THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH

TEL-AVIV UNIVERSITY

RAMAT AVIV ISRAEL

מכון למחקר כלכלי ע״ש ד״ר ישעיהו פורדר ז״ל

ע״י אוניברסיטת תל־אביב

BARGAINING IN INTERNATIONAL TRADE UNDER EXCHANGE RATE UNCERTAINTY

by

Shabtai Donnenfeld* and Itzhak Zilcha**

Working Paper No.13–93

November, 1993

* Stern School of Business, Dept. of Economics, New York University

** The Eitan Berglas School of Economics, Tel–Aviv University, Tel–Aviv, Israel

Donnenfeld acknowledges financial support from SSHRC of Canada and Zilcha acknowledges financial support from the Foerder Institute for Economic Research and the Lion Foundation.

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH
Faculty of Social Sciences
Tel–Aviv University, Ramat Aviv, Israel

I. INTRODUCTION

The common presumption regarding international transactions is that the terms of trade and the quantities traded are determined in anonymous exchange. Furthermore, it is also presumed that decisions on prices and quantities are immutable and irrevocable. In a deterministic environment these assumptions do not seem to be innocuous, since at the time when the terms of the transaction are determined all the information needed to be known is readily available. However, since international transactions spread over longer periods of time than domestic transactions, and thus the exchange rates are likely to fluctuate, there are important gains for both exporters and importers to trade in *customer markets.* As pointed out by Mckinnon (1979) this type of relationship between exporters and importers is common for *Tradable I* goods, i.e., manufactured products with distinctive characteristics and brand name products. In many of these cases the trading parties are large agents, and thus the anonymity and irrevocability of contracts signed by exporters and importers do not seem to be a realistic assumptions. Consequently, we believe that the contracts between exporters and importers that fall in this category are negotiated rather than determined in anonymous market transactions.

This paper investigates the implications of various contracting alternatives between exporting and importing firms on the volume of international transactions, on prices and on the distribution of the gains generated by these transactions. The contracts that we study are determined in a bargaining situation under exchange rate uncertainty. We look at three types of contracts that can be signed between exporters and importers:

Contracts which entail an ex-ante commitment on price and quantity of exports without the possibility of renegotiation ex-post.

A second type of contracts are spot contracts, i.e., the exporter and the importer decide to negotiate the price and the quantity of exports only after the exchange rate is known.

A third type of contracts consist of ex-ante commitment and ex-post renegotiation (if both sides agree to that).

Our analysis focuses on the differences that arise in the level of production and the quantities of goods traded across the three types of contracts negotiated by exporters and importers. These differences are of particular importance for empirical investigations of the effects of changes in exchange rates on the current account of the balance of payments. In particular many empirical studies attempt to measure the effects of depreciation (appreciation) of the currency on size of the deficit (surplus) of the current account, which has become known as the *J curve.* The measurements of changes in the current account are conducted based on the working assumption that in the short run the prices of traded goods are rigid, whereas for the longer run both prices and quantities adjust. (See Magee (1973), Mckinnon (1979), Wood (1990)).

Short run is interpreted as the life time of contracts between exporters and importers. Life time of a contract starts when the importers place orders at a mutually agreed price with the exporters and ends at the time when the actual payment is made. The common assumption in these empirical studies is that both the quantity and the price remain constant during the contract period eventhough the contract period spreads over many months.[2] Our analysis indicates that the quantities and the prices of these traded goods will differ depending on the type of contract signed by exporters and importers. Hence, the measurement of the effects of changes in the current account which are based on the assumption that prices and quantities remain fixed during the contract period may overstate or understate the size of the deficit or surplus due to currency changes.[3]

[2]Magee (1984) investigated the length of the period of contract for exports from Japan and Germany to the U.S. and found that on average it takes 141 days from the time of the exporters acceptance of orders to delivery of imports from Japan and 96 days for imports from Germany. The distribution of contracts length were skewed to the right with a maximum length of 22 months for imports from Japan and 10 months for imports from Germany. Carse, Williamson and Wood (1980)looked at the overall average length of the period of contract of exports and imports of U.K. and found it to be six months for exports and four months for imports.

[3]Magee (1974) points out that in an effort to avoid capital losses in a currency contract

During the last decade we observed a significant increase in the volatility of the exchange rate. The effects a exchange rate risk on the pricing of exports and level of trade by an exporting monopolist were investigated by Baron (1976), Cushman (1983), De Grawe (1988), Knetter (1989), Donnenfeld and Zilcha (1991), and Gagnon (1992). In this paper we extend the analysis further by explicitly incorporating an importer who is purchasing the goods from the exporting firm, and investigate the effects of exchange rate uncertainty on the terms of transactions, price and quantity traded, given that each party has some bargaining power. We highlight the differences that arise in prices and the volume of trade in the case where the firms negotiate the contract versus the case where the exporter himself conducts the foreign sales directly, and thus bypasses the importing firm. The former case corresponds to a non-integrated firm and the latter corresponds to a vertically integrated firm. This interpretation allows us to compare the results that we obtain with the results known from the theory of vertical integration. We show that in some cases the non-integrated firm may produce and sell more than the integrated firm, despite the inefficiencies induced by the double marginalization.

The remainder of the paper is organized as following. In the next section we present the case where the exporter and the importer sign an ex-ante contract which can be renegotiated after the realization of the exchange rate, if both parties wish to do so. In section III we present the benchmark case, where the monopolist bypasses the importer and sells himself the product in the foreign market. In section IV we examine the case where the firms sign a contract ex-ante which cannot be renegotiated ex-post. The effects of increasing exchange rate volatility on prices and the volume of trade in the three cases are examined in section V. In the final section we summarize the main results.

---

period exporters (importers) may renegotiate contracts in order to raise (hold down) the price of exports (the cost of imports) or cancel contracts.

II. The Model

There are two firms. An exporting firm that produces a good that is sold in the domestic market and for exports as well. The second firm, the importer is purchasing the good from the exporting firm and sells it solely in the foreign market.

