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TECHNOLOGY REVOLUTIONS AND THE GESTATION OF NEW TECHNOLOGIES

by

Chien–Fu Chou and Oz Shy

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TECHNOLOGY REVOLUTIONS AND THE GESTATION OF NEW TECHNOLOGIES

Chien-fu Chou* and Oz Shy** †

March, 1991

Abstract

We formalize Schumpeter's explanation of technological progress and growth cycles in a model where consumers and firms benefit from periodic changes in technology which result in the development and marketing of new generations of products. We develop a general equilibrium dynamic differentiated products model in order to explain technological progress via cyclical changes in investment, output, and interest rates as well as the introduction of new products. We characterize the equilibrium and analyze the effect of changes in the rate of technology growth, resource endowment, and costs of R&D and production on the duration of generations of products and the frequency of technology revolutions, and hence of growth cycles.

Keywords: Technology Revolutions, Gestation of New Technologies

JEL Classification Number: 111, 621

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1. Introduction

We propose a general equilibrium cyclical growth model for analyzing the economic factors which affect the frequency of introductions of new generations of products as well as the time gap between the introduction and marketing dates of new generations of products. It is observed that periodically new generations of products are introduced, and many new generation products are incompatible with older generation products. In addition, we observe that there is a time lag between the introduction date of a new generation and the period in which the new products are marketed (and replace old generation products).[1] Over four decades ago Schumpeter suggested that economic growth is not governed by a continuous capital accumulation but occurs through a sequence of discrete technology revolutions. In his classic book, Schumpeter (1975, p. 83) asserted that:

> Those revolutions are not strictly incessant; they occur in discrete rushes which are separated from each other by spans of comparative quiet. The process as a whole works incessantly however, in the sense that there always is either revolution or absorption of the results of revolution, both together forming what are known as business cycles. ... the same process of industrial mutation ... incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism.

In this paper we develop a framework for modelling endogenous technology revolutions and their implications for cyclical fluctuations of output, investment and

[1] Take for example the microcomputer industry. Although new chips are introduced very frequently, computer firms are reluctant to adopt a chip which is incompatible with older models. Thus, compatibility seems to be a major consideration of whether or not and when to introduce a new machine. Moreover, once a new incompatible machine is introduced, most consumers do not purchase it until it is supported by a large variety of compatible products.

growth. We provide a Schumpeterian model of development in which the developers of new technologies play the key role in moving the economy towards the production of more valuable goods. When innovators believe that a totally new set of technologies will be introduced in the future, all new product developers stop developing products based on old technologies and start developing new generation of products to be marketed in the future. Thus, our contribution here is that we are able to model development not only as a continuous increase in the set of available goods but, following Schumpeter, as a process of creating new technologies for producing new products through the elimination of old products.[2]

In this paper, we emphasize the distinction between the exogenous accumulation of experience and the actual adoption of new technologies for producing new generation products. Assuming the incompatibility of new generation products and old generation products, we are able to obtain that adoptions of new technologies occur only in discrete points of time rather than continuously. Our model endogenously determines the number of new products (new firms) in the economy at each point in time. In addition, we are able to determine the periods in which innovators start developing a new generation of products. In order for a technology revolution to succeed, all innovators developing technologies for a new generation of products must be convinced that consumers will adopt the new generation products at a certain date in the future. In our framework, a single or a small group of innovators alone cannot bring a technological change into the economy if there are no followers.[3] Thus, our model can capture the failure of some of the developing countries to bring into a complete change in production patterns through isolated investment projects.

[2]For discussions of technology change see Dasgupta (1986), Kamien and Schwartz (1982), and Tirole (1988, Ch. 10).

[3]A particular feature of the present framework is that if innovators expect a technology revolution to come, then it will come. Hence, our approach is somewhat similar to the sunspots models of Azariadis (1981) and Azariadis and Guesnerie (1986).

We develop a general equilibrium dynamic monopolistic competition model where firms are constantly engaged in product innovation. Once the total number of products reaches a certain level, firms find it profitable to switch to a new generation of products which is incompatible with the old generation of products. From that point of time and on, firms develop only new generation products until they find it profitable to start developing products for a newer generation of products, and so on. At each point in time, new consumers enter the market, make purchase of each of the existing goods and exit the market instantaneously. Over time, the variety of existing products gets to be so large that each existing firm makes a small amount of profit. In addition, firms expect consumers to switch to a new (incompatible) generation of products in a given date in the future. Altogether, the value of an old generation firm declines until innovators find it profitable to develop and construct only firms producing new generation products. We call this stage a technology revolution. Since the variety of the new generation of products is very low, consumers purchase old generation products until the variety of the new generation of products reaches a certain level in which new generation products become more attractive to consumers.

