution is, then, characterized by $m^a, m^b > 0$, $(x^a, y^a, z^a, n^a) \neq (x^b, y^b, z^b, n^b)$, where, in particular, $n^a \neq n^b$. Since, by assumption, there is only one type of optimal **autarkic** community, the optimal communities, a and b, must trade X for Y.

In the next subsection, I explore the conditions under which each of these two cases prevails.

2.2 A Reduced Form of the Optimal Allocation Problem

In order to illuminate the underlying conditions of Case I and Case II, it is useful to simplify the optimal allocation problem. To that end, define

$$V(X, y, n) \equiv \max_z u(x, y, z, n)$$

subject to

$$x + c(z, n)/n = X \tag{13}$$

and:

$$W(p, n) \equiv \left\{ \max_{X, y, l^x, n^x} V(X, y, n) \text{ s.t. } [nX - f(n^x, l^x) + (ny - g(n - n^x, 1 - l^x))p] = 0 \right\}$$

$$= \max_{y, l^x, n^x} V([f(l^x, n^x) + (g(n - n^x, 1 - l^x) - ny)p]/n, y, n) \tag{14}$$

In (13), I eliminate the public good from the utility function, reducing it to a function of the y, n, and the resources available for providing either the private consumption x or the public good z. The problem is, thus, reduced to a format similar to the example provided by Gilles and Scotchmer (1996, 1997).

In (14), I further reduce the problem to an indirect utility function.
The solution to (14) yields $n^x(p,n)$, $l^x(p,n)$, $y(p,n)$, and

$$X(p,n) = [f(n^x(p,n), l^x(p,n)) + g(n - n^x(p,n), 1 - l^x(p,n))\,p]\,/n - y(p,n)\,p.$$

Of course,

$$W(p,n) = V(X(p,n), y(p,n), n). \tag{15}$$

In order to investigate the conditions under which Case II prevails, the function $W(p,n)$ and its representation on the $p-n$ plane in Figure 1 and Figure 2 are characterized.

(a) Denoting the utility maximizing variables when the community is constrained to be autarkic by $\overset{*}{x}$, $\overset{*}{y}$, $\overset{*}{z}$, $\overset{*}{n}$, $\overset{*}{n^x}$, $\overset{*}{l^x}$, $\overset{*}{p}$, we arrive at

$$W\left(\overset{*}{p},\overset{*}{n}\right) = u\left(\overset{*}{x},\overset{*}{y},\overset{*}{z},\overset{*}{n}\right).$$

Likewise, denoting the utility maximizing variables of the unconstrained maximization problem by $\overset{\circ}{x^i}$, $\overset{\circ}{y^i}$, $\overset{\circ}{z^i}$, $\overset{\circ}{n^i}$, $\overset{\circ}{n^{xi}}$, $\overset{\circ}{l^{xi}}$, $\overset{\circ}{p}$, then,

$$\max_n W\left(\overset{\circ}{p},n\right) = W\left(\overset{\circ}{p},\overset{\circ}{n^i}\right) = u\left(\overset{\circ}{x^i},\overset{\circ}{y^i},\overset{\circ}{z^i},\overset{\circ}{n^i}\right) \qquad \text{for } i = a, b.$$

Case I implies $\left(\overset{\circ}{p},\overset{\circ}{n^i}\right) = \left(\overset{*}{p},\overset{*}{n}\right)$ for $i = a, b$; Case II implies $\left(\overset{\circ}{p},\overset{\circ}{n^i}\right) \neq \left(\overset{*}{p},\overset{*}{n}\right)$ and $W\left(\overset{\circ}{p},\overset{\circ}{n^i}\right) > W\left(\overset{*}{p},\overset{*}{n}\right)$ for $i = a, b$.

(b) Starting from the autarkic allocation, any p deviating from $\overset{*}{p}$ allows gains from trade, that is, $p \neq \overset{*}{p} \Longrightarrow W(p,\overset{*}{n}) > W(\overset{*}{p},\overset{*}{n}) = u\left(\overset{*}{x},\overset{*}{y},\overset{*}{z},\overset{*}{n}\right)$. This is precisely the case of a consumer who has a fixed amount of two goods. $\overset{*}{p}$ is the consumer's MRS at the endowment point and, therefore, any other price ratio allows the consumer to be better off. This property is reflected in Figures 1 and 2 by the upward slopes extending to the east and west of $(\overset{*}{p},\overset{*}{n})$.

(c) A locus of autarky can be defined by any combination of p and n satisfying

$$y(p,n) = g\left(n - n^x(p,n), 1 - l^x(p,n)\right)/n.$$

I assume that X is labor-intensive so that an increase in labor supply reduces its relative price, that is, it increases p. It follows that the locus of autarky, represented by the bold line in Figures 1 and 2, is upward sloping.[8]

(d) Since I assume that the maximum utility attainable in an autarkic community with population n, $U(n)$, has an inverted "u" shape, the utility decreases with the deviation of n from $\overset{*}{n}$ along this loci. More precisely, $dW(\cdot)/dn < 0$ for $n > \overset{*}{n}$ and $dW(\cdot)/dn > 0$ for $n < \overset{*}{n}$. To the right (left) of the locus of autarky, the community becomes a net exporter (importer) of Y and, therefore, for a given n, $W(p,n)$ increases (decreases) with p.

[8] p is the slope of the transformation and the indifference curves at an efficient autarchic allocation with population n (that is, an allocation achieving $U(n)$). In making the above assumption, I ignore the possibility that the relative price of X may increase due to its strong complementary relations with n. The following analysis would not change if the locus is decreasing but it would change if it is vertical, as explained below.

(e) In drawing the two figures, I extend the assumption regarding the shape of $U(n)$ to $W(p,n)$: given any p, $\lim_{n\to 0} W(p,n) = \lim_{n\to\infty} W(p,n) < \left(\overset{*}{p},\overset{*}{n}\right)$. These conditions may be explained by (i) a minimal level of collective good, say defense, is required for survival and (ii) dominance of the negative crowding effect when the population increases beyond any bound.

Properties (a)-(e) explain the pattern of the iso-utility loci and the locus of autarky in Figures 1 and 2. The loci of autarky, $y(\cdot) = g(\cdot)/n$, are portrayed as a trough, sloping down as you go away from $\left(\overset{*}{n},\overset{*}{p}\right)$. The arrows indicate the direction of the upstream. In addition, as you go from South to North, or vice versa, you cross at least one ridge line.

Figure 1 here

Figure 2 here

Figure 2 differs, however, from Figure 1 in one important respect. In Figure 2, $W\left(\overset{*}{p},n\right)$ assumes a local **minimum** while in Figure 1, it assumes a local **maximum** at $\left(\overset{*}{p},\overset{*}{n}\right)$. Furthermore, in Figure 2, a range of prices exists such that for any given price within this range, there are elevations higher than $W\left(\overset{*}{p},\overset{*}{n}\right)$ both below and above the locus of autarky.

Observe that Case II requires feasible trade and, therefore, two optimal population sizes, $\overset{\circ}{n}{}^a$ and $\overset{\circ}{n}{}^b$, corresponding to some $\overset{\circ}{p}$, one above the locus of autarky and the other below it. It follows that Figure 2 is consistent with Case II while Figure 1 is not.

It remains to be shown that, the equal-treatment requirement can also be satisfied by Figure 2. In other words, in Figure 2, there exist a set $\left(\overset{\circ}{p},\overset{\circ}{n}{}^a,\overset{\circ}{n}{}^b\right)$ such that $W\left(\overset{\circ}{p},\overset{\circ}{n}{}^a\right) = W\left(\overset{\circ}{p},\overset{\circ}{n}{}^b\right) > W\left(\overset{*}{p},\overset{*}{n}\right)$. The existence of ridge lines both above and below the locus of autarky ensures that there exists some range of p, $(\underline{p},\overline{p})$, containing $\overset{*}{p}$, such that, for any

$p \in (\underline{p}, \overline{p})$, $W(p,n)$ attains maxima at some $n > 0$ below the locus of autarky and at a finite n above the locus of autarky. It follows that, corresponding to $(\underline{p}, \overline{p})$, there are two sections of ridge lines with elevations higher than $W\left(\overset{*}{p}, \overset{*}{n}\right)$, one located above and one below the locus of autarky. This property is then used in Figure 3 to illustrate the determination of $\overset{\circ}{p}, \overset{\circ}{n}{}^a$ and $\overset{\circ}{n}{}^b$ for the case where $\overset{\circ}{p}$ is larger than $\overset{*}{p}$.[9]