The following notation will be used: Q is the exporter's total output, q is the quantity of exports, p is the price per unit of exports stated in the exporter's currency, $R(Q-q)$ is the total revenue that the exporting firm receives from domestic sales, $C(Q)$ is the firm's total cost of production, $R^*(q)$ is the total revenue from sales in the foreign market, and e is the exchange rate i.e., units of exporter's currency per unit of the importer's currency.

The profit functions for the case that the transaction is invoiced in the exporter's currency are:

$$\pi^E = R(Q-q) + pq - C(Q) \tag{1}$$

$$\pi^I = R^*(q) - pq/\tilde{e} \tag{2}$$

The sequence of events and moves is as follows: Ex-ante all firms have expectations regarding the exchange rate. The priors are common to all firms. At this initial stage the exporter selects the level of output to be produced for both domestic and foreign sales. Next, the exporter confronts the importer with a price per unit of exports p_0. The price is quoted in the exporter's currency. Facing the quoted price the importer decides what quantity q_0, to purchase. Ex-post, after the realization of the exchange rate is known, the exporter and the importer can renegotiate the initial contract $< p_0, q_0 >$.

We start from the last stage, i.e., after the realization of the exchange rate is known, and proceed backwards toward the initial stage. Let the actual exchange rate be e, and given $< p_0, q_0, Q >$, that were determined in the previous stage, the firms renegotiate the price and the quantity of exports. We assume that the outcome of the renegotiation is determined by the Nash bargaining solution:

$$\max_{p(e),q(e)} \left[R(Q^* - q_1) + p_1(e)q_1(e) - \bar{U}\right]\left[R^*(q_1(e)) - \frac{p_1(e)q_1(e)}{e}) - \bar{V}\right] \qquad (3)$$

where the "threat point", or the status quo profits are

$$\bar{U}(Q, e, p_0, q_0) = R(Q - q_0) + p_0 q_0 - C(Q)$$

$$\bar{V}(e, p_0, q_0) = R^*(q_0) - \frac{p_0 q_0}{e}$$

If in the initial stage the firms did not contract for price and quantity, they will bargain ex-post; in this case the relevant threat point is: $\bar{U}_0 = R(Q) - C(Q)$, $\bar{V}_0 = R^*(0)$. The first order conditions for the Nash bargaining solution are:

$$[-R'(Q^* - q_1) + p_1][eR^*(q_1) - p_1 q_1 - e\bar{V}] + [eR^{*'}(q_1) - p_1][R(Q^* - q_1) + p_1 q_1 - \bar{U}] = 0 \qquad (4)$$

$$p_1 = \frac{1}{2q_1}[eR^*(q_1) - R(Q^* - q_1) + \bar{U} - e\bar{V}] \qquad (5)$$

For each e let $< p_1(p_0, q_0, Q, e), q_1(p_0, q_0, Q, e) >$ be the Nash bargaining solution.

Claim 1: *Due to the efficiency of the Nash bargaining solution the final contract* $< p_1(e), q_1(e) >$ *will satisfy the condition*

$$R'(Q - q_1) = eR^{*'}(q_1).$$

Hence,

$$q_1 = q_1(Q, e).$$

Using the property that the Nash bargaining solution is ex-post efficient the condition stated in (4) becomes:

$$R(Q-q_1)+p_1q_1-\bar{U}=e[R^*(q_1)-\frac{p_1q_1}{e}-\bar{V}] \tag{6}$$

We proceed now to the stage where the importer selects q_0, the quantity to be purchased from the exporter when confronted with the price p_0. The importer's choice of q_0 is determined by

$$\max_{q_0\geq 0} E[R^*(q_0)-\frac{p_0q_0}{e}] \tag{7}$$

The necessary and sufficient condition for optimality is:

$$\bar{e}R^{*'}(q_0)=p_0 \quad when \quad q_0>0 \tag{8}$$

$$If \quad R^{*'}(0)\leq 0 \quad then \quad q_0=0 \tag{9}$$

where $\bar{e}$ is defined by $\frac{1}{\bar{e}}=E[\tilde{e}^{-1}]$. Note that the importer's best quantity response $q_0=q_0(p_0)$ depends on the distribution of the exchange rate and the exporter's price only.

To complete the determination of the initial contract we proceed now to the stage where the exporter selects the price p_0. While doing so the exporter takes into account the importer's best response as given in (8). Concomitantly, the exporter also selects the level of total production Q. The optimal Q and p_0, and hence q_0, are determined by the following maximization problem:

$$\max_{Q,p_0\geq 0} E[R(Q-q_1(Q,\tilde{e}))+p_1(p_0,q_0(p_0),Q,\tilde{e})q_1(Q,\tilde{e})-C(Q)] \tag{10}$$

$$subject\, to \ (8)\, and \ (9)$$

Using the definition of the threat point $(\bar{U},\bar{V})$ and denoting by $y_1(\cdot)=p_1q_1$ we can rewrite the condition stated in (4) as:

$$R(Q-q_1)+y_1-\bar{U}=e[R^*(q_1)-\frac{p_1q_1}{e}-\bar{V}] \tag{11}$$

Substituting in (11) the expression for $\bar{U}$ and $\bar{V}$ we obtain an expression for the importer's total payments:

$$y_1=\frac{1}{2}[R(Q-q_0)-R(Q-q_1)]+\frac{e}{2}[R^*(q_1)+p_0q_0-\frac{C(Q)}{2}] \tag{12}$$

After substituting for y_1 in the maximization problem stated in (10) we obtain the following necessary conditions for optimality for Q and q_0. The optimality conditions pertain to two cases: (*i*) $q_0>0$ and (*ii*) $q_0=0$.

(*i*) $q_0>0$:

$$E[R'(Q-q_1(Q,\tilde{e}))+R'(Q-q_0)-3C'(Q)]=0 \tag{13}$$

$$-R'(Q-q_0)+\Delta R^{*'}(q_0)+2q_0\bar{e}R^{*''}(q_0)=0 \tag{14}$$

where $\Delta=2\bar{e}-E[\tilde{e}]$ and $\frac{1}{\bar{e}}=E[\frac{1}{\tilde{e}}]$. Let $<Q^*,q_0^*>$ be the solution to (13) and (14).

