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IMPERFECTLY INFORMATIVE EQUILIBRIA FOR SIGNALLING GAMES

by

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ABSTRACT

I analyze the equilibria of signalling games in which initially uninformed players may choose to become informed from an external source at a cost. It is shown that the lower this cost, the greater the extent to which the informed player's actions reveal its private information and the more the outcome resembles the symmetric information equilibrium.

1 Introduction

Since their introduction by Michael Spence (1973, 1974), signalling models have been extensively applied in virtually every field of economic analysis. In its simplest form, the signalling model has the following (extensive form, game theoretic) structure. A privately informed player acts first. The imperfectly informed player chooses its action only after having observed the action of the former.

In a pooling equilibrium, the informed player's action is independent of its type. In a separating equilibrium, the informed player's action depends on, and hence reveals, its private information. For example, in Spence's model of the labor market, employers cannot directly observe workers' ability. In the separating equilibrium, more efficient workers (who have a relatively lower cost of becoming educated) become better educated, to distinguish themselves from their less efficient counterparts. To be persuasive, the amount of education which the efficient type acquires must be costly enough to deter emulation by the less efficient type. The resulting need to 'overinvest' in education - relative to the case in which ability is directly observable - represents the efficient type's 'signalling cost'.

Intuitively, that cost should reflect the uninformed player's initial scepticism. More specifically, it is natural to expect the cost of signalling to be higher, the higher the prior probability that is assigned to low ability. Unfortunately, this is not the case; in separating equilibria, the difference between the efficient type's payoff under symmetric and asymmetric information is independent of the prior probabilities. This fact also implies that, in separating equilibria, even a small informational asymmetry dramatically and discontinuously changes the outcome, relative to the case of perfectly symmetric information.

This goal of this paper is therefore to develop an equilibrium for signalling games which, like separating equilibria, allows for information to be disclosed but in which the importance of asymmetric information is proportional to its severity. The underlying idea is to measure the extent of asymmetric information by the cost to (ex ante) uninformed players of becoming perfectly informed. In particular, it seems natural to expect the effects of asymmetric information to be more pronounced when it is very costly (or impossible) to become better informed than when information is available at a low cost.

For concreteness, this theme is developed in the context of a specific example. A monopoly seller is privately informed about the quality of its product. Buyers are ex ante imperfectly informed about quality. The question is whether in this context, prices may credibly reveal quality. This is an issue which has generated a very large literature, beginning with Akerlof 's (1970) seminal paper. The added ingredient of this paper is that buyers have the opportunity to become better informed from a reliable, independent source, before buying. For example, it is possible to study consumer literature, or have the product tested by an independent expert.

I derive an equilibrium, termed, *imperfectly informative,* (IIE), with the following character. When the cost of information is above a critical value, there is complete pooling: the low and high quality types price identically and no information is disclosed. Below the critical level, information is disclosed in proportion to this cost. Specifically, the high quality seller deterministically charges a "high" price while the low quality seller randomizes between that price and the low quality price - which is perfectly revealing. The frequency with which the low quality seller charges the high price is lower, the smaller the cost of information. Thus the high price is a noisy signal, which becomes less noisy, the lower the cost of information, and becomes perfectly informative as this cost goes to zero. Correspondingly, the high quality seller's profit is greater, the smaller the cost of information and approaches its symmetric information level as information becomes costless. Thus the limit IIE is not the separating equilibrium but the full information outcome.

In section 4, the model is applied to the case in which quality is chosen endogenously. It is shown that quality improvements occur in the IIE but never in conventional equilibria - separating or pooled.

The following section introduces and analyzes the basic signalling model. Section 3 introduces the information option and develops the IIE. Section 4 extends the model to the case of endogenously chosen quality. Section 5 discusses multiplicity of IIE. Section 6 compares the IIE to equilibrium search models.