Figure 3 here

Once the optimal levels of $\overset{\circ}{p}$, $\overset{\circ}{n}{}^a$ and $\overset{\circ}{n}{}^b$, are determined, we can ascertain the population of an optimal economy—in Gilles and Scotchmer's (1996, 1997) terms—which allows the trade in Y implied by $\overset{\circ}{p}$, $\overset{\circ}{n}{}^a$, and $\overset{\circ}{n}{}^b$. To that end, we solve m by

$$(16)\quad \begin{gathered} m\left[\overset{\circ}{n}{}^a\, y(\overset{\circ}{p}, \overset{\circ}{n}{}^a) - g\left(\overset{\circ}{n}{}^a - n^{xa}(\overset{\circ}{p}, \overset{\circ}{n}{}^a), 1 - l^{xa}(\overset{\circ}{p}, \overset{\circ}{n}{}^a)\right)\right] \\ + \\ \left[\overset{\circ}{n}{}^b\, y(\overset{\circ}{p}, \overset{\circ}{n}{}^b) - g\left(n^b - n^{xb}(\overset{\circ}{p}, \overset{\circ}{n}{}^b), 1 - l^{xb}(\overset{\circ}{p}, \overset{\circ}{n}{}^b)\right)\right] = 0. \end{gathered}$$

Denote the solution of (16) for m by $\overset{\circ}{m}$ and assume that $\overset{\circ}{m}$ is a rational number, such that $\overset{\circ}{m} = \overset{\circ}{m}{}^a \,/\, \overset{\circ}{m}{}^b$, where $\overset{\circ}{m}{}^a$ and $\overset{\circ}{m}{}^b$ are integers. It follows that an economy with population of $\overset{\circ}{m}{}^a \overset{\circ}{n}{}^a + \overset{\circ}{m}{}^b \overset{\circ}{n}{}^b$ is an optimal economy, one allowing the maximization of $W(p,n)$ with a feasible trade pattern. In other words, in an economy having $\overset{\circ}{m}{}^a$ communities with $\overset{\circ}{n}{}^a$ residents, each consuming $y(\overset{\circ}{n}{}^a, \overset{\circ}{p})$, and $\overset{\circ}{m}{}^b$ communities with $\overset{\circ}{n}{}^b$ members, each consuming

[9] Observe that $\overset{\circ}{p}$ can also be less or even equal to $\overset{*}{p}$. Figure 3, however, illustrates the case where it exceeds $\overset{*}{p}$.

$y(\overset{\circ}{n}{}^b, \overset{\circ}{p})$, the market for Y is cleared. Therefore, according to Walras' law, the market for X is also cleared:

$$\begin{array}{c} \overset{\circ}{m}{}^a \left[\overset{\circ}{n}{}^a X(\overset{\circ}{p}, \overset{\circ}{n}{}^a) - f\left(n^{xa}(\overset{\circ}{p}, \overset{\circ}{n}{}^a), l^{xa}(\overset{\circ}{p}, \overset{\circ}{n}{}^a) \right) \right] \\ + \\ \overset{\circ}{m}{}^b \left[\overset{\circ}{n}{}^b X(\overset{\circ}{p}, \overset{\circ}{n}{}^b) - f\left(n^{xb}(\overset{\circ}{p}, \overset{\circ}{n}{}^b), l^{xb}(\overset{\circ}{p}, \overset{\circ}{n}{}^b) \right) \right] = 0. \end{array} \tag{17}$$

So far, it has been shown that case I (and its corresponding Figure 1) is obtained when $W\left(\overset{*}{p}, n\right)$ assumes a maximum at $\overset{*}{n}$; Case II (and the corresponding Figure 2) is obtained when $W\left(\overset{*}{p}, n\right)$ attains a (local) minimum at that point. The main question is: when does $W\left(\overset{*}{p}, n\right)$ assume a maximum and when does it assume a minimum? This issue is elaborated in the next subsection.

2.3 The Gains from Trade Versus the Loss from Inefficient community Size

Using the envelope theorem, one realizes that $\left(\overset{*}{p}, \overset{*}{n}\right)$ is either a local extremum or an inflection point of $W\left(\overset{*}{p}, n\right)$. Disregarding the second possibility, this subsection investigates the factors determining whether $\left(\overset{*}{p}, \overset{*}{n}\right)$ is a maximum or a minimum of $W\left(\overset{*}{p}, n\right)$. Consider a shift from A to C in Figure 2. Such a shift denotes a deviation from the (unique) optimal autarkic community to a more populated community that can trade Y at a price of $\overset{*}{p}$. It can also be represented by two other shifts: first, a shift from A to B, which still leaves the community autarkic but with a larger population, n^C; second, a shift from B to C, which leaves the population size unchanged but reduces the relative price of Y from p^B to $\overset{*}{p}$.[10] The shift from A to B is associated with a decline in $W(p,n)$ resulting from the increased crowding. I define this decrease as **the loss from inefficient group size.** It reflects net negative crowding effects, both on production and on consumption. The shift from B to C is associated with an increase in $W(p,n)$, which I define as **the gains from trade.** The sources of these losses and gains are further elaborated below.

Beginning with the net loss from inefficient group size (the shift from A to B), one can discern three effects. First, the utility is directly affected by the increased crowding. This effect can be represented by $u_n \Delta n$, where the derivative is evaluated somewhere between A and B. Second, the average product of labor in producing both X and Y diminishes in response to increasing labor intensity. Third, the per-capita cost of the public good is affected (it may increase or decrease, depending on the net crowding effect on the average provision cost, $c(z,n)/n)$).[11]

The gains from trade, reflected in the shift from B to C, follow from adjusting the production and the consumption to the change in the price of Y.[12] The total benefit (the aggregate consumer and producer surplus), derived from a marginal change in the price of Y, is obtained by differentiating (14) with respect to p:

[10]With some inconsequential modifications, the following analysis equally applies if B' and C' are substituted for B and C, respectively.

[11]Observe that no negative crowding effect is imposed either on the utility or on the per-capita provision cost. The only requirements are that the outcome of the three crowding effects implies that $\overset{*}{n}$ is a unique optimum to $U(n)$ and that the net efffect on the utility of the three sources of crowding in an autarkic jurisdiction is positive for populations smaller than $\overset{*}{n}$ and negative for population larger than $\overset{*}{n}$.

[12]Be aware that gains result from the **change** in p, rather than the **decrease** from p^B to $\overset{*}{p}$. Equally, gains are realized when the price **increases** from $p^{B'}$ to $\overset{*}{p}$.

$$\frac{\partial W(\cdot)}{\partial p} = V_X(\cdot)\left(g(\cdot)/n - y\right). \tag{18}$$

In view of the autarky, the RHS of (18) vanishes at B. However, it is negative between B and C; therefore, the benefit from the trade is obtained by integrating the negative value of the RHS of (18) over the interval BC :

$$-\int_{\overset{*}{p}}^{p^B} \frac{\partial W(\cdot)}{\partial p} dp = \int_{\overset{*}{p}}^{p^B} V_X(\cdot)\left[y\left(p, n^C\right) - g\left(n^C - n^X\left(p, n^C\right), 1 - l^X\left(p, n^C\right)\right)\right] dp. \tag{19}$$ [13]

It follows from the above analysis that the net effect of the gains and losses cannot be ascertained a priori. More precisely, both cases, one where the gains dominate the losses and the other where the losses dominate the gains, can prevail.

2.4 Examples

2.4.1 *Consumption* and *Production* Groups with a *Pure* Collective Good

An important case in which the gains from trade always dominate the losses from inefficient group size, is provided by Wilson (1987). According to his basic specification, the crowding effect on the cost function of the collective good is either ignored or assumed to be mild. As a consequence, the crowding effects associated with a shift from A to B are restricted to a decreasing average productivity of labor (and, perhaps, a **moderate** decline in utility). These adverse effects, however, can always be more than offset by the gains from trade associated with the shift from B to C. To demonstrate this assertion, the outcome of a

[13] See Varian (1984, 2d ed. pages 263-266) for a discussion about the change in the consumer surplus associated with a change in the price of a single commodity.

direct shift from A to C can be evaluated. As long as there is no full specialization, the Rybzynski theorem tells us that such a shift leaves the wage **rate**, $g_n p$, and the **aggregate** rent, $g_l p$, unchanged. Consequently, at C, the resources available in the community allow its population, n^C, to consume the same quantities of X, Y, and z which are realized at A (recall that the crowding effect on the provision cost is ruled out). It follows that, under Wilson's (1987) specification, the gains from trade associated with modifying the production composition, allow the same consumption pattern $\left(x\left(\overset{*}{p},\overset{*}{n}\right),y\left(\overset{*}{p},\overset{*}{n}\right),z\left(\overset{*}{p},\overset{*}{n}\right)\right)$ at C as at A. However, this is not the end of the story. The attainable utility at C must exceed that at A. This is true because the consumption bundle $\overset{*}{x},\overset{*}{y},\overset{*}{z}$, which is attainable at C and which allows the same utility as at A, is inefficient for a population of size n^C. The reason is that the Samuelson condition, (6) , which holds for $\overset{*}{n}$, must be violated for the altered population, n^C. Hence, starting from the bundle $\overset{*}{x},\overset{*}{y},\overset{*}{z}$, a preferable allocation can be achieved by adjusting consumption to satisfy the Samuelson rule. This explains why the homogenous community structure in Wilson (1987) can never be efficient and why an efficient allocation requires that each community specializes in the production of one private good only.[14]

However, once we remove the restrictions on the crowding effects on the utility and cost functions, Wilson's (1987) results change: No specialization at all or only partial specialization in the production of the private goods may characterize the optimal allocation.

