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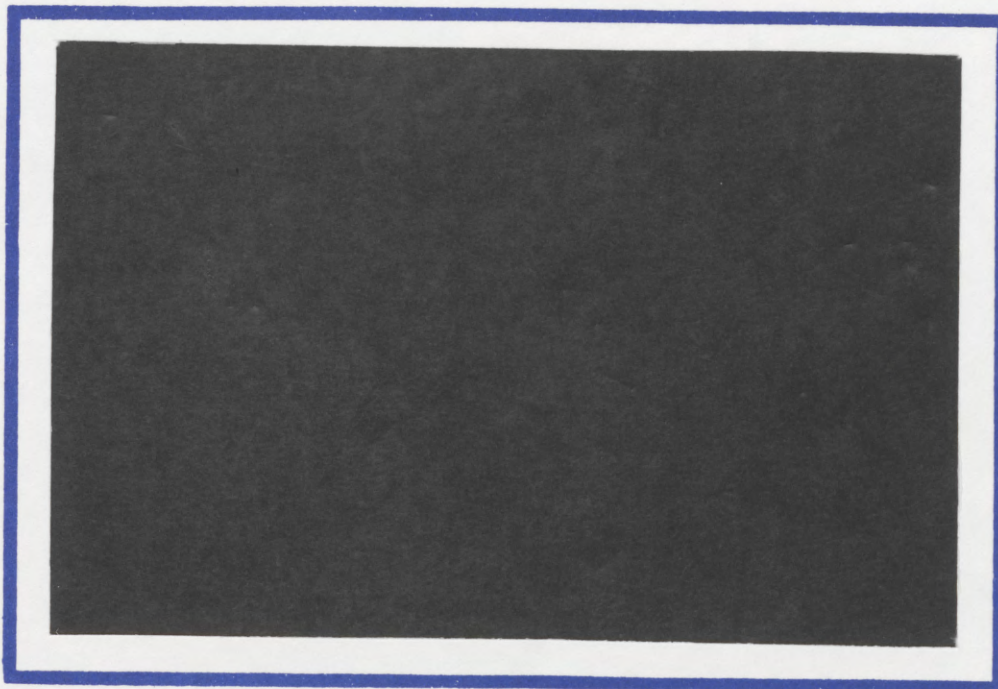
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INTERNATIONAL LINKS OF INNOVATION RATES

by

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## 1. Introduction

The traditional theory of economic growth has concentrated on capital accumulation, exogenous population growth, and exogenous technical progress. Capital accumulation and labor growth proved to explain an important part of observed variations in output growth, but they cannot account for a large remaining residual. The residual has been attributed to technical progress (see Solow (1957) for the original contribution and Maddison (1977) for a recent discussion of growth accounting).

It has been clear, however, that exogenous technical progress cannot be a satisfactory working hypothesis. Major productivity gains require a deliberate effort of invention and innovation, especially in modern times (see, for example, Freeman (1982)). The latter implies that resources need to be devoted to these activities. In order for these resources to be forthcoming, however, the economic system has to properly reward inventors and innovators, thereby providing incentives to engage in these activities. All this is rather self evident, but direct measurement of the contribution of inventions and innovations to economic growth proved to be very difficult (see Griliches (1979)). Part of the difficulty results from the lack of satisfactory data for the problem at hand. But part of the difficulty also rests with the lack of a satisfactory theory.

Following the general slowdown of productivity growth in the 1970s the interest in economic growth has been renewed. This has been manifested in the publication of numerous empirical studies that attempt to explain the events of the seventies, and more recently, in a reexamination of the theory of economic growth with the new concerns in

mind. The 'new growth theory' emphasizes factors that lead to sustained long-run growth at rates that are endogenously determined. Some recent studies have emphasized new features while others have incorporated old features in novel ways. Examples of this research line include the following: (i) Endogenous population growth (Becker and Murphy (1990); (ii) The role of public services (Barro (1989)); (iii) Accumulation of human capital (Lucas (1988), Ohyama (1989), Stokey (1990)); (iv) Learning by doing and knowledge spillovers (Romer (1986,1990), Lucas (1988), Aghion and Howitt (1990), Grossman and Helpman (1989d)). Although learning by doing has been explored in the earlier literature (see Arrow (1962) for the original contribution as well as Uzawa (1965), Levhari (1966) and Sheshinski (1967)), its recent combination with an explicit treatment of product innovation — in the form of development of new products or the improvement of existing products — has yielded important new insights on endogenous technical progress (see, for example, Romer (1990), Aghion and Howitt (1989), and Grossman and Helpman (1989d)).

This study is concerned with international links of innovation patterns, where innovation drives economic growth. Such links have been explored in a series of papers and a forthcoming book by Grossman and Helpman (1989a–e, 1990). (Grossman (1989) has also used it to explain Japan's performance.) They showed in a variety of models that build on endogenous product innovation how the long-run growth rate of a country depends on its own features, features of its trading partners, its own trade policy, trade policy of its trading partners, its own policy towards innovation and imitation and its trading partner's policies towards innovation and imitation. These links proved to be rather involved. For example, a less developed country that encourages imitation of products that have been originally developed in an advanced industrial country may thereby speed up growth around the globe.

In what follows I explore cross country links between innovation rates, with an emphasis on out of steady state dynamics. This type of investigation is necessary in order to identify comovements in innovation rates in the short-run as opposed to the long-run. Indeed, a major finding of this analysis is that even when long-run rates of innovation converge, short-run responses can differ significantly across countries. In particular, we find that in response to changes in the resource base short-run changes of the innovation rates differ between the resource gaining country and its trading partner. While the latter typically declines (especially when comparative advantage in R&D is no very large), the former typically increases. This happens notwithstanding the fact that long-run rates of innovation increase in both countries.

Moreover, we shall see that the world's average rate of innovation may follow elaborate patterns. Its short-run response may differ from its long-run response and short-run values may undershoot or overshoot long-run values. These results are particularly interesting in view of the fact that we shall develop a rather minimal model for the investigation of these questions.

In order to make the paper self contained I present in the next section a simple one country model in which profit seeking entrepreneurs develop new products. An expansion of the menu of available products raises productivity in manufacturing via the refinement of specialization (as in Ethier (1982)) or it raises productivity in consumption via an increase in the available product choice. For this reason a larger product choice is desirable per se. In addition current product development reduces costs of future product innovation. This feature captures the idea that even when targeted at particular products R&D also generates broader knowledge that can be applied to other products. It thereby leads to the accumulation of non-appropriable knowledge capital. Knowledge capital is particularly useful for R&D. Thus, current innovative activities reduce costs of future

innovations. This specification leads to endogenous innovation and growth, with their rates depending on available resources and other parameters. In the one-country case the rates of innovation and growth settle down immediately on steady state levels. Therefore in this case there exist no out of steady state dynamics.