2 A Simple Signalling Model

A market consists of a profit maximizing monopoly seller and identical buyers. The number of buyers is normalized to one. Each buyer demands a single unit. The quality of the product may be either low or high. The value of a low and high quality unit to buyers is V_L and V_H respectively, $V_H > V_L \geq 0$. The high (low) quality seller is able to supply any quantity at a constant marginal cost of $c_H(c_L)$, $c_H \geq c_L$. I normalize $c_L = 0$. The seller is costlessly and perfectly informed about quality. Buyers are imperfectly informed and assign a prior probability of β, $0 < \beta < 1$, to high quality and the probability $1 - \beta$ to low quality. I shall assume that the likelihood of high quality is at least as high as that of low quality, i.e. $\beta \geq \frac{1}{2}$.[1]

Events unfold as follows. First the seller sets a price. Buyers then observe the price and either buy at that price or do without the product. The seller's objective is to maximize its expected profit and a buyer's objective is to maximize its expected surplus. These actions and objectives define an extensive form game of incomplete information. In

[1] This assumption ensures that the equilibrium of proposition 1 is unique.

a Bayesian perfect equilibrium, the seller and buyers act in a sequentially rational manner and buyers update beliefs using Baye's rule on the equilibrium path.

In the pooling equilibrium for this game, both types of seller charge:

$$\overline{V} = \beta V_H + (1-\beta) V_L.$$

which represents the average value of a unit, calculated on the basis of the buyers' prior [2].

The separating equilibrium for this game is easily derived. Prices are the same as under symmetric information: V_L for the low quality seller and V_H for the high quality seller [3]. Thus prices are perfectly informative. Buyers accept V_L with probability 1 and reject any price between V_L and V_H. They accept V_H with probability V_L/V_H and reject it with the complementary probability.[4] It is easy to verify that given this behavior, each type of seller prices optimally.

What is the effect of asymmetric information on the high quality seller's profit in the separating equilibrium?[5] For its price be unattractive to the low quality seller, the high

[2]There also exist other pooling equilibria in which the price is less than $\overline{V}$. However, these equilibria require that buyers associate higher prices with inferior quality. See footnote 12.

[3]It is obvious that the low quality price is V_L. If the high quality price were less than V_H, it would provide buyers with stricly positive surplus. All buyers would then accept the high quality price with probability one, making it more attractive than V_L to the low quality seller. Thus the high quality price must be V_H.

[4]Buyers believe that a unit priced at less than V_H is low quality and therefore reject any price between V_L and V_H. Because a high quality unit is worth V_H to a buyer, she is indifferent between buying at that price and going without the product. That is why buyers are willing to randomize between accepting the price and doing without the product.

The probability V_L/V_H is chosen to make the low quality seller indifferent between the two prices. This probability is not unique, however. There are also separating equilibria in which the probability of making a sale at the high price is less than V_L/V_H , reducing the high quality seller's profit even further.

[5]Of course, asymmetric information does not affect the low quality seller's profit in the separating equilibrium.

quality seller must "sacrifice" sales to $1-V_L/V_H$ buyers, [6] reducing its profit by $(V_H-c_H)(1-V_L/V_H)$. This difference between its profits under symmetric and asymmetric information constitutes the high quality seller's "signalling cost".

Remark: Because all buyers have an identical reservation price, the high quality seller charges the same price whether information is symmetric or asymmetric. Its diminished sales under asymmetric information results from the buyers' autonomous behavior; Because V_H extracts all the surplus from high quality, buyers are willing to randomize between accepting and rejecting that price.

More generally, suppose all consumers have the same reservation price for low quality, V_L, but differ with respect to the premium (over V_L) that they are willing to pay for high quality. Then the high quality seller faces a downward sloping demand curve, $q(p)$, when buyers are perfectly informed. (The number of consumers in the market is again normalized to 1; thus the demand for both low quality and high quality when the price is V_L is 1). Let p^m be the high quality seller's optimal (monopoly) price under symmetric information and $q(p^m) < 1$ the quantity corresponding to that price. Assume further that $V_L - c_L < (p^m - c_L)q(p^m)$. This implies that p^m cannot be the separating high quality price under asymmetric condition; otherwise it would be chosen by the low quality seller as well (since p^m offers positive surplus to all consumers whose reservation price for high quality exceeds it, those consumers must accept p^m with probability 1). The high quality price is $\tilde{p} > p^m$, (with $q(\tilde{p}) < q(p^m)$), such that $(\tilde{p} - c_L)q(\tilde{p}) \leq (V_L - c_L)$ and $(\tilde{p} - c_H)q(\tilde{p}) \geq V_L - c_H$; see figure 1. That is, the separating price reduces the number of buyers who obtain positive surplus from high quality by enough that the low quality seller prefers the higher sales volume that goes

