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**INTERGENERATIONAL TRANSFERS,
ECONOMIC GROWTH AND
INCOME DISTRIBUTION[†]**

by

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[†]This research has been done while I was visiting The Department of Economics at Johns Hopkins University (Spring 1996) and the Tinbergen Institute, Rotterdam. I have benefitted from discussions with Larry Ball, Edi Karni, Jean-Marie Viaene and Caspar de-Vries.

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Abstract

We consider an Overlapping-generations economy where the aggregative production process uses physical capital and human capital. The human capital level of each individual is determined by the direct investment in education and some random "ability". The parents' investment in the education of their offspring is motivated by altruism. We distinguish between two types of transfers: Investment of parents in the education of their offspring, which affects her future income, and the direct capital transfer (the 'bequest motive'). We show that the intensity of each type of altruism plays an important role on the equilibrium growth and the income distributions, but the results differ significantly. Comparing competitive equilibria from the same initial capital and human capital distributions we derive the following results: (a) When altruism is more 'education-inclined', then economic growth is higher and the intragenerational income distributions are more equal (less equal), in all periods, if the aggregate production function's elasticity of substitution is larger (smaller) than 1; (b) When altruism is more 'bequest-inclined', the growth rate is lower and the impact on the intragenerational distributions of income depends on the size of the elasticity of substitution. Public provision of education (financed by taxing wage incomes) reverses the results in (b).

Intergenerational Transfers, Economic Growth and Income Distribution

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1. Introduction

Intergenerational transfers, in their various forms, are among the major factors affecting the degree of inequality in the distribution of income. Other important factors are the level of education and innate abilities. These factors interact with the prices of capital and labor to produce the dispersion of incomes.

In the last two decades we have witnessed sharp disagreement between economists with regard to the quantitative importance of intergenerational transfers in enhancing savings and capital accumulation and regarding their motives. Some estimates find these transfers responsible for as much as three quarters of aggregate capital accumulation (Kotlikoff and Summers (1981), Kotlikoff (1988)), while others claim that it is 20-30% (see, for example, Modigliani (1988), Gale and Scholz (1994)). Kotlikoff demonstrates that "there is strong evidence that intergenerational transfers play a very important role and perhaps dominant role in US wealth accumulation." This claim is also supported by Bernheim (1991) who

presents empirical evidence supporting the view that private savings are strongly influenced by the desire for bequest. Bernheim's evidence is based on the presumption that bequest motives significantly alter the individual's attitude towards annuities markets. Gale and Scholz (1994) claim that we cannot use the life-cycle model to estimate properly the intergenerational transfers since most transfers are *inter vivos* which, in this case, are from living parents to adult children living in separate households; such transfers are estimated at 20% of aggregate wealth in the U.S., while total bequests transfers (including intentional and unintentional transfers) are 31% of the net worth. Lee and Lapkoff (1988) consider the issue of intergenerational transfers and the level of fertility, and estimate the "between-household" transfers and "within-household" transfers. Some argue that most transfers are accidental while savings are precautionary and insurance markets are imperfect (Hurd (1987)) while others insist that bequests reflect altruism or self-interested exchange and are primarily intentional (Tomes (1981), Bernheim (1991), Bernheim, Schleifer and Summers (1985)). Weil (1994) shows that intergenerational transfers explain the substantial gap in the elderly saving rate between household (micro) data and the aggregate (macro) data.

In recent years we witness a vast body of economic literature dealing with endogenous growth generated by accumulation of human capital and its effects on technological progress (see Jovanovic (1995) for a survey). It is frequently assumed in economic models that parents' investment in their offsprings' education is motivated by the parents' preferences. The economic implications of investment in human capital to growth and income distribution, the issue of the inherent externality which exists in these models and the efficiency of government intervention have been recently widely examined (see, for example, Loury (1981), Lucas (1988), Bernheim (1989), Azariadis and Drazen (1990), Laitner (1992), Glomm and Ravikumar (1992), Eckstein and Zilcha (1994), Alsina and Rodrik (1994), Galor and Tsiddon (1994), Fershtman, Murphy and Weiss (1996) and many others).

The economy considered in this paper is an overlapping generations framework where individuals have a two-period lifetime. The production process applies physical capital as well as human capital, which determines the "quality" of labor. The human capital level of each individual is determined by the direct investment in her education when "young" (we assume that parents make the investment if there is no public provision of education) and some random element called "ability." Since parent's investment in the education of their offspring, motivated by altruism, depends on their wealth, the evolution process of human capital in a family (or a dynasty) depends on the intergenerational transfers, and hence on the history of realizations of the random abilities as well; clearly, for each individual her wage income and the transfer to offspring are positively correlated to her human capital level.

We shall distinguish between two kinds of transfers from parents to children: (a) The investment of parents in the education of their child, which affects her future earnings (called 'Education-inclined altruism') and (b) the direct non-human capital transfers motivated by the "bequest motive" and/or the "joy of giving" (called 'Bequest-inclined altruism'). We do not distinguish here between *inter vivos* transfers (namely, transfers between living people) and transfers that occur at the death of the parent since in our model there is no lifetime uncertainty. Both types of transfers are assumed to exist but their intensities can differ between economies. Although in both cases such transfers are altruistically motivated their economic implications on growth and income distribution *are different*. Our main goal in this work is to study the role played by the intensity of each type of altruism, and the consequences of the resulting intergenerational transfers to economic growth and the distribution of incomes over time. In other words, we shall compare the competitive equilibria of two economies starting from the same initial human capital and transfer distributions which differ only in the intensity of each type of altruism.

We show that there exist significant differences in the role played by each of these

two types of altruisms on growth and income distribution: A more altruistic economy, where altruism is motivated mainly by the offspring's human capital, and hence future wage income, has an equilibrium with a *higher* growth. However, the *influence* of this type of altruism on the intragenerational income distribution depends heavily on the production function's elasticity of substitution σ . When $\sigma > 1$ ($\sigma < 1$) it results in a more (less) equal distribution of incomes. On the other hand, a more altruistic economy, where altruism is motivated mainly by bequest, or the "joy of giving", has an equilibrium with a *lower* growth, and its effect on the intragenerational distribution of income (in all generations) depends crucially on the size of the elasticity of substitution of the production function as well. Thus, when we compare growth rates or income distribution in various countries we should pay attention to the mode in which transfers from parents to children are carried out: Is it more in the form of enhancing human capital formation or it is mainly a direct transfer of some sort of capital.