In the literature, some of the earlier models of vintage capital are surveyed in Allen (1967). Recently, Aghion and Howitt (1989) provide an alternative approach in which a random successful outcome of innovation is translated into cost reduction and a complete replacement of the intermediate goods. Chari and Hopenhayn (1990) develop an overlapping generations model in which each technology requires vintage specific skills. Their model predicts a lag between the time when a technology appears and the peak of its usage. The aim of the present paper is to provide a different meaning to technological changes by formalizing the concept of generations of products. In this paper the number of products belonging to a specific generation is increasing over time until producers start developing a new generation of products and stop developing old generation products. At this date, (called the introduction date) the number of products belonging to the old generation of products is maximized. Following the introduction date, there is a gestation period in which consumers purchase a fixed number of old generation products and firms develop only new generation products which are still not sold to consumers. The gestation period ends at a date (called the marketing date) when consumers start buying only new generation products.

The paper is organized as follows. In section 2 we develop a technology revolution model. In section 3 we characterize a stationary revolution equilibrium in which new generations of products are introduced in constant time intervals and have constant durations. We also analyze the factors which influence the duration and gestation of new generations of products. Section 4 deals with non-stationary equilibria. Section 5 concludes.

2. The model

We consider an infinitely lived economy producing differentiated products which are indexed on the real line. We say that technology revolution occurs in period t^*, if all products developed in periods $t \geq t^*$ are incompatible with products developed before period t^*. Thus, we divide time into sub-intervals called generations. A generation is defined as the time interval between two successive revolutions. We index generations by g, where g is an integer.

We first consider a stationary situation where a new generation of products is introduced every fixed (endogenously determined) length of time which is denoted by Δ. With no loss of generality, assume that generation 0 is introduced in period $t = 0$. Thus, the introduction of generation g products begins in period $t = g\Delta$. Even when a new generation of products is introduced, consumers may still prefer to purchase only old generation products because of the low variety of the new products. We call the (stationary) time interval between the introduction date and the first date in which generation g products are sold to consumers the gestation period of generation g, and denote its duration by G, $G < \Delta$. Thus, sales of generation g products begin in period $t = g\Delta + G$, which is called the marketing date of generation g.

Figure 1 illustrates the innovation and the consumption patterns over time. Between $t = g\Delta$ and $t = (g+1)\Delta$, only generation g products are developed. However, during generation g gestation period (between $t = g\Delta$ and $t = g\Delta + G$) only generation $g-1$ products are consumed (but new generation $g-1$ products are no longer being developed). Starting from $t = g\Delta + G$ consumers purchase only generation g products.

INSERT FIGURE 1

Each product is indexed by x, where x is a real number. The set of all available generation g products in period t is denoted by X_t^g which is Lebesgue measurable in $(-\infty, \infty)$. The period t number of actually produced generation g goods is the Lebesgue measure of X_t^g and is denoted by $\mu_t^g = \mu(X_t^g)$. We associate each product with a single firm. Each existing product is produced with a constant marginal cost of m units of labor per unit of output. To develop a new generation g product in period t, the innovator has to spend a sunk cost of F_t^g units of labor. We assume that F_t^g declines with the number of existing generation g products. Formally, let[4]

$$F_t^g = F(\mu_t^g) = \left(c^2 + \theta^2 \mu_t^g\right)^{-1/2} \tag{1}$$

This specification captures the fact that the development cost is high at the beginning of a new generation and is decreasing as more products (within the same generation) are introduced. In other words, the cost of developing one additional generation g product declines with an increase in the variety of already developed generation g products. Here, c^{-1} measures the development cost of the first product of a generation, and θ is the cost reduction coefficient. We assume a dynamic monopolistic competition in the product market. Once a fixed development cost is invested, the firm becomes a monopoly. All firms have perfect foresight regarding future demand and interest rates.[5] We denote by $\pi_t^g(x)$ the period t profit of a firm producing good x of generation g.

At each point in time, a new consumer endowed with L units of labor enters the market, sells its labor endowment, makes purchase of each of the existing products and exits the market instantaneously. We denote by $c_t(x)$, $x \in \cup_g X_t^g$, the period t consumption level of good x. Consumers derive utility from all (existing) products. Formally, period t consumer's utility function is given by

$$U(\{c_t(x)\}) = \sum_g e^{hg\Delta} \left\{ \int_{X_t^g} [c_t(x)]^\alpha dx \right\}^{1/\alpha}, \quad 0 < \alpha < 1, \tag{2}$$

[4]All the results hold for a more general class of F functions satisfying $F' < 0$ and $FF''/3(F')^2 \geq 1$. The present specification enables us to solve for a closed form solution.

