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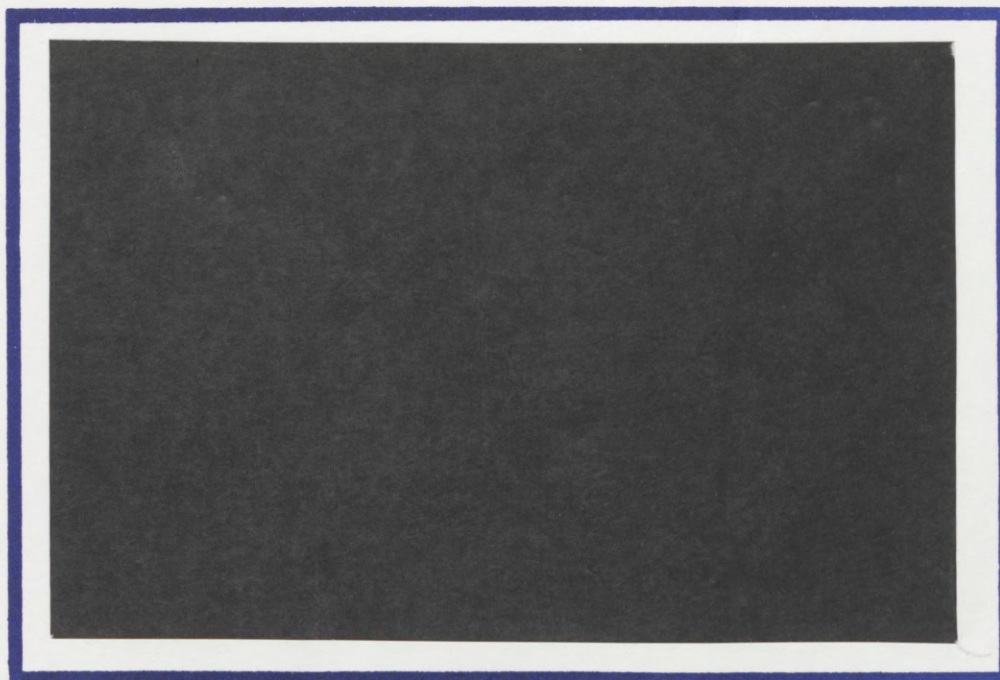
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THE EFFECTS OF COMPULSORY SCHOOLING ON GROWTH
INCOME DISTRIBUTION AND WELFARE

by

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1. INTRODUCTION

Compulsory basic education is taken for granted in modern societies. The principle of placing all children in school for a specified period of time, which began in the eighteenth and nineteenth centuries was later adopted by European governments.¹ During this century and, in particular, after the second World War all countries adopted the practice of compulsory education (OECD, 1983). The exact form of compulsory education varies across countries, but despite differences in most other areas there is a unanimous acceptance of this policy as an essential basic public service. The compulsory education law is widely observed and enforced in all countries, and the main part of public expenditures for education is devoted to the institutions providing education for children.

Why is it that compulsory education is widely accepted as an important basic public service? Why most of the public support to education is provided by this institution and not by a tax-transfer method even in economies where intervention by the state avoids compulsory rules?²

All the classical economists (Smith, Malthus, Bentham and J.S.Mill, see West (1970, pp.111–112)) claimed that an increase in the education level of the poor would result in a decrease in crime and disorder (negative externality)³ and since the state

¹ Frederic the Great of Prussia and Maria Theresa of Austria started compulsory elementary education as early as 1763 and 1773, respectively (Melton, 1988). In England compulsory education was imposed by the Elementary Education Act of 1870 (West (1970)).

² Angrist and Krueger (1990, 1991) have found that compulsory schooling in the U.S. is effective so that a significant larger fraction of students were constrained to enter school early and dropout was delayed such that the duration of school attendance was increased due to compulsory school laws.

³ M. Friedman (1962) emphasized the need of a minimum level of education for the better functioning of a democratic society.

imposes the law and order, a cost benefit calculation can justify the introduction of compulsory education. The classical economists realized the economic gains from education but thought that the division of labor in the free market would internalize all the economic benefits.

As pointed out by J.S.Mill, in the case of children's education the principle of self interest breaks down since "the person most interested is not the best judge of the matter", and it is not clear that parents make the best judgement for their children (West, 1970, pp.7-11). Mill, however, thought that "a general state education is ... moulding people to be exactly like one another", and given his support for variety and liberty he rejected the idea of compulsory state education (West, 1970, p.124).

However, the argument that the state should protect children from parents who do not value education has been viewed as a sufficiently negative externality to justify the law of compulsory minimum education. Educators emphasize the principle of "equal opportunities" for children, which seems to us to stem from both the need to protect children who otherwise would have received a very low level of education, and the social goal of a more equal distribution of income. This work provides some support to this view.

In this paper we consider an economy where heterogeneity among individuals in each generation is a consequence of the different preferences of parents with respect to the education level of their children. Parents' preferences depend on their child's level of education, or human capital, but are independent of the child's income or lifetime utility. On the other hand, the level of education obtained by each child has an important impact on his ability as a worker and in the continuation of the learning process (Becker, 1975). As a result, when parents are homogeneous (as in section 2), the education level is less than the optimal level. The first-best policy which corrects this negative externality constitutes

intergenerational taxes and transfers which would guarantee the optimal investment in children's education. This program involves the transfer of all of the children's income to their parents and a lump-sum tax to balance the children's and parents' incomes.⁴

To guarantee nondegenerate income distribution we assume that parents' preferences regarding their offspring's education are different⁵ hence, investment in their children's human capital differ. The only way to identify an individual is by observing his investment in his child's human capital. As a result, the implementation of such a policy would be 'too late'; furthermore, the actual tax-transfer program should be implemented specifically for each individual at each date. Therefore, such a policy cannot be considered seriously (high implementation costs) in an economy where individuals are heterogenous with respect to their preferences and their earnings (human capital).

Compulsory education can be viewed as a potential second best alternative. The fact that this particular method of intervention has been implemented in all countries independent of other aspects, such as the political environment, implies that it should

⁴ This policy is equivalent to a model that assumes that parents' utility function is an increasing function of the children's income (Becker and Tomes (1979) and Saint-Paul and Verdier (1990)). We assume that preferences of the current population depend only on variables that are directly influenced by the parents' decisions so that full neutrality of future policies is not attained (Bernheim and Bagwell (1988)).