We also consider a model where education is (equally) provided by the government. The cost of education is financed by taxing the wage-income. We demonstrate that the level of education-inclined altruism has no effect on the growth rate of this economy and the intragenerational income distribution at all dates. However, a higher bequest-inclined altruism results in a higher growth if $\sigma \geq 1$, which is in contrast to the case of privately provided education. It also results in less equal income distributions (when $\sigma \geq 1$). Thus under public provision of education, the effects differ significantly from the case of private provision of education. Introducing tax on bequest transfers will not change these results.

In the literature there are many studies comparing private provision of education vs. public provision and the voucher system (where parents can supplement the education expenditure). However, most of these studies deal with different issues such as integration and growth (Benabou (1995)), the size of the public education system vs. the private one

(Glomm and Ravikumar (1995)), financing education (Fernandez and Rogerson (1993)) and others.

The paper is organized as follows. In section 2 we present the OLG model with altruistic individuals. Section 3 compares the economic growth in two economies with a different degree of altruism. Section 4 compares the intragenerational distribution of incomes in two economies which differ only in the degree of altruism. In section 5 we consider the case where education is provided publicly. Section 6 concludes the paper.

2. The Model

2.1. Preferences and Technology

Consider an overlapping generations economy with no population growth. Each individual in this economy lives for two periods - a working period followed by a retirement period. At the end of the first period, every individual gives birth to one offspring. Denote by G_t the set of individuals born at the outset of period t and refer to these individuals as generation t . The economy starts at $t = 0$, where G_{-1} live their retirement period; their only source of income is their savings. Denote by Ω the set of families in each generation; Ω is time independent since there is no population growth. Although our analysis can be carried out for any finite Ω , to simplify our notations we assume there is a continuum of individuals (or families) in each generation, hence we may assume that $\Omega = [0, 1]$. Each individual in G_t is characterized by its family name $\omega \in [0, 1]$. We denote by μ the Lebesgue measure on Ω .

In this economy parents derive utility from the human capital level of their child as well as from bequest, or "joy of giving". This motivates the transfer of wealth to their offspring in the form of human and nonhuman capital. The parent's investment in their offspring's education in conjunction with random ability of the offspring determines the

offspring's level of human capital. The stocks of human and nonhuman capital together with the relevant interest rate and wages determine the offspring's total income.

Let $b_t(\omega)$ be the transfer of nonhuman capital (such as bequest) of dynasty $\omega \in G_t$ to their offspring and denote by $e_t(\omega)$ their investment in her/his education. The human capital of the offspring, $h_{t+1}(\omega)$ is determined by $e_t(\omega)$ and a random process of ability $\tilde{\theta}_{t+1}(\omega)$ assigned by nature to each person. To simplify our subsequent analysis, we shall take this process to be:

$$h_{t+1}(\omega) = e_t(\omega) \tilde{\theta}_{t+1}(\omega), \quad t = 0, 1, 2, \dots \quad (2.1)$$

where the random variables $\tilde{\theta}_t(\omega)$, $t = 1, 2, 3, \dots$; $\omega \in \Omega$ are identically distributed with support $[a, b]$, $0 < a < b < \infty$, and are independent. We shall denote the mean of (each) $\tilde{\theta}_t(\omega)$ by $\bar{\theta}$. It is not difficult to relax the independence assumption and to allow some correlation between $\tilde{\theta}_t(\omega)$ and $\tilde{\theta}_{t+1}(\omega)$ for each ω . However, we shall not consider this case in the sequel. The labor supply of each individual in his first period is assumed to be fixed and normalized to equal 1. We also assume that parents are aware of the native abilities of their offspring, i.e., family $\omega \in G_t$ observes the realization $\tilde{\theta}_{t+1}(\omega)$ prior to making their decision on how much to invest in the offspring's education. In this framework we shall not consider the possibility where the individual can increase investment in her own education.

Preferences of individual $\omega \in G_t$ are represented by a Cobb-Douglas utility function:

$$U = c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} [g_t(\omega)]^{\alpha_3} [h_{t+1}(\omega)]^{\alpha_4} \quad (2.2)$$

where α_i are known parameters, $\sum \alpha_i = \alpha^*$, and $\alpha_i > 0$ for $i = 1, 2, 3, 4$; c_{1t} and c_{2t} denote, respectively, consumption in the first and second periods of the individual's life; g_t denotes the transfer or bequest of (nonhuman) capital while h_{t+1} denotes the human capital level of

the offspring. Let r_{t+1} be the interest rate in period $t + 1$, then,

$$g_t(\omega) = b_t(\omega)(1 + r_{t+1})$$

where $b_t(\omega)$ is the direct transfer (or bequest) .

The dependence of human capital upon the realizations of $\tilde{\theta}_{t+1}(\omega)$ has an important contribution to the heterogeneity in this economy. The heterogeneity in earnings, coupled with the inequality in the intergenerational transfers, results in intragenerational dispersion in incomes along the equilibrium path and in the steady state.

Production in this economy is carried out by competitive firms that use labor and capital to produce a single commodity. The commodity serves for consumption and for investment in nonhuman as well as human capital. For expository convenience and to avoid having to deal with details that are not essential for the main issue, we choose to disregard investment in human capital as a separate production processes. Consequently, our formulation abstracts from possible effects of changes in the relative price of education on the evolution of the economy. Following Diamond (1965), we assume that the aggregate stock of capital in each period, K_t , is determined by the aggregate savings in the preceding period. The aggregate production function $F(K_t, L_t)$ is assumed to exhibit constant returns-to-scale, where L_t denotes the aggregate "effective labor" or, since labor is supplied inelastically, the stock of human capital. We assume that $F_{KK} < 0$, $F_{LL} < 0$, and $F_{KL} > 0$.

2.2. Equilibrium

In each period the economy features three markets: two factor markets (labor and capital), and one commodity market. To define a competitive equilibrium, we begin by considering the state of the economy at the outset of period t . Each family $\omega \in \Omega$ consists of two members,

the "old" member belonging to G_{t-1} and the "young" member belong to G_t . Suppose that the distribution of the bequests received by individuals of generation G_t is given by the function $b_{t-1} : \Omega \rightarrow [0, m]$, where $m < \infty$. The human capital distribution in G_t is $h_t(\omega)$ where the initial $b_{-1}(\omega)$ and $h_0(\omega)$ are given. Clearly, both functions $b_{t-1}(\omega)$ and $h_t(\omega)$ depend upon the history up to date t .

