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COMPLEXITY CONSIDERATIONS AND MARKET BEHAVIOR*

by

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ABSTRACT

The paper is concerned with market behavior when firms have limited ability to handle effectively the complexity of changing market conditions and strategic interaction. Modelling the managerial bounded rationality by using the concept of strategic complexity as measured by finite automaton, we show that market behavior can be considerably altered once there is a limit on the complexity of strategies. In particular the paper demonstrates that when an incumbent firm operates in several markets an entry to one market may induce the incumbent to exit from another market (divestiture) in order to "concentrate" on the competition it faces. For different parameters the incumbent may react to such an entry by exit from the same market creating specialization. The paper also demonstrates that bounded complexity can serve as an entry barrier giving an advantage to the established incumbent firm.

1. INTRODUCTION

Managing corporations is a complex and time-consuming task. Managers need to evaluate changing market conditions, contemplating competitive strategies, deciding on new products, production technologies, new markets, etc. The ability to handle such complex situations effectively is clearly a key ingredient of a successful management. As corporations are run by managers, one cannot ignore the human factor and the unavoidable boundedness of the ability and rationality of human beings (see Simon, 1972 and 1978 for the original introduction of bounded rationality into the economic literature). In analyzing the behavior of firms, the economic literature usually assumes that there is no limit to the ability of management to calculate, to remember, to foresee, or to plan. Thus, in evaluating the contributions of this literature one should remember that it relies heavily on the existence of "super-managers."

A casual observation of the modern business world indicates that, indeed, firms are aware of their inability to handle effectively and costlessly complex situations. As an example consider the recent sale of Fisher-Price by Quaker Oats. The main motive for this sale was stated by **The Wall Street Journal** (Wednesday April 25, 1990) as "Quaker Oats Co. decided to spin off its troubled Fisher Price toy business to stockholders allowing management to concentrate on other difficulties at what, for the first time in two decades, is a "pure" food company." Another example is the recent sale by Zenith Electronics of its computer operations such that "In focusing on consumer goods, Zenith returns to its core business and could strengthen its ability to complete."[1] The above statements indicate the advantage that management might have when they concentrate on one main business particularly when this business is difficult to manage.[2] It is the explanation and ramifcations of these business strategies which are the main concern of this paper.

The general belief of the existence of managerial diseconomies of scale is the main driving force of the literature that claim that "small is beautiful." The claim that a larger firm is harder to manager can be traced back to Kaldor (1934). In his seminal paper Coase (1937) argued that the optimal size of a firm is determined by the comparative cost of internal transaction versus the cost of market transaction and more importantly that at some point internal transaction becomes more expensive. There are several sources of such organization diseconomies. Calvo and Wellisz (1978), Mirrlees (1976), and Rosen (1982) discuss the cost of monitoring and its effect on effort; Williamson (1967), Geanakoplos and Milgrom (1985), and Guenerie and Oddon (1988) consider the limit capacity of managers to process information. McAfee and McMillan (1990) discuss the suboptimal activities of firms as a result of distorted incentives given by principals which implies that when

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[1]See the statement of Zenith's president in the **Washington Post**, October 3, 1990.

[2]See also Lichtenberg's (1990) empirical work in which it is pointed out that "the 1960's conglomerate boom may have contributed to the slowdown in U.S. productivity growth that began at or slightly after that time." Lichtenberg pointed out that the slowdown was partly due to the existence of "unmanageable empires."

there are more layers of hierarchy it results in larger distortions. This type of inefficiencies can also be regarded as a special case of the "influence cost" identified by Milgrom (1988) and Milgrom and Roberts (1988, 1989) as one of the main costs of centralization. Such a cost describes the workers' incentives to influence their superiors' decisions and the inefficiencies associated with the need to impose mechanisms that offset such a behavior.

The main purpose of this work is to study the behavior of a firm when it has bounded capacity to handle complex situations. We are interested in examining the relationship between having bounded ability to handle complexity, the scope of businesses in which the firm operates and in particular the effects of competition on the optimal scope of the businesses. We consider two aspects of market complexity: The complexity of operating simultaneously in different markets when the market conditions are uncertain and the complexity of competing with other firms. These two sources of complexity compete on the firm's limited ability to handle complex situations. Thus when the firm becomes more responsive to changing market conditions it can use less complex strategies in the games with its competitors. Using this setting the paper demonstrates the importance of incorporating bounded rationality into the study of industrial organization and a formal way of doing so.

