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Indirect Hedging of Exchange Rate Risk

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Indirect Hedging of Exchange Rate Risk

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Since the inception of floating exchange rates, firms engaged in international operations have been highly interested in developing ways to protect themselves from exchange rate risk. Since price (or exchange rate) uncertainty results in a reduction of production and exports, the main role attached to financial markets which allow firms to reduce price (or currency) risks lies in their impact on production and export level. In recent years we witness a growing body of literature dealing with optimal hedging by firms facing price or exchange rate uncertainty (see, for example, Feder, Just, and Schmitz (1980), Anderson and Danthine (1981), and Kawai and Zilcha (1986)).

Consider a competitive risk–averse firm producing certain commodity in the domestic market and exporting it under exchange rate uncertainty. This firm maximizes profits denominated in domestic currency. We assume that the firm has no access to currency forward (futures) market but can hedge its currency risk through a futures market for an asset correlated to the domestic currency. Thus no direct hedging is possible but rather "indirect hedging" is available.

It has been shown in the literature (see, e.g., Kawai and Zilcha (1986)) that a firm facing random exchange rate can eliminate this risk altogether if it can use an unbiased currency futures market, or another financial asset which is perfectly correlated to the exchange rate. In the absence of such a market the firm can, usually, reduce its risk by engaging in a hedging activity of assets correlated to the exchange rate. Clearly, in such a case the "Full Hedging theorem" does not hold, i.e., the firm cannot eliminate all the risk by using unbiased futures markets for such assets unless there exists one asset which is perfectly correlated to this currency.

Most commodities and most currencies have no futures market (for the case of currency see Buckley (1992), pp. 216–218 for a whole list). In such a case firms look for another hedging vehicle which provides only partial coverage against fluctuations in either the commodity price or the foreign exchange rate. Since we analyze here the case where there is no currency futures market for this exporting firm, we assume that there exist certain assets whose prices are highly correlated with the foreign currency. Moreover, we assume that such assets have markets for futures contracts and the firm has access to such market. For simplicity we shall assume that the firm utilizes one such asset only. In Section IV we discuss the extension of the hedging to many such assets and its impact on our results. Hence, there is evidence that in many cases hedging is accomplished through alternative futures contracts (see, e.g., Powers and Castelino (1991)). This type of hedging is called *indirect* hedging. A special case is the so–called cross hedging (see, e.g., Anderson and Danthine (1981)), in which a firm dealing in foreign exchange wants to obtain an exchange rate between two or more currencies from their common relationship with another currency. By an indirect hedge the exporter uses a forward contract in which the underlying asset is highly correlated with the foreign exchange rate. If the stochastic relationship remains stable over time, the firm may realize an acceptable hedge of the foreign currency risk using this indirect instrument.

In this paper we analyze the behavior of a risk–averse exporting firm under various hedging regimes. Particularly, the effect of each hedging device on the optimal export and production. We extend previous results attained for a framework with currency futures market to the case where only indirect hedging is available. First we consider a competitive exporting firm facing exchange rate uncertainty, where no foreign exchange futures market is available. However, the firm has access to a forward market of some other domestic financial asset correlated to the exchange rate. In this case the well–known property called "Separation theorem" (for international firms) breaks down if the correlation is not perfect. Furthermore, introducing indirect hedging with *unbiased* futures market does *not* necessarily increase output as in the unbiased market for *direct* hedging (see, e.g., Benninga, Eldor, and Zilcha (1985), Kawai and Zilcha (1986), Broll and Wahl (1992)). Such a phenomenon may occur although the firm benefits from such indirect hedging market.

Later we compare the economic implications of indirect vs. direct hedging for a price discriminating international firm with monopoly power in the domestic market but is a price taker on the world market. The firm can allocate its production between the domestic and the foreign market *after* the observation of the exchange rate. Interestingly, replacing direct hedging by indirect hedging may lead to a decline of total output of the monopoly firm. Furthermore, export may decline while the domestic sales remain unchanged.