[5]Rob (1990) analyzes sequential entry under demand uncertainty.

where the summation is over existing (previously developed) generations, and $g\Delta$ is the introduction date of the first product of generation g. The utility function (2) implies that different generation products are perfect substitutes, and therefore each consumer buys all the existing variety of products which belong only to one generation. By the factor $\exp(hg\Delta)$, we capture the effect of a technological progress on welfare. We can think of a continuous technology progress due to the accumulation of knowledge and experience over time. The rate of technology growth is given by the parameter h. The main feature of the paper is that without technology revolutions the new technologies are not being adopted. Once a revolution occurs, the technology embodied in the new generation of products is marked by its introduction date ($g\Delta$) and its efficiency or utility enhancement is measured by $\exp(hg\Delta)$.[6] The use of the exponential function allows the possibility of stationary equilibria.

[6]At the introduction date of a new generation, the innovators adopt the most advanced technology available at that time. However, all the products of the same generation use the same technology as the product developed at the introduction date of the generation. Thus, during the life time of a generation the technology embodied into products does not improve. For example, the first generation of computers used a vacuum tube even after the integrated circuit became available. Once consumers switched to the second generation (IC) computers, firms stopped using the vacuum tube.

3. The stationary revolution equilibrium

Let w_t denote the period t wage rate. Since the consumer buys products of the same generation, equation (2) implies that the period t profit of a firm producing a generation g product is given by[7]

$$\pi_t(x) = [p_t(x) - w_t m]c_t(x) = \frac{(1-\alpha)w_t L}{\mu_t^g}, \quad t \in [g\Delta + G, \, (g+1)\Delta + G). \qquad (3)$$

Thus, the utility level of period t consumer purchasing all the available generation g products is given by

$$U_t^g = e^{hg\Delta}\left(\frac{\alpha L}{m}\right)(\mu_t^g)^{\frac{1-\alpha}{\alpha}}. \qquad (4)$$

We denote by $\dot{\mu}_t^g$ the number of g generation firms constructed in period t. Labor market equilibrium means that the period t labor demanded for innovation equals period t aggregate profits in terms of period t labor. Hence,[8]

$$\dot{\mu}_t^g F_t^g = \dot{\mu}_t^g \left(c^2 + \theta^2 \mu_t^g\right)^{-1/2} = (1-\alpha)L, \quad \text{for } g\Delta \le t < (g+1)\Delta. \qquad (5)$$

Observe that (5) yields identical product expansion paths and identical fixed development cost paths for all generations. Thus, given any generation g, for $t \in [g\Delta, \, (g+1)\Delta]$,[9]

$$\mu_t^g = \mu_{t-g\Delta} \text{ and } F_t^g = F(\mu_{t-g\Delta}), \text{ where } \mu_s \equiv \left[\frac{\theta(1-\alpha)L\,s}{2} + \frac{c}{\theta}\right]^2 - \left(\frac{c}{\theta}\right)^2, \; s \in [0, \Delta]. \qquad (6)$$

---

[7]Equations (3) and (4) can be derived as follows. In a symmetric CES monopolistic competition market structure, the price of each product is given by $w_t m/\alpha$. The consumption level of each brand is found by dividing the income ($w_t L$) by the price and the variety of existing generation g products and is given by $c_t(x) = \alpha L/m\mu_t^g$. See Dixit and Stiglitz (1977).

[8]The aggregate real profit is found from (3) by multiplying $\pi_t(x)$ by the number of existing firms (μ_t^g) and dividing by w_t.

[9]The last equation in (6) is the solution to the differential equation (5) for generation $g = 0$. For any other generation g, the stationary paths of μ_t^g and F_t^g are translations of the generation 0 paths $\mu_t^0 = \mu_t$ and $F_t^0 = F(\mu_t)$.

In equilibrium, the cost of constructing a new firm (new product) should equal the present value of its discounted future profit. Hence,

$$F_t^g = PV_t^g \equiv \int_{\max\{t, g\Delta+G\}}^{(g+1)\Delta+G} \frac{(1-\alpha)L}{\mu_\tau^g} \exp\left(-\int_t^\tau r_s ds\right) d\tau \quad \text{for } g\Delta \leq t \leq (g+1)\Delta. \quad (7)$$

In the above, r_t is the instantaneous interest rate in period t and is equal to $-\dot{w}_t/w_t$.