In Section III the model is extended to a world of two countries. It represents a simplified version of Grossman and Helpman (1989a). With two countries in place there exist non-trivial out of steady state dynamics. In a stationary environment the rate of innovation may rise or decline over time (depending on initial conditions) until it converges to a steady state. Innovation rates differ across countries except for the steady state. In that section I analyze the response of innovation rates to changes in available resources in the country with comparative advantage in R&D as well as in its trading partner. This analysis illustrates the relationship between innovation rates that can emerge in such economies. The paper closes with a summary of the main findings and concluding comments on alternative specifications.



## II. The Basic One-Country Model

We consider an isolated economy that is populated by identical individuals with time additive preferences

$$(1) \quad U = \int_0^{\infty} e^{-\rho t} \log D(t) dt,$$

where  $t$  represents a time index,  $\rho$  stands for the subjective discount rate, and  $D(t)$  represents a consumption index. There are two alternative interpretations of the consumption index: (a) there exists a single homogeneous final consumer good and  $D(t)$  represents its consumption level; and (b)  $D(t)$  represents an index of consumed differentiated products. Under both interpretations

$$(2) \quad D(t) = \left[ \int_0^{n(t)} x(\omega)^\alpha d\omega \right]^{1/\alpha}, \quad 0 < \alpha < 1,$$

where  $\omega$  is an index of differentiated products,  $\omega \in [0, \infty)$ , and the set of available products at time  $t$  equals  $[0, n(t)]$ ;  $n(t)$  also represents the measure of differentiated products available at time  $t$ ; and  $x(\omega)$  represents the quantity of variety  $\omega$ . Under the former interpretation  $x$  is a differentiated input and (2) represents a production function. Under the latter interpretation  $x$  is a differentiated consumption good and (2) represents a sub-utility function. The following analysis applies to both cases.

The representative consumer maximizes (1) subject to an intertemporal budget constraint

$$(3) \quad \int_0^{\infty} e^{-R(t)} P(t) D(t) dt \leq \Omega(0),$$



where  $R(t)$  stands for the discount factor from  $t$  to  $0$ ;  $P(t)$  represents the ideal price index associated with  $D(t)$  (which equals the price of the consumption good under the first interpretation);  $\Omega(0)$  equals the present value of income plus the value of initial asset holdings (to be determined in equilibrium). As is well known the solution to this problem yields

$$(4) \quad \dot{E}/E = r - \rho,$$

where  $E=PD$  represents consumer spending and  $r$  the interest rate (equal to  $\dot{R}$ ). Hence, the rate of growth of spending equals the difference between the interest rate and the subjective discount rate.

In addition, at every point in time the distribution of spending across different varieties is given by

$$(5) \quad x(\omega) = p(\omega)^{-\epsilon} E / \int_0^n p(\omega')^{1-\epsilon} d\omega', \quad \omega \in [0, n], \quad \epsilon = 1/(1-\alpha) > 1,$$

where  $p(\omega)$  stands for the price of variety  $\omega$ . Under the first interpretation (5) represents consumer demand functions; under the second it represents the demand functions of profit maximizing atomistic producers of the homogeneous consumption good. The last point becomes evident when one recognizes that under (2) manufacturers of the homogeneous consumption good face constant returns to scale and each one of them takes the measure of products  $n$  and total consumer spending  $E$  as given.

We are free to choose the time pattern of a nominal variable (the choice of numeraire). It proves convenient to choose  $E=1$  for all  $t$ . In this case (4) implies

$$(6) \quad r(t) = \rho,$$

i.e., the nominal interest rate is constant and equal to the subjective discount rate.

The manufacturing know-how of an existing variety  $\omega$  belongs to an atomistic entrepreneur that developed it in the past. He needs one unit of labor per unit output for every  $\omega$ . Thus, his marginal manufacturing costs equal the wage rate  $w$ . Facing the demand function (5) the supplier of  $\omega$  maximizes operating profits  $p(\omega)x(\omega) - wx(\omega)$  by charging price  $p(\omega)=w/\alpha$ . Hence, in equilibrium all varieties are equally priced, and we have

$$(7) \quad p = w/\alpha.$$

This pricing strategy yields operating profits (recall that  $E=1$ )

$$(8) \quad \pi = (1-\alpha)/n.$$

Clearly, operating profits of a representative firm vary over time with the measure of available products.

Let  $v(t)$  be the time  $t$  stock market value of a firm that has the manufacturing know-how of a variety of  $x$ . This know-how entitles the firm to a stream of operating profits  $\pi(\tau)$ ,  $\tau \geq t$ , where  $\pi$  is given in (8). Therefore the value of the firm equals the present value of operating profits. Taking account of (6) this condition reads

$$v(t) = \int_t^{\infty} e^{-\rho\tau} \pi(\tau) d\tau,$$

which implies (via differentiation with respect to time) the no arbitrage condition

$$(9) \quad \pi + \dot{v} = \rho v.$$

The left hand side represents the instantaneous reward for owning the firm. It consists of operating profits plus capital gains. The right hand side represents instantaneous costs, that equal foregone interest on the value of the firm.

Ownership of a firm can be acquired in two ways: by purchase of an existing company on the stock market at cost  $v$  or by the establishment of a new firm. The latter possibility requires to develop a new variety of  $x$ . Let  $a/K$  be the labor output ratio in product development, where  $a$  is a parameter and  $K$  represents the stock of knowledge capital in innovation. The interpretation of this coefficient is as follows. Employment of  $L_n$  labor units in R&D for a time interval of length  $dt$  increases the measure of available products by  $dn = (L_n K/a)dt$ . Consequently  $\dot{n} = L_n K/a$ , and the per product cost of product development equals  $wa/K$ . In equilibrium the cost of forming a new firm cannot fall short of the price of an existing firm, because if it did entrepreneurs would have had an unbounded demand for labor for R&D purposes. Therefore  $wa/K \geq v$ . On the other hand, if product development costs exceed the value of an existing firm there can be no innovation. Consequently, as long as there exists active R&D  $wa/K = v$ .

Knowledge capital grows as a result of experience in R&D. For the purpose of this paper I also assume that this knowledge becomes instantly available to all entrepreneurs. This assumption is rather extreme. Indeed, scientific knowledge spreads very fast through congresses and publications, but it affects manufacturing with very long lags (see Adams (1990)). In addition, experience in product innovation provides an advantage to the innovator for at least a limited period of time. For simplicity, however, I choose the simpler specification and assume  $K = n$ . Using this equation together with (7) and (8), the no arbitrage condition (9) becomes

$$(10) \quad (1-\alpha)/\alpha p a + \dot{p}/p - \dot{n}/n = \rho.$$

Finally we come to the resource constraint. Our specification of the R&D production function implies that employment in product development equals  $\dot{a}n/K$ , or  $\dot{a}n/n$ . Employment in manufacturing equals  $n x$ , where output per product is equal for all varieties. In fact,  $x=1/np$  (see (5) and recall that  $E=1$ ). These considerations imply the following labor market clearing condition

$$(11) \quad \dot{a}n/n + 1/p = L,$$

where  $L$  represents the available labor force. The first term on the left hand side represents employment in R&D. The second term represents employment in manufacturing. Our equation states that total employment equals the available labor force (it represents a resource constraint).