[6]For the case $c_L = c_H$,the high quality seller is indifferent between the two prices. If $c_L < c_H$, it strictly prefers the high price.

with the low price. So in the case of heterogeneous buyers, the high quality seller *actively* separates itself through its choice of price. This is more in line with the spirit of the signalling parable a la Spence than the case of homogenous buyers discussed in the text. All of the following analysis applies without substantial change to the case of heterogeneous buyers as well. I have only chosen the case of homogeneous consumers to economize on notation. ■

The unintuitive feature of the separating equilibrium is that the signalling cost is unrelated to the the buyers' prior beliefs; The high quality seller's profit is reduced to $(V_H - c_H)V_L/V_H$ whenever $\beta < 1$, regardless of whether β is near zero or one. Moreover, this implies that even a small change in the buyers' prior, from $\beta = 0$ to $\beta > 0$ has a discontinuously large effect on its profit.

It is also instructive to compare the high quality seller's profit under the separating and pooling regimes. It is easy to verify that its profit in the pooling equilibrium is greater than in the separating equilibrium whenever $\beta \geq \frac{c_H}{V_H}$. Thus, paradoxically, the opportunity to successfully separate is a *liability* for the high quality individual, relative to the pooling regime, precisely when the uninformed players are most confident that quality is high.

Another disturbing feature of the separating equilibrium is that the high quality seller's profit is *less* than that of the low quality seller if $c_H > c_L$ (profits are equal only if $c_H = c_L$). Consider a two stage game in which quality is chosen endogenously. At the first stage, the seller (irrevocably) chooses its quality, with high quality requiring a larger investment in, say R&D, than low quality. Buyers are uninformed about which quality was chosen. The second stage is the pricing game described above. A subgame perfect equilibrium for this game specifies the quality the seller chooses and the price its sets. I argue that the only subgame perfect equilibrium for this game is that the seller chooses low quality and charges V_L. Suppose the separating equilibrium is expected to obtain for the (second stage) pricing game. Then the seller cannot recoup its investment in high quality and must choose not to invest, a choice which consumers must anticipate. If the pooling equilibrium obtains for the pricing game, the seller has no reason to invest in high quality either. So regardless of which equilibrium obtains at the pricing stage, buyers must expect quality to be low and the equilibrium price must be V_L. Thus, if quality is chosen endogenously, even a small amount of asymmetric information prevents high quality from being chosen.

The objective of the following section is to extend the basic model in a way which alleviates these unintuitive features.

3 Imperfectly Informative Equilibria

I now expand the game by affording buyers the opportunity to become informed by consulting an external, independent source of information (such as consumer literature or an independent testing agency) at a cost of $s > 0$[7]. Formally, the order of events in the game is modified so that after observing the price, and before buying, a buyer may either become perfectly informed by consulting the external source, buy without becoming informed or leave the market without buying or becoming informed.

It is easy to see that the conventional separating equilibrium described in the previous section continues to hold for the expanded game for any positive s, however small. That is, introducing the information option does not eliminate the conventional separating equilibrium in which this option is irrelevant[8]. However, there also exists an alternative equilibrium,

[7] A related idea in the principal agent literature is analyzed by, for example, Guasch and Weiss (1980, 1982Aa, 1982B), Nalebuff and Scharfstein (1987), Polinsky and Shavell (1979)

[8] If prices perfectly signal quality, buyers will not invest in costly information. The information option

in which the outcome is determined by the cost of information to buyers. In contrast to the conventional separating equilibrium, in which information is perfectly revealed, information revelation in this equilibrium is not perfect. I shall refer to it as an *imperfectly informative equilibrium* (IIE).