⁵ Heterogeneity among parents' investment in their children's education is a major source for the observed income distribution (Becker and Tomes (1979) and Loury (1981)). In particular, optimal investment in children's education requires that parents take into consideration the income gain to their children due to their education and the ability of at least some parents to take loans in order to educate their children. The inability of the market to implement a contract in which children pay back the loans of their parents imply an underinvestment in human capital. A possible policy that seems natural for such an environment is that the government would enforce such contracts. Is such a policy less reasonable than a compulsory education? Note that we could introduce heterogeneity by assuming that only part of the population is not fully altruistic (Aiyagari (1989)). Hence only a few, randomly chosen, parents do not consider the marginal income gain to their children. As a result, the current investment in the education of these parents implies an important source of negative externality on the level of investment in human capital.

combine several positive elements that are widely shared by people. To investigate this observation we compare the dynamic allocation of our overlapping generation model without intervention to a model where compulsory elementary schooling is financed by a proportional tax on wage income.

We show that a certain minimum level of compulsory schooling, financed by a proportional tax rate on wage income, increases the aggregate output and, at the same time, reduces the level of inequality in income distribution. Furthermore, the majority of individuals in each generation are better off in the long run under some level of compulsory education, hence a wide public support for this policy is guaranteed as well.⁶ We show that compulsory education induces more investment in schooling and "improves" the human capital distribution. As a result, the output level increases at each date vis-a-vis an economy without this policy. Income distribution becomes more equal since the program involves a transfer of resources from the rich to the poor and eliminates the frequency of the population with very low investment in human capital. The median individual is better off since his/her income is affected by the equality aspects of this policy and the economy-wide additional growth. It seems to us that the externality embedded in parents' decisions combined with the heterogeneity of parents' preferences regarding the quality (human capital) of their children, provide the main argument for compulsory elementary education.

Recently we witness a renewed interest in income distribution, growth and the investment in human capital. Lucas (1988) and Azariadis and Drazen (1990) analyzed the role of the investment in human capital and externalities on the long run growth rate of the

⁶ Loury (1981) and Saint-Paul and Verdier (1990) have similar results using models with altruistic parents.

economy. We have adopted a similar function for the accumulation of human capital but we emphasize the role of the parents.⁷ Saint-Paul and Verdier (1991) analyze the relationship between public education, growth and income distribution in a model which is related to ours, but has different features and motivation. Persson and Tabellini (1991) provide some evidence that growth rates are positively associated with more equality in income in a cross section of nations. This evidence is consistent with their model of income distribution and growth as well as with most other recent papers on this subject. In a different framework Grossman and Helpman (1991) consider the role of aggregate "knowledge" and increasing returns on economic growth.

The paper is organized as follows. In the next section we discuss a benchmark model with a representative agent in order to show the inefficiency aspect of the equilibrium allocation and point out the optimal corrective policy. In section 3 we present the model with heterogenous population. The positive implications of compulsory education are discussed in section 4 and welfare implications in section 5. Section 6 concludes the paper and section 7 contains the proofs.

2. A BENCHMARK MODEL

Individuals live for two periods in an OLG economy. We assume that each individual works during her first period ("young") and only consumes in her second period, i.e., when "old". During the first period, i.e., when young, she gives birth to one offspring and thus has to allocate some of her time during this period (where endowment of time is

⁷ Perotti (1991) and Fernandez and Rogerson (1991) analyze the role of tax subsidies to education through a majority voting model in the determination of income distribution and growth.

1) to educating her offspring. This has a direct effect upon her child's human capital, or "knowledge", which is assumed to be a source of utility to the parent. The lifetime utility function of $i \in G_t$ (the t^{th} generation, i.e., all individuals born at date t) is $u_i: \mathbb{R}_+^4 \rightarrow \mathbb{R}^1$, $u_i(c_t^{yi}, c_{t+1}^{oi}, z_t^i, h_{t+1}^i)$, where c_t^y and c_{t+1}^o are the consumption when young (date t) and when old (date $t+1$), z_t^i represents the leisure, h_{t+1}^i represents the human capital of i 's offspring. Parents can affect the human capital of their offspring only by the time devoted in educating this child, hence we disregard random factors. We assume that each utility function u is strictly concave, increasing, continuously differentiable and $\frac{\partial u}{\partial c_j}(x) = \infty$ if $x_j = 0$, $j = 1, 2, 3$. The evolution process of human capital over time depends upon the parents level of human capital and the effort (measured in time) invested by them in raising their offspring. We assume that for some $\frac{1}{2} < \beta \leq 1$, the evolution of human capital is according to:

$$h_{t+1}^i = A(e_t^i)[h_t^i]^\beta \quad (1)$$

where e_t^i is the parents investment (in time) at education;⁸ where the function $A(\cdot)$ satisfies: $A(0) = \theta$, $0 < \theta \leq 1$, $A' > 0$, $A'' \leq 0$. We assume that there is no population growth, the set of individuals in each generation t is given by $[0, 1]$.

The aggregate level of human capital at each date t has a direct effect upon the production possibilities at that period. In particular we take

$$Y_t = F(K_t, L_t^*) \quad (2)$$

⁸This specification is similar to that of Loury (1981, and Azariadis and Drazen (1990).

to be the aggregate production function, where $L_t^* = \ell_t h_t$ is the effective aggregate labor and K_t the aggregate capital stock. $F(\cdot, \cdot)$ is assumed to exhibit constant returns to scale, it is strictly increasing, concave, continuously differentiable and satisfies: $F_1(0, L) = \infty$, $F_2(K, 0) = \infty$, $F(0, L) = F(K, 0) = 0$.

Our assumption basically provides some type of Harrod neutral technological progress due to accumulation in knowledge or human capital. We assume that for each individual $\ell_t = 1$, namely, labor supply is inelastic.⁹ Production at date t takes place by competitive firms who borrow capital at date $t-1$ and hire labor services at date t . The factor prices are given by the marginal product. Since the human capital of a worker is observable the wage payments will depend upon the effective labor supply of the worker, i.e., $W_t^i = w_t h_t^i$ where $w_t = F_2(K_t, L_t^*)$ is the wage rate. The competitive interest rate is $R_{t+1} = F_1(K_t, L_t^*)$.

Let us consider first an economy with homogeneous population. The optimal choice of the "young" in G_t is derived from maximizing,

$$\text{Max}_{s_t, e_t} u(c_t^y, c_{t+1}^o, z_t, h_{t+1}) \quad (3)$$

subject to,

$$c_t^y = w_t h_t - s_t \quad (4)$$

$$c_{t+1}^o = s_t R_{t+1} \quad (5)$$

$$z_t = 1 - e_t \quad (6)$$

$$h_{t+1} = A(e_t) h_t^\beta \quad (7)$$

⁹This assumption simplifies the model. A further simplification would be to ignore the accumulation of physical capital to emphasize the role of human capital. On the other hand it is possible to extend the model by introducing increasing returns to scale due to external effects of human and physical capital (see, e.g., Romer (1986) and Lucas (1988)).