We are mostly interested in the growth rate. But the question is the growth rate of what? There are two variables in whose growth we might be interested: the consumption index  $D$  and the measure of available products; i.e., the rate of innovation. In what follows we concentrate on the latter, defined by  $g=\dot{n}/n$ , which is a basic dynamic driving force in this economy. It can be shown that the growth rate of consumption  $\dot{D}/D$  equals  $\bar{g}/\alpha - g$ , where  $\bar{g}$  represents the steady state innovation rate. It follows that the short and long-run rates of innovation determine the growth rate of consumption at each point in time. Consumption grows faster the larger the long-run rate of innovation and the smaller the short-run rate of innovation.

Equations (10) and (11) can be rewritten as

$$(12) \quad \dot{p}/p = \rho + g - (1-\alpha)/\alpha p,$$

$$(13) \quad g + 1/ap = H,$$

where  $H = L/a$  stands for the effective labor force in terms of R&D. This system represents a differential equation in price plus a side condition that describes the resource constraint. Curve HH in Figure 1 represents the resource constraint (13). It slopes upwards and approaches infinity as  $g$  approaches  $H$ . The downward sloping curve  $\dot{p}=0$  describes stationary points of  $p$ . Its properties are derived from (12). The intersection point 1 identifies a steady state equilibrium. Out of steady state the system follows the arrowed path trajectory. Perfect foresight, the consumer's transversality condition, and the lack of profit opportunities in product development imply that the economy has to converge to the steady state. Since point 1 is a source (i.e., an unstable equilibrium point), it implies that a perfect foresight trajectory coincides with the steady state point. Namely, the economy jumps immediately to the steady state.

Now we can examine two determinants of the innovation rate. First observe that an economy with a larger labor force has an HH curve further to the right. Consequently, countries with a larger resource base feature higher rates of innovation and growth than smaller countries. Next observe that an economy with a lower discount rate has a  $\dot{p}=0$  curve further to the right. It follows that more patient countries, that also have higher saving rates, innovate and grow faster than less patient countries. These two examples demonstrate the dependence of the rate of innovation and the growth rate on economic characteristics.

Before closing this section it is worth pointing out that (12)–(13) can be used to derive an autonomous differential equation in the rate of innovation. To do this

differentiate (13) with respect to time and substitute (12) and (13) into the resulting equation in order to obtain

$$(14) \quad \dot{g} = (H - g)[\rho - (1-\alpha)H/\alpha + g/\alpha] \quad \text{for } 0 \leq g \leq H.$$

(The domain restriction guarantees non-negative employment in innovation and manufacturing.) It implies a steady state rate of innovation

$$(15) \quad \bar{g} = (1-\alpha)H - \alpha\rho.$$

The steady state is again a source and therefore immediately attained in a perfect foresight equilibrium. It is evident from (15) that indeed the rate of innovation is larger the larger the resource base and the smaller the subjective discount rate, as we have argued above.

### III. A Two Country World

We have seen in the previous section how the growth rate and the rate of innovation of an isolated economy depend on resources and its subjective discount rate (i.e., its saving rate). Those rates did not vary over time. In this section we provide a two-country extension for a world in which innovation of a country contributes to the stock of knowledge of its trading partner to the same extent that it contributes to its own stock of knowledge. This represents an extreme case of instantaneous world wide dissemination of knowledge. We discuss an alternative specification in the closing section.

Our two country world does not attain immediately a steady state; country specific as well as the world's average growth and innovation rates vary over time. We shall consider the time pattern of innovation and cross country difference in innovation rates.

Preferences are as before and apply to both countries. I assume free international (financial) capital mobility. Therefore the same interest rate prevails in both countries. In this case (4) applies to each country separately and to the world at large. For current purposes  $E$  represents world spending and our numeraire will be  $E = 1$  for all  $t$ . Consequently (6) remains valid. The demand functions (5) also remain valid.

Now consider country  $i$ . At time  $t$  its firms possess the know-how to manufacture a measure  $n_i(t)$  of differentiated products. The measure of products available in the world equals  $n = \sum_i n_i$ . A typical manufacturer maximizes profits by choosing a price that exceeds marginal costs by a factor of  $1/\alpha$ . Assume that a unit of output requires one unit of labor in every country. This assumption is inconsequential; it represents a normalization of labor units and saves on notation. Therefore marginal manufacturing costs in country  $i$  equal the wage rate in country  $i$ ,  $w_i$ , and we replace (7) by

$$(7') \quad p_i = w_i / \alpha;$$



i.e., all country- $i$  manufactured varieties are equally priced. On the other hand product prices differ across countries as long as wage rates differ. Output of a variety of country  $i$  equals in this case to (see (5))

$$(16) \quad x_i = p_i^{-\epsilon} / \sum_j n_j p_j^{1-\epsilon}.$$

Using this representation profits per product in country  $i$  equal

$$(8') \quad \pi_i = (1-\alpha) p_i^{1-\epsilon} / \sum_j n_j p_j^{1-\epsilon}.$$

Now the no arbitrage condition (9) reads

$$(9') \quad \pi_i + \dot{v}_i = \rho v_i,$$

where  $v_i$  stands for the value of a country- $i$  firm. Given (9') the return on equity holdings is the same in both countries.

As before, the value of a firm equals the cost of product development as long as product innovation takes place. Recall that we assumed equal labor input per unit output in manufacturing in both countries. In order to allow for comparative advantage we now assume that innovation costs per product equal  $w_i a_i / K$  in country  $i$ . The first thing to observe about this formulation is that country  $i$  has a comparative advantage in R&D relative to manufacturing if and only if  $a_i < a_j$ ,  $i \neq j$ . The second thing to observe is that in this formulation the stock of knowledge capital is the same in both countries. As has been explained above, in this section we assume that not only does knowledge that has been acquired through product innovation spread to other domestic firms, but it also

spreads at an equal pace to foreign firms. We shall in fact assume that knowledge spreads instantaneously and  $K=n$ . (The consequences for steady states of lags in the dissemination of knowledge that differ within and across countries have been studied by Grossman and Helpman (1989a).) Using this specification as well as (7')–(9'), the no arbitrage condition can be rewritten in a form similar to (10);

$$(10') \quad n \frac{(1-\alpha)}{\alpha} \frac{p_i^{-\epsilon}/a_i}{\sum_j n_j p_j^{1-\epsilon}} + \dot{p}_i/p_i - g = \rho.$$

Finally, the resource constraint becomes

$$(11') \quad a_i \dot{n}_i/n + n_i p_i^{-\epsilon} / \sum_j n_j p_j^{1-\epsilon} = L_i,$$

where  $L_i$  stands for the labor force of country  $i$ .

This completes the description of the two-country model and we proceed to analyze its dynamics. First note from (10') that the relative price  $p_i/p_j$ ,  $i \neq j$ , converges to zero or infinity unless  $p_i^{-\epsilon}/a_i = p_j^{-\epsilon}/a_j$  initially, and therefore at each point in time thereafter. Consequently, on a perfect foresight equilibrium trajectory relative prices are constant, which implies that there exists a function  $q(t)$  such that

$$(17) \quad p_i = q a_i^{-1/\epsilon} \quad \text{for all } t.$$

Hence, the rate of change of every price equals the rate of change of  $q$ , and using (10') and

(17)

$$(18) \quad \dot{q}/q = \rho + g - (1-\alpha)/\alpha q \sum_j \sigma_j a_j^\alpha,$$

where  $\sigma_j = n_j/n$  represents the share of country  $j$  in the available products.