The IIE is characterized by two prices. The low price is V_L and the (yet-to-be-determined) high price is $p_H > V_L$. The high quality seller always charges p_H. The low quality seller randomizes between V_L, which is perfectly revealing, and p_H. Buyers who encounter p_H randomize between buying while imperfectly informed and becoming informed before buying.

Let α, $0 \leq \alpha \leq 1$, be the probability with which a buyer who observes p_H becomes informed and γ, $0 \leq \gamma \leq 1$, the probability with which the low quality seller charges V_L. Since the high quality seller always charges p_H, a buyer who observes p_H revises her probability of high quality to ξ:

$$\xi = \beta[\beta + (1-\beta)(1-\gamma)]^{-1}. \tag{1}$$

When $\gamma > 0$, $\xi > \beta$; Observing the high price then increases the likelihood of high quality.

Let:

$$\widehat{p} = \xi V_H + (1-\xi)V_L. \tag{2}$$

be the average quality, calculated on the basis of the buyer's posterior beliefs. A buyer who believes that quality is high with probability ξ is willing to pay up to $\widehat{p}$. Note that $\widehat{p} > \overline{V}$ if $\xi > \beta$ (and $\gamma > 0$) and that $\widehat{p} = \overline{V}$ if $\xi = \beta$ (and $\gamma = 0$).

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may eliminate the pure pooling equilibrium, however, as is discussed below.

The following proposition states that when the cost of information is below a critical value, the low quality seller randomizes between the low and high price and the high quality seller charges the high price only. The high price then serves as a noisy signal; observing that price increases the likelihood of high quality but does not guarantee it.

Proposition 1 : *Corresponding to each $s > 0$ there is a unique IIE in which $p_H = \widehat{p}$*[9]. *Let $s^* = \beta(1-\beta)(V_H - V_L)$. $p_H > \overline{V}$ and $0 < \gamma < 1$ if $s < s^*$, and $p_H = \overline{V}$ and $\gamma = 0$ if $s \geq s^*$.*

Proof. In order to be willing to randomize between the low and high price, the low quality seller's profit from charging V_L must equal its expected profit from charging p_H. When its price is V_L, it sells with probability 1. When its price is p_H, it sells only to uninformed buyers. Thus the low quality seller randomizes only if:

$$V_L = (1-\alpha)p_H. \tag{3}$$

To be willing to randomize between becoming informed and buying without becoming informed, the buyer's expected surplus from becoming informed must equal her expected surplus from buying at the high price while imperfectly uninformed. Since an informed buyer buys only if quality is high, the expected surplus from becoming informed is $-s+\xi(V_H - p_H)$. The expected surplus from buying while imperfectly informed is $\xi(V_H - p_H) + (1-\xi)(V_L - p_H)$. Thus it is required that:

$$-s + \xi(V_H - p_H) = \xi(V_H - p_H) + (1-\xi)(V_L - p_H).$$

[9]However, other IIE, in which $p_H \neq \widehat{p}$, may also exist for the same value of s, as is discussed below.

Rearranging gives the condition:

$$s = (1 - \xi)(p_H - V_L). \tag{4}$$

Let s^* be the cost of information at which this individual's surplus from becoming informed before buying equals her surplus from buying without becoming informed, when the price is $\overline{V}$ and she assigns the prior probability β to high quality. That is, s^* satisfies:

$$-s^* + \beta(V_H - \overline{V}) = \beta(V_H - \overline{V}) + (1 - \beta)(V_L - \overline{V})) \tag{5}$$

Substituting for $\overline{V}$ and rearranging gives:

$$s^* = \beta(1 - \beta)(V_H - V_L). \tag{6}$$

Solve for ξ as a function of s by substituting $\widehat{p}$ in (4):

$$\xi = (2\Delta)^{-1}(\Delta + \sqrt{\Delta^2 - 4s\Delta}\,) \tag{7}$$

where $\Delta = V_H - V_L$. It is required that $\Delta^2 - 4s\Delta \geq 0$, i.e. that $s \leq \Delta/4$. For each $s \leq \Delta/4$, there are two solutions for ξ, corresponding to the positive and negative roots of the RHS of (7). Since $0 < \gamma < 1$, (1) implies that $1 > \xi > \beta$. Consider the positive root. For this root, the RHS of (7) is monotonically decreasing in s and goes from 1 to $\frac{1}{2}$ as s goes from zero to $\Delta/4$.