(*ii*) $q_0=0$

In this case condition (14) should be replaced by the condition

$$E[\frac{\partial y_1}{\partial q_0}]\leq 0.$$

That is,

$$R'(Q-q_0(p_0))+\Delta R^{*'}(q_0)+2\bar{e}q_0R^{*''}(q_0)\leq 0 \tag{15}$$

Denote by Q^{**} the level of production which satisfies the equation below

$$E[R'(Q^{**} - q_1) - R'(Q^{**}) - 3C'(Q^{**})] = 0$$

Then

$$\frac{\partial y_1}{\partial q_0}|_{Q=Q^{**},q_0=0} = -\frac{1}{2}R'(Q^{**}) - \frac{e}{2}R^{*'}(0) + \bar{e}R^{*'}(0)$$

Thus,

$$R'(Q^{**}) \geq R^{*'}(0)E[\tilde{e}] \ \ implies \ \ q^* = 0$$

Eventhough in such a case an initial contract will not be signed this does not imply that there are no gains from ex-post trade between the exporter and the importer. There are some realizations of the exchange rate for which $R'(Q^{**}) < eR^{*'}(0)$ and still ex-post trade is beneficial to both parties.

III. Monopoly

In this section we present the benchmark for the evaluation of the renegotiation outcome, presented in the preceding section. The exporter is a monopoly in the home and in the foreign market. Hence, there is no ex-ante contract $< p_0, q_0 >$. The exporting monopolist selects ex-ante the total level of production Q and after the realization of the exchange rate he allocates output across markets. The exporting monopolist maximizes,

$$\max_Q E[R(Q - q_1) + \tilde{e}R^*(q_1) - C(Q)] \tag{16}$$

The optimal level of production is determined by

$$E[R'(Q - q_1)] = C'(Q) \tag{17}$$

and ex-post the firm chooses q_1 to equalize marginal revenues,

$$R'(Q - q_1) = eR^{*'}(q_1) \ \ for\, all\, e. \tag{18}$$

Let $< \hat{Q}, \hat{q}_1 >$ be the solution to the monopoly case examined above.

We turn now to compare the solution of the Nash bargaining problem stated in the previous section with the monopoly case.

Proposition 1. *The exporting firm's total production and the expected level of exports are larger when sales in the foreign market are conducted by a local importer rather than directly by the exporter. That is, $Q^* > \hat{Q}$ and $E[q_1^*] > E[\hat{q}_1]$.*

Proof: We first show that the exporter's profit function in the case where renegotiation takes place, $\pi^B(\cdot)$, is an increasing function of Q when evaluated at $\hat{Q}$. It is simpler to establish this for a fixed level of $q_0 = q_0^*$. Using equation (10) we can write the exporter's profit function as

$$\pi^B(Q, q_0^*) = \frac{1}{2}E[R(Q-q_1(Q,\tilde{e}))+R(Q-q_0^*)+\tilde{e}R^*(q_1(Q,\tilde{e}))-\tilde{e}R^*(q_0^*)+p_0q_0^*-C(Q)]$$

Then,

$$2\frac{d\pi^B}{dQ} = E[R'(Q-q_1(Q,e))(1-\frac{dq_1}{dQ}) + R'(Q-q_0^*) + eR^{*'}(q_1(Q,\tilde{e})\frac{dq_1}{dQ} - C'(Q)]$$

Using (18) the above expression simplifies to

$$2\frac{d\pi^B}{dQ} = E[R'(Q-q_1(Q,\tilde{e})) + R'(Q-q_0^*) - C'(Q)]$$

Hence, $\pi^B(\cdot)$ is concave in Q since

$$\frac{d^2\pi^B}{dQ^2} = E[R''(Q-q_1(Q,\tilde{e}))(1-\frac{dq_1}{dQ})] + R''(Q-q_0^*) - C''(Q) < 0.$$

To prove that total output is larger when foreign sales are

conducted by the importer suppose the contrary holds, that $Q^* < \hat{Q}$. The concavity of $\pi^B(Q, q_0^*)$ in conjunction with (18) implies that

$$E[R'(\hat{Q} - q_1(\hat{Q}, \tilde{e})) + R'(\hat{Q} - q_0^*) - C'(\hat{Q})] < 0.$$

Applying again the first order condition (17), it follows that $R'(\hat{Q} - q_0^*) < 0$, which is a contradiction. Thus, we have established the first part of *Proposition 1*, that $Q^* > \hat{Q}$.
To establish the second part of the proposition we use (18) to obtain

$$\frac{dq_1}{dQ} = \frac{R''(Q - q_1)}{R''(Q - q_1) + R^{*''}(q_1)} \tag{19}$$

Since $0 < \frac{dq_1}{dQ} < 1$ for all e, and since $Q^* > \hat{Q}$ it follows that $q_1(Q^*, e) > q_1(\hat{Q}, e)$ for all e. Hence, $E[q_1(Q^*, e)] > E[q_1(\hat{Q}, e)]$. This completes the proof of *Proposition 1*. □

The intuitive explanation of the result stated in *Proposition 1* is the following: by expanding production beyond the level produced in the benchmark case (monopoly in both markets) the exporter confronts himself with a trade-off between a reduction in the marginal profitability from domestic sales and an increase in profitability from sales to the importing firm. Specifically, by increasing Q the exporter strengthens his bargaining power at the ex-post renegotiation stage. This is reflected in the larger total payment that is extracted from the importer, since $y_1(Q)$ increases at $Q = \hat{Q}$.

IV. No Renegotiation

In the preceding analysis we presumed that the exporter and the importer can, if it is desirable to both parties, contract ex-ante for some $< p_0, q_0 >$. In order to understand the implications of the possibility to renegotiate this contract ex-post, we shall derive the level of production, the terms of trade and the volume of trade

when renegotiation is not allowed.
It is assumed now that the sequence of events is the same as in the case of renegotiation save the last stage.