Next, using (17), we calculate from the resource constraint (11') the rate of innovation in country  $i$ ,  $g_i = \dot{n}_i/n_i$ , and the world's (average) rate of innovation  $g$ :

$$(19) \quad g_i = H_i/\sigma_i - 1/q \sum_j \sigma_j a_j^\alpha,$$

$$(20) \quad g = H - 1/q \sum_j \sigma_j a_j^\alpha,$$

where  $H_i = L_i/a_i$  stands for effective labor in terms of R&D in country  $i$  and  $H = \sum_j H_j$ . Naturally,  $g$  is a weighted average of  $g_i$ ;  $g = \sum_j \sigma_j g_j$ . By definition, the rate of change of the share  $\sigma_i$  equals  $g_i$  minus  $g$ . Therefore (19) and (20) imply

$$(21) \quad \dot{\sigma}_i = H_i - \sigma_i H, \quad 0 \leq \sigma_i \leq 1.$$

This is a simple, stable, linear and autonomous differential equation in  $\sigma_i$ . From every initial condition the shares converge monotonically to  $\sigma_i = H_i/H$ . In steady state the shares are proportional to effective labor. Our argument shows clearly that in this two-country world the steady state is approached gradually, unless the initial ownership of products by countries happens to be proportional to their effective labor values.

Equations (18) and (21) form a system of differential equations with side condition (20). This system represents an extension of (12)–(13). It does not lend itself, however, to an easy analysis. For this reason we proceed as follows. Differentiate (20) with respect to time to obtain

$$\dot{g} = (\dot{q}/q + \Sigma_j \dot{\sigma}_j a_j^\alpha / \Sigma_j \sigma_j a_j^\alpha) / q \Sigma_j \sigma_j a_j^\alpha.$$

Now use (18), (20) and (21) to obtain

$$(22) \quad \dot{g} = (H - g)(\alpha\rho - H + \alpha \Sigma_j H_j a_j^\alpha / \Sigma_j \sigma_j a_j^\alpha + g) / \alpha \quad \text{for } 0 \leq g \leq H.$$

This is the new counterpart of (14). It is straightforward to see that (22) reduces to (14) whenever no country has comparative advantage in R&D. In this special case (22) implies that the world's average rate of innovation does not vary over time, independently of whether the cross country composition of products varies over time. In particular, in this special case the world's innovation rate is given by (15) and therefore equals the innovation rate that obtains in an integrated world economy with effective labor equal to  $H = \Sigma_j H_j$ .

Now let us consider more carefully the case in which countries differ in relative efficiency with which they perform product innovation. In this case (21) and (22), which form a system of autonomous differential equations, can be used to analyze equilibrium trajectories of innovation and product shares. Since the sum of product shares equals identically to 1, we may use a two equation system consisting of (22) and one equation from (21). For concreteness assume that country  $i=2$  has comparative advantage in R&D; i.e.,  $a_1 > a_2$ . Figure 2 depicts the phase diagram of  $(\sigma_1, g)$ , where  $1-\sigma_1$  has been substituted for  $\sigma_2$  in (22). Along the vertical line  $\dot{\sigma}_1 = 0$  and along the upward slopping curve  $\dot{g} = 0$ . Their intersection at point 1 identifies the steady state. Evidently, the steady state is saddle path stable. The arrowed path through point 1 describes the perfect foresight equilibrium trajectory.

The time pattern of innovation is apparent from the figure. First consider the case in which the initial composition of products is such that country 1 — which has comparative disadvantage in R&D — has a disproportionately small share of products as

compared to its relative effective size. Then country 1 innovates faster than its trading partner, thereby increasing its product share. The average rate of innovation in the world economy falls short of its steady state value, but increases over time. If, on the other hand, country 1 has initially a disproportionately large share of products as compared to its effective relative size, it innovates at a slower rate than its trading partner. The average rate of innovation exceeds its steady state value and declines over time. The steady state innovation rate is the same as in (15); namely,  $\bar{g} = (1-\alpha)H - \alpha\rho$ . It follows that cross country differences in relative costs affect out of steady state but not the steady state world innovation rate (as long as  $H$  is the same in both cases). In addition, the larger the world economy in terms of effective labor, the faster steady state innovation. One should, however, bear in mind that more resources are not conducive to innovation in all cases (see Grossman and Helpman (1989a,d) on this point).

In order to demonstrate cross country links of innovation rates we now consider changes in country size as measured by their effective labor  $H_1$ . Suppose that initially the world is in a steady state, say at point 1 in Figure 2. The horizontal trajectory  $A$  in Figure 3 describes the time pattern of its innovation rate in the absence of disturbances. Now suppose, however, that country 1 — which has a comparative disadvantage in R&D — experiences an unexpected permanent increase in labor supply  $L_1$ . In this case the  $\dot{\sigma}_1 = 0$  line shifts to the right. The  $\dot{g} = 0$  curve shifts upwards or downwards at the initial value of  $\sigma_1$ , depending on whether  $\alpha < \Sigma_j H_j a_j^\alpha / H a_1^\alpha$  or  $\alpha > \Sigma_j H_j a_j^\alpha / H a_1^\alpha$ . Nevertheless, in either case the new steady state point is to the North-East of 1 (because the long-run rate of innovation increases).

Figure 4 depicts the case  $\alpha < \Sigma_j H_j a_j^\alpha / H a_1^\alpha$ . In this case the  $\dot{g} = 0$  curve shifts upwards, and so does the saddle path equilibrium trajectory. The rate of innovation rises on impact from point 1 to 3, and increases gradually thereafter together with the share of

country 1 in the available products, until they attain the new steady state values that are represented by point 2. Hence, in this case the innovation rate is higher and increasing monotonically over time, as depicted by trajectory B in Figure 3. The expansion of country 1's resource base is followed by a time interval in which this country innovates faster than its trading partner, as can be seen from the fact that  $\sigma_1$  increases over time.

The facts that country 1 innovates faster than country 2 and  $g$  is larger imply that country 1's rate of innovation,  $g_1$ , increases on impact. What happens to the rate of innovation in country 2? I show in the appendix that a resource expansion in country 1 leads to a decline in  $q$ . It follows from (19) that in the case under discussion the rate of innovation in country 2 declines on impact. Hence, the resource expansion in country 1 leads to slower innovation in its trading partner for a limited period of time. As time goes by, however, the rate of innovation in country 2 rises until it reaches the new steady state level that is represented by point 2 in Figure 4. We conclude that in this case there exists a negative international transmission of the structural change in the short-run and a positive transmission in the long-run. Namely, the structural change in one country affects the rate of innovation of its trading partner in the same direction as its own in the long-run but in the opposite direction in the short-run.