By (6), $s^* = \beta(1 - \beta)\Delta \leq \Delta/4$. Substituting $s^* = \beta(1 - \beta)\Delta$ in (5) gives:

$$\xi(s^*) = \frac{1 + \sqrt{(1 - 2\beta)^2}}{2}. \tag{8}$$

Since $\beta \geq \dfrac{1}{2}$, the positive root of (8) is $2\beta - 1$, giving $\xi(s^*) = \beta$. Thus corresponding to this root, there is a unique $0 < \xi < 1$ for each $0 < s < s^*$, while for $s > s^*$, $\xi = 1$.

Now consider the negative root. For this root, ξ is monotonically increasing in s and goes from $\frac{1}{2}$ to zero as s goes from zero to $\Delta/4$. So for this root, there does not exist $\xi \geq \beta$ for $\beta > \frac{1}{2}$. Thus corresponding to each $0 < s < s^*$, there is a unique $\xi > \beta$ and $0 < \gamma < 1$ while corresponding to s $\geq$ s*, $\xi = \beta$ and $\gamma = 0$. Solving for $\widehat{p}$ gives:

$$\widehat{p} = V_H - \frac{s}{\xi} \tag{9}$$

which, by the above, defines $\widehat{p}$ uniquely for each s. Finally, for each $s < s^*$, given $\widehat{p}$, α is uniquely defined by (3).

It remains to show that neither type of seller and no buyer can profit by changing its behavior. Let the buyers' out-of-equilibrium beliefs be as follows: If the price is less than $\widehat{p}$, she assigns probability 1 to low quality. If the price is $\geq \widehat{p}$, she assigns probability ξ to high quality. A buyer with these beliefs will reject any price greater than V_L and less than $\widehat{p}$ without becoming informed. If the price is $p > p_H$, the buyer's expected surplus from becoming informed is:

$$-s + (1-\xi)(p - V_L) < -s + \xi(V_H - \widehat{p}) = 0.$$

where the equality in the preceding expression is obtained by substituting for $\widehat{p}$ from (2) and for s from (3). Her surplus from paying p without becoming informed is also negative, by (2). Thus buyers optimally leave the market without becoming informed or buying if the price is $> \widehat{p}$. A low quality seller therefore earns zero profit if its price is between V_L and p_H or greater than p_H and earns identical positive profit from V_L and p_H. Therefore the low quality seller has no incentive to deviate from its equilibrium strategy. Since both informed and uninformed buyers accept $\widehat{p}$ from the high quality seller, and all sellers reject prices greater than $\widehat{p}$, the high quality seller's optimal price is $\widehat{p}$. ■

The next proposition describes how, when $s < s^*$, the cost of information determines the amount of noise in the price signal. The lower s, the less noisy the information conveyed by the price signal. As the cost of information goes to zero, the noise vanishes and the price becomes perfectly informative.

Proposition 2 *: For $s < s^*$, the following obtains:*

(i) $d\xi/ds < 0$. (ii) $d\gamma/ds < 0$. (iii) $dp_H/ds < 0$.
As $s \to 0$, $\xi \to 1$, $\gamma \to 1$ and $p_H \to V_H$. As $s \to s^*$, $\xi \to \beta$, $\gamma \to 0$ and $p_H \to \overline{V}$.