After substituting the constraints the first order conditions with respect to s_t and e_t are,

$$-u_1(x_t) + R_{t+1}u_2(x_t) = 0 \quad (8)$$

$$\begin{aligned} -u_3(x_t) + u_4(x_t) A'(e_t) h_t^\beta &\leq 0 \\ &= 0 \text{ if } e_t > 0 \end{aligned} \quad (9)$$

where $x_t = [c_t^y, c_{t+1}^o, z_t, h_{t+1}]$.

It is clear that the optimal decision regarding the time invested by the representative consumer in the human capital of his children disregards the gain of the child from this investment. This is a result of our assumption that neither the child's income nor his utility enter the parent's objective function. Hence, the wage increase in the next period due to an additional investment in human capital does not affect the parents allocation of time.

Efficiency and Optimal Policy

In order to analyze the efficiency of the competitive allocations attained in the above economy we shall consider first the case of a planner (or a dynastic model) where the problem at $t = 0$ is to maximize the present value of a discounted stream of future utilities. Thus for some positive monotone decreasing sequence (λ_t) , $\sum_{t=0}^{\infty} \lambda_t < \infty$. Given k_0 and h_0 the objective of the planner is to maximize

$$\text{Max}_{\{c_t^o, e_t, k_{t+1}\}} \sum_{t=0}^{\infty} \lambda_t u(x_t) \quad (10)$$

s.t.: For $t = 0, 1, 2, \dots$

$$c_t^y + c_t^o + k_{t+1} = F(k_t, h_t) \quad (11)$$

$$z_t = 1 - e_t \geq 0 \quad (12)$$

$$h_{t+1} = A(e_t) h_t^\beta \quad (13)$$

By inserting the constraints (11)–(13) into (10) and differentiating with respect to k_{t+1} , c_t^o and e_t we obtain the first-order conditions for the planner's problem:

$$\lambda_{t+1} u_1(x_{t+1}) F_1(k_{t+1}, h_{t+1}) - \lambda_t u_1(x_t) = 0 \quad (14)$$

$$\lambda_t u_2(x_t) - \lambda_{t+1} u_1(x_{t+1}) = 0 \quad (15)$$

$$\begin{aligned} & -\lambda_t u_3(x_t) + \lambda_t u_4(x_t) A'(e_t) h_t^\beta + \\ & \lambda_{t+1} u_1(x_{t+1}) F_2(k_{t+1}, h_{t+1}) A'(e_t) h_t^\beta \leq 0 \end{aligned} \quad (16)$$

We can rewrite equations (14) to (16) by eliminating the λ 's, and assuming that $e_t > 0$ for all t , as follows:

$$u_1(x_t)/u_2(x_t) = F_1(k_{t+1}, h_{t+1}) \quad (17)$$

$$u_3(x_t) = u_4(x_t) A'(e_t) h_t^\beta + u_2(x_t) A'(e_t) h_t^\beta F_2(k_{t+1}, h_{t+1}) \quad (18)$$

Now we are ready to show that the competitive allocation is not optimal since parents underinvest in their children human capital. The reason for that is that they under-value the benefits of this investment by ignoring its impact upon the production function and the increase in the wages of the coming generation due to higher investment

in human capital.

Let us denote the competitive allocation by " $*$ ", and let the planner's optimum be denoted by a " $\bar{}$ " (bar).

PROPOSITION 1: The competitive equilibrium allocation is not optimal. Moreover, assuming that u is homothetic and that $u_{43} \geq 0$ imply that the level of e_t^* is below the Pareto optimal level \bar{e}_t for all t .

PROOF: For the competitive economy we derived condition (17) as in the optimal allocation case, while condition (9) can be written as (assuming e_t^* is positive for all t):

$$u_3(x_t^*)/u_4(x_t^*) = A'(e_t^*)h_t^{*\beta} \quad (19)$$

Homotheticity of u implies that $\frac{u_3(x_t)}{u_4(x_t)}$ does not depend explicitly on (c_t^y, c_{t+1}^0) .

Comparing (19) and (17) with conditions (18) and (17) we conclude that the two paths cannot coincide for any choice of (λ_t) . Now, assuming that $u_{43} \geq 0$ we can show that $\frac{u_3(x_t)}{u_4(x_t)}$ is increasing in e_t . Given k_0 and h_0 we find from (18) and (19) (since the right-hand-side in (18) is larger than the right-hand-side in (19) at $t = 0$) that $e_0^* < \bar{e}_0$. Therefore, $h_1^* < \bar{h}_1$ and hence, by (18) and (19) for $t = 1$, we obtain that $e_1^* < \bar{e}_1$. This process can be continued for $t = 2, 3, \dots$.

□

Given the separability, or complementarities, between leisure and human capital of children, the economy without any government intervention is characterized by under investment in human capital.

Pareto Improving Policies

It is possible to construct a policy that internalizes the externality in investments in human capital by the representative consumer in this competitive economy. The policy rule is to add the wage earnings of the young to the parents' income in their second period of life. To make this policy feasible it is necessary to impose a lump sum tax on the parents second period income and to transfer the income as a lump sum to their child's first period consumption. Specifically, let equations (4) and (5) of the representative young person be written as

$$c_t^y = T_t - s_t \quad (4)'$$

$$c_{t+1}^o = s_t R_{t+1} + w_{t+1} h_{t+1} \quad (5)'$$

where T_t is the tax transfer to the young satisfying $T_t = w_t h_t$ for all t .

Now it is immediate to see that the first order conditions for problem (3) are the same as the planning problem, where $w_{t+1} = F_2(k_{t+1}, h_{t+1})$. Note that it is necessary that the entire wage income be transferred between the generations to guarantee the optimal human capital investment.

It seems completely unreasonable to assume that there might be an institution that would transfer income between children and parents in the way that the optimal policy requires. If we assume that parents have a bequest motive such as in Barro (1974), then each parent solves an infinite horizon social planning problem, but we encounter the same problems raised by Bernheim and Bagwell (1988).

However, suppose that parents are heterogenous with respect to their preferences on the quality of their children. Furthermore, suppose that the attitude of an individual towards investment in his child's human capital is a random variable which is known to the

agent when he is young but was not known to his parents. In that case the tax/transfer of the optimal policy should be individual specific and since preferences are not observable, there is no way to implement such a policy. Moreover, it is unclear how reasonable it is to assume that each parent solves the dynastic optimal problem (see Bernheim and Bagwell (1988)) when each child's preferences are unknown to his parent.

3. HETEROGENEOUS POPULATION

Following the above discussion we introduce heterogeneity to our economy by assuming that each agent's taste for human capital of his/her child is a random draw from an independent process. That is, in each generation t individuals are alike except in the intensity of their utility from the human capital (of their offspring) h_{t+1} .