Let the bequest transfer, $b_{t-1}(\omega)$, the stock of human capital, $h_t(\omega)$, the effective wage rate, w_t , and interest rate r_t for date t , the interest rate r_{t+1} , for date $t + 1$, and the realization of $\tilde{\theta}_{t+1}(\omega)$ be given. An individual $\omega \in G_t$ chooses the levels of savings, $s_t(\omega)$, bequest transfer, $b_t(\omega)$, and investment in his offspring's education, $e_t(\omega)$, so as to maximize:

$$\text{Max } c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} [(b_t(\omega)(1 + r_{t+1}))^{\alpha_3} [h_{t+1}(\omega)]^{\alpha_4} \quad (2.3)$$

subject to constraints:

$$c_{1t}(\omega) = (1 + r_t)b_{t-1}(\omega) + w_t h_t(\omega) - s_t(\omega) - b_t(\omega) - e_t(\omega) \geq 0 \quad (2.4)$$

$$c_{2t}(\omega) = (1 + r_{t+1})s_t(\omega) \quad (2.5)$$

$$h_{t+1}(\omega) = e_t(\omega) \tilde{\theta}_{t+1}(\omega) \quad (2.6)$$

Given the capital stock K_0 at the outset of period 0 and $b_{-1}(\omega)$, $h_0(\omega)$ for all $\omega \in G_0$, a *competitive equilibrium* is a sequence of functions $[c_{1t}(\omega), c_{2t}(\omega), s_t(\omega), b_t(\omega), e_t(\omega)]_{t=0}^{\infty}$ and a sequence of prices $[w_t, r_t]_{t=0}^{\infty}$ such that for $t = 0, 1, 2, \dots$:

(a) Given the above prices for all $\omega \in G_t$, $[c_{1t}(\omega), c_{2t}(\omega), s_t(\omega), b_t(\omega), e_t(\omega)]$ is the solution to the maximization problem (2.3)-(2.6);

$$(b) \int_{\Omega} h_t(\omega) d\mu(\omega) \equiv L_t = F_L^{-1}(K_t, w_t);$$

$$(c) K_t = F_K^{-1}(L_t, r_t);$$

$$(d) K_{t+1} = \int_{\Omega} [s_t(\omega) + b_t(\omega)] d\mu(\omega)$$

where F_L^{-1} and F_K^{-1} denote the inverse functions corresponding to F_L and F_K .

Condition (a) asserts that the various demand functions in the economy are derived from optimal consumer behavior assuming that all consumers are price takers. Conditions (b) and (c) are the equilibrium conditions in the labor and capital markets, respectively. The specification of the demand functions is based on the assumption that firms are price takers in the factor markets. Condition (d) describes the dynamic adjustment of the aggregate nonhuman capital stock in the economy assuming full depreciation of the capital stock in each period. Note that the aggregate capital stock is determined by aggregate savings and nonhuman capital transfers, while the aggregate labor supply depends only on the intergenerational transfers of human capital in the form of education. These conditions, in conjunction with constraints (2.4) and (2.5) imply the material balance condition:

$$\int [c_{1t}(\omega) + c_{2(t-1)}(\omega) + e_t(\omega)] d\mu(\omega) + K_{t+1} = F(K_t, L_t) \quad \text{for } t = 0, 1, \dots \quad (2.7)$$

Necessary and sufficient conditions for optimum (given the concavity of the utility function) are:

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha_1}{\alpha_2(1 + r_{t+1})} \quad (2.8)$$

$$\frac{c_{1t}}{e_t} = \frac{\alpha_1}{\alpha_4} \quad (2.9)$$

$$\frac{e_t}{b_t} = \frac{\alpha_4}{\alpha_3} \quad (2.10)$$

From (2.8) and (2.9) we also obtain that

$$\frac{b_t}{s_t} = \frac{\alpha_3}{\alpha_2}. \quad (2.11)$$

Given the realization of $\tilde{\theta}(\omega)$, the lifetime income of $\omega \in G_t$, $y_t(\omega)$, is

$$\begin{aligned} y_t(\omega) &= (1 + r_t)b_{t-1}(\omega) + w_t[e_{t-1}(\omega)\theta_t(\omega)] = \\ &= (1 + r_t)b_{t-1}(\omega) \left[1 + \frac{w_t}{1 + r_t} \frac{\alpha_4}{\alpha_3} \theta_t(\omega) \right] \quad t = 1, 2, \dots \end{aligned} \quad (2.12)$$

Using equation (2.10), the aggregate stock of human capital in period t is given by:

$$L_t = \bar{\theta} \frac{\alpha_4}{\alpha_3} \int_{\Omega} b_{t-1}(\omega) d\mu(\omega) = \bar{\theta} \frac{\alpha_4}{\alpha_3} B_{t-1}, \quad t = 1, 2, \dots \quad (2.13)$$

where $B_{t-1} = \int_{\Omega} b_{t-1}(\omega) d\mu(\omega)$ is the aggregate transfer of nonhuman capital to generation t . Similarly, using equation (2.11), the aggregate of physical capital stock in period t is:

$$K_t = \int_{\Omega} [s_{t-1}(\omega) + b_{t-1}(\omega)] d\mu(\omega) = \left(1 + \frac{\alpha_2}{\alpha_3} \right) B_{t-1}. \quad (2.14)$$

3. Altruism and Economic Growth

Our main purpose in this work is to compare such economies with respect to their degree of altruism. Intergenerational transfers are driven by two main motivations: (a) Improving the earning capability of the offspring via her education; (b) the "joy of giving" and the

bequest motive. Thus in (b) we include *inter vivos* transfers as well as bequest transfers (whether intentional bequest or unintentional bequest (see Gale and Scholz (1994))). We shall distinguish between these two types of altruism since their economic impacts are different. Consider two economies with the same structure (as the above one) which differ only in the preferences of the individuals (although each one has the form given by (2.2)) as follows: For each given level of income $y(\omega)$ the individuals in the first economy invest more in human capital of their offspring than in the second one. In this case we say that the first economy is **more Education-inclined (EI) altruistic** than the second one. Similarly, if for each level of income $y(\omega)$ individuals in the first economy transfer more (nonhuman) capital to their offspring than in the second economy, we say that the first economy is **more Bequest-inclined (BI) altruistic** than the second one. Consequently, higher EI-altruism implies higher $e_t(\omega)$, while higher BI-altruism implies higher $b_t(\omega)$, given the initial distribution of savings and human capital.

We shall assume now that aggregate production function in our economy has a constant elasticity of substitution denoted by σ . This is done in order to simplify the proofs of our results.

In comparing two economies from the same initial distributions of capital and human capital we shall consider the competitive equilibria (from these initial conditions) period by period. Thus we shall compare along these two dynamic paths capital and effective labor stocks in each date.

Given the above framework we now prove:

Theorem 3.1. *An economy with higher EI-altruism grows faster in equilibrium if $\sigma \geq 1$.*

Thus, comparing the competitive equilibrium paths from the same initial conditions of two economies which differ only in EI-altruism we obtain that more altruism results in

higher growth in all periods, if the production function's elasticity of substitution is greater or equal to 1.