The organization of the paper is as follows. In Section I, we present a competitive firm involved in international trade under random exchange rate. With a numerical example we show in Section II that the availability of an unbiased indirect hedging instrument does not generally imply an incentive for the firm to increase production. Thus when indirect hedging becomes available the firm's optimal production may decline compared to the case *without any* hedging device. In Section III, we consider a discriminating monopoly selling in the domestic and foreign market. We study the impact of direct and indirect hedging of the exchange risk in such a framework. Section IV has a discussion of possible extensions and conclusions.

I. Competitive firm and indirect hedging

Consider a competitive firm with a given production technology which produces certain commodity and exports all its output to a foreign country. We assume that the firm faces a random exchange rate $\tilde{e}$, and it is risk–averse.

The firm cannot hedge its foreign currency risk directly in a given currency futures market, since such a market does not exist. However, there is a forward market for some domestic financial asset correlated to the exchange rate $\tilde{e}$ which can be utilized by this firm; namely, there exists an indirect hedging device for this foreign exchange uncertainty.

Let $\tilde{\eta}$ be the random spot value of this financial asset (at the time where the firm receives its foreign proceeds), and let η_f be the forward price for this asset at the time where the firm makes its production decision. To study the role of indirect hedging upon the firm's economic behavior, such as the production level, it is enough to consider forward market with zero risk premium. Thus we shall assume throughout this paper, that this forward market is unbiased, i.e., $\eta_f = E\tilde{\eta} \equiv \bar{\eta}$. Moreover, we shall assume that for some $b > 0$ and some random variable $\tilde{\epsilon}$, independent of $\tilde{e}$ and $\bar{\epsilon} \equiv E\tilde{\epsilon} = a \geq 0$, we have

$$\tilde{\eta} = b\tilde{e} + \tilde{\epsilon}.$$

When $\tilde{\epsilon}$ is not a constant (hence $\sigma_\epsilon^2 > 0$) we say that the firm hedges *indirectly.* If $\tilde{\epsilon} = a$, with probability 1, it is called *direct* hedging. We assume that only indirect hedging is available to this competitive firm and we shall compare the economic implications of indirect hedging vs. the direct hedging case.

The firm chooses at time 0 its optimal production x^* and optimal forward contracting z^* in a way that maximizes its expected utility of profits, where profits are denominated in domestic currency. Denote by $C(x)$ the firm's cost function and assume the usual properties: $C(x)$ is increasing and (weakly) convex function of x, i.e., $C'(x) > 0, C''(x) \geq 0$ for all x. Let U be the firm's von Neumann–Morgenstern utility function. We assume that it is a strictly concave function, i.e., $U' > 0, U'' < 0$. Thus we assume risk aversion in the sequel. The assumption of differentiability of the cost and utility functions is widespread in the economic–finance literature and most of the results can be obtained without such assumptions – however, at a high cost of clumsy analysis which uses left and right subderivatives (which exist for concave and convex functions). We decided to follow the earlier papers on these topics and to assume differentiability.

Let the price of this commodity in the foreign country be p and assume that it is fixed. The joint distribution of $\tilde{e}$ and $\tilde{\epsilon}$ is known and let E be the expected value operator. The forward contracting in the market for the asset $\tilde{\eta}$ is denoted by z. Without loss of generality we assume that it is positively correlated to $\tilde{e}$. However, since the firm can buy or sell forward this asset the same analysis is valid for negative correlation.

Before we write down the optimization problem of this firm, let us note that although the cost function $C(x)$ might be linear, the assumption of risk aversion, i.e., that $U'' < 0$ makes the maximand in problem (1) strictly concave in x, z.

The firm's optimization problem is:

$$\max_{x,z} EU(\tilde{\Pi}), \tag{1}$$

where the random profit $\tilde{\Pi}$ is

$$\tilde{\Pi} = \tilde{e}px - C(x) + z(\eta_f - \tilde{\eta}). \tag{2}$$

Since the maximand is strictly concave in x and z the optimum (x^*, z^*) is the unique solution of the necessary and sufficient conditions:

$$E[\tilde{e}p - C'(x^*)]U'(\tilde{\Pi}^*) = 0, \tag{3}$$

$$E(\eta_f - \tilde{\eta})U'(\tilde{\Pi}^*) = 0, \tag{4}$$

where ($\tilde{\Pi}^*$ is $\tilde{\Pi}$, defined in (2), at the optimum values $x = x^*$ and $z = z^*$). Although we assume that the futures market is unbiased, i.e., $\eta_f = E\tilde{\eta}$, we demonstrate that the firm underhedges in this case.