Next consider the case  $\alpha > \sum_j H_j a_j^\alpha / H a_1^\alpha$ . In this case the  $\dot{g} = 0$  curve shifts downwards, but not enough to reduce the long-run rate of innovation  $g$ . The rate of innovation may rise or decline on impact. Figure 5 depicts the case in which the rate of innovation rises on impact, which is qualitatively similar to the case discussed with the help of Figure 4. Figure 6 depicts a case in which the world's rate of innovation declines on impact from point 1 to point 3. When the rate of innovation declines initially, it remains lower than the original innovation rate for some time. However, since it increases over time until it attains the new steady state that features faster innovation, it eventually

becomes higher than the original rate of innovation and remains higher thereafter. Curve C in Figure 3 describes a trajectory of this nature. Despite the decline in the average rate of innovation, the fact that the share of country 1 in the available products increases over time implies that country 1 innovates faster than 2 until the steady state is reached. Due to the fact that  $q$  declines on impact (see appendix), it follows from (19) that  $g_2$  declines on impact, while  $g_1$  may increase or decline.

When the labor force of country 2 increases the  $\dot{\sigma} = 0$  line shifts to the left and the  $\dot{g} = 0$  curve shifts upwards. The new steady state point is North-West of 1, say at point 2 in Figure 7. Consequently, the rate of innovation rises on impact to point 3, overshoots its long-run level which is at point 2, and declines thereafter together with  $\sigma_1$  until the new steady state is reached (see the arrowed path leading to point 2). Trajectory D in Figure 3 describes its time pattern. Since  $q$  may decline or increase on impact (see appendix), it follows from (19) that in the short-run  $g_1$  increases or declines, but that it increases over time to reach the higher steady state level. We show in the appendix that  $q$  declines on impact whenever  $a_1$  is not much larger than  $a_2$ . In this case  $g_1$  declines on impact. We therefore have again a case in which the international transmission of the disturbance is negative in the short-run but positive in the long-run.

Our discussion shows clearly that as simple as this two-country model may be it generates rich dynamics. In particular, it shows that short-run rates of innovation may overshoot or undershoot long-run values, and therefore their time series may exhibit cyclical movements in response to structural shifts.



## V. Summery and Conclusions

Much of economic growth has been driven by inventions and innovations. It is therefore important to study their determinants. In an international environment in which countries trade and learn from each other these activities affect their R&D performance through various channels, such as through intentional and unintentional technology transfers. They also affect their resource allocation, including the allocation of resources to R&D. For these as well as other reasons the rate of innovation of a country depends on its own features as well as on features of its trading partners (see Grossman and Helpman (1990)).

In this paper we have studied the determinants of innovation rates in a simple international environment. The emphasis has been on the comovement of innovation rates across countries out of steady state. We have seen that the world's average rate of innovation may increase or decline over time, depending on whether the country with comparative advantage in R&D has a disproportionately large share of existing products (where the disproportionality is measured relative to the effective resource base). In the former case its innovation rate falls short of the innovation rate of its trading partner, thereby changing over time the composition of products in favor of the country with comparative disadvantage in R&D. In the latter case the country with comparative advantage in R&D innovates faster, changing the composition of products in its own favor.

Starting from a steady state, an increase in the resource base of the country with comparative advantage in R&D brings about an immediate upward jump in the world's rate of innovation and to its gradual decline thereafter. Nevertheless the average rate of innovation is larger at each point in time than in the initial steady state. Not only does the larger resource base of a country with comparative advantage in R&D speed up the world's innovation, but its own innovation rate becomes larger than its trading partner's.

In fact, the trading partner's rate of innovation declines temporarily whenever the comparative advantage in R&D is not very large. This leads to a gradual increase in the share of available products of the country with comparative advantage in R&D. In the long-run both countries innovate faster.

An increase in the resource base of a country with comparative disadvantage in R&D brings about an increase in the long-run average rate of innovation. The short-run rate may rise or fall, however. In either case, following the adjustment on impact, the world's innovation rate gradually increases. In the process the country with comparative disadvantage in R&D innovates at a faster pace than its trading partner and increases its share in available products. The trading partner's innovation rate declines temporarily. If the average innovation rate rises on impact it remains higher forever. Otherwise it remains lower for a limited period of time and becomes higher thereafter.

These results show that rates of innovation can exhibit complicated patterns. Even in the simple model employed in this paper innovation rates can respond to structural shifts by undershooting or overshooting in the short-run their long-run values. They may also rise or decline over time as the case may be.

A country with comparative advantage in R&D is not guaranteed to innovate faster than its trading partner; relative speeds of innovation depend on the relationship between the shares of existing products and relative resources. A country whose relative resource base exceeds the relative number of its available products innovates faster than its trading partner independently of whether it has comparative advantage in R&D.

The results reported in this paper are of course not conclusive. We have employed a strong assumption concerning the international transmission of knowledge which ensures long-run convergence of innovation rates. Under an alternative assumption that knowledge is country specific rather than international in scope, such as  $K_i = n_i$ , country

specific innovation rates do not converge in the long-run (see Feenstra (1990) and Grossman and Helpman (1990, chapter 9)). In the latter case cross country links of innovation rates can differ significantly from the results reported in this paper. Our results do demonstrate, however, that these links are far from being transparent.

### Appendix

We first establish in this appendix that a resource expansion in country 1 reduces  $q$  on impact. For this purpose we develop a differential equation in  $q$  by substituting (20) into (18). The result is

$$(A1) \quad \dot{q}/q = \rho + H - 1/\alpha q \sum_j \sigma_j a_j^\alpha.$$

This equation can be used jointly with (21) for  $i=1$  as an autonomous system of two differential equations in  $q$  and  $\sigma_1$  (with  $\sigma_2 = 1 - \sigma_1$ ). Figure A1 describes the phase diagram of this system. The arrowed path depicts the saddle path stable equilibrium trajectory. An increase in  $H_1$  shifts the  $\dot{\sigma}_1=0$  line to the right and the  $\dot{q}=0$  curve down. Hence, the new steady state point is to the South-East of the original steady state point and the new saddle path stable trajectory is lower around the original value of  $\sigma_1$ , at least for small changes in  $H_1$ . Since the system jumps down immediately to the new saddle path, it follows that  $q$  declines on impact when the system begins in the original steady state ( $\sigma_1$  is a state variable while  $q$  is a jump variable).

Next consider an increase in  $H_2$ . In this case line  $\dot{\sigma}=0$  shifts leftwards and curve  $\dot{q}=0$  shifts downwards. Consequently the new steady state value of  $q$  may increase or decline. A direct calculation from (21) and (A1) implies that the steady state value of  $q$  equals

$$(A2) \quad \bar{q} = \frac{H}{(\rho + H)\alpha \sum_i H_i a_i^\alpha}.$$

It follows from this equation that  $\bar{q}$  declines with  $H_1$  but that it may increase or decline with  $H_2$ . The steady state value of  $q$  declines with  $H_2$  if and only if

$\Sigma_i H_i a_i^\alpha / H a_2^\alpha < (\rho + H)/H$ . On the other hand, the preceding inequality is satisfied whenever  $a_1$  is not much larger than  $a_2$ . We conclude that an increase in  $H_2$  reduces  $\bar{q}$  when country 1 does not have a large comparative advantage in innovation. Otherwise an increase in  $H_2$  may increase the steady state value of  $q$ . Whenever  $\bar{q}$  declines with  $H_2$  an expansion of resources in country 2 reduces  $q$  on impact. When  $\bar{q}$  increases with  $H_2$ , however, an expansion of resources in country 2 may reduce or increase  $q$  on impact.