We assume that each generation G_t has continuum of individuals, say the interval $[0,1]$, hence there is no population growth in this economy. The utility function of each individual is determined at the outset of his lifetime by some random process. These random variables will be independent and identically distributed in each generation and across generations. To state this more precisely let us denote by $\theta \in [0,1]$ a "dynasty", i.e., an infinite sequence of individuals related to each other as a family (i.e., "parent" and "child"). Let \tilde{x} be a random variable with a given distribution on $[a,b]$, $0 < a < b < \infty$. For each $i \in G_t$ who belongs to the family (or dynasty) named θ , $\theta \in [0,1]$, there corresponds a random variable $\tilde{\omega}_t^\theta$ distributed as \tilde{x} . Moreover, these random variables are i.i.d. with respect to t and θ .¹⁰ The realization of $\tilde{\omega}_t^\theta$ will affect each individual's taste regarding the choice between leisure and human capital of his offspring.

¹⁰ One should be more careful in making such an assumption since there are continuum of families in each generation, see Judd (1985).

Denote by Ω the set of all doubly infinite sequences $(\omega_t)_{t=-\infty}^{\infty}$, $\omega_t \in [a, b]$ for all t . For a member θ in G_t the history of his family up to date t is $\omega^{\theta t} = (\dots, \omega_0^{\theta}, \omega_1^{\theta}, \dots, \omega_t^{\theta})$. Denote by Z_t the set of all functions $f: \Omega \rightarrow (-\infty, \infty)$ which depend upon coordinates of ω until date t only, i.e., ω^t . Notice that ω_t^{θ} is revealed at the outset of date t , hence each individual knows his utility function when he makes his decisions about consumption leisure, and investment in his child's human capital. The utility of $\theta \in G_t$ is given by (note that $z_t = 1 - e_t$),

$$U_t^{\theta} = (c_t^y)^{\alpha_1} (c_{t+1}^0)^{\alpha_2} (z_t)^{\alpha_3} (h_{t+1})^{\alpha_4(\omega_t^{\theta})} \quad (20)$$

Moreover, we assume that the functions $h_t, z_t \in Z_t$ and

$$h_t(\omega) = A(e_t(\omega^{\theta})) [h_{t-1}(\omega^{\theta})]^{\beta} \quad t = 1, 2, \dots \quad (21)$$

where $\frac{1}{2} < \beta \leq 1$ and $e_t(\omega^{\theta})$ is the (time) investment in educating the offspring. We assume that $\alpha_4(\omega_t^{\theta})$ is a continuous and increasing function on $[a, b]$. Denote by

$$\underline{\alpha}_4 = \min_{a \leq x \leq b} \alpha_4(x) > 0 \quad \text{and} \quad \bar{\alpha}_4 = \max_{a \leq x \leq b} \alpha_4(x) < \infty.$$

Since we assume that $\ell_t(\omega) = 1$ for all ω the aggregate effective labor at each date t is given by

$$L_t = \int h_t(\omega) d\omega \quad (22)$$

The human capital of a worker is observable, hence the wage payment will depend upon the effective labor supply, i.e., $W_t^i = w_t h_t^i$ where $w_t = F_2(K_t, L_t)$ is the wage rate.

We assume that the government provides compulsory education financed by taxes on income in the following manner. In each period we take the human capital of the publicly provided education to be the *average* human capital of the population at that period, denoted by H_t^G ; thus $H_t^G = \int h_t(\omega) d\omega$.¹¹ The level of this compulsory education (provided to all young members of generation t) at period t is denoted by e_t^G . In this case we assume that the evolution of the human capital process is given by:

$$h_{t+1}(\omega) = A(e_t^G + e_t(\omega)) [\bar{h}_t(\omega)]^\beta$$

where the "relevant" human capital level which affects h_{t+1} is given in this case by

$$\bar{h}_t(\omega) = \frac{e_t^G H_t^G + e_t(\omega) h_t(\omega)}{e_t^G + e_t(\omega)}$$

The compulsory education is financed by proportional taxes on wage income and we denote by τ_t the tax rate at date t . Each $\theta \in G_t$ pays, given the wage rate w_t at date t ,

$$T_t = \tau_t w_t h_t(\omega^\theta) \tag{23}$$

Thus, given w_t and R_{t+1} the interest on savings, the tax rate τ_t , e_t^G and H_t^G each

¹¹Loury (1981) assumes that public education provides the average investment in human capital of a CE. One could consider alternative assumptions on the quality of public education. Note that changes in quality affects taxes and/or total level of compulsory schooling.

individual maximizes her lifetime utility function (20), under these conditions. That is, she chooses saving s_t , and an additional time invested in educating her own offspring e_t , such that she solves the problem:

$$\max_{s_t, e_t} [w_t h_t(\omega)(1-\tau_t) - s_t]^{\alpha_1} [s_t R_{t+1}]^{\alpha_2} [1-e_t]^{\alpha_3} [A(e_t^g + e_t) \bar{h}_t^\beta(\omega)]^{\alpha_4} (\omega_t^\theta) \quad (24)$$

Necessary and sufficient conditions for optimum are:

$$\frac{s_t(\omega^\theta)}{w_t h_t(\omega^\theta)(1-\tau_t) - s_t(\omega^\theta)} = \frac{\alpha_2}{\alpha_1} \quad (25)$$

$$\frac{A(e_t^g + e_t(\omega^\theta)) [\bar{h}_t(\omega^\theta)]^\beta}{1-e_t(\omega^\theta)} \geq \frac{\alpha_4(\omega_t^\theta)}{\alpha_3} (A'(\bar{h}_t)^\beta + \beta A(\bar{h}_t') [\bar{h}_t(\omega)]^{\beta-1}) \quad (26)$$

With equality in (26) whenever $e_t(\omega^\theta) > 0$. Note that if $e_t^g = 0$, $\bar{h}_t' = \frac{\partial \bar{h}_t}{\partial e_t} = 0$ and $\bar{h}_t(\omega^\theta) \equiv h_t(\omega^\theta)$.

Denote the optimum for (24) by a "*", hence,

$$c_t^y^*(\omega) = w_t h_t^*(\omega)(1-\tau_t) - s_t^*(\omega) \quad (27)$$

$$c_{t+1}^o^*(\omega) = s_t^*(\omega) R_{t+1} \quad (28)$$

$$z_t^*(\omega) = 1 - e_t^*(\omega) \quad (29)$$

$$h_{t+1}^*(\omega) = A(e_t^g + e_t^*(\omega))[\bar{h}_t(\omega)]^\beta \quad (30)$$

We shall consider compulsory education plans which satisfy the following two properties: (a) the human capital level of the educators is the average of the population for that generation and (b) the compulsory education is fully financed by the taxes at each date. Namely, for each period t , the expenditure should equal the total amount of taxes collected,

$$w_t e_t^g H_t^g = \int \tau_t w_t h_t(\omega) d\omega$$

which, basically, implies that $e_t^g = \tau_t$ for all t .¹² As a result the effective labor supply (i.e., the labor supply used in the production process) with $\tau_t > 0$ is given by $L_t = (1 - \tau_t) \int h_t^*(\omega) d\omega$.

