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# THE VALUE OF INFORMATION: THE CASE OF SIGNAL-DEPENDENT OPPORTUNITY SETS 

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#### Abstract

We generalize the economic decision problem considered by Blackwell (1953) in which a decision maker chooses an action after observing a signal correlated to the state of nature. Unlike Blackwell's case where the feasible set is fixed, in our framework, the feasible set of actions depends on the signal and the information system. As we indicate such a framework has more significance to economic models. We show that in this case, contrary to Blackwell's well-known result, more information may be disadvantageous. We derive conditions for this general model which guarantee that more information is beneficial.


## 1 Introduction

The significance of information in various models in economics is already well-established. In the last two decades we witnessed an abundant literature demonstrating the crucial role that information plays in the decision making process of individual agents facing uncertainty and upon the existence of markets and their operation.

An important issue studied in various frameworks is the value of information. Unlike the existing statistical decision theory case, in most economic models additional publicly dissemenated information may change the economic environment in which agents are operating, hence in some cases more information may be disadvantageous. Such results have been obtained by Hirshleifer (1971, 1975), Radner and Stiglitz (1974), Wilson (1978), Green (1981), Wakker (1988) and others. Hirshleifer (1971) showed that more information can be detrimental (to all agents) in a contingent contracts exchange economy. Green (1981) studies the effect, in equilibrium, of an improvement in the information structure in various systems of incomplete markets. Green (1981) showed in a model with futures markets that an improvement in information does not necessarily imply more desirable change from the economic standpoint.

Our main goal is to study the case where the opportunity sets (or the sets of "feasible actions") are signal-dependent, that is, it varies as information is obtained. We derive conditions under which, in such a framework, more information is valuable. Obviously, this model includes cases where economic agents in a market observe a signal which is correlated to the true economic situation (e.g., economic indicators regarding the state of the economy) and, consequently, after they update their beliefs, prices vary.

In his seminal works, comparing statistical experiments, Blackwell (1953) introduced several equivalent definitions of the important notion that one information system is "more informative" than another. These criteria, which were characterized by Blackwell, turned out to
be important for information economics, and were introduced to economic theory by Marschak and Miyasawa (1968), Marschak and Radner (1972) and others. However, although this issue of information valuation was a major one in statistical decision theory, surprisingly, its application to economic theory was very restricted. Some examples where these criteria were applied in economics are: Grossman, Kihlstrom and Mirman (1978) in a learning-by-doing dynamic model; Jones and Ostroy (1984) have related "more informative" to "more flexible"; Grossman and Hart (1980) applied this notion to a Principal-Agent problem.

We shall consider here a generalization of the problem considered by Blackwell (1953). In the case studied by Blackwell, and many others, the decision-maker observes a signal correlated to the state of nature, and afterwards chooses an optimal action following the updating of his/her probability distribution. The set of feasible actions does not change after the signal is revealed. In our model, the set of feasible actions which the decision-maker faces, may depend upon the signal that has been revealed and possibly the information system as well. For example, consider a manager who receives a signal about his firm's prospects, then his opportunities to trade are restricted as insider trading is prohibited in many countries (e.g., Britain, Germany and the U.S.). A situation where more information increases the opportunity set can be found in the agreement between the manager and shareholders of a firm where conflicting interests exist. Signals indicating improvement in the financial situation of a firm can remove some restrictions imposed on the manager by the shareholders and thus the set of actions that the manager can take may expand.

Note that in all the above-mentioned literature there is no explicit consideration of such a generalization of Blackwell's model, namely, the explicit dependence of the opportunity sets on the signal and information system. Our model can be applied to a single person decision problem as well as to a multi-person decision problem. In our view, one of the main reasons for the narrow applicability in Blackwell's results lies in the fact that in many economic circumstances, once the
signal is observed, the opportunity sets faced by the agent change. More specifically, following the observation of a signal, agents update their probability distributions and the "oppportunity set" may vary. For example, due to variations in the probability distributions (due to the signal observed), prices may change, and hence the budget sets may differ. Therefore, Blackwell's result that "more information" will be preferred by all decision-makers may not hold for the signal-dependent opportunity sets case. In fact this is the case in Hirschleifer's (1971) contingent contracts examples and Green's (1981) futures markets examples. Also, Schlee (1994) considers the value of information on product quality where "more information" varies the prices which, in turn, affects the consumers' budget sets.