2.4.2 *Consumption* and *Production* Groups with an *Impure* Public Good

Introducing impure local public goods may upset Wilson's results because the Rybzynski theorem is no longer sufficient to guarantee the superiority of the heterogenous community structure. The Rybzynski theorem guarantees that the *aggregate* land rent and the wage *rate*

[14] As long as the specialization is not complete, the Rybzynski theorem applies and, therefore, a further improvement is possible.

do not change with population size. If the local public good were pure, we know from (12) that the aggregate rent would be sufficient to finance it so that, as the population changes, it would still be feasible to finance a constant level of private consumption by the fixed wage rate, and a constant level of local public good by the aggregate land rent. With an impure local public good, the aggregate land rent and the wage rate remain the same as the population increases, but there are other effects which may make the initial utility infeasible. On the one hand, as the population increases the toll revenue increases by $dn\left(c_n - n\frac{u_n}{u_x}\right)$. On the other hand, the cost of providing the local public good and the utility are adversely affected at a rate of -$n\left(c_n - n\frac{u_n}{u_x}\right)$. It follows that the simple reasoning based on Rybzynski theorem no longer applies.

This argument is illustrated in the following example:

$$
\begin{aligned}
u(x,y,z) &= x^{1/3}y^{1/3}(z-10)^{1/3}, \\
f(n^x,l^x) &= n^x, \\
g(n-n^x,1-l^x) &= g(n-n^x,1) = (n-n^x)^{1/2}, \\
c(z,n) &= z + .01n^2
\end{aligned}
$$

This specification implies the presence of two opposing effects on utility as population size increases. First, there are two factors contributing to an increase in utility with population size: A required minimum of 10 units of the public good and a reduced per-capita burden of the local public good, z/n. Second, two factors contribute to a decrease in utility with population size: The diminishing returns in the production of Y and the impurity of the local public good represented by $.01n^2$ in the cost function. It can be shown that the outcome of these two opposing forces is an inverted u shape of the function $U(n)$.[15]

The reduced model in terms of the function $W(p,n)$, corresponding to this example is illustrated in Figure 4. It has a feature common with Figure 2: For any given price, p, all the elevations higher than $W\left(\overset{*}{p},\overset{*}{n}\right)$ are either above or below the bold line that represents the locus of autarky. Similarly, $\overset{*}{n}$ is not only the optimal population size for an autarkic group structure, it also maximizes $W\left(\overset{*}{p},n\right)$ for aggregate income achievable through trading at the optimal imputed price under autarky, $\overset{*}{p}$. We conclude, therefore, that with constant returns to scale in the production of the two private goods and an *impure* public good, efficiency may require a homogenous group structure rather than heterogenous one.

Figure 4 here

Now, consider another example where the impurity of the local public good, reflected by η, is reduced from 2 to 1.2, leaving the other parameters in the above example unchanged. The inverted u shape of $U(n)$ still holds,[16] but the superiority of the heterogenous club structure, as shown by Wilson (1987), emerges. This is reflected in Figure 5 which has features common with Figure 2: A range of prices exists such that for any given price within this range, elevations higher than $W\left(\overset{*}{p},\overset{*}{n}\right)$ are located both below and above the bold line which represents the locus of autarky. Another aspect of this property is that while $\overset{*}{n}$ maximizes the function $U(n)$, it minimizes the function $W\left(\overset{*}{p},n\right)$.

Furthermore, in Figure 5, there exist two sections of ridge lines, one above and one below the locus of autarky, in the vicinity of $\overset{*}{p}$, with elevations exceeding $W\left(\overset{*}{p},\overset{*}{n}\right)$ This permits

[15]Details of the calculations by "Mathematica" are available from the author upon request.

[16]See the preceding footnote.

construction of Figure 3 and the determination of the set $(\overset{\circ}{n}{}^a, \overset{\circ}{n}{}^b, \overset{\circ}{p})$. The upper ridge line is obtained because, for sufficiently large population sizes, the crowding effect, represented by $.1n^{1.2}$, eventually dominates the reduction in the per-capita burden of the local public good so that the utility begins to decline. What about the lower ridge line? Such a line must exist for a sufficiently small population size. To see this, observe that for any given price, p, the utility cannot be positive unless the population size strictly exceeds 10. Since, in the vicinity of $\left(\overset{*}{p}, \overset{*}{n}\right)$, the utility is positive, there must exist a positive population level below which the attainable utility is lower than $W\left(\overset{*}{p}, \overset{*}{n}\right)$. This implies that a ridge line passing below the autarkic locus in some range of p sufficiently close to $\overset{*}{p}$ exists.

<u>Figure 5 here</u>

As in the case of a pure public good, Wilson's (1987) conclusion regarding the superiority of the heterogenous club structure holds also for cases of mild impurity of the public good. Nonetheless, his conclusion regarding complete specialization does not. Starting at any given p, say $\tilde{p}$, and the corresponding n on the locus of autarky, say, $\tilde{n}$, both X and Y are produced. As the population increases beyond $\tilde{n}$, the output of Y remains constant at a level of $\epsilon\left(\tilde{n} - n_x\left(\tilde{p}, \tilde{n}\right)\right)^{\epsilon} = \left(\epsilon\,\tilde{p}\right)^{\epsilon/(1-\epsilon)}$ since all labor increments are allocated to the production of X. It follows, therefore, that at the optimal allocation of the large community, labor is allocated to the production of both X and Y, more specifically, $\overset{\circ}{y}{}^b = \left(\epsilon\,\overset{\circ}{p}\right)^{\epsilon/(1-\epsilon)}$. That is, there is no complete specialization in the large community.

I conclude, therefore, that Wilson's (1987) results regarding the heterogenous club structure and full specialization are not sufficiently robust to withstand impurity of the local public good.

2.4.3 *Consumption* Groups with Crowding Effect (Club Economies)

Gilles and Scotchmer's (1996, 1997) specification refers to a standard club model where there is no production of private goods but, rather, each household is endowed with some quantities of private goods, $\overline{x}$ and $\overline{y}$.[17] Hence, while the crowding effect in Wilson (1987) is caused mainly by diminishing marginal productivity in production, in Gilles and Scotchmer (1996, 1997), the crowding affects utility directly and may be attributed to the impurity of the local public good.[18] This difference, however, is not crucial and, as in Wilson, (1987) the two opposing effects, inefficiency of group size versus the gains from trade, play the main role in determining whether a heterogenous club structure is superior or inferior to a homogenous one. Both models can be reduced to the terms represented in Figures 1-3. We illustrate this assertion by using two examples representing the economies defined by Gilles and Scotchmer (1996, 1997). In both examples, the following specification is used:

$$\begin{aligned} u(x,y,z) &= \left\{\left[x^{\alpha}(z-K)^{(1-\alpha)}\right]^{\rho}+y^{\rho}\right\}^{1/\rho} \\ c(z,n) &= z+\xi n^{\eta} \end{aligned} \tag{20}$$

The parameters of the two examples are

[17] It appears as if their example No 3 (1996, 1997) does not represent the club economy issue since there is no collective good (club good). However, as we have already shown in Subsection 2.2, while using the Samuelson condition, (6), the economy can be reduced to one where only two private goods, X and y, are consumed.

[18] I view their example as a reduced form of a club model with an impure club good (see the preceding footnote).