In order to illustrate the problem consider the following example: a player plays a simultaneous chess game with several players. Every game has a different weight and final payoffs are the sum of the weights of the games he wins. The player needs to decide which of the games he wishes to play. Clearly one can always devise an optimal strategy for (a chess) game as there is a finite set of feasible positions. Note that, in 1912, Zeremelo proved that for rational players chess is a trivial game. In reality, however, there are difficulties in implementing such a strategy. As the number of games in which the player participates affects his performance in each particular game, there is a tradeoff between the number of games and the probability of success in each one of them. The optimal number of games depends on the player's skill, speed and practice (level of complexity). Let us imagine now the same simultaneous chess game but with one of the opponents being replaced by a more skilful player. Intuition suggests that two possible scenarios might happen. Either the incumbent will leave the game with the new opponent and continue to play the other games (and possibly enter a new game) or he will leave some of the other games in order to "concentrate" more on the now more complex game with the new opponent. It is this intuitive behavior which we try to capture and model in this paper.

We will follow the literature (e.g., Abreu-Rubinstein (1988), Aumann (1981), Ben Porath (1986), Kalai-Stanford (1988), Neyman (1985), Rubinstein (1986), and the surveys by Kalai (1990)) and formalize the measure of strategic complexity using the framework of finite automata. While most of this literature was concerned with repeated games, in discussing the market behavior of firms we do not wish to concentrate only on the strategic interaction excluding the complexity of reacting to changing market conditions.[3] We thus define a model in which the firm faces different demand conditions in every period and needs to react to such exogenous changes while at the same time it faces competition. We define the complexity of strategy as the minimal number of states in an automaton implementing it. We will then limit the complexity of the firm's strategies and discuss the implication of such a limit on the scope of the firm's activities and on the pattern of competition. In particular we demonstrate the following possible effects: (i) in the face of competition in one market the firm might divest from other markets in order to concentrate on the competition in its main business; (ii) complexity considerations might lead to market segmentation such that facing a new competition in one of its markets a firm might decide to exit from this market leaving its competition as a single producer; and (iii) bounded complexity might serve as an entry barrier because the incumbent's bounded complexity might make the threat of aggressive response to entry credible.

We would like to note, however, that there are inherent difficulties in modeling and discussing the bounded rationality of managers. It is not yet clear why certain decisions are "difficult" to make within a limited time framework and why other strategic decisions are simple to make. This study, however, is beyond the scope of this

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[3]Formally the game we consider is a stochastic game.

paper. One can also argue that putting an upper bound on the complexity of strategies that the firm can use ignores the ability of firms to hire more managers, decentralizing decisions and thus overcoming the problem of having bounded complexities. Such a view implies that there is no limit to the ability of organization to handle complex situations. Indeed our results do not hold in situations in which a complete decentralization of decision is feasible. It is our basic assumption, however, that even when it is possible to hire more decision makers there is a limited ability to handle complex situations.

2. THE MODEL

2.1. Market Conditions

Let $I = \{1,\ldots,m\}$ be a set of m independent markets. At each market there is a linear demand function $p_i = a_i - q_i$, where p_i and q_i are the price and quantity in market i, respectively. a_i is assumed to be stochastic such that with probability 1/2 there is a high demand and $p_i = h_i - q_i$ and with probability 1/2 there is a low demand and $p_i = \ell_i - q_i$, where $\ell_i < h_i$ for all $i \in I$. The cost of producing one unit of product i is assumed to be $c_i < h_i$. We divide the set I into two subsets as follows:

$$G = \{i \in I | c_i < \ell_i\}$$

$$B = I \backslash G.$$

At every period there is a signal σ of an m tuple of 0 (low) and 1 (high) that specifies the demand condition in each market. We let Σ be the set of all possible signals and σ_i be the i'th component of the signal σ we let σ^0 be the initial market condition given prior to the start of the game.

Consider now a single firm that faces the above market conditions. Let $Q \equiv \mathbb{R}^m_+$ be the set of all possible output combinations. A strategy for the monopolist is a function $A: \Sigma \to Q$ that specifies an output combination for every possible signal. We further let $\pi^i_h(q_i)$ and $\pi^i_\ell(q_i)$ be the monopolistic profit function from market i. The monopolist's optimal strategy is:

(i) For $i \in G$ to produce $(h_i - c_i)/2$ if $\sigma_i = 1$ and $(\ell_i - c_i)/2$ if $\sigma_i = 0$.

(ii) For $i \in B$ to produce $(h_i - c_i)/2$ if $\sigma_i = 1$ and 0 otherwise.

Such a strategy, however, suggests that the firm adjusts its output vector for every different signal σ and is fully responsive to market conditions. This strategy is optimal as long as being responsive to market conditions does not incur any additional costs. When changing the output level or monitoring market conditions is costly the above strategy may not be optimal. We let π^i denote the monopolist's expected profit in market i when he plays the above optimal strategy.

2.2. **Finite** automaton

Most of the recent literature using the concept of finite automaton discuss the strategic bounded complexity in repeated games setting (see Kalai (1989)). Our model differs as it assumes, besides the strategic interaction, the possibility of having changing market conditions. Thus the definition of automaton needs to acount for complexities induced by nature as well as with those induced by strategic interactions.