Given the initial capital stock K_0^* , the human capital distribution at period 0 $h_0^*(\omega)$ and the tax rates (τ_t) to finance compulsory education (hence $(e_t^g)_{t=0}^\infty$ will be determined accordingly), a competitive equilibrium (CE) is a $\langle c_0^{O*}(\omega), (c_t^{y*}(\omega), c_{t+1}^{O*}(\omega), e_t^*(\omega))_{t=0}^\infty, (w_t, R_{t+1})_{t=0}^\infty \rangle$ which satisfies the following conditions:

- (a) $(c_t^{y*}(\omega), c_{t+1}^{O*}(\omega), e_t^*(\omega))$ is the optimum for (24) for all ω , $t = 0, 1, \dots$.

¹² Note that if τ_t depends on ω (e.g. progressive taxes), then the level of compulsory education would not be equal to the tax rate.

$$(b) \quad L_t^* = (1-\tau_t) \int h_t^*(\omega) d\omega \quad \text{and} \quad K_t^* = \int s_{t-1}^*(\omega) d\omega \quad \text{for all } t = 0, 1, 2, \dots$$

$$(c) \quad w_t = F_2(K_t^*, L_t^*) \quad \text{for } t = 0, 1, 2, \dots \quad (31)$$

$$(d) \quad R_{t+1} = F_1(K_{t+1}^*, L_{t+1}^*) \quad \text{for } t = 0, 1, 2, \dots \quad (32)$$

$$(e) \quad w_t e_t^g H_t^g = \tau_t w_t \int h_t^*(\omega) \quad t = 0, 1, \dots \quad (33)$$

Thus the effective wages are the marginal product of the effective labor L_t^* , i.e., the effective labor applied in the production process (not including efforts used to raise the quality of labor through education $e_t^g H_t^g$). Interest factors are competitive, i.e., the marginal product of capital K_t^* . Condition (33) guarantees that the cost of compulsory education is covered by the taxes collected at equilibrium and it is easy to show that the competitive equilibrium satisfies the material balance conditions:¹³

$$\int c_t^{y*}(\omega) + \int c_t^{o*}(\omega) + K_{t+1}^* = F(K_t^*, L_t^*), \quad t = 0, 1, \dots \quad (34)$$

Without loss of generality we shall assume that $\tau_t = \tau$ for all t and that the equilibrium $\{K_t^*/L_t^*\}_{t=0}^{\infty}$ is bounded.

Let us consider first the case where $e_t^g = 0$ for all t , i.e., $\tau = 0$. From (26), since $\bar{h}_t \equiv h_t$ and $\bar{h}_t' = 0$ in this case we obtain that

¹³Note that due to the externality in production, namely: each individual chooses $e_t^*(\omega)$ regardless of its effect on the aggregate output, we cannot expect the C.E. to be efficient.

$$\frac{\alpha_3}{\alpha_4(\omega)} \frac{A(e_t^*)}{A'(e_t^*)} \geq 1 - e_t^*(\omega) \quad (35)$$

With equality if $e_t^*(\omega) > 0$.

$$e_t^*(\omega) = 0 \Rightarrow \frac{A(0)}{A'(0)} \geq \frac{\alpha_4(\omega)}{\alpha_3}. \quad (36)$$

Hence, from equation (36) we conclude that $e_t^*(\omega)$ is positive if

$$\frac{A(0)}{A'(0)} < \frac{\alpha_4(\omega)}{\alpha_3} \Rightarrow e_t^*(\omega) > 0. \quad (37)$$

If τ is positive then the RHS of (37) should be $\tau + e_t^*(\omega) > 0$ for all ω . By (25) and (26) we education is introduced.

Rewriting (26) for all ω where $e^G + e_t(\omega) > 0$ we obtain

$$s_t^*(\omega) = \frac{\alpha_2(1-\tau)}{\alpha_1 + \alpha_2} w_t h_t^*(\omega). \quad (38)$$

Consider an individual who chooses $e_t^*(\omega) = 0$ when $\tau = 0$. For this individual the change to $\tau > 0$ has no effect on e_t ; that is, the optimum remains $e_t(\omega) = 0$. Hence, an individual who does not invest in his child's human capital when there was no compulsory education will not provide more time for this purpose once compulsory education is introduced.

Rewriting (26) for all ω where $e^G + e_t(\omega) > 0$ we obtain

$$\frac{A(e^g + e_t(\omega))}{A'(e^g + e_t(\omega))} = \frac{\alpha_4(\omega)}{\alpha_3} \left(\frac{1 - e_t(\omega)}{1 - \beta(1 - e_t) \frac{e^g (h_t(\omega) - H_t^g)}{e^g H_t^g + e_t h_t}} \right). \quad (39)$$

In the sequel we shall denote by "''" the CE when $\tau = e^g$ is positive and by "*" the CE when $\tau = 0$. Since $\frac{A(\cdot)}{A'(\cdot)}$ is an increasing function and the R.H.S. of (39) is an increasing function in $(1 - e_t)$, we find that (see (38) as well) if τ is not too large:

$$e^g + e'_t(\omega) > e_t^*(\omega) \quad \text{for all } \omega. \quad (40)$$

Otherwise, if (40) does not hold for a set of ω of positive measure the L.H.S. of (39) declines hence $1 - e'_t(\omega) < 1 - e_t^*(\omega)$. But we assumed that $e'_t(\omega) < e_t^*(\omega)$ which is a contradiction.

4. COMPULSORY SCHOOLING

We first examine the effects of compulsory schooling on the rate of growth in equilibrium. Specifically, compare the CE when $\tau = 0$ with the CE when $\tau > 0$ (not too large) such that a small level of compulsory education is imposed.

The following proposition shows that a small level of compulsory education implies that the economy moves to a higher capital path.

PROPOSITION 2: Suppose that, $\frac{eA'(e)}{A(e)}$ is nonincreasing and that $A(1) > 1$. Let K_0 and $h_0(\omega)$ be given. Let $\langle c_0^{0*}(\omega), (c_t^{y*}, c_{t+1}^{0*}, e_t^*)_{t=0}^\infty, (w_t, R_{t+1})_{t=0}^\infty \rangle$ and $\langle c_0^{0'}, (c_t^{y'}, c_{t+1}^{0'}, e_t')_{t=0}^\infty, (w_t', R_{t+1}')_{t=0}^\infty \rangle$ be the competitive equilibria with $e^g = 0$ and $e^g > 0$

correspondingly. If e^g is not too large, there exists $N(\tau) < \infty$ such that for all $t \geq N(\tau)$, $K'_t > K_t^*$ and $L'_t > L_t^*$. Moreover, $N(\tau) \rightarrow 1$ as $\tau \rightarrow 0$.

We shall relegate all the proofs to the last section.