The importer has to decide ex-ante what quantity to purchase given the per unit price p_0. Behaving as a follower, in a leader-follower relationship, he is chooses q_0 according to:

$$\max_{q_0} E[R^*(q_0) - \frac{p_0 q_0}{\tilde{e}}]$$

This yields:

$$\bar{e}R^{*'}(q_0) = p_0 \qquad (20)$$

Taking into account (20), that the quantity to be demanded by the importer depends on the price p_0, the exporter maximizes,

$$\max_{Q,p_0}[R(Q - q_0) + \bar{e}q_0 R^{*'}(q_0) - C(Q)] \qquad (21)$$

This leads to the following first order conditions:

$$R'(Q - q_0) = C'(Q) \qquad (22)$$

$$R'(Q - q_0) - \bar{e}R^{*'}(q_0) = \bar{e}q_0 R^{*''}(q_0) \qquad (23)$$

Combining (22) and (23) we reach,

$$\bar{e}[R^{*'}(q_0) + q_0 R^{*''}(q_0)] = C'(Q) \qquad (24)$$

Let $< \bar{Q}, \bar{q}_0 >$ be the solution to the maximization problem stated in (21).

Now we turn to the comparison between the exporter's total output and level of exports with and without renegotiation. The outcome of this comparison hinges on the properties of the domestic and foreign marginal revenue functions.

Proposition 2. *When the domestic marginal revenue function, $R'(\cdot)$ is convex* *(or linear) and the foreign marginal revenue function, $R^{*'}(\cdot)$ is linear the total output is larger in the case where there is renegotiation. Moreover, in this case $Q^* > \hat{Q} > \bar{Q}$.*

Proof: First we show that in the benchmark case, (monopoly in both markets) the total output and the level of exports are higher than when renegotiation is ruled out.

Suppose to the contrary that $\hat{Q} < \bar{Q}$. From (18) and the assumption that $R'(\cdot)$ is convex we obtain:

$$C'(\bar{Q}) > C'(\hat{Q}) \geq R'(\hat{Q} - E[q_1(\hat{Q}, \tilde{e}))])$$

The above chain of inequalities is due to the fact that $C'(Q)$ is increasing in Q and $R'(\cdot)$ is convex.

Using (22) we obtain that $R'(\bar{Q} - \bar{q}_0) > R'(\hat{Q} - E[q_1(\hat{Q}, \tilde{e})])$, which implies that $\bar{Q} - \bar{q}_0 < \hat{Q} - E[q_1(\hat{Q}, \tilde{e})]$ and thus $\bar{q}_0 > E[q_1(\hat{Q}, \tilde{e})]$. On the other hand, using equation (19) we find that

$$\begin{aligned} C'(\hat{Q}) = E[R'(\hat{Q} - q_1(\hat{Q}, \tilde{e}))] = E[\tilde{e}R^{*'}(q_1(\hat{Q}, \tilde{e})] = \\ E[\tilde{e}]E[R^{*'}(q_1(\hat{Q}, \tilde{e}))] + cov(\tilde{e}, R^{*'}(q_1(\hat{Q}, \tilde{e}))) \end{aligned} \tag{25}$$

For the remainder of the proof of this proposition we shall assume that the linear foreign marginal function is, $R^{*'}(\cdot) = \alpha - \beta q$. Making use of (22) and (23) in conjunction with the linear specification of the foreign marginal revenue function we obtain

$$C'(\bar{Q}) = \bar{e}(\alpha - \beta\bar{q}_0) - \bar{e}\beta\bar{q}_0 \tag{26}$$

Furthermore,

$$cov(\tilde{e}, \alpha - \beta q_1(\hat{Q}, \tilde{e})) = -\beta cov(\tilde{e}, q_1(\hat{Q}, \tilde{e})).$$

Since $E[\tilde{e}]E[q_1(\hat{Q}, \tilde{e})] \geq cov(\tilde{e}, q_1(\hat{Q}, \tilde{e})) > 0$, and the assumption that $\bar{q}_0 > E[q_1(\hat{Q}, \tilde{e})]$ it follows that $-\beta cov(\tilde{e}, q_1(\hat{Q}, \tilde{e})) > -\beta\bar{e}\bar{q}_0$. In addition, since $E[\tilde{e}] \geq \bar{e}$, we obtain

$$E[\tilde{e}]E[R^{*'}(q_1(\hat{Q},\tilde{e}))] = E[\tilde{e}R^{*'}(E[q_1(\hat{Q},\tilde{e})])] > \bar{e}R^{*'}(\bar{q}_0)$$

Hence from (25) and (26) we reach the inequality $C'(\hat{Q}) > C'(\bar{Q})$. This in turn implies that $\hat{Q} > \bar{Q}$, which is a contradiction. This completes the proof that $\bar{Q} < \hat{Q}$. Together with the results stated in *Proposition* 1 and under the assumptions made in *Proposition* 2, this leads to the following ranking of the level of total production and exports $Q^* > \hat{Q} > \bar{Q}$ and $q_1^* > \hat{q}_1$ for all realizations of the exchange rate.□

This ranking however, will be different under different assumptions about the marginal revenue functions. In particular, the ranking of the output produced in the benchmark case and in the case of no-renegotiation may be reversed.

Proposition 3. *(i) If the domestic marginal revenue function $R'(\cdot)$ is (strictly) concave and $qR^{*'}(q)$ is concave then: total output is higher under no renegotiation, i.e., $\bar{Q} > \hat{Q}$ and exports are lower under no renegotiation, i.e., $\bar{q}_0 < E[\hat{q}_1]$.*
(ii) If the domestic marginal revenue function $R'(\cdot)$ is linear and $qR^{*'}(q)$ is (strictly) convex then $\bar{Q} < \hat{Q}$ while exports are larger under no renegotiation $\bar{q}_0 > E[\hat{q}_1]$.