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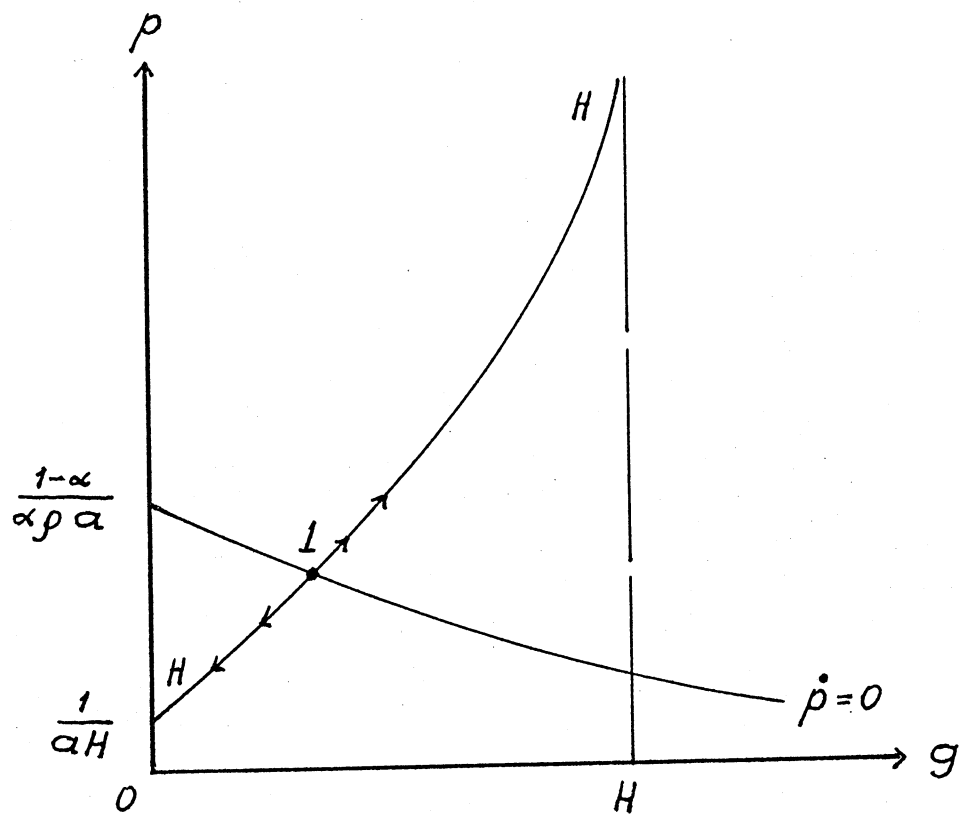