In section 3 we first demonstrate the following surprising result: Consider any two information systems $P, Q, \quad P$ being "more informative" than $Q$ and for each signal $y_{k}$ the feasible set of actions corresponding to $P$ and $y_{k}$, denoted $B\left(P, y_{k}\right)$ strictly contains the feasible set of actions corresponding to $Q$ and $y_{k}$, denoted $B\left(Q, y_{k}\right)$, i.e., $B\left(Q, y_{k}\right) \subset B\left(P, y_{k}\right)$ for all signals $y_{k}$. Then, under some mild conditions there are decision makers who prefer $Q$ to $P$. We also show, as opposed to Blackwell's model, that in our extended model the value function of information is generally non-convex in the information system (note that convexity implies preference for early resolution of uncertainty). However, we derive some conditions which guarantee convexity of the value function in this framework. In section 4 we bring the main result of this paper: Sufficient conditions regarding the signal-dependent opportunity sets that guarantee that "more information" is always desirable. Such a result has not been attained yet in the literature. In fact, Green (1981) shows that in his futures markets example, introducing markets for options will "almost" (but not certainly) guarantee such preference for more informative systems.

## 2 The Blackwell Result

Consider decision makers under uncertainty. Let $S=\left\{s_{1}, \ldots, s_{n}\right\}$ be the set of states of nature, and $\pi=\left(\pi_{1}, \ldots, \pi_{n}\right)$ be a prior probability distribution over S . We assume that each decision maker is an expected utility maximizer where her von-Neumann Morgenstern utility function is state-dependent $U=\left\{u\left(., s_{i}\right)\right\}$ each defined over the set of feasible actions $B \subseteq R^{k}$. Before taking an action the decision maker observes a signal $y$ which is correlated to the state of nature. Denote by $Y=\left\{y_{1}, \ldots, y_{m}\right\}$ the set of possible signals. We take $m=n$ for simplicity.

An information system $P$ is an $n \times n$ row stochastic matrix specifying for each state of nature a probability vector over the set of signals. In our model a decision maker does not observe the true state of nature but rather observes signals which are generated by those states. Upon receiving a signal the a priori probability vector is updated, using Bayes rule, and then actions are chosen in a way that maximizes expected utility.

Let $P$ and $Q$ be two information systems. We say that $P$ is more informative than $Q$, denoted by $P \succeq Q$, if there exists an $n \mathrm{x} n$ row stochastic matrix R such that $Q=P R$. Multiplying by R adds some noise (randomization) to the information contained in $P$.

Denote by $V(P, \pi, U)$ the value function of information given the utility function U and the probabilities vector $q=\left(q_{1}, \ldots, q_{n}\right)$ where $q_{j}$ is the probability (derived from $\pi$ and $P$ ) that the signal $y_{j}$ would be received. More precisely,

$$
V(P, \pi, U)=\sum_{y_{j} \in Y} q_{j} \max _{a \in B} \sum_{i=1}^{n} \pi_{i}\left(y_{j}\right) u\left(a, s_{i}\right)
$$

where $\pi_{i}\left(y_{j}\right)$ is the posterior, i.e., the updated probability of state $s_{i}$ given the signal $y_{j}$.

Theorem (Blackwell): $P \succeq Q$ if and only if $V(P, \pi, U) \geq V(Q, \pi, U)$ for all $\pi, U$.

Blackwell's theorem states that an information system $P$ is "more informative" than an information system $Q$ if and only if every expected utility decision maker prefers (weakly) using $P$ to using $Q$. A key element of Blackwell's model is the assumption that the set of feasible actions does not vary with the signals. That is, given any signal and any information system, the decision maker faces the same feasible set of actions.

## 3 Signal-Dependent Opportunity Sets

Consider the following extension of the model described above; assume now that the set of feasible actions $B$ would not remain the same regardless which information system is available and which signal was received. Instead, assume that to every signal $y$ and information system $P$ there corresponds a feasible set, to be denoted by $B(P, y)$; the notation $B(P, y)$ emphasizes the dependence on both elements: the information system and the specific signal.Examples in economics where a set of feasible actions may either expand or contract as a result of the revelation of some signal, are widespread. One exampe (single decision-maker) can be found in the manager vs. stockholders relationship (Principal-Agent framework). "Positive signal" (which indicates "good news") may allow the manager of a firm to take certain actions which were not permissive beforehand (e.g., concerning the debt structure or risky ventures), while a "bad signal" will result in more cautious behavior (due to earlier commitment to the stockholders). Another type of example can be generated by the impact that signals may have on prices in the market (hence the budget sets of consumers and firms' behavior). In fact, these are the cases considered, for example, by Hirschleifer (1971), Green (1981) and Schlee (1994). The dependence of the feasible set of actions on the information system can be justified as well.