	Example 1 (Figure 6)	Example 2 (Figure 7)
α	1/2	1/2
ρ	.1	.95
η	2	2
ξ	1	1
K	100	5000
$\overline{x}$	1000	1000
$\overline{y}$	1000	1000

Thus the two examples differ from one another mainly by the elasticity of substitution: While the first example exhibits close to unitary elasticity of substitution (more precisely, 1.1), the second example exhibits much higher elasticity, that is, 20. The higher elasticity of substitution allows the gain from trade to mitigate the crowding effect of increased population size by substituting greater amount of X for Y through trade; it also allows tolerating more crowding effect, relative to autarky, by substituting Y for X when the population decreases. This explains why, in the second example, we obtain case II.

Figure 6 here

Figure 7 here

Observe that the slope of the autarky loci in Figures 6 and 7 differ from those in Figures 4 and 5. In Figures 4 and 5, the slope of this locus is dominated by the higher labor intensity in the production of X relative to that of Y which more than offsets the complementarity effect in consumption. In Figures 6 and 7, which represents an economy without production, the slope of the autarky locus, reflects only the complementarity between the population size and X: As the population increases, the marginal rate of substitution of Y for X increases.

Summing up, this subsection illustrates the general observation regarding the two opposing effects determining whether the homogenous or the heterogenous group structure is superior: The loss associated with deviation from optimal group size versus the gains from trade allowed by heterogenous group structure. This subsection also shows that in their reduced form, Wilson (1987) and Gilles and Scotchmer (1996, 1997) are identical.

3 Decentralization

I follow Berglas and Pines (1980) and Pines (1991) in defining a price-taking equilibrium as a case of commodities with variable qualities.[19] First, I define the concept of price-taking equilibrium in the present context. Then I show that if such an equilibrium exists, the resulting allocation is optimal. Last, I characterize the supporting price systems.

Consider four types of agents: individuals who are consumers and suppliers of labor, producers of X, producers of Y, and profit-maximizing developers. Taking a wage function, $\omega(z,n)$, each individual chooses x, y, z, and n to maximize a utility function $u(x,y,z,n)$ subject to a budget constraint $x + yp = \omega(z,n)$. With the same initial holdings and free

[19] In essense, this concept is very similar to that of Scotchmer and Wooders (1987), Scotchmer (1994), Barham and Wooders (1997), and Conley and Wooders (1997). Barham and Wooders (1997) and Conley and Wooders (1997) refer to it as "admission price equilibrium". The concept expresses the property that the price of a commodity is a function of its attributes. In the context of local public good or club economies, the list of a non-tradable commodity's attributes may include the characteristics of the community.

migration, the attainable utility is equalized across communities. The developer chooses z and n which maximize the bid price for land (demand price for land) of the producers of X and Y. Taking the wage as given, the producers choose inputs so as to maximize the bid rent (see below). The developer supplies the land to the highest bidder. There is free entry into the "communities' business", so that net surplus (rent minus improvement cost) vanishes. Accordingly, an equilibrium is an allocation $(U, x^i, y^i, z^i, x^i, n^i, n^{xi}, l^{xi}, m^i;\ i = a, b)$ and a price system $(p, \omega(z, n))$, such that, for $i = a, b$,

I. y^i, z^i, n^i maximize $u(\omega(z,n) - yp, y, z, n)$,

II. n^{xi}, l^{xi} maximize $f(n^x, l^x) - n^x\omega(z^i, n^i)$,

III. $n^i - n^{xi}$, $1 - l^{xi}$ maximize $g(n^y, l^y)p - n^y\omega(z^i, n^i)$,

IV. z^i and n^i maximize

$$\left\{\max_{n^{xi}, l^{xi}} f(n^{xi}, l^{xi}) - n^{xi}\omega(z^i, n^i)\right\}$$

$$+\left\{\max_{n^{yi}, l^{yi}} g(n^i - n^{xi}, 1 - l^x)p - (n^i - n^{xi})\omega(z^i, n^i)\right\}$$

$$-c(z^i, n^i) = 0,$$

V. $f(n^{xi}, l^{xi}) + g(n^i - n^{xi}, 1 - l^{xi})p - n^i\omega(z^i, n^i) - c(z^i, n^i) = 0,$

VI. (1) - (5) are satisfied.

It is shown in the appendix that the first-order conditions for these maximization problems yield (6), (11), and (12). These conditions and the constraints, (1)-(5), are also the necessary conditions for an optimal allocation. I conclude that decentralization through a price-taking equilibrium, if it exists, yields the optimal allocation.

Next, I show that such a price system exists and that it induces the individuals to choose $\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right)$; the producers of X and Y to choose $\left(\overset{\circ}{n^{xi}}, \overset{\circ}{l^{xi}}\right)$ and $\left(\overset{\circ}{n^i} - \overset{\circ}{n^{xi}}, 1 - \overset{\circ}{l^{xi}}\right)$, respectively; and the developers to choose $\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right)$. To this end, I define an upper and a lower bound on $\omega(z,n)$, showing that **any price function** strictly bounded by them is satisfactory. The upper bound, $\overline{\omega}(z,n)$ is:

$$\overline{\omega}(z,n) \equiv e\left(\overset{\circ}{p}, z, n, \overset{\circ}{U}\right) \quad \text{for all } z, n,$$

$$\overline{\omega}\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right) = e\left(\overset{\circ}{p}, \overset{\circ}{z^i}, \overset{\circ}{n^i}, \overset{\circ}{U}\right),$$

where $e\left(\overset{\circ}{p}, z, n, \overset{\circ}{U}\right)$ is the minimum expenditure function. It is clear that, with a wage function satisfying $\omega(z,n) < \overline{\omega}(z,n)$ for all $(z,n) \neq \left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right)$ and $\omega\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right) = \overline{\omega}\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right) = e\left(\overset{\circ}{p}, \overset{\circ}{z^i}, \overset{\circ}{n^i}, \overset{\circ}{U}\right)$, the individual is induced to choose $\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right)$ because, otherwise, he/she cannot afford $\overset{\circ}{U}$.

The lower bound, $\underline{\omega}(z,n)$, is

$$\underline{\omega}(z,n) \equiv \max_{n^x, l^x} \left[f(n^x, l^x) + g(n - n^x, 1 - l^x)\, \overset{\circ}{p} - c(z,n) \right] / n \qquad \text{for all } z, n.$$

$$\underline{\omega}\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right) = \left[f\left(\overset{\circ}{n^{xi}}, \overset{\circ}{l^{xi}}\right) + g\left(\overset{\circ}{n^i} - \overset{\circ}{n^{xi}}, 1 - \overset{\circ}{l^{xi}}\right) \overset{\circ}{p} - c\left(\overset{\circ}{z^i}, \overset{\circ}{n^i}\right) \right] / \overset{\circ}{n^i}.$$

It is evident that a wage function satisfying $\omega(z,n) > \underline{\omega}(z,n)$ for all $(z,n) \neq \left(\overset{\circ}{z}, \overset{\circ}{n}\right)$ and

$\omega\left(\overset{\circ}{z^i},\overset{\circ}{n^i}\right)=\overline{\omega}\left(\overset{\circ}{z^i},\overset{\circ}{n^i}\right)$ induces the developer to choose $\left(\overset{\circ}{z^i},\overset{\circ}{n^i}\right)$ and, consequently, the producers of X and Y to choose $\left(\overset{\circ}{n^{xi}},\overset{\circ}{l^{xi}}\right)$ and $\left(\overset{\circ}{n^i}-\overset{\circ}{n^{xi}},1-\overset{\circ}{l^{xi}}\right)$, respectively.[20]