PROPOSITION 1 *Assume that unbiased future market exists for indirect hedging. Then*

(a) The Separation property does not hold in this case; namely, the optimal production of this firm depends upon the distribution of $\tilde{\eta}$ and upon the firm's attitude towards risk.

(b) If $\tilde{\eta}$ is not perfectly correlated to $\tilde{e}$, the firm underhedges, i.e., $px^* > bz^*$, where $z^* > 0$.

(c) Under perfect correlation between $\tilde{\eta}$ and $\tilde{e}$ (i.e., $\tilde{\epsilon} = \bar{\epsilon} =$ fixed), the firm fully hedges, i.e., after hedging its profits are nonrandom.

Note that perfect positive correlation does not imply that px^* is completely sold in the forward market, unless $b = 1$.[1]

Proof. (a) Combining equations (3) and (4) we derive

$$C'(x^*) = \frac{\eta_f p E\tilde{e}U'(\tilde{\Pi}^*)}{E\tilde{\eta}U'(\tilde{\Pi}^*)},$$

which clearly proves that the production decision cannot be separated from expectations and risk behavior, unless $\tilde{\eta}$ and $\tilde{e}$ are the same.[2]

(b) From equation (4) we have:

$$(\eta_f - \bar{\eta})EU'(\tilde{\Pi}^*) - cov(\tilde{\eta}, U'(\tilde{\Pi}^*)) = 0. \quad (5)$$

Using the assumption that the hedging market is unbiased, namely, $\eta_f = E\tilde{\eta} = \bar{\eta}$ we derive that $cov(\tilde{\eta}, U'(\tilde{\Pi}^*)) = 0$. Writing $\tilde{\eta} = b\tilde{e} + \tilde{\epsilon}$ we find that,

$$b\, cov(\tilde{e}, U'(\tilde{\Pi}^*)) + cov(\tilde{\epsilon}, U'(\tilde{\Pi}^*)) = 0, \quad (6)$$

since

$$\tilde{\Pi}^* = \tilde{e}(px^* - bz^*) - \tilde{\epsilon}z^* - C(x^*) + \eta_f z^*.$$

If $z^* \leq 0$ then $cov(\tilde{\epsilon}, U'(\tilde{\Pi}^*)) \leq 0$, so that $cov(\tilde{e}, U'(\tilde{\Pi}^*)) \geq 0$ which implies $px^* - bz^* \leq 0$ which is impossible. Therefore $z^* > 0$. This implies that $cov(\tilde{e}, U'(\tilde{\Pi}^*)) < 0$, hence $px^* - bz^* > 0$.

(c) When hedging occurs in an unbiased market, then $cov(\tilde{\eta}, U'(\tilde{\Pi}^*)) = 0$. By assumption $\tilde{\eta}$ mimics $\tilde{e}$ without noise, therefore,

$$cov(\tilde{e}, U'(\tilde{e}[px^* - bz^*] + k)) = 0, \tag{7}$$

where $k \equiv -\bar{\epsilon}z^* - C(x^*) + \eta_f z^*$. If $px^* - bz^* > 0$ profit increases in $\tilde{e}$ while marginal utility is strictly decreasing in contradiction to (7). Equation (7) holds if and only if $px^* = bz^*$. Hence the profits are nonrandom. □

Contrary to the usual direct hedging case the wording "full hedge" does not entail a short forward position of total export revenue but rather that the firm is not exposed to foreign exchange risk.

Comparing the firm's production level under indirect hedging with the direct hedging case we can prove

PROPOSITION 2 *Assume that $\eta_f = E\tilde{\eta} = a + bE\tilde{e}$, then the firm's production, when using indirect hedging, is lower than its production when (unbiased) direct hedging is available.*