We define an **automaton** as a triple $((M,m_0),A,T)$ with the following interpretation. M is the set of states of the automaton with $m_0 \in M$ being the initial state. $A: M \to Q$ is a behavioral function that prescribes for each state an output combination $q \in Q$. The transition function $T: M \times \Sigma \times Q \to M$ governs the transition of the automaton from one state to another. Thus the input that the automaton receives at every period consists of a signal that describes the new market conditions and the output that was produced at the previous period.[4]

In the single firm case an efficient automaton corresponds to a partition of the signal space Σ such that every subset in the partition can be viewed as a state of the automaton and the transition function is degenerate, i.e. does not depend on the current state of the

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[4]We have chosen the more general notion of automation, the one allowing for mistakes (the intput to the automation can be actions not consistent with the automaton's earlier prescriptions). In addition to the greater generality this would enable us to discuss subgame perfection when we switch to the multiplayer model.

automaton or on the last period production vector. We choose however the more general formulation, described above, as it can be easily extended to the multi firm case in which there is strategic interaction.

OBSERVATION 1: Every strategy of the firm can be described by such an automaton.

PROOF: (We use a generalized version of the construction in Kalai-Stanford (1988).) Given a game with an initial condition σ^0 we define the set of histories of length zero $H^0 = \{e\}$ with e denoting the empty history. The set of histories of length ℓ, H^ℓ, consists of all vectors $q^1\sigma^1,\ldots,q^\ell\sigma^\ell$ with $\sigma^t \in \Sigma$ and $q^t \in Q$ for $t = 0,1,\ldots,\ell$. We let $H = \cup_{t=0}^{\infty} H^t$ denote the set of all (finite length) histories. Now we formally define a strategy to be a function $f:H \to Q$.

Given a history $h \in H$ it is useful to discuss entities defined on the subgame induced by h. h is of the form $h = q^1\sigma^1,\ldots,q^\ell\sigma^\ell$ and we define the game induced by h, G_h, to be the one with the initial condition σ^ℓ. The set of induced histories is $H_h^0 = \{e\}$, and H_h^r is be the set of all vectors $\bar{q}^1\bar{\sigma}^1,\ldots,\bar{q}^r\bar{\sigma}^r$. Starting with a strategy f and a history $h \in H$ we define the **strategy induced by** f **and** h on the game G_h to be the function $f_h:H_h \to Q$ such that for every t and every $\bar{h} \in H_h^t$, $f_h(\bar{h}) = f(h\bar{h})$ with $h\bar{h}$ denoting the concatenation of the two histories, i.e., $h\bar{h} = q^1\sigma^1,\ldots,q^\ell\sigma^\ell,\bar{q}^1\bar{\sigma}^1,\ldots,\bar{q}^t\bar{\sigma}^t$ (we make the convention that $eh = he = h$). We use $f_H \equiv \{f_h: h \in H\}$ to denote the set of all strategies induced by f. Notice that even if $G_h \neq G_{\bar{h}}$ (when $\sigma^{\ell} \neq \bar{\sigma}^{\ell}$) the set of strategies of the two games coincide. Thus the set f_H induces all the induced strategies from all different induced games and it thus may be small even if the number of induced games is larger. Suppose, for example, that a constant strategy is used with $q^t = q^*$ for all t. Then f_H includes only one element (the constant strategy).

Now for a given strategy we will exhibit an automaton implementing it. Its states will correspond to the different strategies it induced, i.e., $M = f_H$, with the initial state corresponding to f itself, i.e., $m_0 = f$. The behavior function assigns to each state the initial action taken by the corresponding induced strategy, i.e., $A(m) = f_h(e)$. The transition function is defined by

$$T(f_h,\sigma,q) = f_{\bar{h}} \text{ with } \bar{h} = q^1\sigma^1,\ldots,q^{\ell}\sigma^{\ell}q\sigma.$$

It is easy to check that the automaton just describes is well defined and that it implements f.

It is worth noting that the above construction used number of states equals the cardinality of the set of strategies induced by f. This shows that the number of states needed to implement f does not exceed the number of strategies induced by f. It is easy to see that the converse is also true, i.e., the number of states needed to implement a strategy equals at least the number of different strategies it induces. Thus, we can conclude that the number of states needed to implement a strategy equals the number of strategies it produces.

□

We now define the **complexity of a strategy** to be the number of states of the smallest (in the number of states) automaton describing it, or, equivalently, as was discussed in the last proof, the number of different strategies it induced. We will model bounded rationality by assuming that firms will use strategies not exceeding a certain finite complexity.[5] We will denote by k the bound on the complexity of strategies.