Provided that $\overline{\omega}(z,n)>\underline{\omega}(z,n)$ for all $(z,n)\neq\left(\overset{\circ}{z^i},\overset{\circ}{n^i}\right)$, any price system satisfying $\delta\underline{\omega}(z,n)+(1-\delta)\overline{\omega}(z,n)$ where $0<\delta<1$, supports the optimal allocation; that is, it induces both the individuals and the developers to choose $\overset{\circ}{z^i},\overset{\circ}{n^i}$ and the producers of X and Y to choose $\left(\overset{\circ}{n^{xi}},\overset{\circ}{l^{xi}}\right)$ and $\left(\overset{\circ}{n^{xi}}-\overset{\circ}{n^{xi}},1-\overset{\circ}{l^{xi}}\right)$, respectively. Hence, the existence of an equilibrium hinges on whether, for all $(z,n)\neq\left(\overset{\circ}{z^i},\overset{\circ}{n^i}\right)$, the relationship $\overline{\omega}(z,n)>\underline{\omega}(z,n)$ is satisfied. It turns out that this relationship must hold, as can be verified from Figure 2 and the associated analysis. The reason is that, for $\overset{\circ}{p}$ and $n\neq\overset{\circ}{n^i}$, we have

$$\overset{\circ}{U}=W\left(\overset{\circ}{p},\overset{\circ}{n^i}\right)>W\left(\overset{\circ}{p},n\right)$$
$$=$$
$$\max_{y,z,n^x,l^x} u\left(\left[f\left(n^x,l^x\right)+g\left(n-n^x,1-l^x\right)\overset{\circ}{p}-c\left(z,n\right)\right]/n-y\,\overset{\circ}{p},y,z,n\right)$$
$$\geq$$
$$\max_{y,n^x,l^x} u\left(\left[f\left(n^x,l^x\right)+g\left(n-n^x,1-l^x\right)\overset{\circ}{p}-c\left(z,n\right)\right]/n-y\,\overset{\circ}{p},y,z,n\right)$$
$$=$$
$$\max_{y} u\left(\underline{\omega}\left(z,n\right)-y\,\overset{\circ}{p},y,z,n\right).$$

Since

[20]The profits of the developer, π, are equal to

$$\pi=f(\cdot)+g(\cdot)p-n\omega(\cdot)-c(\cdot).$$

Then, $\partial\pi/\partial\omega=-n<0$. Thus, the profits achievable with $\omega(z,n)>\underline{\omega}(z,n)$ are certainly lower than those achievable with $\overline{\omega}(\cdot)$.

$$\max_{y} u\left(\overline{\omega}(z,n) - y\,\mathring{p}, y, z, n\right) = \max_{y} u\left(e\left(\mathring{p}, z, n, \mathring{U}\right) - y\,\mathring{p}, y, z, n\right) = \mathring{U},$$

it follows that

$$\overline{\omega}(z,n) = e\left(\mathring{p}, z, n, \mathring{U}\right) > \underline{\omega}(z,n),$$

which proves the assertion. It also follows that any price system $\delta\underline{\omega}(z,n) + (1-\delta)\overline{\omega}(z,n)$, where $0 < \delta < 1$, supports the optimal allocation.[21]

4 Summary and Concluding Comments

Using common terms, this paper explains Wilson's (1987) and Gilles and Scotchmer's (1996, 1997) results that, with multiple private goods, even when the population is homogenous, the optimal consumption-production groups need not be homogenous. It follows that, in economies with clubs or local governments, specialization and trade need not be outcomes of differentiated tastes or diseconomies of scope—the standard explanations for prevailing specialization and trade. In both Wilson's (1987) and Gilles and Scotchmer's (1996, 1997) specifications, the reason for the superiority of the heterogenous structure is that the gains from trade, associated with the heterogenous group composition, may dominate the losses from inefficient group size. Wilson's (1987) specification concentrates on the gains from trade and losses from inefficient group size associated with the production of private goods and a pure local public good. This specification leads to the conclusion that the heterogenous

[21] Observe that in our specification, the supporting price system is $\delta\underline{\omega}(z,n) + (1-\delta)\overline{\omega}(z,n)$, which is a generalization of Berglas and Pines (1980) and Scotchmer and Wooders (1987). In the former, the supporting price system traces the indifference surface; in the latter, it traces the average cost function. Here, it is any intermediate surface.

always dominates the homogenous structure. Gilles and Scotchmer's (1996, 1997) specification is focused on the gains from trade and the crowding effects-the outcome of the direct effect on the utility and the cost-sharing of the collective good. This specification implies that the heterogenous may, but need not always, dominate the homogenous structure. It turns out that this weaker result is valid in the case of economies with production as well, once the local public good is impure. Furthermore, when the local public good is impure, the communities need not fully specialize in production, as implied in the case of pure public good discussed by Wilson (1987).

The framework suggested in this paper allows elaboration on the decentralization questions: Can a competitive equilibrium that yields the optimal allocation be defined? Can its existence be assured, and what characterizes the supporting price system? The paper elaborates on a price system that includes some *wage function* of the community's attributes (rather than a single parameter). This concept corresponds to the case of goods with continuous qualities, so that the price per unit is a function of the quality of the good. Such a price system may appear problematic from the informational point of view because there are an infinite "number" of prices (see Barham and Wooders (1995) and Conley and Wooders (1995)). However, the problem is not inherent in the prices but in the continuum of the qualities. If one restricts the number of alternative qualities, one can, then, restrict the corresponding number of prices to be considered.

One may wonder whether the definition of equilibrium and the resulting conclusions hinge on confining the analysis to a homogenous population, given that the definition of the upper bound of the supporting price system is associated with information on preferences. Accordingly, one can doubt whether the approach suggested here is applicable to more than one type of individual. In fact, however, the application of this bound to the case of a heterogenous population is straightforward. It is possible to define the lower envelope of the lower bounds of each type. The supporting price system is, then, the weighted average of this envelope and $\overline{\omega}(\cdot)$.

Finally, my conclusions are based on completely ignoring the integer problem (the possibility that the core is empty). This, however, is not unique to the multiple private goods case vis-a-vis the one private good case. Nor is this problem specific to economies with collective goods. Thus, I prefer to concentrate on what is specific to collective good economies with multiple private goods.

5 Appendix

The first-order conditions for the maximization problems are:

Consumers:

(21) $$p - \frac{u_{y^i}}{u_{x^i}} = 0; \quad i = a, b,$$

(22) $$-\frac{u_{z^i}}{u_{x^i}} = \omega_z \left(z^i, n^i\right); \quad i = a, b,$$

(23) $$-\frac{u_{n^i}}{u_{x^i}} = \omega_n \left(z^i, n^i\right); \quad i = a, b,$$

Producers of X:

(24) $$f_n(\cdot) - \omega\left(z^i, n^i\right) = 0; \quad i = a, b,$$

Producers of Y:

(25) $$g_n(\cdot)\, p - \omega\left(z^i, n^i\right) = 0; \quad i = a, b,$$

Developers

(26) $$-n^i \omega_z \left(z^i, n^i\right) - c_z \left(z^i, n^i\right) = 0; \quad i = a, b,$$

(27) $$g_n(\cdot)\,p - n^i\omega_n\left(z^i,n^i\right) - \omega\left(z^i,n^i\right) - c_n\left(z^i,n^i\right) = 0; \quad i = a,b,$$

Free entry to the communities' business:

(28) $$f(\cdot) + g(\cdot)\,p - n^i\omega\left(z^i,n^i\right) - c\left(z^i,n^i\right) = 0; \quad i = a,b,$$

With (21), we only have to derive the Samuelson and the Henry George conditions, (6) and (12), to obtain fully isomorphic conditions for the optimal and the decentralized allocations. Equation (6) follows from (22) and (26); (12) follows from (23), (25), (27), (28), and the Euler theorem.