Proof. (i) follows from (7). (ii) then follows immediately from equation (1). (iii) and the first limit statement follow immediately from (8). The second limit statement follows from substituting $s^* = \beta(1-\beta)\Delta$ and $\xi(s^*) = \beta$ in (8). ■

Proposition 2 also links the level of the high price to the quality of the information it conveys. The lower the cost of information, the nearer the high price is to its level under symmetric information. In the limit, as $s \to 0$, the high price becomes perfectly informative and goes to V_H. Similarly, since $\gamma \to 1$ as $s \to 0$, the limit average price of the low quality seller also approaches its level under symmetric information, V_L.

In the other direction, as $s \to s^*, \xi \to \beta$ and $\gamma \to 0$. Therefore the price is completely uninformative and is set at the buyer's ex ante reservation value, $\overline{V}$.[10]

[10] By (2) $\alpha = 1 - V_L/p_H$. Thus, by proposition 2, $d\alpha/ds < 0$ for $s < s^*$. That is, the frequency with which buyers become informed increases as s decreases. Increased monitoring behavior restricts the frequency with which the low quality seller charges the high price, making it more informative. Interestingly, α is bounded away from 1 (i.e., never exceeds $1 - V_L/V_H$) no matter how small s is. This is because prices provide increasingly precise information as s grows smaller. Thus as external information becomes cheaper, the need to acquire it is correspondingly diminished.

The following is an immediate implication of proposition 2:

Corollary: For $s < s^*$, the high quality seller's profit increases as s decreases and approaches its profit under symmetric information as $s \to 0$.

Thus, in the IIE, the high quality seller's loss from asymmetric information depends on the cost of independent information. The smaller that cost, the nearer its profit to its level under symmetric information.

The IIE provides a natural criterion of transition from complete pooling to increasingly informative behavior as a function of s. When $s \geq s^*$, there is perfect pooling. Partial separation occurs in the range $0 < s < s^*$, in which the two seller types price differently. In this range, separation, and hence information, is imperfect. The degree of separation increases as s decreases, as the high price becomes an increasingly reliable indicator of high quality. The high quality seller's profit is higher the greater the degree of separation that is achieved (the more precise is the information conveyed by the price) and is lowest in the pooling region. Thus, in the IIE, the ability to separate is always an asset for the high quality individual.[11]

Proposition 2 describes the comparative statics of the IIE with respect to the cost of information. The IIE is also sensitive to the prior probability, β. $\overline{V}$ is increasing in β and s^* is decreasing in β. Consider $\beta_1 > \beta_0$. Then $s^*(\beta_1) < s^*(\beta_0)$ and $\overline{V}(\beta_1) > \overline{V}(\beta_0)$. For $s < s^*(\beta_1)$, the high price is identical for both priors. For $s^*(\beta_1) < s < s^*(\beta_0)$, however, the high price is less under β_0 than that under β_1, with the difference between the two prices increasing in s. For $s > s^*(\beta_0)$, the difference between the high price under the two priors is

[11] It is interesting to note that the low quality seller's expected profit is V_L for $s < s^*$ (otherwise it would not be willing to randomize) but increases discontinuously to $\overline{V}$ when $s > s^*$. In the separating region of information costs, the high quality seller's ability to separate prevents the low quality seller from benefitting from the buyers' uncertainty.

just the constant difference between the two pooled prices, $\overline{V}(\beta_1) - \overline{V}(\beta_0)$.

We observed above that introducing the information option does not eliminate the standard separating equilibrium in which that option is irrelevant, i.e., the outcome is independent of s and β. However, the information option does eliminate the pure pooling equilibrium, in which each seller's price is $\overline{V}$, irrespective of s. If $s < s^*$, and the price were pooled at $\overline{V}$, it is easy to verify that consumers would optimally become informed, with probability 1, before buying. The low quality seller would then not make any sales and would profitably deviate to V_L. Thus, when the information option is explicitly accounted for, pooling at $\overline{V}$ is possible only if $s \geq s^*$.[12].