The automaton measure of complexity, and Observation 1 can be applied to all extensive form games. For example, later we replace the monopoly game described above by an oligopoly. Then we assume that the automaton was modified by replacing the number q of the input to be the vector consisting of all firms' production levels in all markets. If the game was played with imperfect monitoring one would require that the input is the information revealed to the player prior to his making a decision. For example, in the Abreu-Pearce-Stachetti (1986) (APS) game,

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[5]An alternative formulation will be to assign a certain cost associated with the number of states in the automation (see Abreu-Rubinstein (1988)). We choose our formulation for simplicity sake. We believe, however, that our main results can be obtained with a model of costly states as long as this cost function is convex with the number of states. In this case our formulation is a special case as we assume zero cost until k states and infinite cost for every state beyond k.

the input to the automaton at every stage will be the firm's own previous production level and the resulting market price. Indeed, the claim of APS that their strategies are "simple" is verified using the formal definition of complexity by the fact that they can be implemented by automata of complexity two. See Figure 1 for a diagram of the strategy for player 1.

Figure 1 to be inserted here

In this diagram, circles represent states. Entries in the circle represent production levels (low or high). Arrows represent transitions as functions of states and inputs (self production levels and observed market prices). In this simple strategy one may think of the initial state as the cooperative one where the player keeps his production level low. The other state is for the punishment mode where production level is high. Notice that the player moves into the cooperative state from a cooperative state provided that the market as a whole was cooperative, i.e., the price is higher than the threshold level $\bar{p}$, or from a noncooperative state provided that the market as a whole punishes, i.e., market price is below the threshold level $\bar{\bar{p}}$.

Before proceeding it is important to note that the existence of multi markets does not imply immediately that the strategy used is complex. One can, for example, adopt a strategy in which constant quantities (possibly zero) are produced in all markets. The complexity of such a strategy is 1. In general, however, the complexity of a strategy must equal at least the number of different actions it may prescribe among the play of the game which implies the following:

OBSERVATION 2: The optimal strategy of the monopoly facing the multi-market problem studied here has the complexity of 2^m.

Observation 2 implies that the complexity required to implement the monopolist optimal strategy is potentially huge due to the many possible production combinations. In particular, note that we restrict the demand function to take one of two values. If we change this assumption and the demand function can take many possible values, it will increase the complexity of the monopolist's optimal strategy dramatically.[6] The main question is, of course: What happens if the firm is restricted to use strategies of complexity not exceeding $k < 2^m$.

2.3. The optimal Automaton for $m = 2$

Consider a monopolist operating in two markets. Let $q_h^i(q_\ell^i)$ be the optimal quantities for the i'th market for a high (low) demand. Further, let z_i be the optimal constant level of production in market i when the firm does not distinguish between a high and a low signal,

[6]Indeed, one reasonable way that firms deal with this problem is by decentralizing into divisions dealing with submarkets. Such a decentralized process could be modeled by a central automaton coordinating the actions of several automata, each dealing with few products. Complexity decentralization is an important issue left for future research.

i.e., z_i maximizes $(1/2)(h_i - c_i - z_i)z_i + (1/2)(\ell_i - c_i - z_i)z_i$. Let

$$\gamma_i = (1/2)[\pi_h^i(q_h^i) + \pi_\ell^i(q_\ell^i) - \pi_h^i(z_i) - \pi_\ell^i(z_i)].$$

γ_i is thus the gain from being fully responsive to demand conditions in market i versus producing the optimal constant quantity. Without loss of generality let $\gamma_1 > \gamma_2$. We now describe the optimal automata for this case:

CLAIM 1: For a monopolist who operates in two markets, the optimal automaton is as follows:

(i) For $k = 1$, there is one state of the automaton. Output is (z_1, z_2) regardless of the signal.

(ii) For $k = 2$ the optimal automaton moves to state 1 whenever the market conditions input is either (1,1) or (1,0). In this state production is (q_h^1, z_2). The automaton moves to the second state if the market condition is either (0,0) or (0,1) and in this case production is (q_ℓ^1, z_2).

(iii) For $k = 3$ the optimal automaton moves to state 1 when the market condition is (1,1) in this state it produces (q_h^1, q_h^2). It moves to state 2 when the market condition is (1,0) and it produces (q_h^1, q_ℓ^2). It moves to state 3 whenever the market condition is either (0,1) or (0,0). In this state it produces (q_ℓ^1, z_2).

(iv) For $k = 4$ the automaton implements the optimal monopoly strategy as discussed in the previous section.

Proof: $k = 1$ is trivial. For $k = 2$ let us consider all possible partitions. Since $\gamma_1 > \gamma_2$ it is evident that having the partition $\{(1,1),\ (0,1)\},\ \{(1,0),\ (0,0)\}$ is not optimal. For the partition $\{(1,1),\ (0,0)\},\ \{(1,0),\ (0,1)\}$ the optimal production levels are (z_1,z_2) for all states of the automaton and thus the monopolist does not exploit his ability to be partially responsive to market conditions. Consider now the partition $\Sigma_1 = \{(1,1)\}$ and $\Sigma_2 = \{(1,0),\ (0,0),\ (0,1)\}$. For such a partition the optimal