Proof. When the firm can hedge directly on the exchange rate $\tilde{e}$, i.e., $\tilde{\eta} = b\tilde{e} + a$ (hence $\eta_f = a + b\bar{e}$, $\bar{e} = E\tilde{e}$) it can be shown from equations (3) and (4) that its optimal output x^{**} is given by the equation

$$C'(x^{**}) = \frac{\eta_f - a}{b} p = \bar{e}p. \tag{8}$$

This demonstrates that the Separation theorem holds (see, e.g., Danthine (1978), Feder, Just, and Schmitz (1980)). Now assume that only indirect hedging is possible through $\tilde{\eta} = b\tilde{e} + \tilde{\epsilon}$ and $\sigma_\epsilon^2 > 0$, thus $\tilde{e}$ and $\tilde{\eta}$ are not perfectly correlated. Since we consider positive correlation only, using the same argument as in the proof of Proposition 1 we can show that,

$$cov(\tilde{e}, U'(\tilde{\Pi}^*)) < 0.$$

But this implies from equation (3) that

$$E(\tilde{e}p - C'(x^*)) > 0. \tag{9}$$

Since C' is nondecreasing comparing (8) with (9) we find that $x^{**} > x^*$. □

Note that when $\tilde{\epsilon}$ is nonrandom $\tilde{\eta}$ is a linear function of $\tilde{e}$. The optimum output in this case is determined by equation (8), hence the Separation theorem holds.

Denote by $\tilde{\Pi}^\circ$ the profits of the firm when no hedging is available (and by x°, z° the corresponding output and hedging amount). Comparing the firm's welfare level under indirect hedging with the case where no hedging markets exist, we can prove,

CLAIM *The firm prefers $\tilde{\Pi}^*$ to $\tilde{\Pi}^\circ$ regardless whether it produces more or less, i.e., regardless whether $x^* > x^\circ$ or $x^* \leq x^\circ$.*

Proof. We shall use the strict concavity of U and the convexity of C as follows: Since $\tilde{\Pi}^* \neq \tilde{\Pi}^\circ$ we can write

$$E[U(\tilde{\Pi}^*) - U(\tilde{\Pi}^\circ)] > EU'(\tilde{\Pi}^*)[\tilde{\Pi}^* - \tilde{\Pi}^\circ] =$$
$$EU'(\tilde{\Pi}^*)[\tilde{e}p(x^* - x^\circ) + C(x^\circ) - C(x^*) + z^*(\eta_f - \tilde{\eta})].$$

But $C(x^\circ) - C(x^*) \geq C'(x^*)(x^\circ - x^*)$ whenever $x^\circ \neq x^*$, hence

$$E[U(\tilde{\Pi}^*) - U(\tilde{\Pi}^\circ)] > E[U'(\tilde{\Pi}^*)(C'(x^*) - \tilde{e}p)](x^\circ - x^*) + z^* E(\eta_f - \tilde{\eta})U'(\tilde{\Pi}^*) = 0,$$

due to equations (3) and (4). Note that it is possible that $x^\circ > x^*$ since in this case we may have $Var(\tilde{\Pi}^\circ) > Var(\tilde{\Pi}^*)$ but also $E\tilde{\Pi}^\circ \geq E\tilde{\Pi}^*$. This follows from the concavity of the function $\bar{e}px - C(x)$. Since $E\tilde{\Pi}^* = \bar{e}px^* - C(x^*)$ and $E\tilde{\Pi}^\circ = \bar{e}px^\circ - C(x^\circ)$ if $x^\circ > x^*$ *and* $x^\circ < argmax[\bar{e}px - C(x)]$, implies that $E\tilde{\Pi}^\circ \geq E\tilde{\Pi}^*$. This shows that in the case where no direct hedging exists production does not necessarily increase as the indirect hedging device becomes available. □

The fact that the firm will always prefer indirect hedging devices to none does not exclude the possibility of an adverse effect of such a device on production. This phenomenon is demonstrated in the following section.

II. Decreasing exports with indirect hedging: an example

In following numerical example we show that the availability of an indirect hedging does not generally imply that the exporting firm will increase its production. Hence, contrary to the well–known case where (unbiased) direct hedging causes a higher output, introducing (unbiased) indirect hedging market may lower the firm's optimal output compared to the case without any hedging at all. Therefore, an indirect hedging instrument can be counterproductive from this point of view.