Proof: Assume first that the conditions stated in part (ii) of *Proposition 3* hold. Recall that the optimality conditions for profit maximization in the no-renegotiation case and in the benchmark case respectively are $C'(\bar{Q}) = R'(\bar{Q}-\bar{q}_0)$ and $C'(\hat{Q}) = E[R'(\hat{Q} - q_1(\hat{Q},\tilde{e}))]$. Suppose to the contrary that $\bar{Q} > \hat{Q}$. Then $C'(\bar{Q}) > C'(\hat{Q})$ and by using (18) we get

$$\begin{aligned} R'(\bar{Q}-\bar{q}_0) > E[R'(\hat{Q}-\hat{q}_1(\hat{Q},\tilde{e}))] = \\ E[\tilde{e}R^{*'}(q_1(\hat{Q},\tilde{e}))] > E[\tilde{e}]R^{*'}(E[q_1(\hat{Q},\tilde{e})]) > \bar{e}R^{*'}(E[q_1(\hat{Q},\tilde{e})]). \end{aligned} \tag{27}$$

The above inequalities stem from the strict convexity of $qR^{*'}(q)$ and $E[\tilde{e}] > \bar{e}$.

Therefore,

$$R'(\bar{Q}-\bar{q}_0)-\bar{e}R^{*'}(E[q_1(\hat{Q},\tilde{e})])>0 \tag{28}$$

From equation (23) we know that

$$R'(\bar{Q}-\bar{q}_0)-\bar{e}R^{*'}(\bar{q}_0)\leq 0 \tag{29}$$

Expressions (28) and (29) imply that $E[q_1(\hat{Q},\tilde{e})]>\bar{q}_0$. Due to the linearity of $R'(\cdot)$ we also have $C'(\bar{Q})=R'(\bar{Q}-\bar{q}_0)>C'(\hat{Q})=R'(\hat{Q}-E[q_1(\hat{Q},\tilde{e})])$. Hence, $\bar{Q}-\bar{q}_0<\hat{Q}-E[q_1(\hat{Q},\tilde{e})]$. The last inequality contradicts the presumption that $\bar{Q}>\hat{Q}$. We will show that it also contradicts that $\bar{q}_0>E[q_1(\hat{Q},\tilde{e})]$. To establish this note that $R'(\hat{Q}-E[\hat{q}_1])>R'(\bar{Q}-\bar{q}_0)$, and hence $\hat{Q}-E[\hat{q}_1]<\bar{Q}-\bar{q}_0$.

We turn now to part (*i*) of the proposition. Assume that the conditions stated in (*i*) hold. The assumption that $R^{*'}(\cdot)$ is linear yields:

$$C'(\hat{Q})=E[R'(\hat{Q}-q_1(\hat{Q},\tilde{e}))]=E[\tilde{e}R^{*'}(q_1(\hat{Q},\tilde{e}))]\geq E[\tilde{e}]R^{*'}(E[q_1(\hat{Q},\tilde{e}]) \tag{30}$$

and

$$C'(\bar{Q})=R'(\bar{Q}-\bar{q}_0)\leq\bar{e}R^{*'}(\bar{q}_0) \tag{31}$$

To arrive at (31) we made use of (23) and the assumption that $R^{*''}(\cdot)\leq 0$. Given the assumption that $R'(\cdot)$ is (strictly) concave, this leads to $C'(\hat{Q})<R'(\hat{Q}-E[q_1(\hat{Q},\tilde{e}])$. Hence, together with (31) this inequality leads to

$$\hat{Q}>\bar{Q}\ \text{, implies}\ \hat{Q}-E[q_1(\hat{Q},\tilde{e})]<\bar{Q}-\bar{q}_0\ \text{and}\ E[q_1(\hat{Q},\tilde{e})]>\bar{q}_0. \tag{32}$$

But from (30) and (31) we also conclude that

$$E[q_1(\hat{Q},\tilde{e})]<\bar{q}_0\ ,\ \text{implies that}\quad \hat{Q}>\bar{Q}. \tag{33}$$

To show that the inequality in (33) holds note that $R^{*'}(E[\hat{q}_1])>R^{*'}(\bar{q}_0)$ which implies that $E[\tilde{e}]R^{*'}(E[\hat{q}_1])>\bar{e}R^{*'}(\bar{q}_0)$. The last inequality implies that $C'(\hat{Q})>C'(\bar{Q})$ which in turn implies that $\hat{Q}>\bar{Q}$. However, from (32) and (33) we have a contradiction if $\hat{Q} > \bar{Q}$. Thus, we must have that $\bar{Q} > \hat{Q}$. Furthermore, since $C'(\bar{Q}) > C'(\hat{Q})$, by using (30) and (31) we obtain that $E[\tilde{e}]R^{*\prime}(E[\hat{q}_1]) < \bar{e}R^{*\prime}(\bar{q}_0)$, hence $E[\hat{q}_1] > \bar{q}_0$.

□

The results stated in *Propositions 1-3* indicate that exchange rate uncertainty affects the level of exports in quite different and opposite ways, depending on the mode of sales in the foreign markets. When sales in the foreign markets are undertaken by a a local firm, the importer, rather than directly by the exporting firm and when bargaining determines the terms of trade between the two parties, the level of international trade is higher when the exchange rate is uncertain. The ability to renegotiate initial contracts endows both firms with some flexibility which is conducive to larger transactions than would have taken place when renegotiation of the initial contracts are not allowed. This flexibility stems from the ability to allocate output across markets in view of the value that the exchange rate takes. The initial contract which is signed ex-ante, is a commitment which also has desirable properties since it enables the exporter to produce a higher level of output than would have been produced otherwise. This was shown in *Proposition 3.*

In the next proposition we consider the case where the marginal revenue function is linear in both markets.