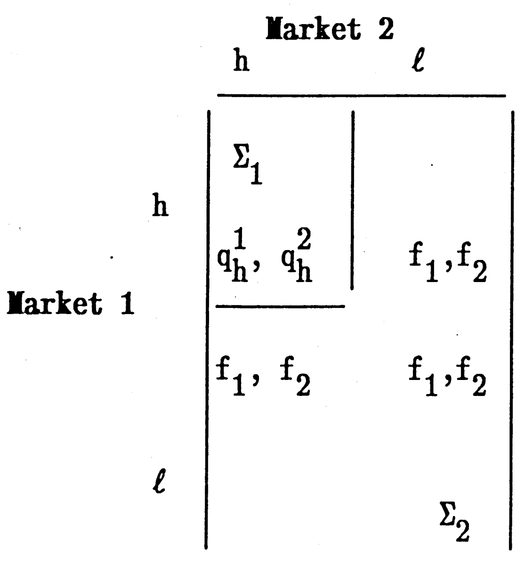

Figure 2A

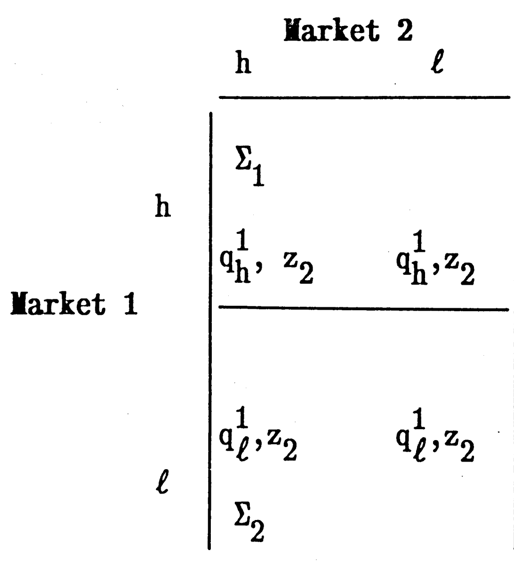

Figure 2B

Market 2

		h	ℓ
Market 1	h	q_h^1, q_h^2	z_1, q_ℓ^2
	ℓ	q_ℓ^1, z_2	z_1, z_2

Figure 2c

production levels are described in Figure 2A, where f_i is the optimal output when the probability of having a low demand is 2/3 . Now observe that transforming the quantities produced at 2A to the ones depicted in Figure 2C yields higher profits for the firm. The quantities described in Figure 2C, however, cannot be implemented by a strategy of complexity 2. It remains to be shown that the automaton suggested in the claim (which are given by Figure 2B) yields even higher payoffs than those in Figure 2C. Changing from 2C to 2B yields the following changes in payoffs:

$$(1) \qquad [-\pi_h^2(q_n^2) + \pi_h^2(z_2) + \pi_h^1(q_h^1) - \pi_h^1(z_1) - \pi_\ell^2(q_\ell^2) + \pi_\ell^2(z_2) + \pi_\ell^1(q_\ell^1) - \pi_\ell^1(z_1)]/4$$

which after rearranging and using our assumption that $\gamma_1 > \gamma_2$ yields that (1) is positive and thus the firm has higher payoffs with 2B than with 2A. The other partitions can be analyzed similarly.

For $k = 3$ the only other partition is the one that perfectly uses the information regarding market 2 and only partially uses the information regarding market 1. Using our assumption that $\gamma_1 > \gamma_2$ and following the above procedure would indicate that such a partition yields lower payoffs. The case of $k = 4$ is trivial.

□

3. MARKET SCOPE WITHOUT COMPETITION

When a monopolist can use strategies of unlimited complexity, it would operate in all m markets. But when $k < 2^m$ operating in all markets is not necessarily optimal. To illustrate the problem consider a firm operating in m markets with a strategy of complexity k. Assume that there is an opportunity to enter another market. Given the bound on the complexity, k, the firm has three options that illustrate the tradeoff it faces: (i) the firm can choose a strategy which is (partially) responsive to demand conditions in the new market and by so doing the firm reduces its responsiveness to demand conditions in the previous m markets; (ii) to enter the new market by producing a constant quantity; (iii) not to enter the new market.

Since for every $i \in G$, $c_i < \ell_i$ it is clear that there is a constant quantity that yields positive expected profits. Thus the firm will always enter to markets of type G. One cannot extend the above argument to markets of type B.

Let

$$MB = \{i \in B \mid [\pi_h^i(z_i) + \pi_\ell^i(z_i)] \geq 0\};$$
$$VB = B \backslash MB.$$

The monopolist enters markets in MB (moderately bad) as, by definition, producing a constant quantity, z_i, yields positive profits. The entry decisions to a market of type VB (very bad) is more complicated. Markets of type VB are markets in which in order to make profits one needs to react to changing market conditions. On the other hand, there are indirect costs associated with being responsive to market conditions. Since the firm can use strategies of bounded complexity, being responsive (even partially) to demand conditions implies that the strategy is less responsive to demand conditions in the other m markets which reduces profits from these markets.