We start from the following data set:

state of nature	s_1	s_2	s_3	s_4
$Prob(s_i)$	.25	.25	.25	.25
e_i	2.20	2.20	1.80	1.80
ϵ_i	–1.00	1.00	–1.00	1.00

Table 1: Probability distribution

Table 1 exhibits states of nature s_i ($i = 1,..,4$) and the corresponding probabilities, foreign exchange rates and disturbances. The asset for indirect hedging exhibits a positive linear stochastic relationship with the exchange rate, i.e., $\tilde{\eta} = b\tilde{e} + \tilde{\epsilon}$. The random variables $\tilde{e}$ and $\tilde{\epsilon}$ are independent, and we assume an unbiased hedging market, i.e., $\eta_f = E\tilde{\eta} = bE\tilde{e} + E\tilde{\epsilon}, b \geq 0$.

The utility function of the firm belongs to the HARA class, i.e.,

$$U(\Pi) = \frac{\alpha}{1-\alpha}\left[\left(A + \frac{\Pi}{\alpha}\right)^{1-\alpha} - 1\right],$$

where $\alpha \neq 0$ and $A + \Pi/\alpha > 0$.[3] Let $A = 55$. The cost function of the firm is given by

$$C(x) = \frac{x^2}{2}.$$

The foreign commodity price is fixed at $p = 1.5$.

Table 2 shows the optimal output for specific values of the parameter α of the utility function. These values imply $U''' < 0$ because they fall in the range $-1 < \alpha < 0$.

α	x°	$x^*(b=0)$	$x^*(b=.12)$	$x^*(b=.48)$
-.098	2.929705	2.929705	2.929642	2.928688
-.100	2.958095	2.958095	2.958088	2.957983
-.102	2.966411	2.966411	2.966414	2.966456

Table 2: Export production without hedging and with indirect hedging

If the hedging vehicle does not covariate with the foreign exchange rate, i.e., $b = 0$, indirect hedging is not effective and optimal output x^* coincides with the output in the case without hedging market, x°. In the cases of $\alpha = -.098$ and $\alpha = -.100$ the firm's optimal output decreases when the hedging vehicle becomes available and $cov(\tilde{\eta}, \tilde{e}) > 0$, i.e., $b > 0$. This result is driven mainly by the concavity of the marginal utility ($U''' < 0$). It is well–known that under such a condition the utility function exhibits increasing absolute risk aversion. However, the concavity of U' is a necessary condition for this type of example but not sufficient. This can be seen from the case of $\alpha = -.102$.

III. Monopoly and indirect hedging

Consider a domestic firm which sells a homogeneous product in the domestic market and on the world market. The firm is a monopoly in the domestic market but a price taker on the world market, where the price (in foreign exchange) p is fixed. The firm faces a domestic revenue function R assumed to be increasing and strictly concave, i.e., $R' > 0$ and $R'' < 0$. Furthermore, we assume that the allocation of output x between the two markets is determined *after* the observation of $\tilde{e}$. Therefore, the firm equates the marginal revenues in both markets. Denote by y the sale of the commodity on the foreign market and by z the forward contracting in the market for the asset $\tilde{\eta}$ (which is positively correlated to $\tilde{e}$); then the firm's profit function, when *indirect* hedging is available, is given by

$$\tilde{\Pi} = \tilde{e}py + R(x-y) - C(x) + z(\eta_f - \tilde{\eta}). \tag{10}$$

Assumption: the firm sells in both markets in all realizations of e.

If we take the values of the exchange rate to be in the interval $[\underline{\gamma}, \overline{\gamma}]$, then this assumption holds if we have: $R'(0) > \overline{\gamma}p$ and $R'(x^*) < \underline{\gamma}p$. Therefore we have

$$R'(x^* - y^*(e)) = ep \quad \text{for all } e. \tag{11}$$

Equation (11) defines, implicitly, the amount of commodity sold domestically as a function of the random variable e. Thus let us assume that $R(\cdot)$ and the exchange rate distribution satisfy the stated conditions.