DEFINITION 1 A stationary technology revolution equilibrium is the pair (Δ, G) so that (a) for $t \lesseqqgtr g\Delta$, $\frac{PV_t^g}{F_t^g} \lesseqqgtr \frac{PV_t^{g-1}}{F_t^{g-1}}$ and, (b) for $t \lesseqqgtr g\Delta + G$, $U_t^g \lesseqqgtr U_t^{g-1}$.

Condition (a) states that at the introduction date of generation g, the present value of a dollar invested in developing a product (firm) belonging to the new generation g (defined in (7)) overtakes that of a dollar invested in developing a product belonging to the old generation $g-1$. Condition (b) states that at the marketing date of generation g, the utility of consuming generation g goods (given in (4)) overtakes that of generation $g-1$ goods.

From (6), (7), and condition (a) in definition 1 (evaluated at $t = g\Delta$), we have that[10]

$$\mu_\Delta = \frac{F(0)\mu_G}{F(\mu_\Delta)}. \quad (8)$$

From (4), (6), and condition (b) (evaluated at $t = g\Delta + G$), we have that

$$\mu_\Delta = \exp\left(\frac{\alpha h \Delta}{1-\alpha}\right)\mu_G. \quad (9)$$

Using (8) and (9) to eliminate μ_G, the equilibrium Δ is determined by

$$\exp\left(\frac{\alpha h \Delta}{1-\alpha}\right) = \frac{F(0)}{F(\mu_\Delta)} = 1 + \frac{\theta^2(1-\alpha)L\Delta}{2c}. \quad (10)$$

Proposition 1 *Given that $\theta^2(1-\alpha)^2 L > 2c\alpha h$, the stationary equilibrium exists and is unique. That is, the solution to (10) is uniquely determined.*

[10] A step by step derivation is given in the appendix .

We can now analyze what factors determine the frequency of technology revolutions and how long generations of products last.

Proposition 2 *The duration of each generation (Δ) increases with a decrease in the technology enhancement rate (h), the resource endowment (L), the development cost parameters (θ and c^{-1}), and the degree of intra-generation product substitution (α).*

Proofs of Propositions 1 and 2. The LHS(10) and RHS(10) are both increasing functions of Δ, and are drawn in figure 2. Also, the LHS is convex and the RHS is linear with respect to Δ. The condition of proposition 1 insures the existence of a unique positive Δ solving (10) which corresponds to a unique intersection in figure 2. Next, the RHS(10) increases with θ, L, c^{-1} and $(1-\alpha)$, thereby reducing the equilibrium value of Δ. In terms of figure 2, an increase in α or h implies an upward shift of LHS.[11]

INSERT FIGURE 2

[11]An algebraic proof is given in the appendix.

4. Non-stationary equilibrium and the determination of the first revolution

In this section we analyze the case where at time $t = 0$ the economy produces μ_0 products of generation $g = 0$, introduced at $t = D^0 < 0$. That is, we consider a situation where the initial variety of generation 0 products may be different from its stationary value. Observe that if $\mu_0^0 = \mu_{-D^0}$ then the economy is on the stationary equilibrium path. For $g \geq 1$, denote by D^g and M^g the introduction date and marketing date of generation g, respectively. Also, let Δ^g denote the length of generation g ($\Delta^g \equiv D^{g+1} - D^g$) and let G^g denote the gestation length of generation g ($G^g \equiv M^g - D^g$). Noting that in this general case the utility enhancement factor (see equation 2) for generation g products is given by $\exp(hD^g)$, equation (4) becomes

$$U_t^g = e^{hD^g}\left(\frac{\alpha L}{m}\right)(\mu_t^g)^{\frac{1-\alpha}{\alpha}}. \tag{11}$$

Equation (7) becomes

$$F_t^g = PV_t^g = \int_{\max\{t, M^g\}}^{M^{g+1}} \left[\frac{(1-\alpha)L}{\mu_\tau^g}\right] \exp\left(-\textstyle\int_t^\tau r_s ds\right) d\tau. \tag{12}$$

Similar to definition 1 we state the following definition.