4 An Application: Endogenous Quality

It was argued above that when quality is chosen endogenously, asymmetric information about quality choice prevents the provision of high quality under conventional equilibria. It will now be shown that in the IIE, high quality is chosen if it is not too costly for consumers to learn the realized quality. Specifically, consider the following two stage, quality choice game. A seller who does not invest in quality improvement can only produce low quality. The ability to produce high quality requires an initial R&D investment of $F > 0$. Such investment is successful with probability β and unsuccessful with the complementary probability. In the latter event, the seller can subsequently produce only low quality. At the first stage the seller

[12]There are, however, other pooling equilibria which exist for any s. Specifically, it is not hard to see that V_L is the *only* pooled price for *any* $s > 0$. However, for this price to be an eqm, buyers must believe that any higher price comes from a low quality seller. Otherwise a high quality seller, who does not fear being monitored, would increase its price. Beliefs which associate higher prices with low quality seems contrived and impluasible.

decides whether to invest and is privately informed about its success. Consumers learn only whether or not the seller invested, not if investment was successful[13]. At the second stage, the seller offers the product for sale to asymmetrically informed buyers, who can become perfectly informed about realized quality before buying, as described in section 3. Of course, buyers expect quality to be low with probability 1 if the seller has not invested, and low with probability 1-β if it has.

In the standard separating equilibrium at the pricing stage, the low quality seller's operating profit is at least as high as that of the high quality seller and in any pooling equilibrium, profit is independent of quality. Thus, under conventional equilibria for the pricing stage, high quality is not provided under asymmetric information. Consider the IIE for the pricing stage. If $s < s^*$, the difference between the high and low quality seller's operating profit (gross of the investment outlay) is $p_H - c_H - (V_L - c_L)$, which, by proposition 2, is greater the smaller the cost of information and the smaller the production cost differential, $c_H - c_L$. In the subgame perfect equilibrium, the seller invests in high quality if $\beta(p_H - c_H) - F > (V_L - c_L)$. Thus the IIE does allow for investment in quality improvement under asymmetric information if the buyers' cost of becoming informed is low enough.

5 Other IIE

Proposition 1 restricts attention to an IIE in which the high price is $\widehat{p}$. It is easily seen that no IIE in which $p_H > \widehat{p}$ exists. A buyer who observes p_H must assign probability ξ to high quality. Therefore, by (2), she will not pay $p_H > \widehat{p}$ without becoming informed. The low

[13]Actually, the assumption that consumers are informed about the investment decision is unnecessary. Even if the decision to invest is unobservable, buyers can surmise that investment occurs only if its expected return is positive.

quality seller thus earns zero profit when its price is p_H and could increase profit by charging V_L with probability 1.

There may exist IIE in which the high price is less than $\widehat{p}$. In these equilibria, buyers must associate high prices with lower quality. Specifically, suppose that $p_H < \widehat{p}$ and that buyers who observe a price greater than p_H assign a probability of at least ξ to high quality. Then these buyers, when facing a price $p < \widehat{p}$, strictly prefer to buy or become informed over the prospect of leaving the market without buying the product. Since informed buyers accept $p \leq \widehat{p}$ if quality is high, the high quality seller would profit by deviating to $p_H < p < \widehat{p}$. Thus $p_H < \widehat{p}$ only if buyers assign probability less than ξ to high quality when the price is greater than $\widehat{p}$. That is, in such an equilibrium, buyers must associate higher prices with lower quality. This unintuitive feature contrasts with the IIE of proposition 1. There a deviation to a price greater than $\widehat{p}$ is unprofitable even if it does not diminish the buyers belief in high quality, as explained in the proof of proposition 1. I therefore consider the IIE in which the high price is $\widehat{p}$ to be the natural equilibrium on which to focus.

6 Comparison with the Diamond Search Model

We have seen that in the IIE for our signalling model, the outcome varies continuously as a function of the cost of information, approaching the full information outcome as information becomes costless. Intuitively appealing as this notion is, it is not always so. The Diamond (1971) search model provides a dramatic counterpoint. Diamond considers an imperfectly competitive market for a homogeneous product in which it is costly for buyers to become informed about the prices of different sellers. He shows that if buyers face even a small cost of becoming informed about prices, the only equilibrium is that sellers earn monopoly profit, although the competitive (Bertrand) outcome obtains if the cost of search is zero.