This proposition implies that a small level of compulsory education results in higher levels of output, capital and aggregate human capital beginning at some finite date. This is true from period 1 given a low level of compulsory education, and there exists a tradeoff between the level of compulsory schooling and the time interval until growth becomes higher. If τ is too large there is no increase in output, capital and labor. Therefore, there exists a positive level of τ (or e^g) that maximizes the *steady state* level of output or/and aggregate consumption. The next question is whether higher growth due to compulsory education implies more equal income distribution.

To study the distributional effects of compulsory education we need a formal measure of income inequality. The measure we use here has been introduced by Atkinson (1970) and later nicely characterized by Rothschild and Stiglitz (1973). Given two income distributions $X(\omega)$ and $Y(\omega)$ with the same mean, denote by $s(\alpha, X)$ the share of total income received by the poorest α percentage of the population. As α varies in $[0,1]$ $s(\alpha, X)$ traces the Lorenz curve associated with X . We say that X is more equal (income distribution) than Y if $s(\alpha, X) \geq s(\alpha, Y)$, for all $\alpha \in [0,1]$ with strict inequality for some α . As was shown by Atkinson and by Rothschild–Stiglitz this is equivalent to a second-degree stochastic dominance (SDSD), i.e., $X >_2 Y$.

The next proposition shows that a small level of compulsory education improves the intragenerational distribution of income in equilibrium for all subsequent periods.

PROPOSITION 3: Let $\langle c_0^{0*}, (c_t^{y*}, c_{t+1}^{0*}, e_t^*)_{t=0}^{\omega}, (w_t^*, R_{t+1}^*)_{t=0}^{\omega} \rangle$ and $\langle c_0^{0'}, (c_t^{y'}, c_{t+1}^{0'}, e_t^{\prime})_{t=0}^{\omega}, (w_t^{\prime}, R_{t+1}^{\prime})_{t=0}^{\omega} \rangle$ be the CE with $e_t^g = 0$ and $e_t^g = \tau > 0$ and let $(y_t^*(\omega))_{t=0}^{\omega}$ and $(y_t^{\prime}(\omega))_{t=0}^{\omega}$ be the corresponding income distributions. If τ is not too large, then for each generation t , $t = 0, 1, \dots$ the income distribution, $y_t^{\prime}(\omega)$ is more equal than the income distribution $y_t^*(\omega)$.

An implication of proposition 3 is that the introduction of a compulsory education results in a more equal distribution of the human capital in each generation. We shall claim, without a proof, that the CE with compulsory education τ converges to a steady state. Denote the human capital distribution at this steady state by $h^{\tau}(\omega)$. The initial human capital distribution $h^0(\omega)$ can be considered as the steady state distribution with $\tau = 0$. The next result shows that for τ not "too large" we can guarantee that the left tail of the distribution of $h^0(\omega)$ is shifted to the right when we introduce compulsory education with level τ .

COROLLARY: Assume that the initial steady state distribution of human capital $h^0(\omega)$ has a support $[m, M]$ (where $\inf h^0(\omega) = m$). There exists $\tau^* > 0$ such that for any $0 < \tau < \tau^*$ the support of $h^{\tau}(\omega)$ is $[m + \epsilon^*(\tau), \hat{M}(\tau)]$ where $\epsilon^*(\tau) > 0$ and $\hat{M}(\tau) > M$.

The implication is simply that compulsory schooling is a policy which can improve the situation of the very poor fraction of the population. Social public policy tries to guarantee a minimum standard of living using different intervention methods. The above corollary shows that compulsory schooling is an effective policy to achieve this goal. It is interesting to note that in addition this policy implies a transition to a steady state with better distribution of income and a higher aggregate output for the economy. In the next section we analyze the welfare distribution aspects of the policy.

5. WELFARE IMPLICATIONS OF COMPULSORY SCHOOLING

Since compulsory schooling, financed by tax on income, constitutes, basically, some transfer from individuals with higher human capital to individuals with lower human capital we cannot expect it to result in a Pareto improvement, at least not at the early stages. However, let us demonstrate that under our assumptions the majority of individuals in each generation will prefer the system with the compulsory education.

Given a CE with taxes at rate τ financing compulsory schooling (CS). We say that compulsory schooling is *acceptable* by generation t if the majority of people in G_t prefer the equilibrium with compulsory schooling to the one without it. The compulsory schooling is *acceptable in the long run* if there exists some $T < \infty$ such that CS is acceptable by all generations t for $t \geq T$. Thus, in the long run CS will prevail once it is established.

PROPOSITION 4: *Consider a competitive equilibrium with compulsory education e^g . If e^g is not too large then this CE is acceptable in the long run. That is, for some finite T all generations G_t $t \geq T$ will prefer the compulsory schooling regime when the majority voting rule is applied.*

It is clear that since we consider proportional taxes the compulsory schooling reduces the welfare of the very high income group of the population. However, we show that this policy is supported by most of the population, thus the fact that it has been implemented in all democratic countries is well explained.

6. CONCLUSIONS

The analysis presented here suggests that compulsory schooling, which is financed by proportional taxes on income, is a public policy that enhances growth, makes the distribution of earnings more equal while the majority of the population supports the policy in the long run. As a result, the wide implementation of this policy around the world can be explained by its role in achieving a preferred allocation of resources and more equal distribution of human capital

We show in our framework that a higher growth path for the economy is accompanied by a more equal distribution of income.¹⁴ This result is obtained by comparing two different paths of growth and it is compatible to evidence of a cross-section between countries. The question whether along the growth path income becomes more equal cannot be answered when income distribution is endogenous. The reason is that whenever the income distribution approaches some steady state with positive frequency on several income levels, the change in the distribution may depend on the initial conditions which are given exogenously. On the other hand, a model where the income distribution approaches full equality cannot be considered as a model that endogenously determines the income distribution. Hence, the question of variations in income inequality along a growth path should be studied by using a comparative analysis of equilibrium growth paths and the associated income distributions as done in this paper.

¹⁴Persson and Tabellini (1991) provide another model that has similar results. They also provide cross country evidence that support the implication that equality is positively associated with growth.