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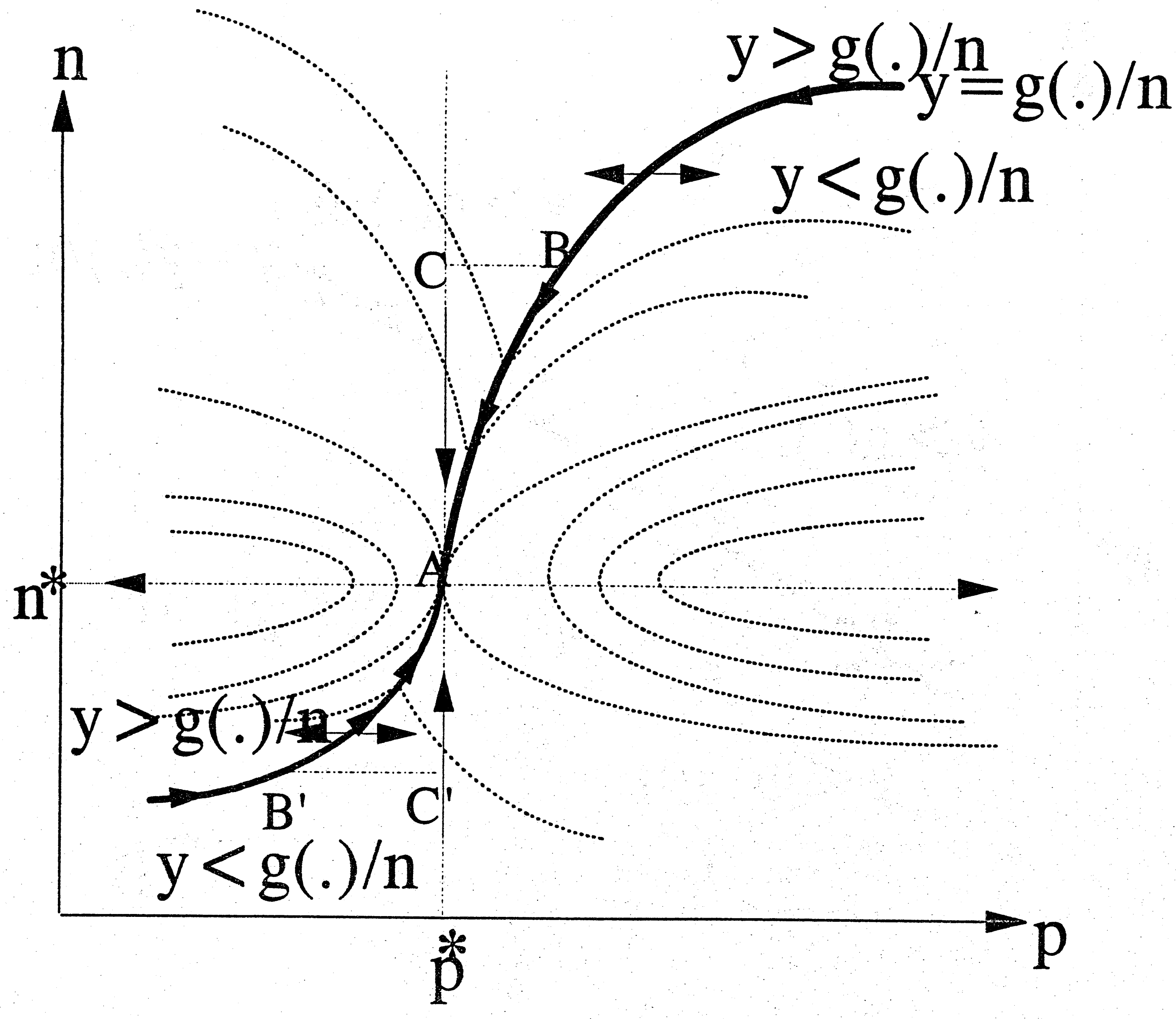
Figure 1



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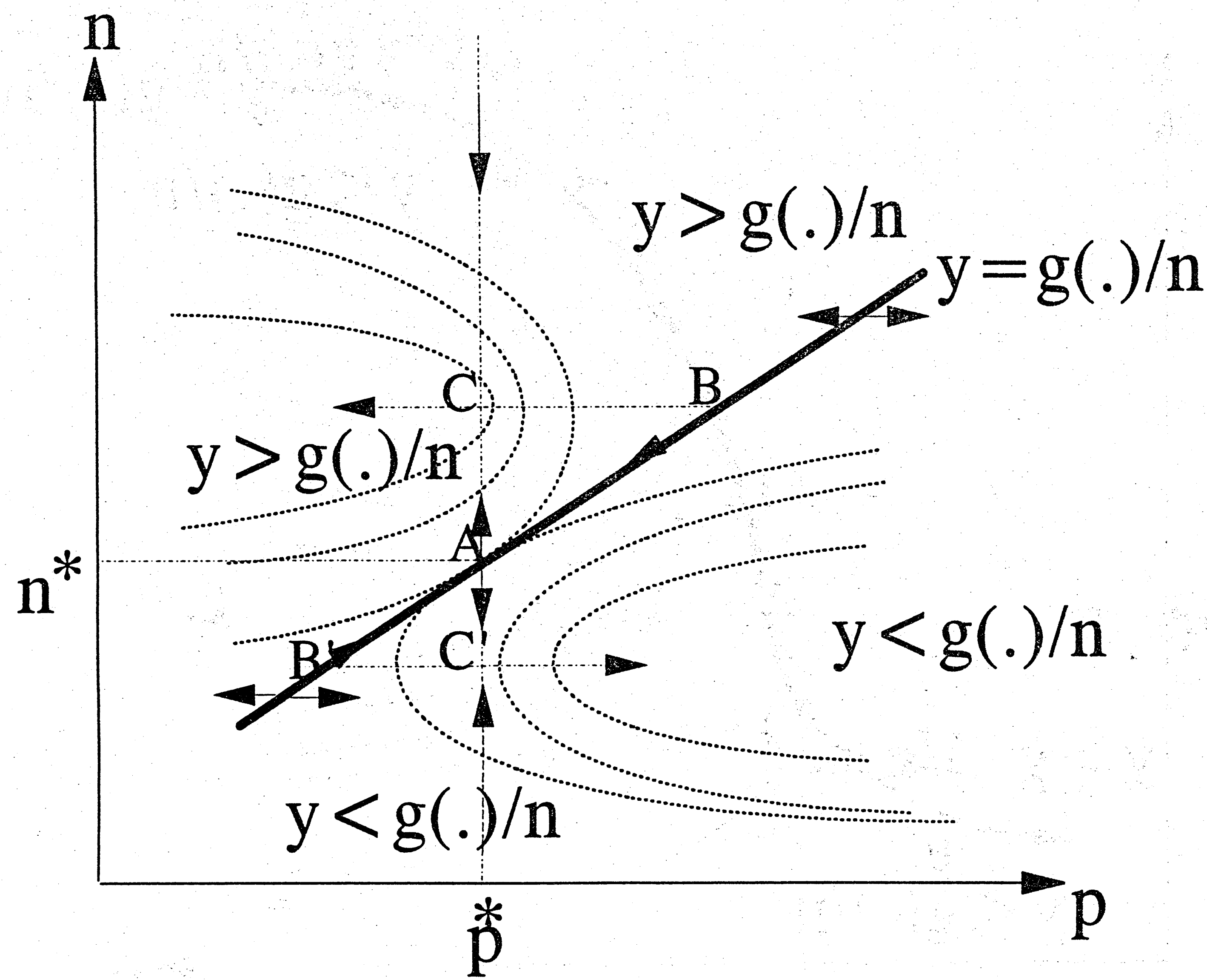

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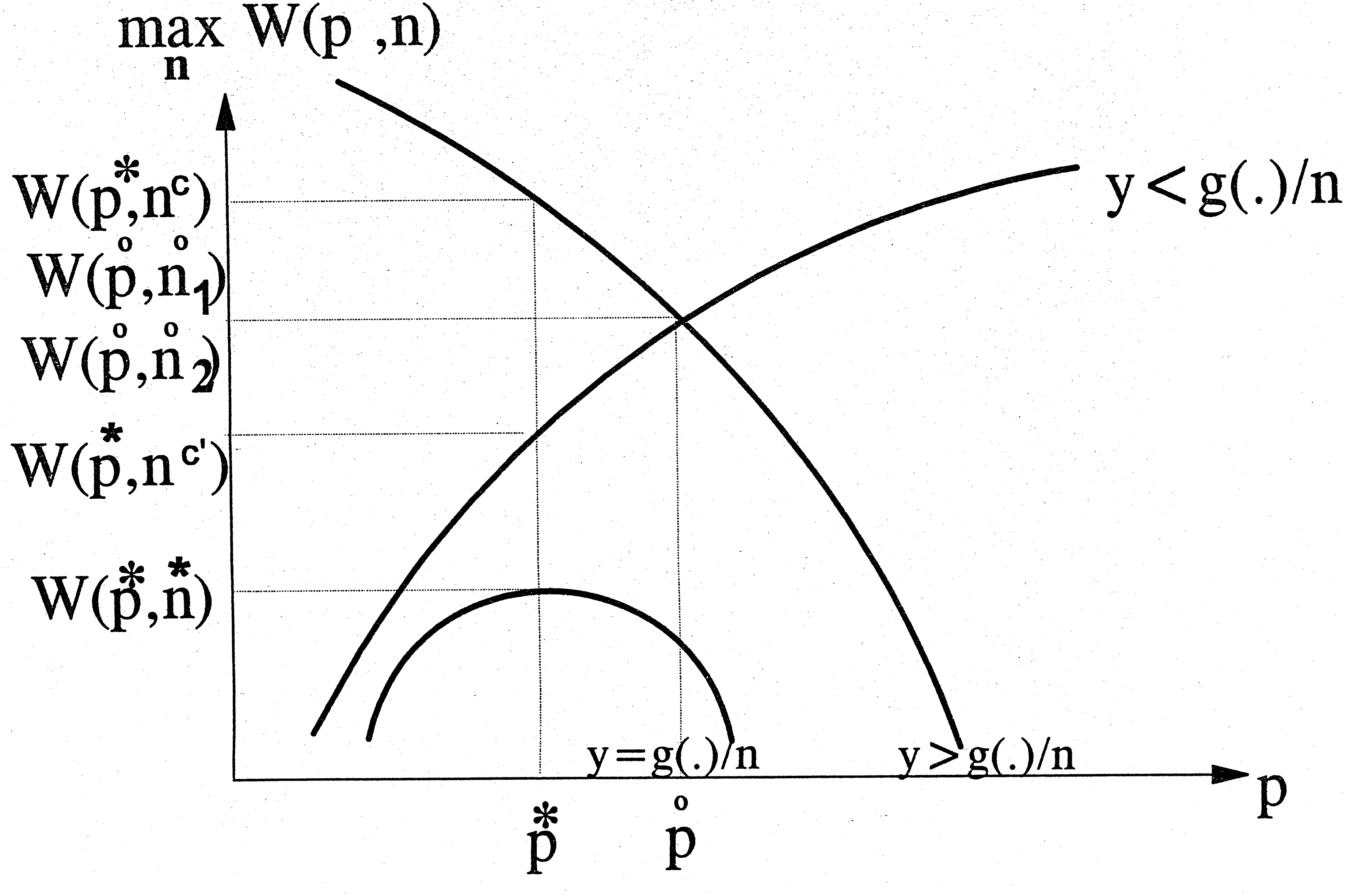

