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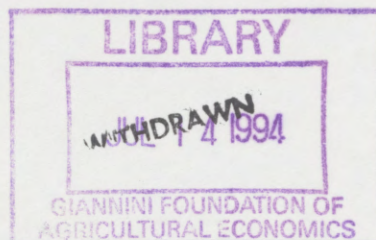
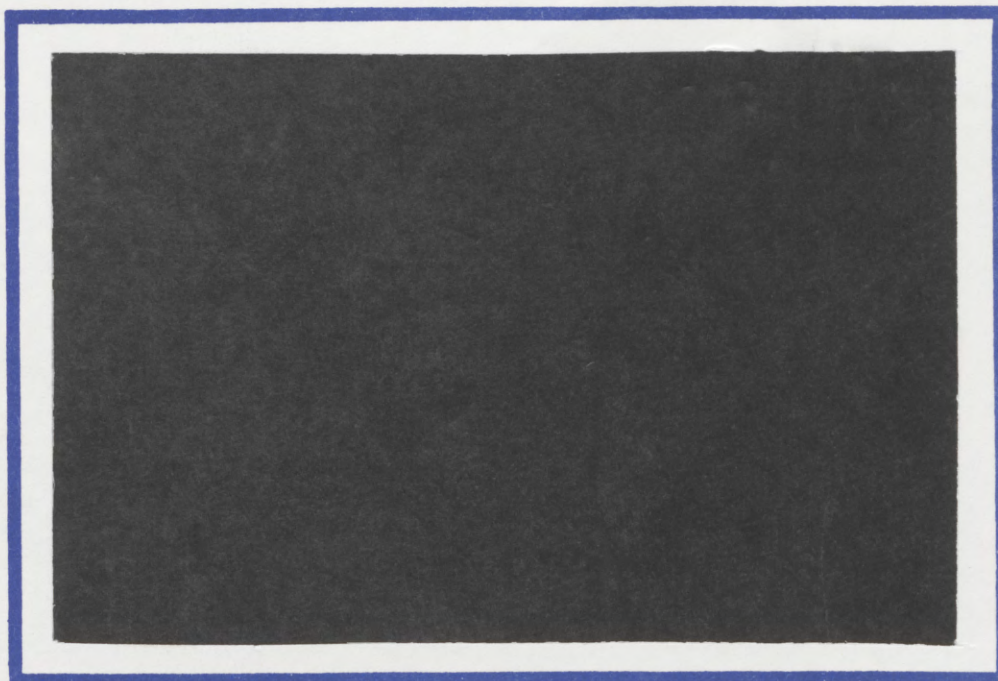
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A SIMPLE MODEL OF EQUILIBRIUM
IN SEARCH PROCEDURES*

by
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Abstract

The paper presents a simple game theoretic model in which players decide on the search procedures for a prize which is located in one of a labeled set of boxes. The prize is awarded to the player who finds it first. A player can decide on the number of (costly) search units he employs and the order in which he conducts the search. It is shown that in equilibrium, the players employ an equal number of search units and conduct a fully random search. The paper demonstrates that the search procedure is intrinsically inefficient in two senses: the players employ a non-optimal number of search units and they may open the same empty box twice.

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1. Introduction

When a decision maker is involved in an interactive situation both his action and the decision procedure is subject to strategic reasoning. In this short paper we present and analyze a simple model of strategic choice of procedures which demonstrates how interactive choice of procedures leads to intrinsic inefficiency.

The model we use is similar to conventional search models. As Herbert Simon pointed out in the fifties (see, for example, Simon (1955)) the model of search can be viewed as a model of decision making in which the decision maker searches for a solution to a problem. Therefore, a game in which the procedures of search are decided strategically can be viewed as a situation in which the procedures of decision making are a result of interactive reasoning. Since the choice of decision making procedures is modeled explicitly, one can view the paper as a modest contribution to the growing literature of economic models of bounded rationality.