In many cases experts (such as laywers or doctors), which constitute part of the information system, not only assess probabilities, but also shape the domain of feasible actions.

Within this extended model the value function of information should be adjusted as follows:

$$
\begin{equation*}
V^{*}(P, \pi, U)=\sum_{y_{j}} q_{y_{j}} \max _{a \in B\left(P, y_{j}\right)} \sum_{i=1}^{n} \pi_{i}\left(y_{j}\right) u\left(a, s_{i}\right) \tag{1}
\end{equation*}
$$

In such a generalized framework the question whether Blackwell's theorem still holds seems natural. Put differently, given a model with signal- dependent feasible set, is it always the case that more information is advantageous. It is not difficult to show that for any two information systems with $P \succ Q$ there exist two families of feasible sets each associated with an information system, $\left\{B\left(P, y_{i}\right)\right\}_{i=1}^{n}$ and $\left\{B\left(Q, y_{i}\right)\right\}_{i=1}^{n}$, a utility function $U$ and an a priori probability vector $\pi$, such that: $V^{*}(P, \pi, U)<V^{*}(Q, \pi, U)$.

Surprisingly, even the following result can be shown,

Proposition 1 Let $P$ and $Q$ be two information systems and assume that $P \succ Q$. Let the labeling of the signals be such that the associated families of feasible sets of actions satisfy:

$$
\begin{equation*}
B\left(Q, y_{i}\right) \subset B\left(P, y_{i}\right) \quad \text { for } i=1,2, \ldots, n \tag{2}
\end{equation*}
$$

If $B\left(P, y_{i}\right) \nsubseteq \bigcup_{k \neq i} B\left(P, y_{k}\right)$ for all $i$, then there exists a utility function $U$ and a prior probability vector $\pi$ such that $V^{*}(P, \pi, U)<V^{*}(Q, \pi, U)$.

To get an intuition for the above result, one should bear in mind the fact that in Blackwell's classical model, more information was always (weakly) preferred, since at the worst it could be ignored. Upon introducing a varying feasible set of actions, information can no longer be ignored since not only posterior probabilities are changed but also the feasible sets. Moreover, informativeness of an information system is not determined by the "shape" of each
signal per se, but rather it constitutes the relationship between the different signals. Therefore, $B\left(P, y_{i}\right) \supset B\left(Q, y_{i}\right), \quad i=1, \ldots, n$, is not sufficient to guarantee preference for $P$. An alternative explanation of Proposition 1 rests upon the tradeoff between more accurate signals allowing better decisions versus more accurate signals possibly making desirable actions infeasible.

Proof of Proposition 1: Let $P=\left[P_{i j}\right]$ and $Q=\left[Q_{i j}\right]$ and let $P \succ Q$. Since $P \succ Q$ there exists $(i, j)$ such that $Q_{i j}>P_{i j}$. If $B\left(P, y_{k}\right) \supset B\left(Q, y_{k}\right)$ for $k=1,2, \ldots, n$, we can find some utility $U$ and a priori probability $\pi$ such that $V^{*}(P, \pi, U)<V^{*}(Q, \pi, U)$ under the following condition: the most desirable action (which is not necessarily unique) with respect to $u\left(a, s_{i}\right)$ on $B\left(P, y_{j}\right)$, denoted $a^{*}$ does not belong to $M_{j}=\bigcup_{k \neq j} B\left(P, y_{k}\right)$. This condition is not very restrictive since we can choose $u\left(a, s_{i}\right)$ to attain its extreme value at some $a^{*}, a^{*} \in B\left(P, y_{j}\right)$ and $a^{*} \notin M_{j}$.

By definition:

$$
\begin{aligned}
V^{*}(P, \pi, U) & =\sum_{j=1}^{n} \max _{a \in B\left(P, y_{j}\right)} \sum_{i=1}^{n} P_{i j} \pi_{i} u\left(a, s_{i}\right) \\
V^{*}(Q, \pi, U) & =\sum_{j=1}^{n} \max _{a \in B\left(Q, y_{j}\right)} \sum_{i=1}^{n} Q_{i j} \pi_{i} u\left(a, s_{i}\right) .
\end{aligned}
$$

Let $\pi_{i}>0$.
Define now $U$ as follows:
(i) $\max _{a \in B\left(P, y_{j}\right)} u\left(a, s_{i}\right)=\max _{a \in B\left(Q, y_{j}\right)} u\left(a, s_{i}\right)>0$
(ii) $\mathrm{u}\left(\mathrm{a}, \mathrm{s}_{k}\right)=0$ for all $k \neq i$.
(iii) $\mathrm{u}\left(\mathrm{a}, \mathrm{s}_{i}\right)=0$ for all $a \notin B\left(P, y_{j}\right)$.