7. PROOFS

Proof of Proposition 2: Given K_0 and $h_0(\omega)$ we have seen that when $e_t^g > 0$ (but not too large) for all ω $e_t'(\omega) + e_t^g > e_t^*(\omega)$ (see (40)). Since $w_0' = F_2(K_0, (1-\tau)L_0) > w_0$ hence $K_1' > (1-\tau)K_1^*$ since by (38) $s_0'(\omega) > (1-\tau)s_0^*(\omega)$ for all ω . For some $\mu_1 > 0$ we can write

$$L_1' = (1-\tau) \int A(e_0'(\omega) + e^g) h_0(\omega)^\beta \geq (1-\tau) (1+\mu_1) \int A(e_t^*(\omega)) h_0^\beta(\omega) = L_1^* (1-\mu_1) (1-\tau).$$

Thus $L_1' \geq (1-\tau)(1+\mu_1)L_1^*$. Thus for some $\lambda^* > 0$

$$F(K_1', L_1') \geq (1-\tau)(1+\lambda^*) F(K_1^*, L_1^*).$$

CLAIM: For some $\hat{\epsilon} > 0$ (which depends upon τ), for all $t \geq 1$,

$$\int A(e_t'(\omega) + \tau) \geq (1+\hat{\epsilon}) \int A(e_t^*(\omega)). \quad (41)$$

PROOF OF THE CLAIM: By (35) and (39) we derive that for all ω where $e_t^*(\omega) > 0$ and $e_t'(\omega) > 0$ we have

$$\frac{A(e_t^*)}{e_t^* A'(e_t^*)} = \frac{\alpha_4}{\alpha_3} (1 - e_t^*).$$

$$\frac{A(e_t' + e^g)}{(e_t' + e^g) A'(e_t' + e^g)} \geq \frac{\alpha_4}{\alpha_3} \frac{1 - e_t' + \hat{\lambda}_\tau (1 - e_t')}{e_t' + e^g}.$$

Thus, by our assumptions about $A(\cdot)$ we conclude that:

$$\left[\frac{\alpha_4(\omega)}{\alpha_3} + \hat{\lambda}_\tau \right] \frac{1 - e'_t(\omega)}{e'_t + e^g} \leq \frac{\alpha_4(\omega)}{\alpha_3} \frac{[1 - e_t^*(\omega)]}{e_t^*(\omega)} \text{ for all } \omega.$$

Hence, noting that $e^g \equiv \tau$

$$\left[\frac{\alpha_4(\omega)}{\alpha_3} + \hat{\lambda}_\tau \right] \frac{1 + \tau}{e'_t + \tau} - \hat{\lambda}_\tau \leq \frac{\alpha_4(\omega)}{\alpha_3} \frac{1}{e_t^*(\omega)}.$$

But $1 + \tau > e'_t + \tau$ implies that whenever $e_t^*(\omega) > 0$, $e'_t(\omega) > 0$,

$$e'_t(\omega) + \tau \geq e_t^*(\omega) (1 + \tau) \tag{42}$$

which by integration proves the claim.

It can also be shown that for some $\mu_j > 0$ $j = 1, 2, 3, \dots$, we have

$$\int h'_t(\omega) \geq \prod_{j=1}^t (1 + \mu_j) \int h_t^*(\omega). \tag{43}$$

Moreover $\{\mu_j\}$ does not converge to 0 for a given $\tau > 0$. Thus for n large

$$(1 - \tau)(1 + \hat{\epsilon}) \prod_{j=1}^n (1 + \mu_j) > 1. \tag{44}$$

To complete the proof of the proposition let us note the following facts: (a) By (35) we see that $e_t^*(\omega)$ is determined (whenever it is positive) regardless of $h_t^*(\omega)$, as long as $\tau = 0$. (b) By (39) it is easy to verify, using the monotonicity of $\frac{A(x)}{A'(x)}$, that when $\tau_t > 0$ (given H_t^S) the optimal $e_t'(\omega)$ increases as $h_t'(\omega)$ increases, thus $\text{Cov}(e_t', h_t') > 0$. Therefore we can derive the following inequalities:

$$\begin{aligned} L_t' &= (1-\tau) \int A(e_t'(\omega) + e^S) [\bar{h}_t'(\omega)]^\beta \geq \\ &\geq (1-\tau) \int A(e_t'(\omega) + e^S) \cdot \int [\bar{h}_t'(\omega)]^\beta. \end{aligned} \quad (45)$$

To get the last inequality we use the fact that $(\bar{h}_t')^\beta \geq_2 (h_t^*)^\beta$ to be proved during the proof of Proposition 3.

By (42) and (43) it can be verified that the RHS of (45) becomes larger than L_t^* for t large enough.

From (38) we derive that:

$$K_{t+1}' = \frac{\alpha_2}{\alpha_1 + \alpha_2} L_t' \quad t = 0, 1, 2, \dots$$

which proves our assertion about the capital stocks. □

PROOF OF PROPOSITION 3: Since there are no explicit intergenerational transfers,

$$y_t^*(\omega) = w_t^* h_t^*(\omega) \quad t = 0, 1, 2, \dots \quad (46)$$

$$y_t'(\omega) = (1-\tau) w_t' h_t'(\omega) \quad t = 0, 1, 2, \dots \quad (47)$$

Let us prove the theorem by induction on t . At $t = 1$ we have $h_0^*(\omega) = h_0'(\omega)$ for all ω . Also by the concavity and strict monotonicity of $A(\cdot)$, using (40) we find that $A(e_0'(\omega) + \tau)[h_0(\omega)]^\beta$ is more equal than $A(e_0^*(\omega))[h_0(\omega)]^\beta$ (since it dominates it in the second degree stochastic dominance, see Rothschild and Stiglitz (1973)). Therefore $y_1'(\omega)$ is more equal than $y_1^*(\omega)$. This clearly implies that $h_1'(\omega)$ is more equally distributed than $h_1^*(\omega)$. To continue this induction let us prove

LEMMA 1: Let $X(\omega)$, $Y(\omega)$, $\bar{X}(\omega)$, $\bar{Y}(\omega)$ be positive random variables with c.d.f.'s F , G , \bar{F} , \bar{G} correspondingly, each having a support $[a, b]$, $0 < a < b < \infty$. Define $Z(\omega) = X(\omega)Y(\omega)$ and $\bar{Z}(\omega) = \bar{X}(\omega)\bar{Y}(\omega)$. If $X \geq_1 \bar{X}$ and $\|Y\| = \|\bar{Y}\| = 1$, $Y >_2 \bar{Y}$ then $Z >_2 \bar{Z}$.