DEFINITION 2 A technology revolution equilibrium is a $\bar{g}$, $1 \leq \bar{g} \leq \infty$, and the sequences $\{D^g, M^g, \mu_t^g\}$, $1 \leq g < \bar{g}$, such that $D^g < M^g < D^{g+1}$, and (a) for $t \stackrel{<}{\stackrel{=}{>}} D^g$, $\frac{PV_t^g}{F_t^g} \stackrel{<}{\stackrel{=}{>}} \frac{PV_t^{g-1}}{F_t^{g-1}}$ and (b) for $t \stackrel{<}{\stackrel{=}{>}} M^g$, $U_t^g \stackrel{<}{\stackrel{=}{>}} U_t^{g-1}$. If $\bar{g} = \infty$, we say that the revolution equilibrium is *endless*. Otherwise, it is called a *terminated* revolution equilibrium.

Similar to (8), condition (a) in definition 2 implies that

$$\mu_{D^g}^{g-1} = \frac{F(0)\mu_{M^g}^g}{F(\mu_{D^g}^{g-1})} \quad (\geq \text{ for } g = 1)\ (> \text{ only if } D^1 = 0). \tag{13}$$

Similar to (9), condition (b) implies that

$$\mu^{g-1}_{D^g} = \mu^{g-1}_{M_g} = \exp\left(\frac{\alpha h}{1-\alpha}(D^g - D^{g-1})\right)\mu^g_{M^g}. \tag{14}$$

Thus, a revolution equilibrium must satisfy (13) and (14).

4.1 The determination of the first revolution

We now analyze generation $g = 1$ and the determination of the first revolution for a given initial variety of generation 0 at $t = 0$ (μ^0_0) and a given introduction date of generation 0 ($D^0 < 0$). Starting from $t = 0$ until $t = D^1$, the differential equation for generation 0 product development path is given in (5). Similar to (6), the particular solution is given by

$$\mu^0_s = \left(\frac{\theta(1-\alpha)Ls}{2} + \sqrt{\mu^0_0 + \frac{c^2}{\theta^2}}\right)^2 - \frac{c^2}{\theta^2},\ \ 0 \leq s \leq D^1. \tag{15}$$

Let t_0 be defined by $\mu_{t_0} = \mu^0_0$ where the function μ_s is given in (6). That is, t_0 is the time it would take for the variety of products to reach the level of μ^0_0 in a stationary equilibrium if generation $g = 0$ had started at $t = 0$. Then, $\mu^0_s = \mu_{t_0+s}$ and in particular $\mu^0_{D^1} = \mu_{t_0+D^1}$. Clearly, $\mu^1_{D^1} = 0$, and similar to (6) $\mu^1_{M^1} = \mu_{M^1-D^1} = \mu_{G^1}$. Now, for generation $g = 1$, (13) and (14) become

$$\mu_{t_0+D^1} \geq \frac{F(0)\mu_{G^1}}{F(\mu_{t_0+D^1})}\ \ (> \text{only if } D^1 = 0), \text{ and }\ \mu_{t_0+D^1} = \exp\left(\frac{\alpha h(D^1 - D^0)}{1-\alpha}\right)\mu_{G^1}. \tag{16}$$

Equation (16) determines the introduction date of generation 1 (D^1) and the marketing date of generation 1 ($M^1 = D^1 + G^1$). Similar to (10), solving (16) by eliminating μ_{G^1} yields

$$\exp\left(\frac{\alpha h(D^1 - D^0)}{1-\alpha}\right) = \frac{F(0)}{F(\mu_{t_0+D^1})}\ \left(= 1 + \frac{\theta^2(1-\alpha)L(t_0 + D^1)}{2c}\right). \tag{17}$$

The RHS and LHS of equation (17) are illustrated in the upper part of figure 3 for the case when $-t_0 < D^0$. That is, the initial variety of generation 0 products μ^0_0 is higher than the stationary value.

INSERT FIGURE 3

The intersection of the two curves determines the first introduction date D^1. Then the generation $g = 1$ gestation length (G^1) is determined graphically on the lower part of figure 3 as explained below. Given the equilibrium D^1 determined on the upper part of figure 3, μ_{G^1} is obtained from the first equation of (16) which is represented by the μ_{G^1}-curve. Then G^1 is solved from the stationary product variety expansion path μ_s given in (6).