Clearly, since $\pi_{i} P_{i j}<\pi_{i} Q_{i j}$ the result follows from properties (i)-(iii).
That is, upon relaxing the assumption that the feasible set of actions is independent of the prevailing information system and which signal is observed, one finds that $P \succ Q$ is
insufficient to guarantee that all decision-makers would prefer using $P$ to $Q$, even if for all signals the feasible set of actions under $P$ strictly contains the feasible set of actions under $Q$. Let us demonstrate this surprising result in an example:

Let $n=2, \quad P=I_{2}=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$ is the full information system and $Q=E_{2}=\left[\begin{array}{cc}\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}\end{array}\right]$ is the null information system.

Let $\pi=\left(\frac{1}{2}, \frac{1}{2}\right)$ and assume the feasible sets of actions are the following:

$$
\begin{gathered}
B\left(I_{2}, y_{1}\right)=\left\{a_{1}, a_{2}\right\} \quad B\left(I_{2}, y_{2}\right)=\left\{a_{3}, a_{4}\right\} \\
B\left(E_{2}, y_{1}\right)=\left\{a_{2}\right\} \quad B\left(E_{2}, y_{2}\right)=\left\{a_{4}\right\} .
\end{gathered}
$$

That is, $I_{2} \succ E_{2}$ and also $B\left(I_{2}, y_{i}\right) \supset B\left(E_{2}, y_{i}\right)$ for $i=1$, 2. Surprisingly, however, there exists, as suggested by Proposition 1, a utility function $U$ such that $V^{*}\left(E_{2}, \pi, U\right)>V^{*}\left(I_{2}, \pi, U\right)$. Indeed, let $U$ be defined as follows:

$$
\begin{aligned}
& u\left(a_{1}, s_{1}\right)=u\left(a_{2}, s_{1}\right)=0=u\left(a_{3}, s_{2}\right)=u\left(a_{4}, s_{2}\right) \\
& u\left(a_{2}, s_{2}\right)>0, u\left(a_{4}, s_{1}\right)>0
\end{aligned}
$$

Then:

$$
\begin{gathered}
V^{*}\left(I_{2}, \pi, U\right)=\frac{1}{2} u\left(a_{1}, s_{1}\right)+\frac{1}{2} u\left(a_{4}, s_{2}\right)=0 \\
V^{*}\left(E_{2}, \pi, U\right)=\frac{1}{2}\left(\frac{1}{2} u\left(a_{2}, s_{1}\right)+\frac{1}{2} u\left(a_{2}, s_{2}\right)\right)+\frac{1}{2}\left(\frac{1}{2} u\left(a_{4}, s_{1}\right)\right. \\
\left.+\frac{1}{2} u\left(a_{4}, s_{2}\right)\right)>0 .
\end{gathered}
$$

This example shows that the condition $B\left(P, y_{i}\right) \cap B\left(P, y_{j}\right)=\phi$ for all $i \neq j$, which guarantees the requirements in the proposition, is not a necessary condition for Proposition 1 since we can take $a_{1} \equiv a_{3}$.

Convexity of the value function $V$ in the information is a desired property exhibited by Blackwell's results. Convexity can be interpreted as implying preference for early resolution of uncertainty: the condition $\alpha V^{*}(P, \pi, U)+(1-\alpha) V^{*}(Q, \pi, U) \geq V^{*}(\alpha P+(1-\alpha) Q, \pi, U)$, implies preference for an early revelation of the prevailing information system $P$ or $Q$. It is therefore interesting to see whether at the root of the above results stands the non-convexity of $V^{*}$ in the information system.

Indeed, let $P, Q$ be two information systems and let $\alpha P+(1-\alpha) Q, \alpha \in[0,1]$ be an information system generated by a convex combination of $P$ and $Q$. If the associated families of feasible sets of actions are arbitrary, then convexity is clearly impaired. However, it turns out that when confining attention to feasible sets of actions that depend only on the signals and do not vary with the information system, one can obtain convexity of $V^{*}(P, \pi, U)$ in $P$. Let us demonstrate that:

Proposition 2 Let the feasible sets of actions be independent of the information system. That is, $B\left(P, y_{i}\right)$ is independent of $P$ for all $P$. Then, $V^{*}(P, \pi, U)$ is convex in $P$.