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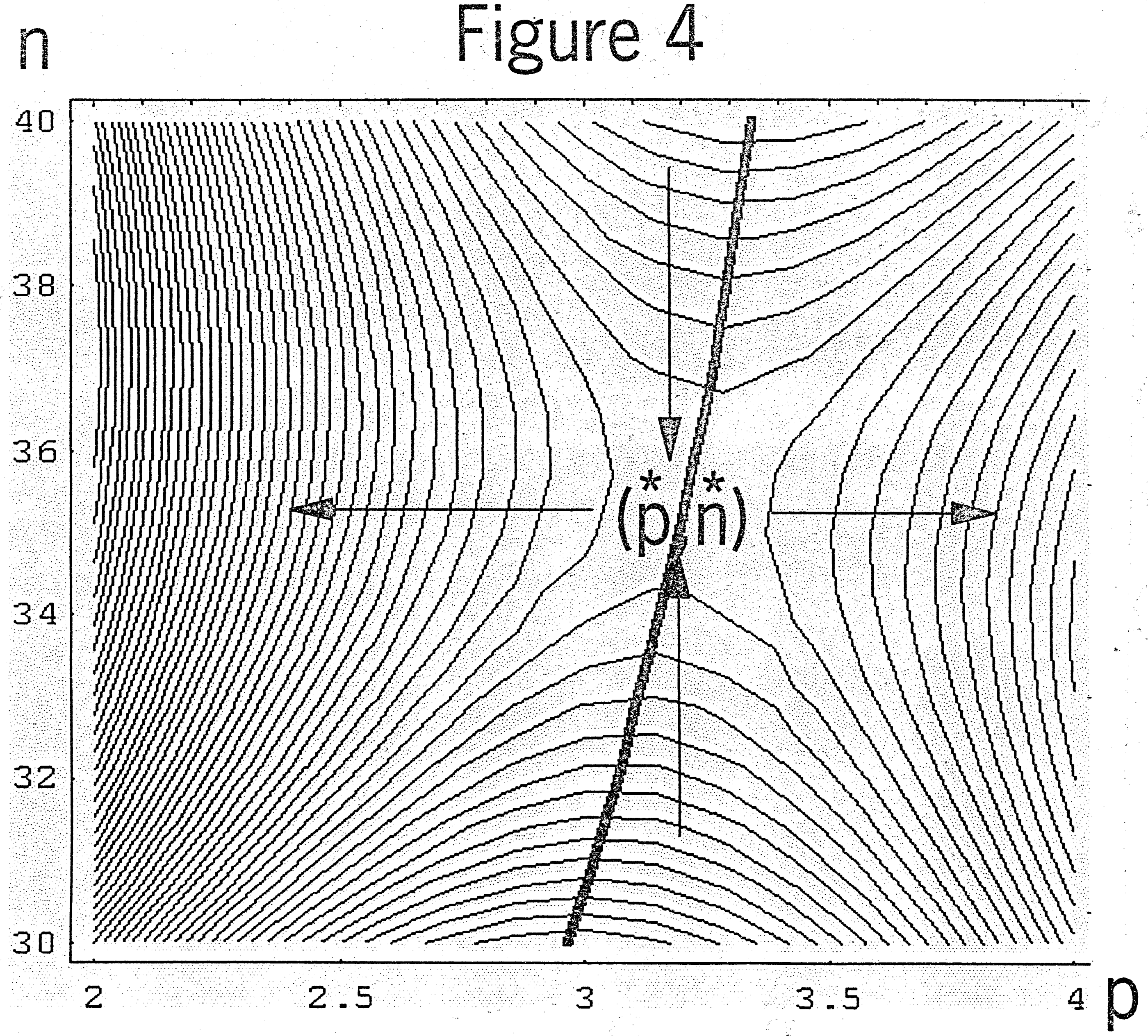