4.2 Revolution equilibria

For $g \geq 2$, $\mu_{D^g}^{g-1} = \mu_{D^g - D^{g-1}} = \mu_{\Delta^{g-1}}$ and $\mu_{M^g}^{g} = \mu_{M^g - D^g} = \mu_{G^g}$. Hence, (13) and (14) become

$$\mu_{\Delta^{g-1}} = \frac{F(0)\mu_{G^g}}{F(\mu_{\Delta^{g-1}})} \quad \text{and} \quad \mu_{\Delta^{g-1}} = \exp\left(\frac{\alpha h \Delta^{g-1}}{1-\alpha}\right)\mu_{G^g}, \; g \geq 2. \tag{18}$$

Observe that the two equations in (18) are identical to (8) and (9), implying that the duration of generations (Δ^g) and the gestation lengths (G^g) for generations $2 \leq g < \bar{g}$ are all equal to the stationary values. Thus,

Proposition 3 *Given a revolution equilibrium, then $\Delta^g = \Delta^*$ and $G^{g+1} = G^*$ for $1 \leq g < \bar{g}$, where Δ^* is the unique stationary duration of a generation solved from (10) and G^* is the unique stationary gestation length solved from (7).*

Proposition 3 says that a revolution equilibrium is characterized by stationary duration and gestation lengths for all generations starting from generation $g = 2$, except perhaps the last generation in the case of a terminated equilibrium.

Finally, we would like to point out two extreme cases when the initial generation $g = 0$ variety (μ_0^0) is either very low or very high. In the first case the calculated D^1 is less than 0 implying that the generation $g = 1$ should have been introduced before $t = 0$. This is the case of *instantaneous revolution*. That is, $D^1 = 0$ in equilibrium. In the second case, since the initial variety is very large it takes a very long time for the utility of consuming generation 1 products (U^1) to overtake the utility of consuming generation 0 products (U^0). That is, the generation $g = 1$ gestation period will be very long. In the extreme case, G^1 will exceed the stationary duration of a generation implying that no revolution can occur in a rational expectations equilibrium. This is the case of a *terminated revolution equilibrium* ($\bar{g} = 1$) where the initial variety is very large so that the old generation technologies (traditional sector) will persist and new generation products will not be introduced. Thus, similar to Rostow (1961), an economic take-off may not be possible if the initial variety of old generation products is very large.

5. Concluding remarks

We develop a dynamic general equilibrium model in order to explain the evolution of technological change and cyclical growth. The main feature of this environment which distinguishes it from previous literature is that a continuous technological progress, measured by (ht) in the utility function, results in a discrete revolution process and persistent growth cycles. We are able to sort out the parameters which affect the duration of each generation of products as well as the frequency of technology revolutions and hence the growth cycles. During the marketing period the initial profit of each firm is high. When the variety of products expands, there is more competition and the profit of existing firms decline. When the new generation of products is marketed, the profit of old generation firms becomes zero, and the profit of new generation firms is at the highest level and so on. At each moment the real interest rate adjusts to equate the present value of all future profit of a newly constructed firm with its development cost. Also, the discussion of the determination of the first revolution and proposition 3 reveal that the present framework is stable in the sense that any small deviation from the stationary path will not affect subsequent generations.

Finally, the term 'generation of products' used in this paper can also be given a different interpretation. It is possible to view the variety of products belonging to a particular generation as the variety of services supporting a particular technology. With this interpretation, the reason why consumers in the economy do not switch to a new technology immediately after it has been introduced is because the new technology is not supported by a sufficient amount of supporting services.[12]

[12]In a different context, Chou and Shy (1990) formalize the notion of supporting services by introducing consumers that choose, say, among different brands of computers by taking into consideration the variety of services supporting each technology (supporting software for each computer brand.

Appendix:

Derivation of equation (8)

Since $\pi_t^g = 0$ for $g\Delta \le t \le g\Delta + G$ (gestation period), by (7) we have that

$$F_t^g = PV_t^g = PV_{g\Delta}^g \exp\left(\int_{g\Delta}^t r_s ds\right) = F(0)\exp\left(\int_{g\Delta}^t r_s ds\right). \tag{19}$$

Therefore,

$$\frac{F(0)}{F(\mu_t^g)} = \exp\left(-\int_{g\Delta}^t r_s ds\right) \quad \text{for } g\Delta \le t \le g\Delta + G. \tag{20}$$