Proof. Let us show that for all $\alpha \in[0,1]$, all information systems $P, Q$ and for all $U$ and $\pi$, the following inequality holds:

$$
\alpha V^{*}(P, \pi, U)+(1-\alpha) V^{*}(Q, \pi, U) \geq V^{*}(\alpha P+(1-\alpha) Q, \pi, U)
$$

By definition:

$$
V^{*}(\alpha P+(1-\alpha) Q, \pi, U) \equiv \sum_{j=1}^{n} \max _{a \in B\left(y_{j}\right)} \sum_{i=1}^{n}\left(\alpha P_{i j}+(1-\alpha) Q_{i j}\right) \pi_{i} u\left(a, s_{i}\right)
$$

$$
\begin{aligned}
\leq & \alpha \sum_{j=1}^{n} \max _{a \in B\left(y_{j}\right)} \sum_{i=1}^{n} P_{i j} \pi_{i} u\left(a, s_{i}\right) \\
& +(1-\alpha) \sum_{j=1}^{n} \max _{a \in B\left(y_{j}\right)} \sum_{i=1}^{n} Q_{i j} \pi_{i} u\left(a, s_{i}\right) \\
= & \alpha V^{*}(P, \pi, U)+(1-\alpha) V^{*}(Q, \pi, U)
\end{aligned}
$$

which proves the proposition.
Let us now consider the convexity of the value function when $B(p, y)$ depends upon $P$ only.

Proposition 3 Let the feasible sets of actions be independent of the signals, i.e., $B\left(P, y_{i}\right)$ depends upon the information system $P$ only. Assume also that for any $P, Q$.

$$
\begin{equation*}
B(\alpha P+(1-\alpha) Q)=B(P) \bigcap B(Q) \text { for any } 0<\alpha<1 \tag{3}
\end{equation*}
$$

Then $V^{*}(P, \pi, U)$ is convex in $P$.
Remark 1 : (a) Condition (3) reflects the lack of knowledge which information system will be available. In such a case only actions in the intersection should be considered.
(b)Observing (3) we need to consider the case of an empty set of actions (i.e., no "a" can be chosen). We assume that in such a case the value function is $-\infty$..

Proof. Given any two information systems $P$ and $Q$ and let $0<\alpha<1$, then:

$$
\begin{aligned}
V^{*}(\alpha P+(1-\alpha) Q, \pi, U)= & \sum_{j} \max _{a \in B(\alpha P+(1-\alpha) Q)} \sum_{i}\left[\left(\alpha P_{i j}+(1-\alpha) Q_{i j}\right) \pi_{i} u\left(a, s_{i}\right)\right] \leq \\
& \alpha \sum_{j} \max _{a \in B(P) \bigcap_{B(Q)}} \sum_{i} P_{i j} \pi_{i} u\left(a, s_{i}\right)+ \\
& (1-\alpha) \sum_{j} \max _{a \in B(P) \bigcap_{B(Q)}} \sum_{i} Q_{i j} \pi_{i} u\left(a, s_{i}\right) \\
\leq & \alpha V(P, \pi, U)+(1-\alpha) V(Q, \pi, U)
\end{aligned}
$$

which proves the convexity of the value function.
Remark 2: If $B\left(P, y_{i}\right)$ is independent of $y_{i}$ for all $P$ (i.e., $B\left(P, y_{i}\right)=B(P)$ ) then:

$$
\begin{gathered}
P \succ Q \text { and } B(P) \supseteq B(Q) \Rightarrow V^{*}(P, \pi, U)> \\
\\
V^{*}(Q, \pi, U) \text { for all } \pi, U .
\end{gathered}
$$

This follows from the fact that for any $(\pi, U) P$ equipped with $B(Q)$ is more desirable than $Q$ equipped with $B(Q)$ (as in Blackwell (1953)), while $P$ with $B(P)$ has at least the same value as $P$ with $B(Q)$.

## 4 Sufficient Criterion

Since in many economic models with uncertainty and signals, signal-dependent opportunity sets arise naturally it would be useful to find some criterion that guarantees that more information is preferable. The next theorem provides sufficient condition for that.