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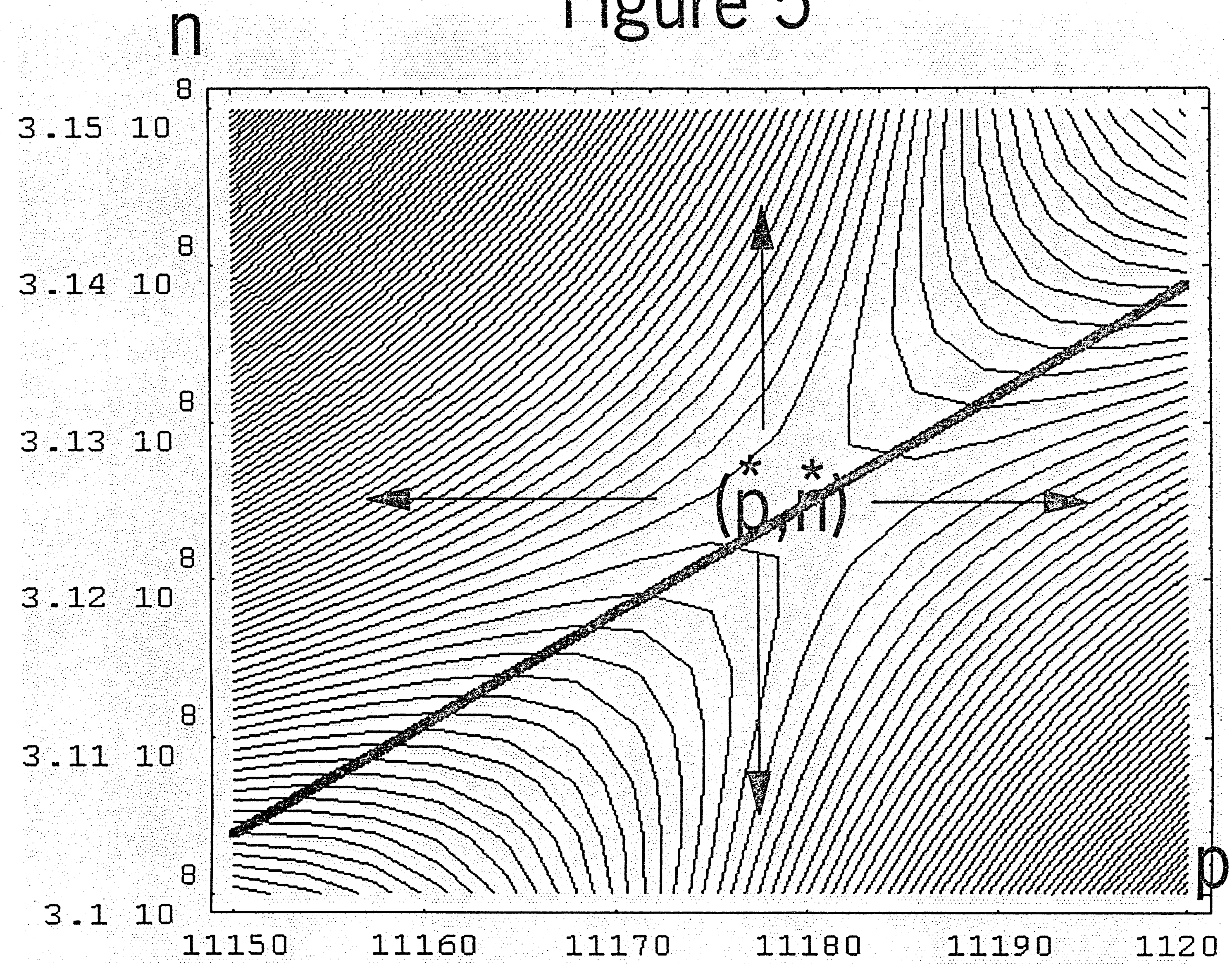
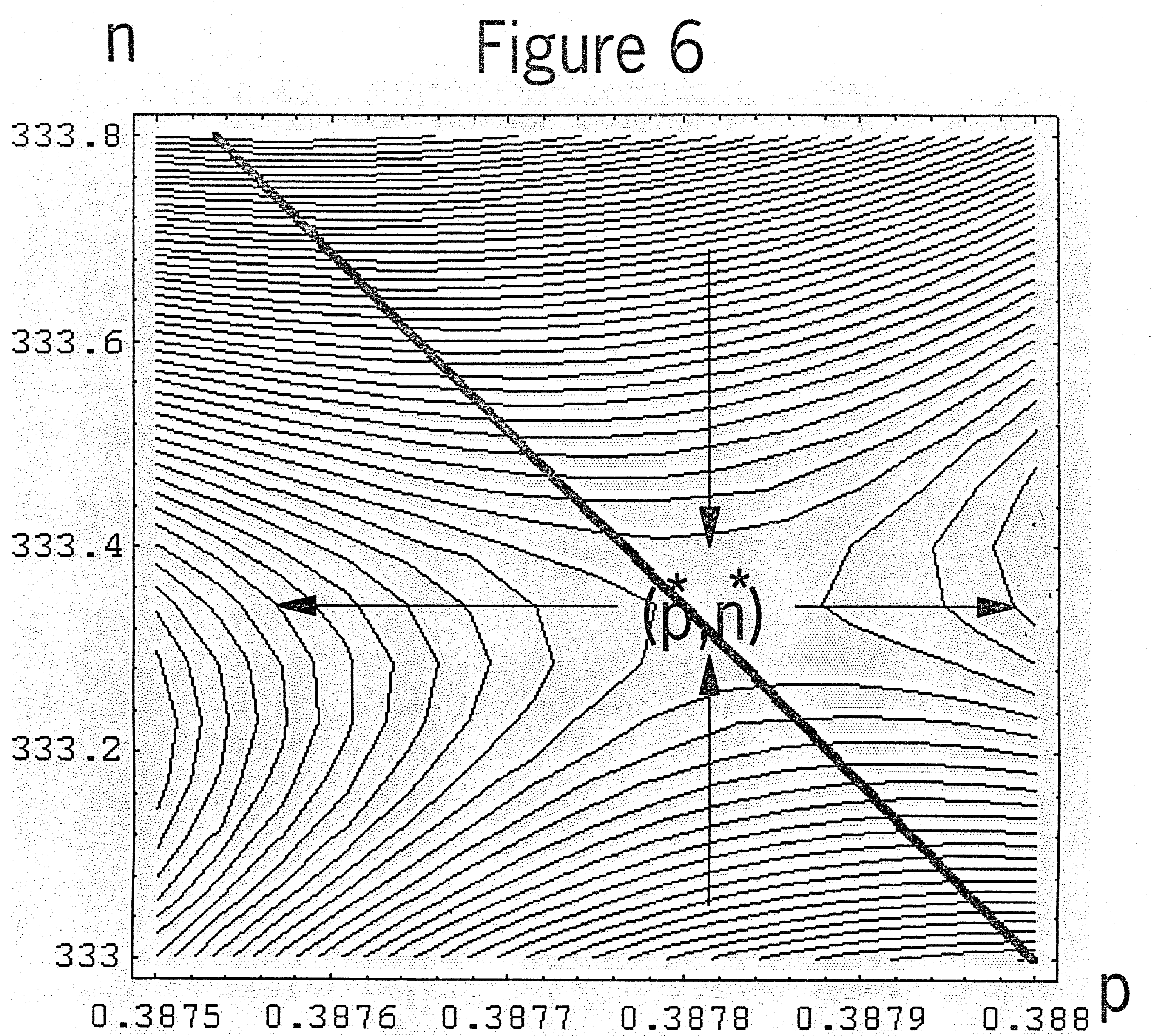


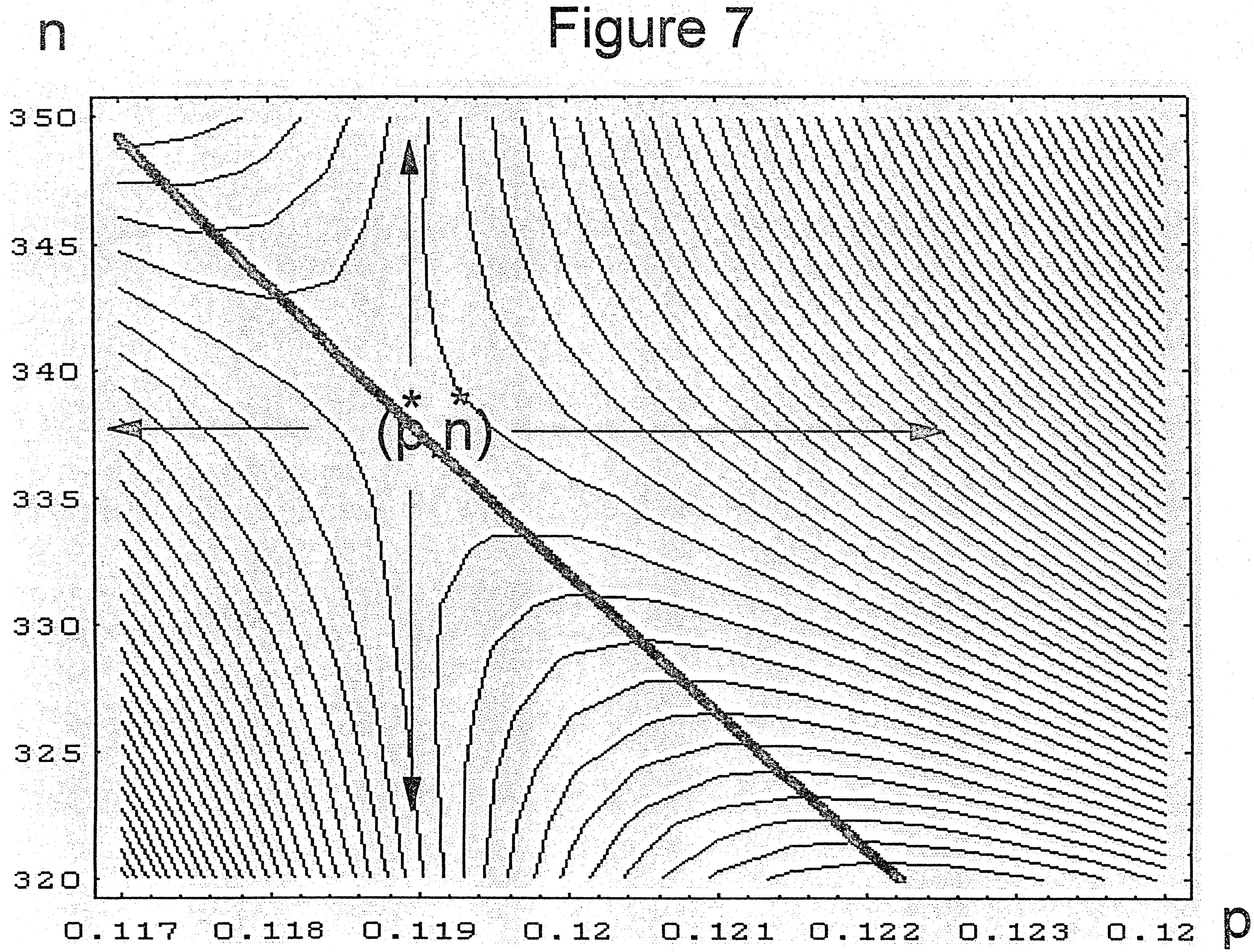
n
3.15 10^8
3.14 10^8
3.13 10^8
3.12 10^8
3.11 10^8
3.1 10^8
(p*,n*)
p
11150
11160
11170
11180
11190
1120

Figure 6



n
333.8
333.6
333.4
333.2
333
(p*, n*)
0.3875
0.3876
0.3877
0.3878
0.3879
0.388
p

Figure 7



n
350
345
340
335
330
325
320
(p*,n*)
0.117
0.118
0.119
0.12
0.121
0.122
0.123
0.12
p

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