Proposition 4. *Assume that $R'(\cdot)$ and $R^{*\prime}(\cdot)$ are linear. Total output is lowest when there is no renegotiation it is intermediate in the monopoly case and is highest when there is renegotiation of the initial contract, i.e., $\overline{Q} = \hat{Q} < Q^*$. The level of exports is in the same order as the level of output, i.e., $\bar{q}_o = E[\hat{q}_1] < E[q_1^*]$.*

Proof: The proof of *Proposition 4* follows from the proof of *Propositions 2 and 3.* Assume that $R^{*\prime}(\cdot)$ is linear. According to *Proposition 2* when $R'(\cdot)$ is strictly convex we have $\bar{Q} < \hat{Q} < Q^*$. On the other hand, from *Proposition 3* part (i) if $R'(\cdot)$ is strictly concave, the linearity of $R^{*\prime}(\cdot)$ implies that $qR^{*\prime}(q)$

is concave. Together this yields that $\bar{Q} > \hat{Q}$. For the case of linear domestic marginal revenue functions, the results stated in *Propositions 2 and 3* are not contradictory if $\bar{Q} = \hat{Q}$.

To prove that the ranking of the level exports is the same as the ranking of the level of production, we use the fact that $\bar{Q} = \hat{Q}$ and we get

$$R'(\bar{Q} - \bar{q}_0) = C'(\bar{Q}) = C'(\hat{Q}) = E[R'(\hat{Q} - \hat{q}_1)] = R'(\hat{Q}) - E[\hat{q}_1]$$

The above equalities yield $\bar{q}_0 = E[\hat{q}_1]$.
□

V. Increasing Volatility

We now turn to examine the effects of increased exchange rate volatility on the firms total production and exports. In particular we investigate the effects of a mean preserving spread increase (*MPS*) of the exchange rate in the benchmark case and compare with the case of no-renegotiation.
For the exporting monopoly we have:

Proposition 5. *Assume that the foreign demand is linear. A mean preserving spread increase of the exchange rate induces the monopolist exporter to: (i) increase production and increase expected exports if the domestic marginal revenue $R'(\cdot)$ is convex, and (ii) lower total output and lower expected exports when the domestic marginal revenue is "sufficiently" concave.*

Proof: First we show that when condition (18) holds $q_1(Q, e)$ is a strictly concave function in e when $R'(Q - q_1)$ is concave and $R^{*\prime}(q_1)$ is linear. Differentiation of (18) with respect e yields:

$$-R''(Q - q_1)\frac{dq_1}{de} = R^{*\prime}(q_1) + eR^{*\prime\prime}(q_1)\frac{dq_1}{de}$$

Hence,

$$\frac{dq_1}{de} = \frac{-R^{*'}(q_1)}{eR^{*''}(q_1) + R''(Q - q_1)} > 0 \tag{34}$$

Differentiating again with respect to e yields

$$\frac{d^2q_1}{de^2} = \frac{1}{\gamma^2}[-\gamma R^{*''}(q_1)\frac{dq_1}{de} + R^{*'}(q_1)(R^{*''}(q_1) - R'''(Q - q_1)\frac{dq_1}{de}] \tag{35}$$

where $\gamma = eR^{*''}(q_1) + R''(Q - q_1)$. Since $\frac{dq_1}{de} > 0$, $R'''(Q - q_1) \geq 0$ and $R^{*''}(Q - q_1) < 0$ it follows that $\frac{d^2q_1}{de^2} < 0$. Thus $R'(Q - q_1))$ is a convex function of e, which implies that a mean preserving spread increase in e, which results in a new random exchange rate $\hat{e}$, leads to a higher expected domestic marginal revenue, $E[R'(Q - q_1(Q, \hat{e}))]$. Substituting $\hat{e}$ for $\tilde{e}$ in (18), while keeping $\hat{Q}$ at the optimal level prior to the *MPS* in e, we obtain

$$E[R'(\hat{Q} - q_1(\hat{Q}, \hat{e})] - C'(\hat{Q}) > 0 \tag{36}$$

To restore the equality in (36) it is required that the solution $Q > \hat{Q}$ since the left hand side in (36) is a strictly concave function of Q. This proves part (i) of the proposition.

To prove part (ii) assume that $R'(Q - q_1)$ is sufficiently concave as stated in the proposition. Thus $-R'''(Q - q_1)$ has a large positive value and from (34) it follows that $\frac{dq_1^2}{de^2} > 0$; that is, $q_1(\cdot, e)$ is convex in e. Consequently, $R'(Q - q_1(Q, e))$ is a strictly concave function in e. Again using (18), a *MPS* in e leads to

$$E[R'(\hat{Q} - q_1(\hat{Q}, \hat{e})) - C'(\hat{Q}] < 0. \tag{37}$$

As before the concavity of the maximand in (16) implies that the optimal Q is lower than $\hat{Q}$. This proves part (ii) of the proposition.

□

In the case of no-renegotiation an increase in exchange rate volatility has an unambiguous effect on total production and exports.[4] Let us define $m^*(q) = \frac{-qR^{*''}(q)}{R^{*'}(q)}$, which is the elasticity of the foreign marginal revenue. Note that when the foreign marginal revenue is linear, $m^*(q)$ is a strictly increasing function.

Proposition 6. *Assume that the foreign marginal revenue function is concave (linear) and that $m^*(q)$ is non-decreasing. In the case of no-renegotiation a MPS in the exchange rate leads to lower total output and lower exports.*

Proof: Recall from (22) – (24) that $< (\bar{Q}, \bar{q}_0) >$ is the solution of the no-renegotiation case. Since a *MPS* in e results in a lower $E[\frac{1}{e}]$, from (22) we obtain that domestic sales decrease as total output becomes larger, i.e.,

$$\frac{d}{dQ}[Q - q_0] = \frac{C''(Q)}{R''(Q - q_0)} \leq 0$$

Consequently, $\frac{dq_0}{dQ} \geq 0$. Now suppose by way of negation, that $\bar{Q}$ increases as the exchange rate becomes more volatile. Hence, $\bar{q}_0(\hat{e}) > \bar{q}_0(\tilde{e})$, whereas $[\bar{Q}(\hat{e}) - \bar{q}_0(\hat{e})]$ declines. From (23) we get

$$R'(Q - q_0) = \bar{e}R^{*'}(q_0) + \bar{e}q_0 = \bar{e}R^{*'}(q_0)[1 - m^*(q_0)] \tag{38}$$

Since $[1 - m^*(q_0)]$ is non-increasing, $R^{*'}(q_0)$ is decreasing in q_0, and $\bar{e}$ declines with a *MPS* increase in e, the right hand side of equation (38) decreases. On the other hand, the left hand side of the same equation increases as Q increases since $R'(Q - q_0)$ is decreasing. Hence the above equation cannot hold. This implies that the supposition that we started with, namely $\bar{Q}$ increase with a *MPS* increase cannot occur. In addition q_0 must also decline.