CLAIM 2 (Market Scope): When $m = 2$, $k = 2$, $\gamma_1 > \gamma_2$, and the second market is of type VB then the firm will not enter the second market.

PROOF: When the firm enters both markets the optimal automaton is specified by Claim 1. For such an automaton the output in the second market is constant and not responsive to demand conditions. Since market 2 is of type VB the firm loses money in this market and therefore it is better off not entering it.

□

Claim 2 illustrates that bounded complexity considerations can determine the scope of activities of firms even in situations in which the only relationship between the different activities is that they are managed by the same firm.

4. BOUNDED COMPLEXITY AND MARKET COMPETITION

Strategic interaction adds another source of complexity to the firm's decision problem. Facing one type of complexity diminishes the firm's ability to handle the other type. This tradeoff between the two types of complexities plays an important role in determining the firm's behavior in oligopolistic markets.

There is a fundamental difficulty in the use of finite automaton framework to model market behavior with bounded complexity. The main question is the ability of players to change the automaton they are using. If there is no limit to such ability then we are back in the world of unbounded complexity as any strategy of any complexity can be implemented. But to assume that players never change the automaton is too restrictive. Moreover, under such an assumption the use of finite automaton as a modeling tool will capture more of the ability of players to commit themselves rather than modeling their bounded rationality. In this work we assume that once a competitor enters a market it is possible for the incumbent firm to react by changing the automaton. Thus only when a "major" event such as entry occurs, is it possible to change the automaton. Without this assumption it would be possible for the incumbent to commit himself to a certain automaton and the use of this automaton will prevent entry or will give the incumbent some advantages in the post entry game.

4.1. Competition in a Single Market

Consider a single duopolistic market in which demand is as specified in section 2 and both firms have the same cost function. Let x_h and x_ℓ be the Cournot equilibrium output for the high and low demand respectively, and $\tilde{\pi}$ be the Cournot equilibrium payoffs. Our symmetry assumption implies that both firms realize the same profits.

OBSERVATION 3: The Cournot equilibrium can be implemented with strategies of complexity 2 (for each firm).

PROOF: Consider the following simple automaton: There are two states, M_1 and M_2 with the initial state M_1 if the market is high initially and M_2 otherwise. The behavioral function is $A(M_1) = x_h$ and $A(M_2) = x_\ell$. The transition function is $T(M_i,1,q_1,q_2) = M_1$ and $T(M_i,0,q_1,q_2) = M_2$ for every q_1, q_2 and M_i. Clearly if for both firms $k = 2$ and one firm uses the above automaton the best response of the other firm is to use the same automatons.

□

Consider now other equilibria of the repeated play of the above duopolistic game and let the two firms maximize discounted profits. Let r_h and r_ℓ be the collusive output level for the high and low demand respectively and $\hat{\pi}^i$ be the expected collusive payoffs from market i.[7] As it is well-documented in the literature, when the discount factor is sufficiently close to one, the collusive outcome can be supported as a noncooperative (subgame perfect) Nash equilibrium. For example, one can use the well-known grim trigger strategies such that firms cooperate until one defect and then they both switch to the Cournot-Nash equilibrium forever.

OBSERVATION 4: The grim trigger strategy equilibrium can be implemented as an equilibrium with strategies of complexity 4.

PROOF: Can be proven by a straightforward construction.

□

One can also verify that it is possible sometimes to economize on the punishment phase of the grim trigger strategies and obtain the collusive outcome by using strategies of complexity 3. The need to have states assigned to the punishment phase leads to the following observation:

OBSERVATION 5: (Collusion is Complex): In a single market duopoly supporting the collusive outcome requires the use of strategy of complexity $k > 2$, i.e., above the complexity of the Cournot-Nash

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[7] Although there might be several collusive levels let us choose the one yielding the highest possible profits among the symmetric outcome.

equilibrium strategies.

PROOF: When an automaton of only two states is used to support the collusive outcome it must be that $A(M_1) = r_h$ and $A(M_2) = r_\ell$. Clearly a pair of such strategies is not an equilibrium.

□

4.2. Multi-Market Competition

Let us move to a multi-market setup. Although we assume that markets are independent with respect to demand and cost conditions, complexity considerations may introduce interdependence among markets as we have already seen in the monopoly case. In particular the introduction of competition in one market may lead to a different behavior in the other. In order to demonstrate this consider the following example: An incumbent firm operates in two markets, $k = 2$ and $\gamma_1 > \gamma_2$. As claim 1 suggests the optimal automaton is to be fully responsive in market one and produce a constant quantity in market two. Assume now that a new firm enters market one. As a response to such an entry the incumbent may decide to exit from one of the markets. If he decides to stay in both he needs to determine whether to continue being responsive in the first market and producing a constant quantity in the second market is still its optimal strategy. It is possible that as a result of an entry to market one it becomes optimal for the firm to be responsive to market conditions in the second market, in which there is no competition and to produce a constant quantity in market one.