Proof. Consider the equations (2.8)-(2.11) derived from the optimality conditions. The income $y_t(\omega)$ is given by (2.12) and we can write:

$$\alpha^* = \sum_{i=1}^4 \alpha_i, \quad b_t(\omega) = \frac{\alpha_3}{\alpha^*} y_t(\omega); \quad e_t(\omega) = \frac{\alpha_4}{\alpha^*} y_t(\omega) \quad (3.1)$$

At date $t = 0$, $K_0, b_{-1}(\omega)$ and $h_0(\omega)$ are given. Denote the equilibrium with individuals having utility functions as in (2.2) by $\{(c_{1t}, c_{2t}, s_t, b_t, e_t), (w_t, r_t)\}$. The equilibrium with the more altruistic individuals, i.e., with $\alpha'_4 > \alpha_4$ (while $\alpha_1, \alpha_2, \alpha_3$ remain the same) is denoted by $\{(c'_{1t}, c'_{2t}, s'_t, b'_t, e'_t), (w'_t, r'_t)\}$. Let us write $\alpha'_4 = (1 + m)\alpha_4$, where $m > 0$.

Since the distribution of savings and human capital $h_0(\omega)$ are given at the initial date 0, $L'_0 = L_0$ and also $K'_0 = K_0$. Thus we also have $w'_0 = F_L(K'_0, L'_0) = F_L(K_0, L_0) = w_0$ and similarly $r'_0 = r_0$. The change in preferences does not affect the level of incomes in period 0, hence $y_0(\omega) = y'_0(\omega)$.

Claim: For all t , $t = 1, 2, \dots$ $y'_t(\omega) > y_t(\omega)$ for all ω .

Proof. We shall prove this claim by induction. Let us show first that under our assumptions:

$$\frac{K'_t}{L'_t} < \frac{K_t}{L_t} \quad t = 1, 2, 3, \dots \quad (3.2)$$

This can be verified, using (3.1), from:

$$L'_t = \int e'_{t-1}(\omega) \theta_t(\omega) d\mu = \bar{\theta} \int \frac{\alpha_4(1+m)}{\alpha^* + m\alpha_4} y'_{t-1}(\omega) d\mu$$

$$K_t' = \int [s_{t-1}^t(\omega) + b_{t-1}^t(\omega)] d\mu = \frac{\alpha_2 + \alpha_3}{\alpha_2 + m\alpha_4} \int y_{t-1}^t(\omega) d\mu.$$

Hence

$$\frac{K_t'}{L_t'} = \frac{\alpha_2 + \alpha_3}{\alpha_2 + m\alpha_4} > \frac{\theta}{\alpha_2 + \alpha_3} = \frac{L_t}{K_t} \quad t = 1, 2, 3, \dots \quad (3.3)$$

Assuming that $\sigma \geq 1$ we proceed with the following expression which appears in equation (2.12) for $t \geq 1$: Let $\hat{\mu}$ be a constant defined by $\frac{F_t(K, L)}{F_t(K, L)} = \hat{\mu} \left(\frac{L}{K} \right)^{1/\sigma}$, then:

$$\begin{aligned} & \frac{K_t'}{L_t'} = \frac{\alpha_2 + \alpha_3}{\alpha_2 + m\alpha_4} \cdot \frac{1 + r_t'}{w_t'} = \hat{\mu} \frac{\alpha_3}{\alpha_4(1+m)} \left(\frac{L_t'}{K_t'} \right)^{1/\sigma} \\ & = \hat{\mu} \left[\frac{\theta(1+m)}{\alpha_2 + \alpha_3} \right]^{1/\sigma} = \frac{\alpha_3}{\alpha_4} \frac{1 + r_t'}{w_t'} \geq \frac{\alpha_3}{\alpha_4} \frac{1 + r_t'}{w_t'} \end{aligned} \quad (3.4)$$

where (3.4) holds with equality if and only if $\sigma = 1$.

Now, using (3.2) we obtain that $r_t' > r_t$ for $t = 1, 2, \dots$. By the induction step

$y_{t-1}^t(\omega) > y_{t-1}^{t-1}(\omega)$ for all ω implies that $b_{t-1}^t(\omega) > b_{t-1}^{t-1}(\omega)$ for all ω . Now we derive from

(2.12) that $y_t^t(\omega) > y_t^{t-1}(\omega)$ for all ω which implies that $b_t^t(\omega) > b_t^{t-1}(\omega)$. This establishes our

claim. Now it is easy to show that $F(K_t', L_t') > F(K_t, L_t)$ for all t , $t = 1, 2, \dots$. Using

equation (2.7) and the fact proven in the above claim, regarding the higher incomes the LHS

of (2.7) increases, hence the RHS increases in the more altruistic case.

We now demonstrate that when considering BI-altruism the result of Theorem 3.1.

is reversed.

Theorem 3.2. *Consider two economies starting from the same initial capital holdings and human capital distribution. If $\sigma \geq 1$ then the economy with the higher Bequest-inclined altruism grows slower in equilibrium*

The contrasting results of Theorems 3.1 and 3.2. demonstrate the gap in the economic implications that each type of altruism has. Thus the way in which altruism is implemented has a major impact on economic development.

Proof. The proof goes along the lines of the proof of Theorem 3.1. However, in this case increasing α_3 to $\alpha'_3 = (1+m)\alpha_3$, keeping $\alpha_1, \alpha_2, \alpha_4$ unchanged, results in the following:

$$K'_t = \frac{\alpha_2 + \alpha_3(1+m)}{\alpha^* + m\alpha_3} \int y'_{t-1}(\omega)$$

$$L'_t = \bar{\theta} \frac{\alpha_4}{\alpha^* + m\alpha_3} \int y'_{t-1}(\omega).$$

Hence

$$\frac{K'_t}{L'_t} = \frac{\alpha_2 + \alpha_3(1+m)}{\bar{\theta} \alpha_4} > \frac{\alpha_2 + \alpha_3}{\bar{\theta} \alpha_4} = \frac{K_t}{L_t} \quad t = 1, 2, \dots \quad (3.5)$$

Now let us show that:

$$\begin{aligned} \frac{w'_t}{1+r'_t} \frac{\alpha_4}{\alpha_3(1+m)} &= \hat{\mu} \left[\frac{\alpha_2 + \alpha_3(1+m)}{\bar{\theta} \alpha_4} \right]^{1/\sigma} \frac{\alpha_4}{(1+m)\alpha_3} < \\ \hat{\mu} \left[\frac{\alpha_2 + \alpha_3}{\bar{\theta} \alpha_4} \right]^{1/\sigma} \frac{\alpha_4}{\alpha_3} &= \frac{w_t}{1+r_t} \cdot \frac{\alpha_4}{\alpha_3} \end{aligned} \quad (3.6)$$

The inequality in (3.6) can be shown from the following argument. Define

$$h(m) = \frac{1}{1+m} \left[\frac{\alpha_2 + \alpha_3}{\bar{\theta} \alpha_4} + m \frac{\alpha_3}{\alpha_4} \right]^{1/\sigma}.$$

Then, differentiating $h(m)$ with respect to m we obtain that:

$$h'(m) < 0 \quad \text{if } \sigma > \frac{\alpha_3(1+m)}{\alpha_2 + \alpha_3(1+m)}.$$

Since this should hold for any positive m assuming $\sigma \geq 1$ guarantees that $h'(m) < 0$. We now conclude that $h(m) < h(0) = \left[\frac{\alpha_2 + \alpha_3}{\theta \alpha_4} \right]^{1/\sigma}$ for any $m > 0$.