Theorem: Let $P, Q$ be two information systems, $\left\{B\left(P, y_{i}\right)\right\}_{i=1}^{n}$ and $\left\{B\left(Q, \dot{y}_{i}\right)\right\}_{i=1}^{n}$ are the associated families of feasible sets. Assume that $P \succ Q$, i.e., there exists a stochastic matrix $R$ such that $P R=Q$. If the following condition holds:

$$
\begin{equation*}
\text { For all } k, j: R_{k j}>0 \Longrightarrow B\left(P, y_{k}\right) \supseteq B\left(Q, y_{j}\right) \text {. } \tag{4}
\end{equation*}
$$

Then, for all $U$ and $\pi, V^{*}(P, \pi, U) \geq V^{*}(Q, \pi, U)$.

Proof. Given some $\pi, U$ consider the value function associated with Q:

$$
\begin{gathered}
V^{*}(Q, \pi, U) \equiv \sum_{y_{j}} \max _{a \in B\left(Q, y_{j}\right)} \sum_{i=1}^{n}\left(Q_{i j} \pi_{i}\right) u\left(a, s_{i}\right) \\
\quad=\sum_{y_{j}} \max _{a \in B\left(Q, y_{j}\right)} \sum_{i=1}^{n}\left(\sum_{k=1}^{n} R_{k j} P_{i k}\right) \pi_{i} u\left(a, s_{i}\right)
\end{gathered}
$$

where the second equality is a consequence of $P R=Q$. Changing the order of the summation, one gets:

$$
\begin{aligned}
& V^{*}(Q, \pi, U)=\sum_{y j} \max _{a \in B\left(Q, y_{j}\right)} \sum_{k=1}^{n} R_{k j} \sum_{i=1}^{n} P_{i k} \pi_{i} u\left(a, s_{i}\right) \\
& \quad \leq \sum_{y_{j}}\left(\sum_{k=1}^{n} R_{k j}\left(\max _{a \in B\left(Q, y_{k}\right)} \sum_{i=1}^{n} P_{i k} \pi_{i} u\left(a, s_{i}\right)\right)\right.
\end{aligned}
$$

where the inequality follows from the "sub-linearity" property of the maximum operator.
Now, since condition (3) holds, it follows that:

$$
V^{*}(Q, \pi, U) \leq \sum_{y_{j}}\left(\sum_{k=1}^{n} R_{k j}\left(\max _{a \in B\left(P, y_{k}\right)} \sum_{i=1}^{n} P_{i k} \pi_{i} u\left(a, s_{i}\right)\right)=V^{*}(P, \pi, U)\right.
$$

where the equality is a consequence of $R$ being a stochastic matrix.
Let us interpret Theorem 1 as follows. If $Q$ is less informative than $P$ in the sense of Blackwell's theorem, then we can think of $Q$ being generated by adding noise to $P$. Thus, after
observing a signal $y_{k}$ under $P$, there is another randomization and signal $y_{j}$ is observed with probability $R_{k j}$. Say that $y_{j}$ (under $Q$ ) follows $y_{k}$ (under $P$ ) if $R_{k j}>0$, i.e., $y_{j}$ is sometimes chosen by the noise process following $y_{k}$. Theorem 1 shows that if less actions are available under $Q$ at all signals following $y_{k}$ than under $P$ at $y_{k}$, then $P$ is more valuable than $Q .{ }^{1}$

## 5 Remarks and Consequences

It is possible to define the signal-dependent feasible set as a set-valued function defined on the posterior probability. However, this way of presentation restricts the scope of the model; particularly, in cases where agents may differ in their information systems and beliefs.

Our framework deals with cases where signals are obtained (according to a given random process) without any cost. A case which yields signal-dependent set of feasible actions is the one where the dependence of opportunity sets on signals is generated by some expense incurred in obtaining the available signal. Moreover, it is possible to describe such a model assuming that different available signals are obtained at different costs.

The phenomenon that introducing risk-sharing markets may hurt risk-averse economic agents when signals provide more information, holds under general circumstances and in a wide range of competitive economies. Hirschleifer (1971) was the first to point out this phenomenon. It has been shown by Green (1981) that in the presence of futures markets (when prices are random) more information may be disadvantageous. Sulganik and Zilcha (1994) have shown that in a life-cycle problem with uncertain lifetime and bequest motives, in the presence of life insurance market more information may be disadvantageous to the individual. On the other hand without life insurance market more information is always beneficial.

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[^0]:    ${ }^{1}$ Note that this type of argument worked in Hirschleifer's (1971) result: revealing information destroys the possibility of insuring against bad times.