In the model there are N boxes and two players searching for a prize with a value of 1 hidden in one of the N boxes. The first player to find the prize keeps it. The elementary device used in the search is called a search unit and is able to check one box per unit of time. We model the players' constraints regarding the procedures of search by treating the search units as costly. A player decides on how many search units to employ as well as the strategy for the search units to follow.

The assumption of a single prize runs contrary to the informational assumptions made in most of the search literature in which the values of alternatives are taken to be stochastic independent. It is, however, suitable for decision problems in situations where an action to be successful it has to be a solution of a problem. The assumption that the prize can be discovered only through "opening boxes" corresponds to a situation in which the solution can only be found through case-by-case testing. For real life examples of the model consider reporters who are looking for a hotel

where a movie star is staying and can only make inquiries by calling one hotel at a time, or, mathematicians who know that the solution to a particular problem can be found by following only one of a given number of routes, or, intelligence agents who use trial and error to break a code.

Clearly, the model is similar to those of the R&D race literature. The main difference lies in our interest in determining the procedure of R&D search. For this reason the boxes in our model have labels and a decision maker can decide the order of search and not merely its intensity. As an R&D model, it resembles the trial and error of research procedure typical to the chemical and pharmaceutical industries. The assumption of only one prize has been extensively used in the theoretical R&D literature (see Reinganum (1989)) and seems to be empirically supported by findings that the private value of patents is extremely skewed (see Griliches, Pakes and Hall (1987)). The search activity in our model is carried out by search units, the number of which is determined at the outset. This assumption is in line with the empirical R&D findings which indicate that firms adjust their R&D expenditures very infrequently (see Pakes (1985)).

The distinction between all labeled boxes permits us to consider a variety of procedures such as, for example, opening the boxes in a certain deterministic order. Since the boxes are originally symmetric, the model would not make much sense if there was only one decision maker. However, when more than one player searches for the prize, one cannot a priori exclude the possibility of an equilibrium with a pair of search procedures, where each player uses different search procedures which is optimal given the other.

Our main conclusion in this paper is that an interactive search situation entails intrinsic inefficiency in two respects: In equilibrium, the players will use an excess of costly search units though the prize could be discovered (though not as quickly) using only one search unit. Secondly, one could argue that the model demonstrates intrinsic inefficiency in that

the players conduct more than the minimum required search since in all equilibria the players fully randomize such that two players may frequently search the same empty box. In addition, this means that the two players do not reach the prize as quickly as they could. (Although time considerations are not embedded in the model explicitly, the equilibrium is robust to the addition of some degree of impatience.)

To appreciate the intuition in our main result, consider the case in which both players employ k search units. Efficiency (in terms of minimizing the number of search operations) requires that the two players split the N boxes into blocks of N/k boxes, half of which are searched by player 1 and half by player 2. However, if $N/k > 2$ this is not an equilibrium since player 2 can deviate profitably by searching those boxes in period t which player 1 plans to search in period $t+1$, assuring a probability of $1-k/N$ of winning the prize. This is greater than $1/2$ unless $N/k \leq 2$.

2. The Model

Two players, 1 and 2, are searching for a prize which has a value of 1. There are N boxes b_1, \dots, b_N , one of which contains the prize. The location of the prize is determined randomly. Player i must choose a number, k_i , interpreted as the number of search units. For simplicity we will confine ourselves to cases where N is divisible by all k_i . This enables us to avoid unnecessary calculations without taking away from the analysis. A (pure) search strategy, P_i , is a partition of the N boxes into sets $S_i^1, \dots, S_i^{T_i}$ such that the number of elements in each of the cells in the partition is k_i (there is no reason to assume that the players prefer to delay the process and keep some search units idle). Thus, $T_i = N/k_i$. A random search strategy is a probability distribution over the set of pure search strategies.

We analyze a two-stage game. In the first stage the players choose the numbers k_i and in the second stage, choose a search strategy P_i (which may be random) after learning of his rival's choice at stage one. This latter choice is constrained by the number of units chosen at the first stage.