Assuming (again the induction step) that $y'_{t-1}(\omega) \leq y_{t-1}(\omega)$ for all ω , we obtain also that $b'_{t-1}(\omega) \leq b_{t-1}(\omega)$ for all ω . From (3.5) and (3.6) we conclude that $r'_t < r_t$, hence by (2.12) we derive that $y'_t(\omega) < y_t(\omega)$ for all ω . This implies, using (3.1), that $b'_t(\omega) < b_t(\omega)$ for all ω . As before, this result demonstrates that the growth is slower when the economy is more bequest-biased altruistic.

4. Intergenerational Transfers and Income Distribution

Before presenting the effect of altruism on intragenerational distribution of incomes in equilibrium let us define here income inequality measurement. Let X and W be two random variables with values in a bounded interval in $\Re$ and let m_x and m_w denote their respective means. Define $\widehat{X} = X/m_x$ and $\widehat{W} = W/m_w$. Denote by F_x and F_w the cumulative distribution functions of $\widehat{X}$ and $\widehat{W}$, respectively. Let $[a, b]$ be the smallest interval containing the supports of $\widehat{X}$ and $\widehat{W}$.

Definition: F_x is more equal than F_w if, for all $t \in [a, b]$, $\int_a^t [F_x(s) - F_w(s)] ds \leq 0$.

This definition, due to Atkinson (1970), is equivalent to the requirement that the Lorenz curve corresponding to X is everywhere above that of W . Thus, if F_x is more equal than F_w , then it has a lower Gini index. We say that X is *more equal* than W if the c.d.f. of $\widehat{X}$ and $\widehat{W}$ satisfy: F_x is more equal than F_w . Henceforth the relation X is more equal than W is denoted $X \gg W$. We say that X is equivalent to W , and denote this relation by

$X \approx W$, if $X \gg W$ and $W \gg X$.

We present now the effect of higher EI-altruism on the income distribution in each generation. Comparing two economies which differ only in the intensity of the EI-altruism, we find that the impact of transfers via education on the equilibrium income distribution depends heavily on the aggregate production function of this economy. Let us state:

Theorem 4.1. (a) *If $\sigma > 1$ then higher EI-altruism will result in a more equal intragenerational income distribution in each generation.*

(b) *If $\sigma = 1$ then higher EI-altruism has no effect on the intragenerational income distribution.*

(c) *If $\sigma < 1$ then higher EI-altruism will result in a less equal intragenerational income distribution.*

Note that for the case where $\sigma > 1$ we obtain that higher economic growth is associated with more equality in income distribution. Such empirical findings were reported by Persson and Tabellini (1994) and Park (1996).

Proof. Denote by $y'_t(\omega)$ the incomes when α_4 increases to $\alpha'_4 = (1 + m)\alpha_4$. We have shown in the proof of Theorem 3.1. that $y'_0(\omega) \approx y_0(\omega)$ and hence $b'_0(\omega) \approx b_0(\omega)$. Define:

$$X_t = \frac{\alpha_4}{\alpha_3} \frac{w_t}{1 + r_t} = \frac{\alpha_4}{\alpha_3} \frac{F_L}{F_K} = \frac{\alpha_4}{\alpha_3} \hat{\mu} \left(\frac{K}{L} \right)^{1/\sigma} \quad (4.1)$$

By equations (2.13) and (2.14) we obtain that:

$$X_t = \hat{\mu} \frac{\alpha_4}{\alpha_3} \left(\frac{\alpha_2 + \alpha_3}{\bar{\theta} \alpha_4} \right)^{1/\sigma} = \hat{\mu} \alpha_4^{1-1/\sigma} \frac{(\alpha_2 + \alpha_3)^{1/\sigma}}{\alpha_3} \quad (4.2)$$

We conclude from (4.2) that:

(i) If $\sigma > 1$, then X_t increases as α_4 increases.

(ii) If $\sigma < 1$ then X_t decreases as α_4 increases.

(iii) If $\sigma = 1$ then X_t remains unchanged as α_4 increases.

To proceed we need the following Lemma.

Lemma 1: Let X and Y be random variables taking values in $(-\infty, \infty)$. Let Z be a positive random variable with compact support and assume that Z is independent of X and Y , then:

- (a) $X \approx Y$ implies that $XZ \approx YZ$
- (b) $X \gg Y$ implies that $XZ \gg YZ$.

The proof of this lemma follows the same lines as in the proof of Lemma 1 in Karni and Zilcha (1995), hence it is omitted.

Rewriting equation (2.12):

$$y_t(\omega) = (1 + r_t)b_{t-1}(\omega) [1 + X_t\theta_t(\omega)]. \quad (4.3)$$

Clearly, when $\sigma = 1$, X_t is unchanged hence, the distribution of $y_1(\omega)$ and $y'_1(\omega)$ satisfy $y'_1 \approx y_1$. This implies that $b'_1 \approx b_1$ and using (4.3) this process can be continued to show that $y'_t(\omega) \approx y_t(\omega)$.

It follows from Lemma 1 in Karni and Zilcha (1995) that if $X'_t > X_t$ then:

$$1 + X_t\theta_t(\omega) \gg 1 + X'_t\theta_t(\omega). \quad (4.4)$$

Since $\theta_t(\omega)$ are i.i.d. across individuals, using the Lemma stated in the proof of Theorem 4.1, we reach from (4.4) the following conclusions: Assuming that $\sigma > 1$, since $b'_0(\omega) \approx b_0(\omega)$ while $X'_1 > X_1$ (X'_1 corresponds to a higher α_4) then $y_1 \gg y'_1$. But for each t $b_t(\omega) = \frac{\alpha_3}{\alpha^*} y_t(\omega)$ while $b'_t(\omega) = \frac{\alpha_3}{\alpha^* + m\alpha_4} y'_t(\omega)$ then $b_1 \gg b'_1$. Using (4.3) for $t = 2$, in a similar way we derive that $y_2 \gg y'_2$ and so on.