PROOF of LEMMA 1: Since the support of all the given four random variables is $[a, b]$ let us compute the c.d.f. of Z as follows.

$$H(\xi) = \text{Prob}\{Z(\omega) \leq \xi\} = \text{Prob}\{Y(\omega) = 0 \text{ and } X(\omega) \leq \xi/\theta \text{ for some } a \leq \theta \leq b\}.$$

Hence we can write: (Assume: $b/a \geq b$)

$$H(\xi) = \int_a^{b/a} G'(x)F\left(\frac{\xi}{x}\right)dx = \int_a^b G'(x)F\left(\frac{\xi}{x}\right)dx.$$

We shall use the Rothschild–Stiglitz (1970) criteria for SDDS to prove our assertion. Let $\bar{H}(\xi)$ be the c.d.f. of $\bar{Z}(\omega)$ and assume that the support of H and \bar{H} is $[a', b']$.

$$\Delta(t) = \int_{a'}^t [H(\xi) - \bar{H}(\xi)] d\xi = \int_{a'}^t \int_a^b [G'(x)F(\frac{\xi}{x}) - \bar{G}'(x)\bar{F}(\frac{\xi}{x})] dx d\xi.$$

Since $F >_1 \bar{F}$ we have $F(\theta) \leq \bar{F}(\theta)$ for all θ and thus we can write

$$\Delta(t) \leq \int_a^b [G'(x) - \bar{G}'(x)] \left[\int_{a'}^t \bar{F}(\frac{\xi}{x}) d\xi \right] dx \quad (48)$$

However, $\bar{F}(\frac{\xi}{x})$ is positive and decreasing in x on $(a, b]$. Hence the function $m(x) = \int_{a'}^t \bar{F}(\frac{\xi}{x}) d\xi$ has the same properties as a function of x for all $a < t < b$. Using integration by parts, noting that $G(a) - \bar{G}(a) = 0$ and $G(b) - \bar{G}(b) = 0$, we obtain from (48) that

$$\Delta(t) \leq - \int_a^b [G(x) - \bar{G}(x)] m'(x) dx.$$

However, since $G >_1 \bar{G}$ we have $G(x) - \bar{G}(x) \leq 0$ and since $m'(x) \leq 0$ we have shown that $\Delta(t) \leq 0$ for all $t \in (a, b)$. This implies that $Z >_2 \bar{Z}$ (see Theorem 2.3 in Brumelle and Vickson (1975)) which completes the proof of the Lemma.

To complete the proof of the Theorem assume that for a given t the income distribution $y'_t(\omega)$ is more equal than $y_t^*(\omega)$. We shall use here Theorem I of Rothschild and Stiglitz (1973, p.191). Thus to prove the induction step notice first that by (46) and (47) our assumption implies that $h'_t \geq_2 h_t^*$. Since \bar{h}'_t is attained from h'_t by averaging it

with its own average \bar{h}'_t (it is similar to a mean preserving squeeze) hence $\bar{h}'_t \geq_2 h_t^*$. Since $0 < \beta \leq 1$ we also obtain that $(\bar{h}'_t)^\beta \geq_2 (h_t^*)^\beta$.

Using (40), $\int (e'_t(\omega) + \tau) > \int e_t^*(\omega)$ and A is concave. Hence $\frac{A(e'_t(\omega) + \tau)}{\lambda'}$ dominates SDD $\frac{A(e_t^*(\omega))}{\lambda}$ where $\lambda' = EA(e'_t(\omega) + \tau)$, $\lambda = EA(e_t^*(\omega))$. In particular this implies first degree stochastic dominance. Using the above Lemma we find that $h'_{t+1}(\omega) = A(e'_t(\omega) + \tau)[\bar{h}'_t(\omega)]^\beta$ is more equal than $h_{t+1}^*(\omega) = A(e_t^*(\omega))[h_t^*(\omega)]^\beta$. This clearly implies that $y'_{t+1}(\omega)$ is more equal than $y_{t+1}^*(\omega)$ thus proving the induction step.

□

PROOF OF THE COROLLARY: Without loss of generality assume that in the no-intervention steady state, i.e., $\tau = 0$ case, $\inf e^*(\omega) = 0$. Thus m solves the equation $h = A(0)h^\alpha$, namely $m = (A(0))^{1/1-\alpha}$. Now consider the steady state distribution $h^\tau(\omega)$ when compulsory schooling at level $0 < \tau < \tau^*$ is introduced. Since $e'_t(\omega) + \tau > e_t^*(\omega)$ for all ω and t this inequality should hold in steady state as well (proved as in the proof of proposition 2, equation (45)). Thus $\text{essinf}[e'_t(\omega) + \tau] = \bar{e} > 0$. The infimum of $h^\tau(\omega)$, to be denoted by h_m , must be the solution of the equation $h_m = A(\bar{e}) \left[\frac{\bar{e}h_m + \tau H^\tau}{\bar{e} + \tau} \right]^\beta$, where H^τ is the average of $h^\tau(\omega)$ (hence $h_m \leq H^\tau$). By our earlier results $H^\tau > \int h^0(\omega)$ and since $A(\bar{e}) > A(0)$ let us prove now that this infimum of $h^\tau(\omega)$, to be denoted by h_m , is strictly larger than m . Let us write:

$$\left[\frac{h_m}{A(\bar{e})} \right]^{1/\beta} - \frac{\bar{e}}{\bar{e} + \tau} h_m = \frac{\tau}{\bar{e} + \tau} H^\tau \geq \frac{\tau}{\bar{e} + \tau} h_m.$$

Define \bar{y} by the equation

$$\bar{y} = A(\bar{e})(\bar{y})^\beta.$$

However, by the above inequality h_m satisfies $h_m \geq A(\bar{e}) h_m^\beta$. Thus $h_m \geq \bar{y}$. But $A(\bar{e}) > A(0)$ hence $\bar{y} > m$ which proves our claim.

PROOF OF PROPOSITION 4: We shall apply Propositions 2 and 3 to show that the majority of each G_t are better off with the CS equilibrium compared to the no intervention case. It was proved in Proposition 2 that, given the CS at level $\tau = e^{\delta}$ (not too large) there exists some $N(\tau) < \infty$ such that for any date t , $t > N(\tau)$, the effective labor supply is higher, i.e., $L_t^e \geq L_t^*$; the total output in the CS equilibrium is higher $F(K_t', L_t^e) > F(K_t^*, L_t^*)$ for all $t > N(\tau)$. Let us show first that the total income of each G_t , $t > N(\tau)$, is higher in the CS case. By our assumptions about the production function as L_t^* increases to L_t^e (and K_t^* increases to K_t') the aggregate income $w_t' L_t^e = w_t' F_L(K_t', L_t^e) \geq w_t F_L(K_t^*, L_t^*)$. Particularly $\int y_t'(\omega) > \int y_t^*(\omega)$ for $t > N(\tau)$. Since the income distribution $y_t'(\omega)$ is more equal than $y_t^*(\omega)$ for all $t \geq 1$ for each α , $0 < \alpha < 1$ the percentage of the total income received by the lower-income 100α percent of G_t is higher in the CS equilibrium. However, for $t > N(\tau)$ the aggregate income of G_t is higher with the CS and hence the income of each individual in the lower income 50 percentage is higher under $y_t'(\omega)$ than under the distribution $y_t^*(\omega)$. As we have seen during the earlier proofs, the distribution of $h_{t+1}'(\omega)$ is more equal than the distribution $h_{t+1}^*(\omega)$, hence it is easy to verify that the lower-income 50 percent of the population in G_t are better off in the CS equilibrium for all $t > N(\tau)$. This means that the CS equilibrium is acceptable for all generations from $N(\tau)$ to ∞ .

□

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