Figure 1

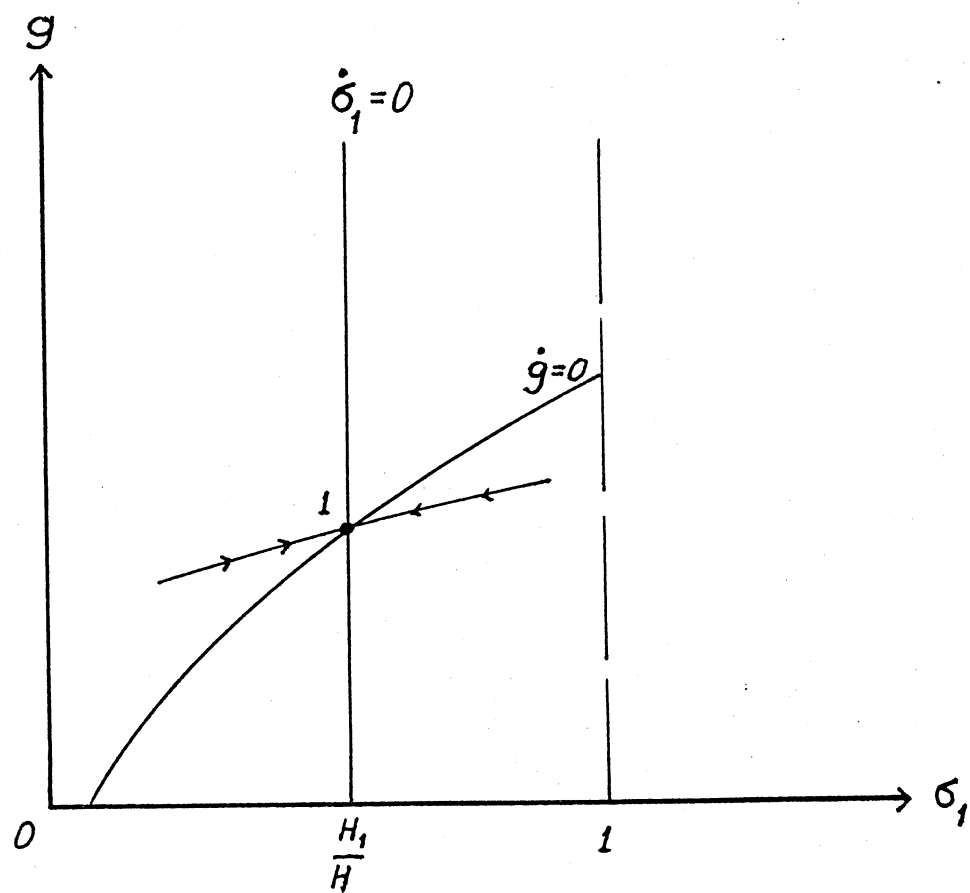


Figure 2

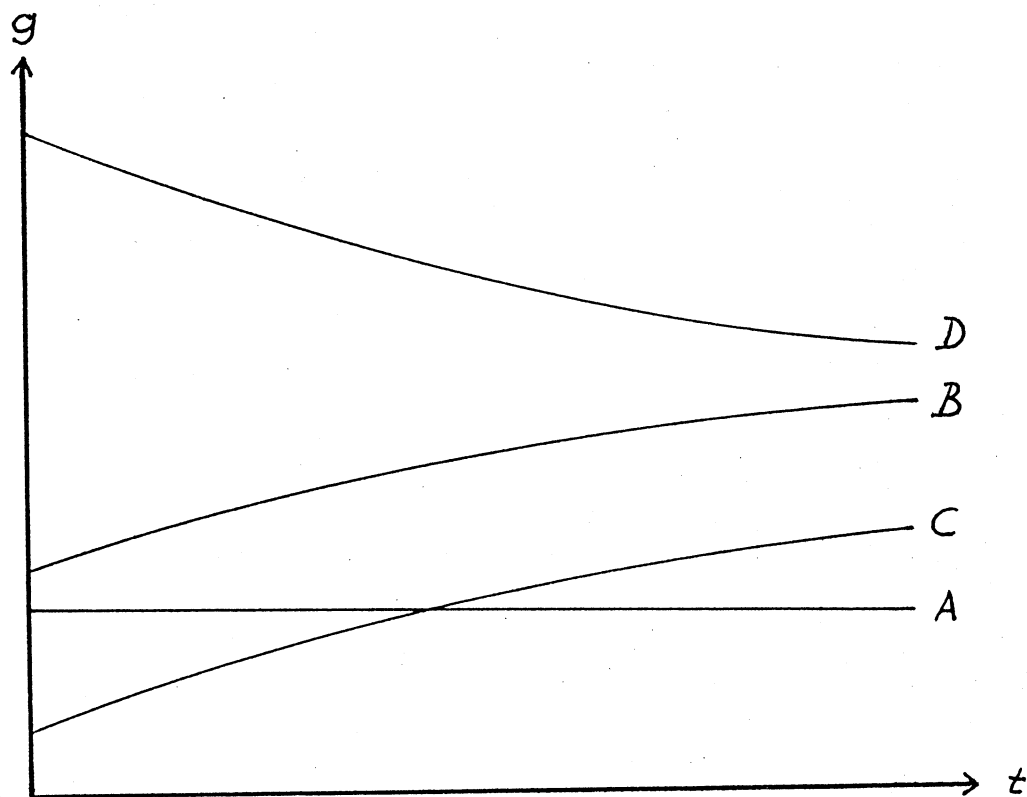


Figure 3

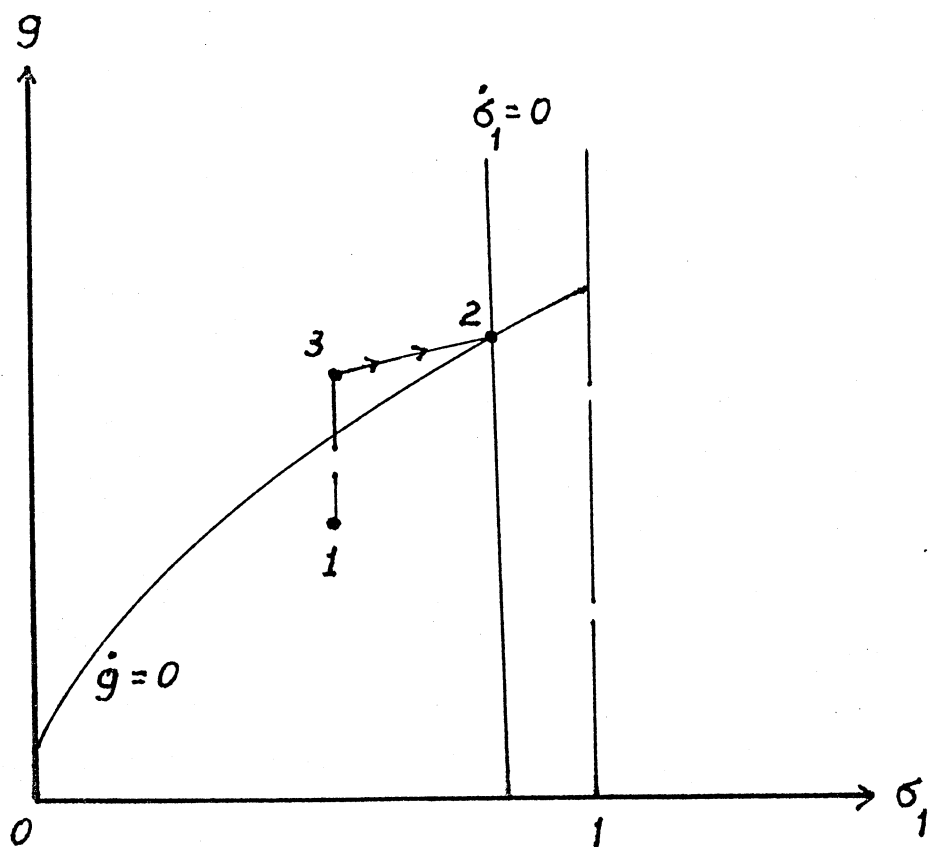


Figure 4

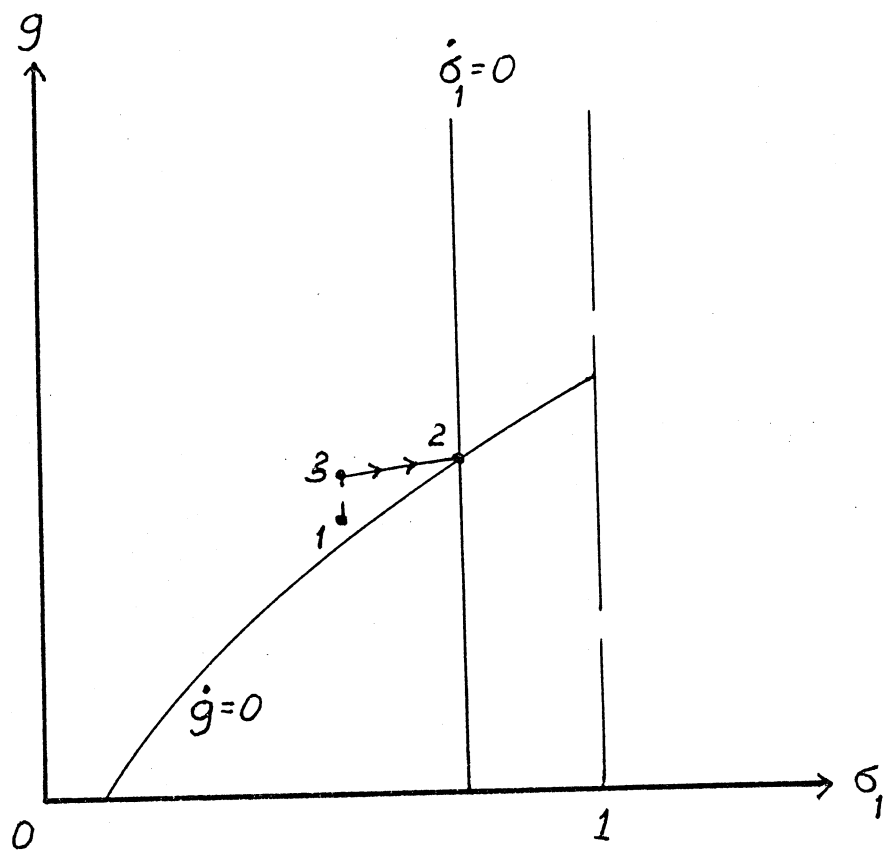