Assume now that $\sigma < 1$; from $b'_0 \approx b_0$ while $X'_1 < X_1$ we reach $y'_1 \gg y_1$. Thus,

similarly, $b'_1 \gg b_1$. But $X'_t < X_t$ for any t , thus (4.4) is "reversed" in this case. Consequently $b'_{t-1}(\omega) \gg b_{t-1}(\omega)$ and $X'_t < X_t$ imply that $y'_t \gg y_t$ for all t .

Now let us show again that higher Bequest-inclined altruism implies different results than the case with higher EI-altruism.

Theorem 4.2. (a) If $0 < \sigma < \frac{\alpha_3}{\alpha_2 + \alpha_3}$ then higher Bequest-inclined altruism results in less equality in the distribution of income in each generation.

(b) If $\sigma > \frac{\alpha_3}{\alpha_2 + \alpha_3}$ higher BI-altruism results in more equality in the distribution of income in each generation.

Proof. As before let us define:

$$X_t = \frac{w_t}{1 + r_t} \frac{\alpha_4}{\alpha_3} = \hat{\mu} \frac{\alpha_4}{\alpha_3} \left(\frac{\alpha_2 + \alpha_3}{\theta \alpha_4} \right)^{1/\sigma}. \quad (4.5)$$

It is straightforward to verify that:

$$\frac{\partial X_t}{\partial \alpha_3} > 0 \quad \text{if } \sigma < \frac{\alpha_3}{\alpha_2 + \alpha_3}. \quad (4.6)$$

$$\frac{\partial X_t}{\partial \alpha_3} < 0 \quad \text{if } \sigma > \frac{\alpha_3}{\alpha_2 + \alpha_3}. \quad (4.7)$$

Using equation (4.3) together with (4.6) and $b'_{-1}(\omega) = b_{-1}(\omega)$ for all ω we conclude that (again we apply Lemma 1 in Karni and Zilcha (1995)) $y_0(\omega) \gg y'_0(\omega)$ and hence $b_0(\omega) \gg b'_0(\omega)$. Applying once again the same arguments we obtain that $y_1(\omega) \gg y'_1(\omega)$ and hence $b_1 \gg b'_1$. This process can be continued to derive $y_t \gg y'_t$ for all t . The proof of part (b) uses the same technique; however, since $X'_t < X_t$ we obtain now $y'_t \gg y_t$ for $t = 0, 1, 2, \dots$.

5. Public Provision of Education

Let us consider now the case where education is provided by the government with equal opportunities to all. To simplify the analysis we shall assume "no discrimination," i.e., the level of investment in education per person is the same, regardless of the talent of that person. We also assume that the education costs are covered by tax on wage-income of the working individuals. Moreover, parents do not complement the investment in education provided by the government. We assume that there is no tax on interest income or bequest transfers. Denote by τ the tax rate on wage incomes, and by $\bar{e}_t$ the investment in education of each young person (by the government) in period t . Then the maximization problem of individual ω in G_t (choosing s_t and b_t only) is the following:

$$\text{Max } c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} [(b_t(\omega)(1 + r_{t+1}))^{\alpha_3} [h_{t+1}(\omega)]^{\alpha_4} \quad (5.1)$$

subject to the constraints:

$$c_{1t}(\omega) = (1 + r_t)b_{t-1}(\omega) + (1 - \tau)w_t h_t(\omega) - s_t(\omega) - b_t(\omega) \geq 0. \quad (5.2)$$

$$c_{2t}(\omega) = (1 + r_{t+1})s_t(\omega) \quad (5.3)$$

$$h_{t+1}(\omega) = \bar{e}_t \tilde{\theta}_{t+1}(\omega) \quad (5.4)$$

The equilibrium conditions will now include the condition that revenues from taxing wage incomes cover the total cost of education, i.e.,

$$\tau \int w_t h_t(\omega) d\mu(\omega) = \int \bar{e}_t d\mu(\omega)$$

or

$$\tau w_t L_t = \bar{e}_t \quad \text{for } t = 0, 1, 2, \dots \quad (5.5)$$

On the other hand the aggregate labor supply can be expressed in this case as follows:

$$L_t = \int h_t(\omega) d\mu(\omega) = \int \bar{e}_{t-1} \tilde{\theta}_t(\omega) d\mu(\omega) = \bar{e}_{t-1} \bar{\theta}. \quad (5.6)$$

From (5.5) and (5.6) we conclude that

$$\bar{e}_t / \bar{e}_{t-1} = \tau \bar{\theta} w_t. \quad (5.7)$$

The income in this case can be written as:

$$\begin{aligned} y_t(\omega) &= (1 + r_t) b_{t-1}(\omega) + (1 - \tau) w_t h_t = \\ &= (1 + r_t) \left[b_{t-1}(\omega) + \frac{w_t}{1 + r_t} (1 - \tau) \bar{e}_{t-1} \tilde{\theta}_t(\omega) \right] \end{aligned} \quad (5.8)$$

Let us examine now the effects of altruism on economic growth and intragenerational income distribution when education is provided equally by the government and financed by taxes on wage incomes.

Theorem 5.1. *Assume that education is provided by the government and financed by tax on wage income. In this case the level of education-inclined altruism has no effect on the equilibrium rate of growth and the intragenerational income distribution.*

Proof. From the first-order conditions in this case for problems (5.1)-(5.4) we obtain that:

$$b_t(\omega) = \frac{\alpha_3}{\alpha_1} c_{1t}(\omega) \text{ and } s_t(\omega) = \frac{\alpha_2}{\alpha_1} c_{1t}(\omega).$$

Thus, from equations (5.2) and (5.8) we obtain:

$$b_t(\omega) = \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} y_t(\omega) \quad (5.9)$$

$$s_t(\omega) = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3} y_t(\omega) \quad (5.10)$$

Let $\alpha'_4 = (1 + m)\alpha_4$ while $\alpha_1, \alpha_2, \alpha_3$ remain unchanged. Given $K_0, b_{-1}(\omega)$ and $h_0(\omega)$ it is clear that $y_0(\omega)$ is not affected by a change in α_4 ; Thus, K_1 and L_1 are not affected by the change in α_4 . Now, since w_1 and r_1 do not vary by (5.8) $y_1(\omega)$ does not change. Hence this process can be continued to all $y_t(\omega)$, K_t , L_t and so on.

Now let us study the effect of Bequest-inclined altruism in this framework. Let us make the following assumption regarding the elasticity (with respect to capital) of F_K and the elasticity (with respect to labor) of F_L :

$$(A.1) \quad -\frac{KF_{KK}}{F_K} \leq 1 \text{ and } -\frac{LF_{LL}}{F_L} \leq 1.$$

Note that for a Cobb-Douglas production function both elasticities are constant and strictly less than 1.