The firm chooses optimal x and z in a way that

$$\max_{x,z,y(e)} EU(\tilde{\Pi}),$$

$$\text{subject to (10) and } 0 \le y(e) \le x, \ \text{ for all } e.$$

Under the above assumption since $0 < y^*(e) < x^*$ for all e equation (11) must hold. Due to the concavity of $EU(\tilde{\Pi})$ in (x, y), necessary and sufficient conditions for the unique optimal solution x^*, z^* and $y^*(e)$ are equation (11) and,

$$E[R'(x^* - y^*(\tilde{e})) - C'(x^*)]U'(\tilde{\Pi}^*) = 0, \tag{12}$$

$$E(\eta_f - \tilde{\eta})U'(\tilde{\Pi}^*) = 0. \tag{13}$$

Contrary to the results attained by Eldor and Zilcha (1987) for direct hedging we show here that, under the above assumption,

PROPOSITION 3 *Assume that only unbiased indirect hedging market exists. Then*

(a) the Separation theorem does not hold,

(b) underhedging occurs: $py^*(e) - bz^* > 0$ *with positive probability, while* $z^* > 0$.

Proof. Let us prove first that $z^* > 0$. Assume to the contrary that $z^* \leq 0$, then from equation (13) we have

$$b\ cov(\tilde{e}, U'(\tilde{\Pi}^*)) + cov(\tilde{\varepsilon}, U'(\tilde{\Pi}^*)) = 0.$$

Under our assumption $cov(\tilde{\varepsilon}, U'(\tilde{\Pi}^*)) \leq 0$, since U' is a decreasing function. It is not hard to verify from equation (11) that $y^*(e)$ increasing in e, hence $R(x^* - y^*(e))$ is a decreasing function of e. But

$$(14) cov(\tilde{e}, U'[\tilde{e}(py^*(e) - bz^*) - \tilde{\varepsilon}z^* + R(x^* - y^*(e)) - C(x^*) + \eta_f z^*]) \geq 0.$$

Hence $e(py^*(e) - bz^*) > 0$ with positive probability. Moreover we also have $epy^*(e) + R(x^* - y^*(e))$ increasing in e (since x^* is fixed, higher exchange rate realization results in higher total revenues). Thus $\tilde{\Pi}^*(e)$ increasing in e when $z^* \leq 0$, but this is in contradiction to (14). This proves that $z^* > 0$ and part (b) of the proposition as well. Equation (12) can be written as

$$E(\tilde{e}p - C'(x^*))U'(\tilde{\Pi}^*) = 0. \tag{15}$$

We have proved that $cov(\tilde{\epsilon}, U'(\tilde{\Pi}^*)) > 0$, while $cov(\tilde{e}, U'(\tilde{\Pi}^*)) < 0$. Thus (15) implies that $[\bar{e}p - C'(x^*)]EU'(\tilde{\Pi}^*) > 0$, namely, $C'(x^*) < \bar{e}p$. This inequality implies that the Separation property does not hold (part (a)). □

In this framework assume now that for institutional reasons no direct forward market for foreign currency exists while indirect hedging instruments are accessible. Comparing both scenarios we show that,

PROPOSITION 4 *If (unbiased) direct hedging is replaced by an (unbiased)* indirect *hedging, then (a) total output of the firm declines, and (b) for all values of* e *the domestic sales are the* same *while the* export declines.

Proof. With direct hedging the Separation theorem in this model holds (see Eldor and Zilcha (1987)). Hence, denoting the optimum in the direct hedging case by " **", from equations (12) and (13) for the direct hedging case,

$$
\begin{aligned}
&(16) \qquad & C'(x^{**}) &= \bar{e}p, \\
&(17) \qquad & R'(x^{**} - y^{**}(e)) &= ep \quad \text{for all } e.
\end{aligned}
$$

Consider the indirect hedging conditions for optimum. From the proof of Proposition 3 we have $C'(x^*) < \bar{e}p = C'(x^{**})$, hence $x^* < x^{**}$. Since sales in *both* markets take place for all values of e, we must have the following equality:

$$(18) \qquad R'(x^* - y^*(e)) = R'(x^{**} - y^{**}(e)) = ep \quad \text{for all } e.$$

Therefore,

$$(19) \qquad x^* - y^*(e) = x^{**} - y^{**}(e) \quad \text{for all } e.$$

Or

$$(20) \qquad y^*(e) = y^{**}(e) - (x^{**} - x^*).$$

Since $x^{**} - x^*$ is a positive constant (18) and (19) prove (b). □

Remark: Note that if $(x^\circ, y^\circ(e))$ are the optimum for the no–hedging case then it was shown that $x^\circ < x^{**}$. On the other hand if the firm sells in both markets for all values of e then by the same argument we must have:

$$y^{\circ}(e) = y^{**}(e) - (x^{**} - x^{\circ}) \quad \text{for all } e. \tag{21}$$

Now we demonstrate another difference between indirect and direct hedging,

PROPOSITION 5 *Consider the above firm without any hedging market. When (unbiased) indirect hedging market becomes available the firm will use this market but in* some *cases (a) its output declines and (b) the* export *will decrease for all values of* e.