A pair of search strategies stochastically determine the winner of the prize. Assume that the prize is in box b . If $b \in S_i^{t(i)} \cap S_j^{t(j)}$ then if $t(i) < t(j)$, player i locates the prize and if $t(i) = t(j)$ each of the players has probability 0.5 of winning it.

For any pair of choices $(k_i, P_i)_{i=1,2}$ player i 's payoff is the expected probability of locating the prize minus k_i times the cost of a search unit, c .

In order to simplify the statement of the results we assume that there is no k for which $c = 1/k$. We further assume that $1/2 > c > 1/2N$. The other cases are degenerate. If $c > 1$ the choice of $k_i = 0$ is obviously a dominating strategy. If $c > 1/2$ the only equilibrium is such that only one player employs one search unit. If $c < 1/2N$, the only equilibrium will involve both players employing N search units. It is only in the range $1/2 > c > 1/2N$ that the model has interesting strategic content.

3. The Analysis

We now turn to analyzing the set of Nash equilibria for the game. The second stage of the game is a zero sum game and thus a pair $(k_i, P_i)_{i=1,2}$ is a Nash equilibrium outcome if and only if it is the outcome of a perfect equilibrium of the game.

The analysis includes three claims. The first claim involves calculating the value of the second stage of the game for every pair of choices (k_1, k_2) :

Claim 1: For any given pair of search unit numbers (k_1, k_2) the value of the second stage of the game for player i is $k_i/2k_j$ if $k_i \leq k_j$ and $(1-k_j/2k_i)$ if $k_i \geq k_j$.

This formula makes it possible to view the choice of the number of search units in the first stage of the game as the outcome of a Nash equilibrium of the game in which each player chooses the number of units and in which the payoffs are determined by Claim 1.

Claim 2: In all Nash equilibria of the game, the two players choose $k_1 = k_2 = k$ where k satisfies $1/2(1+k) < c < 1/2k$.

Finally, we are able to show that when $c > 1/N$ (and thus, $k < N/2$) the players fully randomize the orders of search in the second stage equilibrium so that the probability that a certain box is checked at each period is equal to $1/T_i = k/N$.

Claim 3: For $k < N/2$, the probability that box b is checked at t is precisely k/N , in all Nash equilibria for any box b and period $1 \leq t \leq N/k$.

To prove claim 1, observe first that given $k_i \leq k_j$, the pair of strategies in which both players mix over all possible pure strategies with equal probabilities is an equilibrium. The search lasts for no more than N/k_j periods and the number of boxes which may be searched by player i is Nk_i/k_j . On condition that the prize is in one of these boxes, the probability that player i will reach it first is $1/2$. Thus, the probability that i will find the prize first is $(Nk_i/2k_j)/N = k_i/2k_j$. Since it is a zero sum game the value of player i where $k_i \geq k_j$ is $(1 - k_j/2k_i)$.

To prove claim 2 assume that (k_i, k_j) is an equilibrium choice in the first stage. Observe that if $k_i < k_j$ then player i 's payoff is $k_i/2k_j$, an expression which is linear in k_i . If $k_j \leq k$ then the marginal gain from a search unit is larger than c and player i can profitably deviate by increasing k_i . If $k_j > k$ then the marginal gain from another search unit is $1/2k_j \leq 1/2(k+1) < c$. Thus, for such an equilibrium, k_j must be 0 which implies that k_j must be 1. Since $1 = k_j > k$ this implies that k must be 0. But then player i could deviate by purchasing one search unit and achieving a payoff of $1/2 - c > 0$ in contradiction to (k_i, k_j) being an equilibrium. Thus, $k_i = k_j$. If $k_i = k_j > k$ then a player can profitably deviate by reducing the number of search units (since $1/2k_j \leq 1/2(k+1) < c$). If $k_i = k_j < k$ then by increasing the number of units by one player i increases his payoff by $1 - k_j/2(k_j+1) - 1/2 = [(k_j+2) - (k_j+1)]/[2(k_j+1)] = 1/[2(k_j+1)] \geq 1/2k > c$

which is thus profitable. We are left with $k_i = k_j = k$. It is easy to verify that the choice $(k_1, k_2) = (k, k)$ is indeed an equilibrium.