Theorem 5.2. *Assume a public provision of education and assume that (A.1) holds. Higher level of Bequest-inclined altruism results in higher economic growth.*

This result stands in contrast to that obtained in Theorem 3.2; namely, when the education investment is exogenous and equal to all individuals higher bequest-inclined altru-

ism results in higher capital stock. Since aggregate effective labor supply, given the initial distribution of human capital, evolves according to the level of public provision of education (determined by the government in this case) we obtain higher aggregate output, due to higher investment in education in the more altruistic economy.

Proof. Let $\alpha'_3 = (1+m)\alpha_3$ for $m > 0$. Denote the equilibrium corresponding to α'_3 with " $\prime$ ". Since $y'_0(\omega) = y_0(\omega)$ by (5.9) and (5.10), we can obtain that $b'_0(\omega) > b_0(\omega)$ and $b'_0(\omega) + s'_0(\omega) > b_0(\omega) + s_0(\omega)$ for all ω . Thus $K'_1 > K_1$. Since $w'_0 = w_0$ $\bar{e}_0$ remains unchanged, and hence $L'_1 = L_1$, which implies that $K'_1/L'_1 > K_1/L_1$ and $w'_1 > w_1$.

From (5.9) and (5.10) we obtain that

$$K_{t+1} = \frac{\alpha_2 + \alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \int y_t(\omega) \quad (5.11)$$

Moreover, by (A.1) we also see that $(1+r_t) \int b_{t-1}(\omega)$, which behaves like $(1+r_t)K_t$, increases as K_t increases if L_t does not decline. By the assumption that $-\frac{LF_{LL}}{F_L} \leq 1$ we derive that $w_t L_t$ increases in equilibrium if L_t increases (given that K_t does not decline)..

Since $\int y'_1(\omega) > \int y_1(\omega)$ by (5.11) we obtain that $K'_2 > K_2$ and $(1+r'_2) \int b'_1(\omega) > (1+r_2) \int b_1(\omega)$ if $L'_2 \geq L_2$. But if $L'_2 < L_2$ then $w'_2 > w_2$ and hence, using (5.6) and (5.5), $\bar{e}'_1 < \bar{e}_1$ which implies that $w'_1 L'_1 < w_1 L_1$ which is a contradiction. Thus $L'_2 \geq L_2$. Since $K'_2 > K_2$ by (A1) we obtain that $w'_2 L'_2 > w_2 L_2$, which implies that $\bar{e}'_2 > \bar{e}_2$ (and therefore $L'_3 > L_3$ etc.). In particular, $F(K'_2, L'_2) > F(K_2, L_2)$. Also,

$$y_t(\omega) = (1+r_t)b_{t-1}(\omega) + w_t L_t (\tilde{\theta}_t(\omega)/\bar{\theta})(1-\tau) \quad (5.12)$$

Now, $K'_2 > K_2$ implies that $(1+r'_2)K'_2 > (1+r_2)K_2$ since $L'_2 \geq L_2$. Using (5.12) we can show since $(1+r'_2) \int b'_1 > (1+r_2) \int b_1$ and $\frac{w'_2}{1+r'_2} > \frac{w_2}{1+r_2}$, that $\int y'_2 > \int y_2$. Thus, by (5.11) $K'_3 > K_3$, and also $F(K'_3, L'_3) > F(K_t, L_t)$ for all t . This process can be continued for

all t .

Let us now show that for $\sigma \geq 1$ when education is provided by the government the result of Theorem 4.3 (b) reverses; namely, in this case higher bequest-inclined altruism (hence higher bequest transfers) results in less equal income distribution in all generations.

Theorem 5.3. : Assume that education is publicly provided, (A.1) holds and that $\sigma \geq 1$. Then higher bequest-inclined altruism results in less equal intragenerational income distributions.

Before proceeding with the proof of this theorem let us state the following two Lemmas which will be used in the proof.

Lemma 2: Consider two independent random variables $\tilde{X}$ and $\tilde{Y}$ with values in $[a, b]$, $0 < a < b < \infty$. Let $\bar{X} = E\tilde{X}$, $\bar{Y} = E\tilde{Y}$ and define for $\lambda > 0$, $Z(\lambda) = \frac{\tilde{X} + \lambda\tilde{Y}}{\bar{X} + \lambda\bar{Y}}$. Then, $\lambda > \hat{\lambda}$ implies that $Z(\hat{\lambda}) > Z(\lambda)$.

Lemma 3: Let the nonnegative random variables $\tilde{X}, \tilde{Y}, \hat{X}, \hat{Y}$ satisfy:

- (a) $\tilde{X}, \tilde{Y}$ are independent and $\hat{X}, \hat{Y}$ are independent.
 - (b) $\tilde{X}$ dominates (in the second degree) $\hat{X}$, i.e., $\tilde{X} >_2 \hat{X}$. Also $\tilde{Y} >_2 \hat{Y}$.
- Then $Z = \tilde{X} + \tilde{Y} >_2 \hat{Z} = \hat{X} + \hat{Y}$.

The proofs of Lemmas 2 and 3 are relegated to the Appendix.

Proof. Let us rewrite equation (5.12),

$$y_t(\omega) = (1 + r_t)[b_{t-1}(\omega) + \frac{w_t}{1 + r_t} L_t(\tilde{\theta}_t(\omega)/\bar{\theta})(1 - \tau)] \quad (5.13)$$

However, using the CES production functions we can substitute $\frac{w_t}{1 + r_t}$ by $\hat{\mu}(\frac{K_t}{L_t})^{1/\sigma}$, where $\hat{\mu}$ is a constant. Thus (5.13) can be rewritten as,

$$y_t(\omega) = (1 + r_t)[b_{t-1}(\omega) + \hat{\mu} K_t^{1/\sigma} L_t^{1-1/\sigma} (1 - \tau) \tilde{\theta}_t(\omega)/\bar{\theta}] \quad (5.14)$$

Let us use some facts from the proof of Theorem 5.2. In the more bequest-inclined altruism (the equilibrium is denoted by " ' ") we have: $(K'_t)^{1/\sigma} (L'_t)^{1-1/\sigma} > K_t^{1/\sigma} L_t^{1-1/\sigma}$ for $t = 1, 2, 3, \dots$ while $\tilde{\theta}_t(\omega)$ are i.i.d. in ω for each t . Since $y'_0(\omega) = y_0(\omega)$ we have $b'_0(\omega) \approx b_0(\omega)$.