Substituting (20) into (7) and evaluating (7) for generation $g-1$ yields

$$\begin{aligned}
F(\mu_{g\Delta}^{g-1}) &= PV_{g\Delta}^{g-1} = \int_{g\Delta}^{g\Delta+G} \frac{F(0)(1-\alpha)L}{F(\mu_\tau^g)\mu_\tau^{g-1}} d\tau \\
&= \frac{(1-\alpha)LF(0)}{\mu_\Delta}\int_{g\Delta}^{g\Delta+G}\frac{1}{F(\mu_\tau^g)}d\tau && [\text{since } \mu_\tau^{g-1} = \mu_\Delta \text{ for } \tau \ge g\Delta] \\
&= \frac{(1-\alpha)LF(0)}{\mu_\Delta}\int_{g\Delta}^{g\Delta+G}\frac{\dot{\mu}_\tau^g}{F(\mu_\tau^g)\dot{\mu}_\tau^g}d\tau \\
&= \frac{F(0)}{\mu_\Delta}\int_{g\Delta}^{g\Delta+G} d\mu_\tau^g && [\text{by (5)}] \\
&= \frac{F(0)\mu_G}{\mu_\Delta}. && [\text{first theorem of calculus}]
\end{aligned} \tag{21}$$

An Algebraic Proof of Proposition 2:

Let $A \equiv \theta^2 L/(2c)$ and $B \equiv 1/(1-\alpha)$. Then (10) becomes $\exp((B-1)h\Delta) = 1 + AB^{-1}\Delta$. By implicit function rule, we have

$$\frac{\partial\Delta}{\partial A} = -\frac{B^{-1}\Delta}{E} < 0, \quad \frac{\partial\Delta}{\partial h} = -\frac{(B-1)\Delta e^{(B-1)h\Delta}}{E} < 0, \quad \frac{\partial\Delta}{\partial B} = -\frac{h\Delta e^{(B-1)h\Delta} + AB^{-2}\Delta}{E} < 0,$$

where $E = (B-1)he^{(B-1)h\Delta} - AB^{-1} > 0$ since the slope of the LHS(10) is greater than that of the RHS(10) at the equilibrium point.

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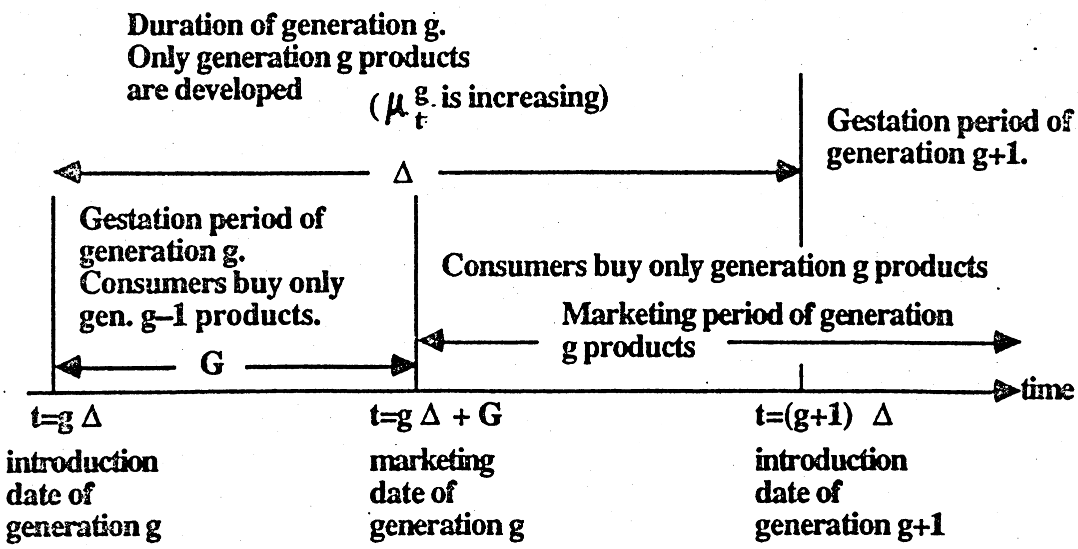
Duration of generation g.
Only generation g products
are developed
(μ_t^g is increasing)
Δ
Gestation period of
generation g+1.
Gestation period of
generation g.
Consumers buy only
gen. g–1 products.
Consumers buy only generation g products
Marketing period of generation
g products
G
time
t=g Δ
introduction
date of
generation g
t=g Δ + G
marketing
date of
generation g
t=(g+1) Δ
introduction
date of
generation g+1