To prove claim 3 consider the following equilibrium. The "uniform strategy" is one which assigns equal probabilities to all possible pure strategies. Notice that since $k_i = k_j$, if a player uses the "uniform strategy" he locates the prize with probability $1/2$, independent of which strategy the other player uses. Now, denote by $p_i(b, t)$ the probability that player i checks box b at period t . We first show that at equilibrium $p_i(b, t) + p_i(b, t+1) = p_i(b', t) + p_i(b', t+1)$ for any boxes b and b' . To see this assume that $p_i(b, t) + p_i(b, t+1) > p_i(b', t) + p_i(b', t+1)$ for some b, b', i and t . Player j can adopt a modified "uniform strategy" in which he transfers the probability mass which is assigned to any pure search strategy which searches b' at period t and b at period $t+1$, to a strategy in which he searches b at period t and b' at $t+1$. His "loss" from the postponement of searching b' is $q[p_i(b', t) + p_i(b', t+1)]/2$ (where q is the proportion of pure strategies in which b' is searched at t and b at $t+1$). His "gain" from advancing the search of box b is $q[p_i(b, t) + p_i(b, t+1)]/2$ which is larger than the loss of postponement.

Now since for all i and t , $\sum_b p_i(b, t) = k$, we obtain $[p_i(b, t) + p_i(b, t+1)] = 2k/N$. In addition to the $k-1$ equations $p_i(b, t) + p_i(b, t+1) = 2k/N$ we have the equation $\sum_{t=1, \dots, N/k} p_i(b, t) = 1$ and thus $p_i(b, t) = k/N$ for all b and t .

Note that claim 3 does not propose that the two players use the "uniform strategies". For example in the case that $k=1$ and $N=3$, both the pair of strategies in which each player randomizes equally between the three orders of search (b_1, b_2, b_3) , (b_2, b_3, b_1) and (b_3, b_1, b_2) as well as the pair of strategies in which they randomize equally among all six orders are equilibria.

Comment: Note that when $k=N/2$, any pair (k_i, P_i) with $k_1 = k_2 = k$ is a

Nash equilibrium, including the pair of strategies in which the players split the N boxes equally between them.

4. Discussion

One important assumption in the search literature is that sampling is a random draw. As we have pointed out, this is not the only search procedure available to players. Stores, projects or boxes may have specific addresses, and the search procedure may utilize these addresses and sample the boxes non randomly. Our main result (Claim 3) indicates that in our setting, players will not use their ability to call up specific boxes and will use a totally randomized search procedure.

We identify two types of inefficiencies in the model. First, the competition between the players prevents them from splitting the set of boxes between them and therefore they may frequently search the same box twice. This is not optimal if the search is costly or if time is valuable. Notice that we did not include time loss explicitly in the model: however, adding this component makes little substantial difference. If time is not valuable then the model exhibits a second source of inefficiency, the excessive use of search units. This second inefficiency result conforms to the R&D literature which has demonstrated the inefficiency arising from excessive search intensities.

An additional inefficiency is demonstrated by considering an asymmetric case in which there is a higher probability, q^+ , that the prize is in one specific box, let us say box 1, than the probability q^- that it is in any other box. Clearly, an efficient search rule will start by searching box 1. However, it is easy to see from the arguments presented in the paper that in equilibrium, unless q^+ is relatively high, there is a positive probability that neither player will search box 1 at the first period.

Admittedly the model is too simple. Initially, we had planned to analyze the equilibrium of games in which players may use more complicated and interesting decision procedures. We failed to prove clear-cut propositions for such models and made do with reporting on the simple model.

Nevertheless, we still believe that this area of research calls for more work which will focus on the equilibrium structure of decision procedures.

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