Let us define

$$\lambda_t = \hat{\mu}(1 - \tau) K_t^{1/\sigma} L_t^{1-1/\sigma} / \bar{\theta} \quad t = 0, 1, 2, \dots \quad (5.15)$$

and λ'_t for the case where K'_t, L'_t replace K_t, L_t in (5.15). Using Lemma 2 and $b'_0 \approx b_0$ $\lambda'_1 > \lambda_1$ we conclude that $y_1(\omega) \gg y'_1(\omega)$. Therefore we obtain that $b_1(\omega) \gg b'_1(\omega)$. Since, by the proof of Theorem 5.2, $\lambda'_t > \lambda_t$ for all t we proceed with the same argument using (5.14) to attain $y_t(\omega) \gg y'_t(\omega)$ for $t \geq 2$.

Remark 1. We have assumed so far that the government taxes wage incomes (only) to finance the cost of education. It is possible to generalize the results of Theorem 5.2 and Theorem 5.3 to the case where the same tax applies to bequest transfers as well. In this case the income $y_t(\omega)$ is given by:

$$y_t(\omega) = (1 - \tau) [(1 + r_t)b_{t-1}(\omega) + w_t h_t(\omega)]$$

while condition (5.5) should be replaced by:

$$\tau [(1 + r_t)B_{t-1} + w_t L_t] = \bar{e}_t \quad t = 0, 1, \dots$$

We shall not prove this claim since the proof follows the same lines.

6. Concluding Remarks

The attempts to explain the persistent differences in growth rates between countries, using new models of growth, were concentrated on the technological aspects (such as increasing returns-to-scale, Romer (1986)) and the technological progress aspects due to human capital accumulation (see, for example Lucas (1988)). We claim here that altruism within families, which is generated by the values and cultural background in each society, plays an important role in shaping the composition and size of intergenerational transfers. To put it differently, the interactions between generations, which are affected greatly by altruistic considerations, have significant impact on economic growth and income distribution. However, as was shown in this work, it is important to distinguish between the intensities of the various ways in which these transfers are executed: Whether the emphasis is on the human capital formation of the children or, perhaps, other direct forms of transferring wealth from parents to children. Whether parents transfer mostly property or capital to their offspring or the transfers are predominantly education and knowledge have different economic implications. Such comparison between countries (or societies) may explain part of the persistent differences in growth rates or income distributions.

Surprisingly, in our model, when education is provided freely by the government (financed by taxes on wages) the impact of altruism (the relevant one to this case) is reversed compared to the private provision case. The resulting redistribution of income *within* each generation in this case, namely, transfers from the rich to the poor, plays an important role. However, it seems to us that the "mixed case", i.e., the case where a certain level of education is provided by the government while parents can increase the investment in the education of their offspring (see Eckstein and Zilcha (1995) and Glomm and Ravikumar (1995)) should be examined as well. In such a case some of the results obtained in section 5 may change. However, results in this case will depend on the initial distributions of human capital as well as wealth holdings at $t = 0$

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APPENDIX

Proof of Lemma 2: Differentiating $Z(\lambda)$ with respect to λ we obtain:

$$Z'(\lambda) = \left(\frac{\bar{X}}{(\bar{X} + \lambda\bar{Y})^2} \right) \left[\tilde{Y}(\omega) - \frac{\bar{Y}}{\bar{X}} \tilde{X}(\omega) \right].$$

Thus, for any $\alpha = \left(\frac{\bar{Y}}{\bar{X}} \right) X(\omega)$ we see that $Z' < 0$ if $Y(\omega) < \alpha$ while $Z' > 0$ if $Y(\omega) > \alpha$. Moreover $|Z'(\lambda)| < |Z'(\hat{\lambda})|$ whenever $Y(\omega)/\bar{X} \neq \alpha$. Thus, for any $s \geq 0$

$$\text{Pr ob} \left\{ Z(\hat{\lambda}) \leq s \mid \tilde{X} = \beta \right\} \leq \text{Pr ob} \left\{ Z(\lambda) \leq s \mid \tilde{X} = \beta \right\}.$$

This inequality holds for any value of $\tilde{X}$. This implies that for any $s \geq 0$,

$$\text{Pr ob} \left\{ Z(\hat{\lambda}) \leq s \right\} \leq \text{Pr ob} \left\{ Z(\lambda) \leq s \right\}.$$

Namely, $Z(\hat{\lambda})$ dominates (SDSD) $Z(\lambda)$ which proves the claim (see Atkinson (1970)).

Proof of Lemma 3: Denote the c.d.f. for $\tilde{X}$, $\tilde{Y}$ etc. by $F_x(\cdot)$, $F_y(\cdot)$ and so on. Then:

$$\begin{aligned} F_z(s) &= \text{Pr ob} \left\{ \tilde{X} + \tilde{Y} \leq s \right\} = \\ &\text{Pr ob} \left\{ \tilde{X} \leq \xi \text{ and } \tilde{Y} \leq s - \xi \text{ for } 0 \leq \xi \leq s \right\} = \\ &\int_0^s F_x(\xi) F_y(s - \xi) d\xi. \end{aligned}$$

Therefore, for $\tau \geq 0$,

$$\begin{aligned}
 \int_0^\tau F_z(s)ds &= \int_0^\tau \int_0^s F_x(\theta)F_y(s-\theta)d\theta ds = \\
 &= \int_0^\tau \left[\int_\theta^\tau F_x(\theta)F_y(s-\theta)ds \right] d\theta = \\
 &= \int_0^\tau F_x(\theta) \left[\int_0^{\tau-\theta} F_y(s')ds' \right] d\theta \leq \\
 &= \int_0^\tau F_x(\theta) \left[\int_0^{\tau-\theta} F_{\hat{y}}(s')ds' \right] d\theta = \int_0^\tau F_x(\theta)\hat{G}(\tau-\theta)d\theta
 \end{aligned}$$

where $H(r) = \int_0^r F_x(\xi)d\xi$, $G(r) = \int_0^r F_y(\xi)d\xi$ and similarly $\hat{H}(r)$ and $\hat{G}(r)$ are defined. Then, using integration by parts we obtain:

$$\begin{aligned}
 \int_0^\tau F_z(s)ds &\leq H(\theta)\hat{G}(\tau-\theta) \Big|_0^\tau + \int_0^\tau H(\theta)F_{\hat{y}}(\tau-\theta)d\theta \\
 &\leq \int_0^\tau \hat{H}(\theta)F_{\hat{y}}(\tau-\theta)d\theta \\
 &= \int_0^\tau \int_0^\theta F_{\hat{x}}(\xi)F_{\hat{y}}(\tau-\xi)d\xi d\theta = \int_0^\tau F_{\hat{z}}(\xi)d\xi
 \end{aligned}$$

which proves that $\tilde{Z} >_2 \hat{Z}$.

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