Thus, a small informational friction has a dramatic and discontinuous effect on the market equilibrium in the search model, in contrast to the results developed here for signalling games.

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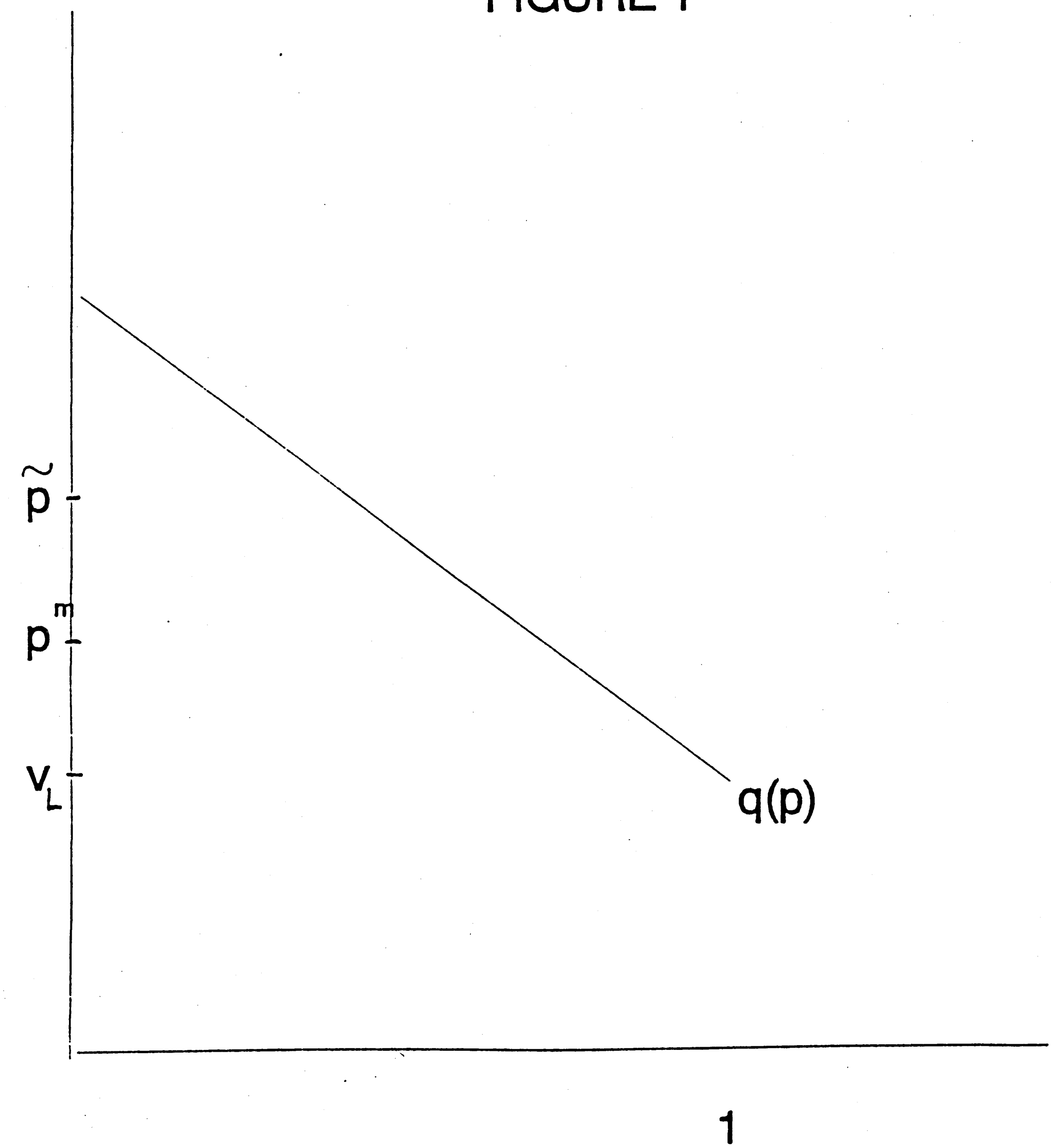
FIGURE 1



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