□

[4] This result is obtained without imposing any restrictions the domestic marginal revenue function.

Finally, we turn to examine the effects of an increase in volatility in the case where the exporter and the importer renegotiate the initial contract. An increase in volatility increases the likelihood that an initial contract will not be signed. From (15) we observe that as e becomes sufficiently volatile, in the sense of a *MPS* increase, the expression Δ becomes negative. Thus the equality in (15) is replaced by

$$-R'(Q^* - q_0^*) + \Delta R^{*'}(q_0^*) + 2q_0^*\hat{e}R^{*''}(q_0^*) < 0 \tag{39}$$

Hence, the optimal initial quantity of exports is $q_0^* = 0$. When $q_0^* = 0$ the first order condition (14) becomes

$$E[R'(Q^* - q_1(Q^*, \hat{e}))] + R'(Q^*) - 3C'(Q^*) = 0 \tag{40}$$

Thus for the case when $q_0^* = 0$, the conclusions that we reached for the benchmark case hold for this case as well. That is when the domestic marginal revenue is convex and the foreign marginal is linear total production and the expected level of exports increase when the exchange rate becomes more volatile.

We now consider the case where the volatility is not too high so that the optimal $q_0^* > 0$. It turns out that in this case the conclusion regarding the optimal output and expected exports are ambiguous. The ambiguity arises because when we substitute $\hat{e}$ for $\tilde{e}$ in (15) the left hand side may take either a positive or a negative value. Furthermore, the left hand side in (14) may take a positive value when $R'(\cdot)$ is strictly convex. Consequently, we cannot determine whether the total output is increasing or is decreasing with a *MPS* in the exchange rate when renegotiation of the initial contract is possible.

The analysis above reveals that the impact of an increase in exchange rate volatility on international transactions differs across the three modes of exporting. Specifically, in the benchmark case, the monopolist expands production and foreign sales, whereas when the foreign sales are conducted by a local importer and there is *no renegotiation* of the initial contract, both production and exports decline. In the monopoly case, there is some flexibility since the allocation of sales across markets takes place after the exchange rate is known. In the case of no-renegotiation this flexibility is absent and thus has an adverse impact on production and exports. In the case when contracts are renegotiated the impact of increased exchange rate volatility is ambiguous. This can be understood by noting that renegotiation of the initial contract, comprises a hybrid of the aspects discussed above; the adverse effect of the no renegotiation and the flexibility of allocating output after knowing the exchange rate.

VI. Concluding Remarks

In this paper we investigated the effects of exchange rate uncertainty on the terms of transactions, price and quantity traded, given that both the exporter and the importer have some bargaining power. We highlighted the differences that arise in prices and the volume of trade in the case where the firms negotiate the contract versus the case where the exporter himself conducts the foreign sales directly, and thus bypasses the importing firm. The former case corresponds to a non-integrated firm and the latter corresponds to a vertically integrated firm.

According to *Propositions 3 and 4* the exporting monopolist who sells in its product domestically and in the foreign market, may produce more, less or the same level of total output and may sell more, less or the same amount in the foreign market relative to the case where the foreign sales are conducted by an importing firm. The outcome depends upon the properties of the marginal revenue functions in each market.

These results differ significantly from the standard result in the literature of economics of vertical/horizontal integration.[5] Our benchmark case corresponds to a vertically integrated firm while the case of no-renegotiation corresponds to the non-integrated relationship between an up-stream and a down stream firm. Due to double marginalization, it is argued that the non-integrated industry will underproduce and sell too little of the final good relative to the integrated firm.

[5]See Tirole (1988) for an extensive analysis and discussion of the theory of vertical integration.

This distortion is further exacerbated if uncertainty about demand or cost of production is prevalent. We identified conditions which lead to opposite results; the non-integrated firm may produce more and sell more than the integrated firm. There are, however, some differences between our model and the basic set up used in the literature dealing with the economics of vertical integration. In our case the monopolist sells its final good in two separate markets, a domestic and a foreign market. Uncertainty about price exists in one market only, the foreign market. Consequently, in the case of no-renegotiation, the exporting firm sets the price ex-ante, before the value of the exchange rate is known, and thus commits itself to a deterministic quantity to be delivered to the importer. In the case where the monopolist sells by itself the product in the foreign market, i.e., the intermediary/importer is absent, the firm does not commit to sell any particular quantity before the exchange rate is known. Hence, it has the flexibility to allocate sales across markets in accordance with the value that exchange rate takes. Depending on the curvature of the marginal revenue functions (linear domestic marginal revenue and a concave foreign marginal revenue) the flexibility in the distribution of output across markets may be conducive to larger exports and thus to a larger of total output. When both marginal revenue functions are linear the integrated monopoly does not have any advantage over the non- integrated monopoly and hence produces and sells the same quantities as the latter.

References

Baron, D., (1976), "Fluctuating Exchange Rates and Pricing of Exports", *Economic Inquiry*, 425-438.

Clark, P. B., (1973), "Uncertainty Exchange Risk, and the Level of International Trade", *Western Economic Journal*, 11, 302-313.

Cushman, D., (1983), "The Effects of Real Exchange Rate Risk on International Trade", *Journal of International Economics*, 15, 45-63.

De Grawe, P., (1988), "Exchange Rate Variability and the Slow-Down of Growth in International Trade", *IMF Staff Papers*, 25, 63-84.

Donnenfeld, S., and I. Zilcha, (1991), "Pricing of Exports and Exchange Rate Uncertainty", *International Economic Review*, 1009-1022.

Gagnon, J., (1992), "Exchange Rate Volatility and the Level of International Trade", *Journal of International Economics.*

Giovannini, A., (1989), "Exchange Rates and Traded Goods Prices", *Journal of International Economics*, 45-68.