Figure 5

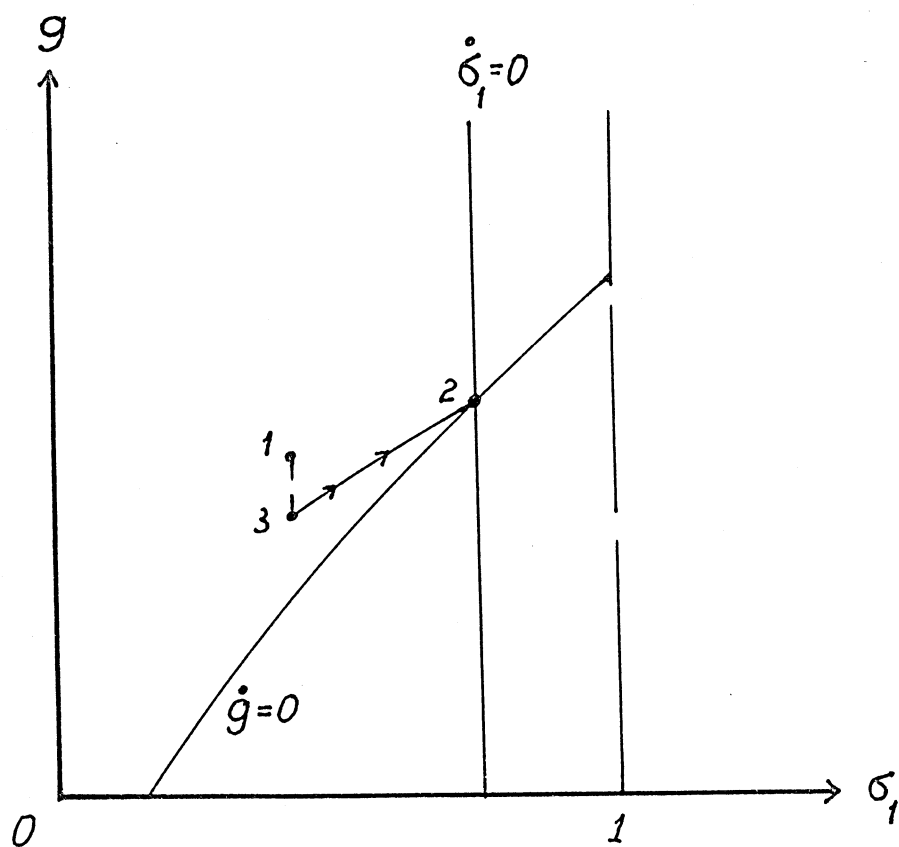


Figure 6



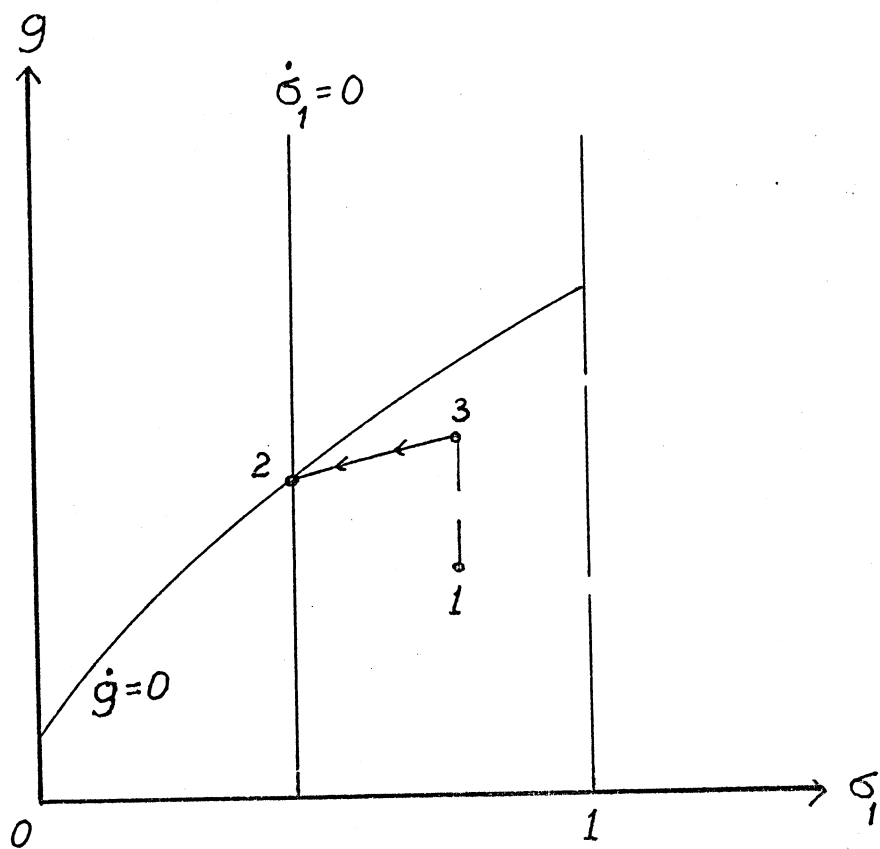


Figure 7

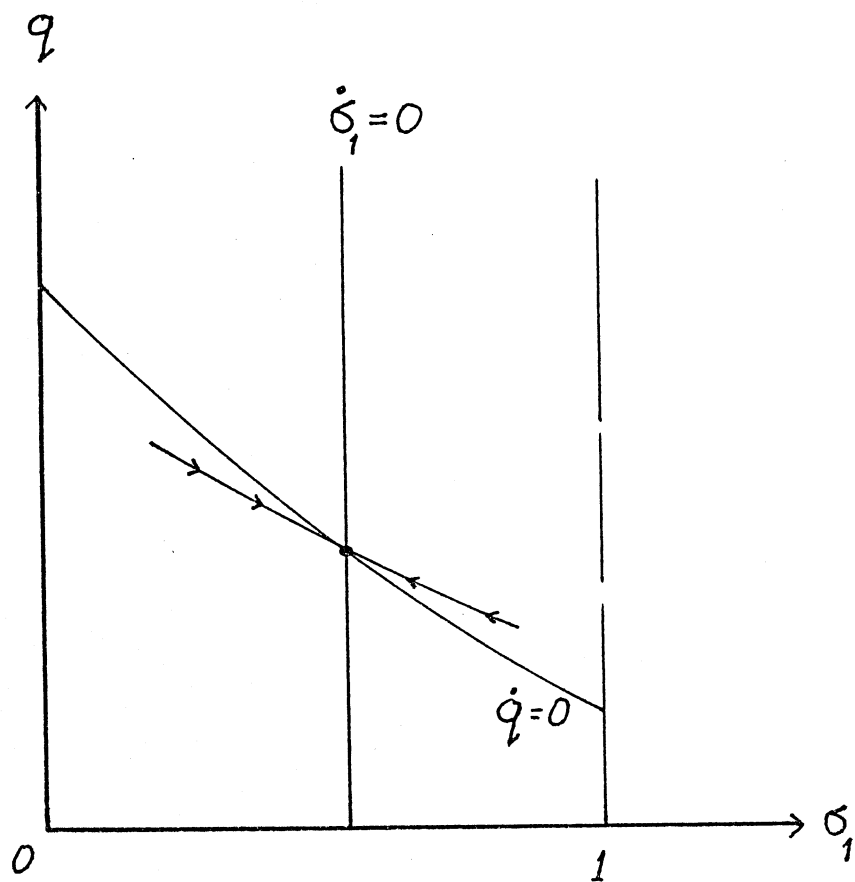


Figure A1