4.2.1. Competition and Divestiture Policy of Firms

Consider now a firm that operates in two markets. Letting $k = 4$ implies that the firm operates in both markets and is fully responsive to demand conditions. We further assume that $\gamma_1 > \gamma_2$ and that the second market is of type VB.

Assume now that a new firm enters the first market. The two firms can now compete and produce the Cournot equilibrium quantities or collude. Note, however, that as indicated by Observation 5 collusion is complex, i.e., in order to support a collusive outcome the firms need to use strategies of complexity exceeding the complexity of the Cournot-Nash equilibrium.

CLAIM 3: Consider an entry to the first market. (i) When $\hat{\pi}^1 > \tilde{\pi}^1 + \pi^2$ and $\tilde{\pi}^1 > 0$ the incumbent's optimal response is to exit from the second market. (ii) When $\tilde{\pi}^1 \leq 0$ and $\hat{\pi}^1 < \pi^2$, the incumbent's optimal response is to exit from the first market.[8]

PROOF: Facing competition in market 1 the incumbent has three options: He can exit from market one and remain in market 2, he can exit from

[8] One of the conditions of (ii) is that the expected Cournot equilibrium profits, $\tilde{\pi}^1$, is negative. This can happen for example when there are sufficiently large fixed costs.

market 2 and cooperate in market one, or he can stay in both markets and play the Cournot strategies in market one. Given that $k = 4$ and that collusion is complex the option of staying in both markets, being responsive to market conditions in market two and yet supporting the collusive outcome in market one is not available to him as it requires strategies of complexity exceeding 4. The conditions $\hat{\pi}^1 > \tilde{\pi}^1 + \pi^2$ and $\tilde{\pi}^1 > 0$ imply that supporting the collusive outcome in the first market is the incumbent's best strategy. In such a case the incumbent cannot be responsive to demand conditions in market two and since this market is of type VB, getting out of this market is part of the optimal strategy. This completes the proof of (i).

When $\hat{\pi}^1 < \pi^2$ and $\tilde{\pi}^1 < 0$, supporting the collusive outcome in market one is not optimal as it requires getting out from market two and losing π^2. Staying in market 2 while colluding in one implies that the firm produces a constant quantity in market 2 and since this market is of type VB the firm will realize losses from such a policy.

□

Claim 3 demonstrates two possible scenarios. Part (i) demonstrates that limited complexity may lead to divestiture while part (ii) demonstrates that it can result in specialization. Divestiture occurs when the competitor enters the market which contributes significantly to the firm's total profits. In such a case the firm decides to exit from markets which are not their main business and to "concentrate" on their main business when by concentrating we mean using a strategy with higher complexity. Specialization occurs when, as a result of entry into one of the markets, the firm decides to leave this market as the strategic interaction is too complicated and thus costly. The outcome of this behavior is a complete specialization such that in both markets there is a monopoly.

Note also the importance of the independence assumption in claim 3. If markets one and two are related such that the conditional probability p (the demand is high in market 2 | the demand is high in market 1) > $\frac{1}{2}$, the optimal behavior might be different. The firm can use the correlation to reduce the complexity of its strategy. For example, when the above conditional probability is 1 such that the two markets are perfectly correlated one can produce the optimal quantity with strategies of complexity 2. Thus our result of divestiture will not hold when the markets are sufficiently correlated. The firm can collude (use a strategy of higher complexity) in the first market and still produce the optimal quantities in the second market. This claim supports our intuition that the divestiture will occur in a conglomerate firm when the businesses are not related and not in a firm producing in related markets.

4.2.2. Bounded Complexity and Entry Deterrence

Intuitively one may think that having bounded complexity is always disadvantageous for the firm as it limits its ability to use complex strategy. But as often happens in strategic interaction one can sometimes benefit from a handicap (see also Gilboa and Samet (1989)), i.e. it is possible that having bounded complexity will enhance profits.

Consider a firm, I, operating in two markets such that $\gamma_1 > \gamma_2$. There is a firm E that considers entering the second market. In the post-entry game the incumbent may choose to cooperate and to get the profits $\hat{\pi}^2_I$ or to use strategies of lower complexity and to get the Cournot-Nash equilibrium profits $\tilde{\pi}^2_I < \hat{\pi}^2_I$. The entrant equilibrium profits are $\tilde{\pi}^2_E < 0$ if the incumbent chooses not to cooperate and $\hat{\pi}^2_E > 0$ if the incumbent cooperates. Thus entry is attractive only when the incumbent cooperates. We assume that the entrant does not have any complexity constraints and thus it is the incumbent who decides upon the type of the post entry game, i.e. cooperation or fighting. Note, however, that since $\tilde{\pi}^2_E < 0$ an entrant with a complexity constraint will not enter the market without the ability to support the collusive outcome.