FIGURE 1

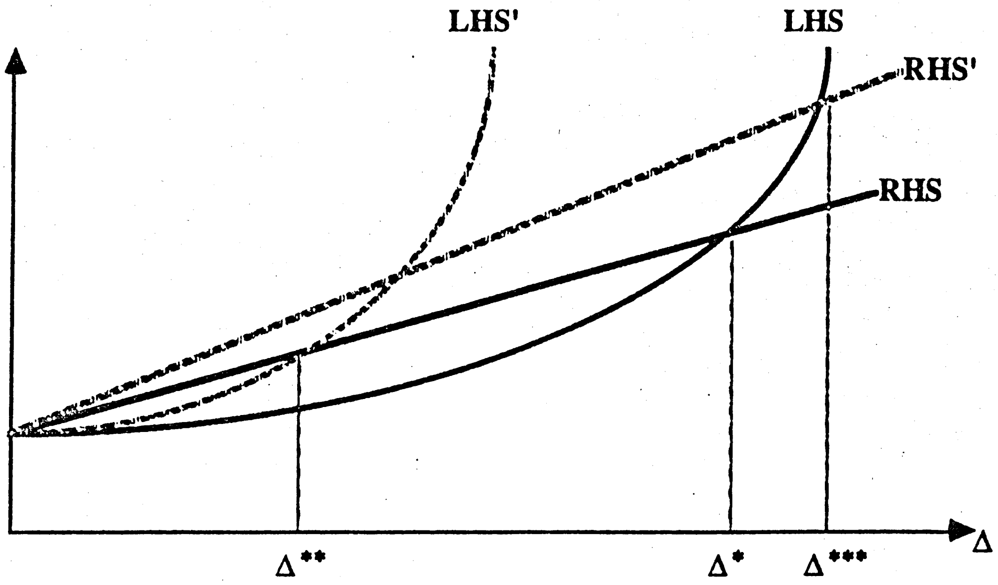

FIGURE 2

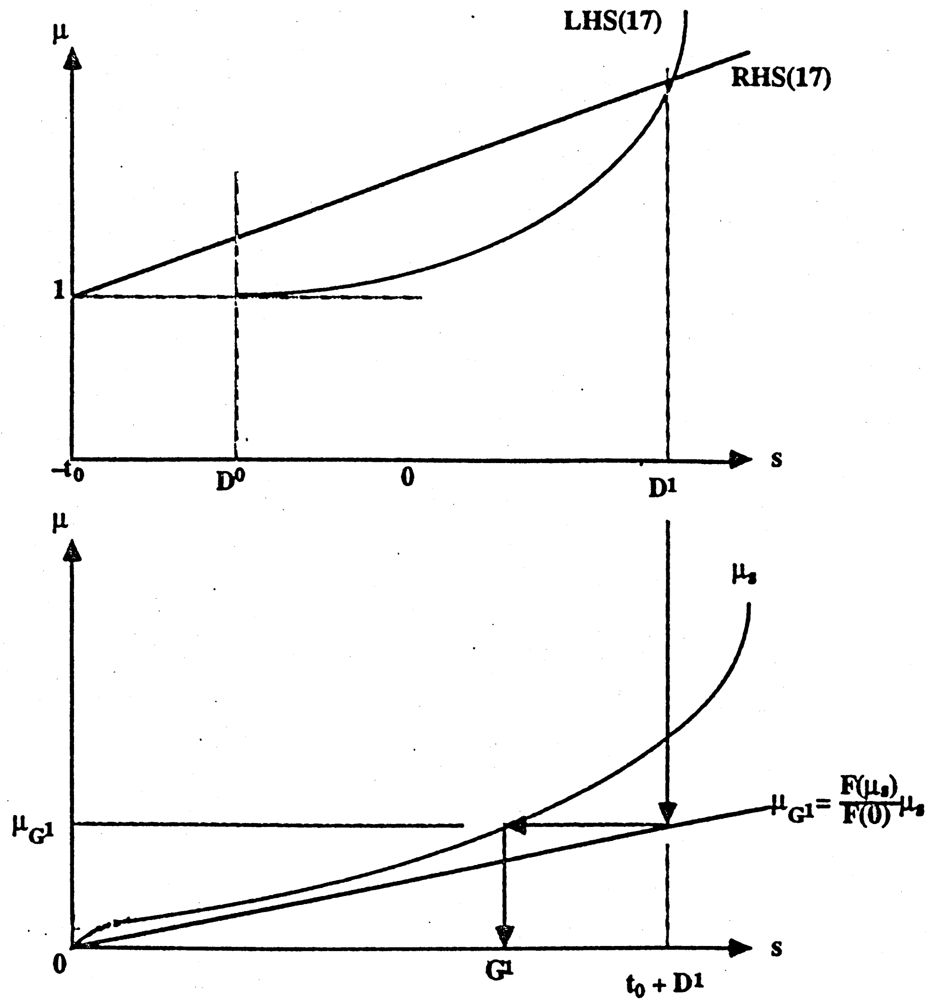

FIGURE 3