Knetter, M., (1989), "Price Discrimination by U.S. and German Exporters", *American Economic Review*, 198-210.

Magee, S., (1973), "Currency Contracts Pass-through and Devaluation", *Brookings Papers in Economic Activity*, 303-325.

Magee, S., (1974), "US Import Prices and the Currency Contract Period", *Brookings Papers in Economic Activity*, 117-164.

Mckinnon, R., (1979), it Money in International Exchange: The Convertible Currency System, (Oxford University Press).

Wood, G., (1991), "Valuation Effects, Currency Contract Impacts and the J-Curve:Empirical Estimates", *The Australian Economic Review*, 148-163.

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH
List of Recent Working Papers*

4.92	Arthur Fishman	The Durability of Information, Market Efficiency and the Size of Firms: Search with Repeated Transactions
5.92	Michele Piccione Ariel Rubinstein	Finite Automata Play a Repeated Extensive Game
6.92	Gil Bufman Leonardo Leiderman	Currency Substitution under NonExpected Utility Some Empirical Evidence
7.92	Arthur Fishman Neil Gandal	Experimentation in a Duopoly
8.92	Oz Shy	A Welfare Evaluation of Anti–Dumping Rules
9.92	Gideon Fishelson	Mortgage Market in Israel: 1950–1990 (Hebrew)
10.92	Arthur Fishman	The Informative Role of Prices in Markets with Endogenously Informed Buyers
11.92	Efraim Sadka	International Capital Flows: Efficiency of Savings and Investments
12.92	Elhanan Helpman	Innovation, Imitation, and Intellectual Property Rights
13.92	Zvi Eckstein and Kenneth I.Wolpin	Duration to First Job and the Return to Schooling: Estimates from a Search–Matching Model.
14.92	Adam B.Jaffe Manuel Trajtenberg Rebecca Henderson	Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations
15.92	Manuel Trajtenberg Rebecca Henderson Adam B. Jaffe	Ivory Tower Versus Corporate Lab: An Empirical Study of Basic Research and Appropriability
16.92	Timothy F.Bresnahan Manuel Trajtenberg	General Purpose Technologies: "Engines of Growth?"

*The list of the Sackler Institute recent working papers follows this list. Copies of the working papers or a complete working paper list of the two Institutes can be obtained from: Mrs.Stella Padeh, The Foerder Institute for Economic Research, Tel–Aviv University, Tel–Aviv, 69978 Israel. Fax: 972–3–640–99082

17.92	Elhanan Helpman Leonardo Leiderman	Israel's Exchange Rate Band
18.92	Neil Gandal	Hedonic Price Indexes for Spreadsheets and an Empirical Test of the Network Externalities Hypothesis
19.92	Eitan Berglas David Pines	Transaction Costs in a Tiebout Setup
20.92	Gideon Fishelson	Endowment Effect and Market Effect: Revisited
21.92	Gene M.Grossman Elhanan Helpman	Protection for Sale
22.92	Eyal Sulganik	On the Structure of Blackwell's Equivalence Classes Of Information Systems
1.93	Gene M.Grossman Elhanan Helpman	Trade Wars and Trade Talks
2.93	Neil Gandal David J.Salant	Hollygopoly: Oligopolistic Competition for (Hollywood) Movies
3.93	Eran Yashiv	On Reputation Formation in Monetary Policy
4.93	Assaf Razin Efraim Sadka	Interactions Between International Migration and International Trade: Positive and Normative Aspects
5.93	David T.Coe Elhanan Helpman	International R&D Spillovers
6.93	Torkild Byg	Simultaneous Bargaining with Two Potential Partners
7.93	Gideon Fishelson	Performance of Mutual Funds (Hebrew)
8.93	Chaim Fershtman Kevin M.Murphy and Yoram Weiss	Social Status, Education and Growth
9.93	Neil Gandal Katharine Rockett	Licensing a Sequence of Innovations
10.93	Gideon Fishelson	Political Events and Economic Trends
11.93	Neil Gandal	Compatibility Standards and Complementary Network Externalities in the PC Software Market
12.93	Gene M.Grossman Elhanan Helpman	Endogenous Innovation in the Theory of Growth
13.93	Shabtai Donnenfeld Itzhak Zilcha	Bargaining in International Trade under Exchange Rate Uncertainty

THE SACKLER INSTITUTE OF ECONOMIC STUDIES
List of Recent Working Papers

4.93	Gil Bufman Leonardo Leiderman	Israel's Stabilization: Some Important Policy Lessons
5.93	Oded Hochman David Pines	Tiebout Setup as a System of Spatial–Clubs Agglomerations
6.93	Torkild Byg	Contracts Under Non–Verifiability
7.93	Chaim Fershtman	On the Value of Incumbency: Managerial Reference Point and Loss Aversion
8.93	Edi Karni Itzhak Zilcha	Technological Progress and Income Inequality: A Model with Human Capital and Bequest
9.93	Jacob Glazer Ariel Rubinstein	Simplicity of Solution Concepts: Subgame Perfect Equilibrium in Extensive Games vs. Iteratively Undominated Strategies in Normal Games
10.93	Chaim Fershtman Ehud Kalai	Unobserved Delegation
11.93	Henrik Horn Oz Shy	Local Services and International Market Integration
12.93	Zvi Hercowitz Nirit Kantor Leora Rubin Meridor	Immigration and Growth Under Imperfect Capital Mobility: The Case of Israel
13.93	Aba Schwartz Menachem Gotlibivski	The Evolution of Wage and Employment in the Israeli Economy 1986–1991
14.93	Ariel Rubinstein	Why are Linear Orderings So Common in Natural Language?
15.93	Elchanan Ben Porath and Michael Kahneman	Communication in Repeated Games with Private Monitoring
16.93	Zvi Eckstein and Ron Shachar	Correcting for Errors in Retrospective Data
17.93	Ariel Rubinstein Amos Tversky	Naive Strategies in Zero–sum Games