Note that this Proposition is in contradiction to the direct hedging case (see Eldor and Zilcha (1987, p. 465)) where introducing unbiased futures market results always in higher output and higher exports.

Proof. An example where the firm's output x^* under indirect hedging is lower than its output x° (without any hedging) can be constructed as in the competitive firm case. Thus assume that under some choice of U and $(\tilde{e}, \tilde{\varepsilon}, b)$ we have $x^* < x^{\circ}$. By the same argument used in the above proof for all values of e,

$$R'(x^* - y^*(e)) = R'(x^{\circ} - y^{\circ}(e)) = ep \quad \text{for all } e.$$

Therefore

$$y^*(e) = y^{\circ}(e) - (x^{\circ} - x^*) \quad \text{for all } e.$$

Since in this case $x^{\circ} - x^* > 0$ it proves (b). □

IV. Discussion

Our analysis can be carried out in a similar way when more than one hedging asset correlated to the exchange rate is available. For example if two such instruments exist with random returns $\tilde{\eta}_1$ and $\tilde{\eta}_2$ the profits become

$$\tilde{\Pi} = \tilde{e}px - C(x) + z_1(\eta_{f_1} - \tilde{\eta}_1) + z_2(\eta_{f_2} - \tilde{\eta}_2),$$

where η_{f_1}, η_{f_2} are futures price of the two assets. The producer must choose x, z_1 and z_2 in this optimization problem. Following the same assumption if

$$\tilde{\eta}_1 = b_1\tilde{e} + \tilde{\epsilon}_1 \qquad \tilde{\eta}_2 = b_2\tilde{e} + \tilde{\epsilon}_2,$$

where $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ are uncorrelated to the exchange rate $\tilde{e}$. Unless some convex combination of $\tilde{\eta}_1$ and $\tilde{\eta}_2$ is perfectly correlated to $\tilde{e}$ such analysis does not add much to the insights of this work, since the absence of direct hedging is the main driving force for our results. However, the existence of many financial instruments correlated to $\tilde{e}$ may, under some circumstances, provide an almost perfect hedging. In such a case we are actually back to the direct hedging model with all the well–known consequences. Since we were interested in a framework where no direct hedging is available we did not pursue such a course.

Another issue to be studied in such a framework is the effect that the correlation between $\tilde{\eta}$ and $\tilde{e}$ has upon the production and the optimal hedging. Is it true that as this (positive) correlation increases the output increases as well?

To sum up, our analysis shows that an international firm can benefit from an indirect hedging in the presence of price uncertainty. However, there is a substantial difference between the case where only indirect hedging is available and the case of direct hedging. Introducing a currency forward market, i.e., allowing direct hedging, results in an increase in production for exports. This may not be the case when indirect hedging market becomes available. Thus the very nature of the hedging device has important implications for international trade. Also, an unbiased futures/forward market in this case will not enable the firm to eliminate exchange risks altogether.

While we believe that indirect hedging possibilities are important for international firms compared with the case where no hedging markets exist, empirically the correlation between the foreign exchange rate and the 'proxy' should be stable over time. Otherwise, the hedging policy has to be adjusted.

Furthermore, contrary to our assumption, the relationship between the two stochastic variables may be non–linear, which would require more complex hedging strategies than a forward hedge.

Notes

1. Since $b = cov(\tilde{\eta}, \tilde{e})/\sigma_e^2$, the term bz^* has the dimension of the foreign currency.
2. More precisely, unless $\tilde{\eta}$ is a deterministic linear function of $\tilde{e}$.
3. Note that $\lim_{\alpha \to 1} U(\Pi) = ln(A + \Pi)$.

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