The above setup can be regarded as the last period problem in the chain store paradox (Selten (1978).) Indeed, without a limit on the complexity of strategies, subgame perfection arguments imply that once entry occurs the incumbent will cooperate and thus entry is profitable. This result holds since supporting cooperation is not costly. This well-known result does not hold, however, if there is a bound on the complexity of strategies of the incumbent firm.

CLAIM 4: (i) When k is sufficiently large the incumbent will react cooperatively and the entrant will enter. (ii) When $k \leq 4$ and $\hat{\pi}_I^2 - \tilde{\pi}_I^2 < \gamma_1$ the incumbent will react aggressively to an entry and thus the entrant will not enter.

PROOF: When k is sufficiently large the bounded complexity is not a binding constraint and subgame perfection implies (i). When $k = 4$ the incumbent has two options: The first is to cooperate in the second market and to produce a constant quantity in the first market. The second is not to react cooperatively and to be responsive to market conditions in the first market. Note that the option of reacting cooperatively in the second market and still be responsive to market conditions in the first market is not available as it requires strategies of complexity exceeding 4. Since $\hat{\pi}_I^2 - \tilde{\pi}_I^2 < \gamma_1$ the second possibility yields higher payoffs, the incumbent will not react cooperatively and thus the entrant will not enter.

□

The bounded complexity serves here as a credibility device to the threat of noncooperation with an entrant. Given the bound on complexity a cooperative behavior becomes costly. If the incumbent reacts cooperatively to an entry it will have to be less responsive to market conditions in the first market which reduces its profits by γ_1.

5. CONCLUDING REMARKS

The transaction cost economics literature emphasizes the need to revise the analysis of markets taking into account that engaging in a contract, changing production level, or generally changing strategies is not without cost. The major claim of this paper is that the economic analysis of markets needs also to account for the limited ability of management to handle effectively the complexity of changing market conditions and strategic interaction with competitors. Modeling the managerial limited rationality by using the concept of strategic complexity as measured by automata we show that the outcome of market behavior and conduct can be considerably altered once there is a limit on the complexity of strategies. We believe that such an analysis can explain different market behavior that the classical industrial organization literature cannot explain. In this paper we discussed only two aspects of market complexities. There are, however, many other aspects of complex market situations unaccounted for in this paper. For example, entry and exit decisions, R&D decisions, contracting complexities, and so on. We believe that positive approach to industrial organization ought to account for the effects of such complexities on the managerial decision and market behavior.

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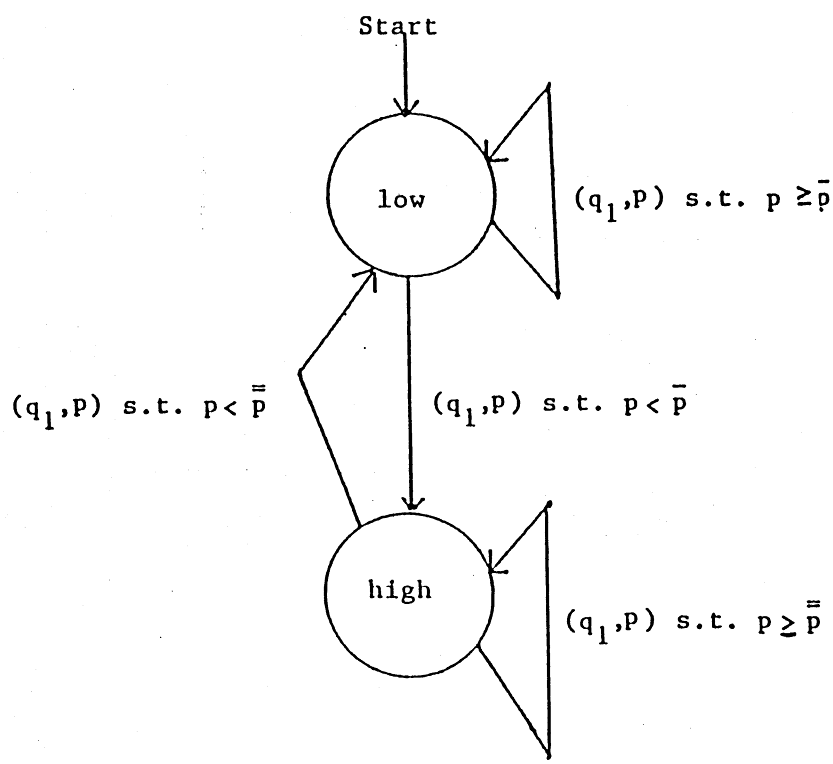
Start
low
(q_1,p) s.t. $p \geq \bar{p}$
(q_1,p) s.t. $p < \bar{\bar{p}}$
(q_1,p) s.t. $p < \bar{p}$
high
(q_1,p) s.t. $p \geq \bar{\bar{p}}$

Figure 1.