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VON THUENEN'S MODEL OF THE DUAL ECONOMY[1]

by

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> Imagine a very large town, at the centre of a fertile plan which is crossed by no navigable river or canal. Throughout the plain the soil is capable of cultivation and of the same fertility. Far from the town, the plain turns into uncultivated wilderness which cuts off all communication between this State and the outside world.
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> There are no other towns on the plain. The central town must therefore supply the rural areas with all manufactured products and in return it will obtain all its provisions from the surrounding countryside.
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> J.H. von Thuenen (1826; 1842, p.11).

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1. Introduction

Models of economic growth and development in the dual economy tend to give short shrift to the role of preferences and to demand.[1] The rationale for this is clear: In the long run, rates of growth of the capital stock or population and labor supply, and the rate of technological progress determine the path which the economy follows. The composition of demand and preferences, except insofar as they affect individuals' allocations between present and future, do not affect the stationary equilibrium paths, provided such exist. Two things must, however, be said in this connection: First, the notion of a stationary path is itself an artificial construct; there is no reason why in the course of development such proportional growth should obtain. Transitions are all important. Second, although dual-economy models of growth do emphasize the allocation of labor between the two sectors and the consequent change in the composition of total product, the effects of changes in the terms of trade between agriculture and industry (or traditional and modern, or between whatever two sectors are distinguished), and the arbitrary nature of assumptions made concerning demand preclude an understanding of the role which relative **commodity prices** may play in the allocation of labor.

Recently Samuelson (1983) has given an extended appreciation of von Thuenen's contributions to location theory and, above all, to the development of the neoclassical theory of marginal factor productivity and distributive shares.[2] In his appreciation, however, Samuelson also brings out very nicely the general equilibrium nature of von Thuenen's location theory. Except by geographers concerned with economic development, the spatial aspects of growth have been generally neglected in the economic literature.[3] In particular, the role of transportation costs in determining the spatial distribution of commodity prices and, therefore, real wages and the spatial distribution of labor is nowhere treated adequately. The general spatial equilibrium model of von Thuenen, as exposited by Samuelson, may be used to fill this gap and to provide a basis for both equilibrium and disequilibrium models of growth of the dual economy. Although von Thuenen is widely regarded as the father of location theory and now appreciated as the independent discoverer of the marginal productivity theory of distribution, he is also, in our view, the author of the first, and in some ways the best, model of the dual economy, a model which, with a little effort can be turned into a model of dual economic development having considerable relevance to the developing economies of the world today in which high costs of transport are pervasive and significant determinants of relative commodity prices. Section 2 provides a general discussion of von Thuenen's general equilibrium model of the location of economic activity following Samuelson's appreciation. Section 3 brings out some spatial features of von Thuenen's model. Section 4 provides some interesting comparative statics results, concerning, in particular, the possibilities of technological progress and growth.

2. Von Thuenen's Equilibrium Model

We make the following assumptions:

(a) All land is homogeneous except for distance from the town. The town exists as a point.

(b) There are only two goods. A manufactured commodity, say cloth, produced in the town, and an agricultural commodity, say corn, produced in the countryside.

(c) Cloth is produced by labor alone (any raw materials used are available at the site of the town). To simplify the exposition, assume constant returns to scale, although this is unnecessary.

(d) Corn is produced by labor and land according to a mainly constant returns production function. It is useful, however, to assume a minimum application of labor per unit of land is necessary to grow anything at all. This translates into a minimal, nonzero land rent at the extensive margin of cultivation.

(e) All commodity markets clear instantaneously at every point; the total demand for cloth equals its supply as does the total demand for and supply of corn; prices adjust accordingly at every point.

(f) Transport costs for both commodities, although different, are proportional to distance (logarithmically linear in distance).

(g) Labor is homogeneous and assumed perfectly mobile. All laborers have identical preferences. The distribution of a fixed labor force between town and country, and in the country at different distances from the town, is determined so as to equalize the **real** wage (or utility) everywhere. Because the prices of cloth and corn may differ from location to location, however, **money** wages vary from point to point. Labor is assumed to live where it works.

(h) Landowners live in town and use all their rent (profit) income to consume corn and cloth. Their wealth consists entirely of the value of the land they own. All landowners have identical, homothetic preferences. This assumption simplifies matters since, as shown below, given relative prices, landowners' demand will be proportional to total rent income and will not depend on the number of landowners among whom it is divided.[4]

Independent, exogenous variables of the analysis are the technology of production, preferences for corn and for cloth, transport costs (or technology), and population or labor force. Subsuming the necessary minimal labor per unit land as a technological parameter, all other variables, such as relative prices at each point, the location of the extensive margin of cultivation, the real wage, money wages at each point, the amount of labor applied per unit of land, land rent at each point, and the total consumptions of cloth and corn, and the distribution of these in space, are all endogenous variables determined within the von Thuenen system by the equilibrium of factor and product markets. To see how the system works, proceed as follows: All distances may be measured in terms of the distance from the town, r. Indeed, all behavior at every point on a circle of radius r, with the town at the centre, is identical. Let the subscript 0 denote cloth (manufactures) and 1 denote corn (agricultural products). Production of cloth, Q_0, may be assumed to be proportional to the labor used in cloth production who live in the town, L_0:

(1) $$Q_0 = f_0 L_0,$$

where $f_0 > 0$ is assumed constant. Production of corn at a distance r from the town, $Q_1(r)$, depends on the labor applied, $L_1(r)$ to the quantity of land planted to corn, $A_1(r)$. We assume that agricultural poroduction takes place according to a constant returns production function, so that the **yield of corn per unit area**, $q_1(r)$, is a function of the labor applied per unit area, $\ell_1(r) \equiv L_1(r)/A_1(r)$, i.e.:

(2) $$q_1(r) = f_1[\ell_1(r)].$$

The function f_1 is strictly increasing ($f_1' > 0$) and strictly concave ($f_1'' < 0$). If R is the location of the extensive margin of cultivation, the distance from the town at which "... the plain turns into uncultivated wilderness," then the total production of corn is

(3) $$Q_1 = 2\pi \int_0^R q_1(r) r \, dr.$$

The total labor force in agriculture is, similarly,

(4) $$L_1 = 2\pi \int_0^R \ell_1(r) r \, dr.$$

If L is the total labor force available, given exogenously, then

(5) $$L = L_0 + L_1.$$

Let $w(r)$, $p_0(r)$, and $p_1(r)$ be, respectively, the wage rate, the price of cloth, and the price of corn, all at a distance r from town. Thus, equating the wages to the values of its marginal product yields:

(6) $$w(0) = p_0(0) f_0 = f_0,$$

and

(7) $$w(r) = p_1(r) f_1'(\ell_1(r)) \quad \text{for all} \quad r > 0,$$

where the price of cloth in town is set as the numeraire $(p_0(0) = 1)$. Wage payments in the town exhaust the total product of cloth. However, wage payments do not exhaust the product of corn at r; there is a positive residual equal to rent per unit of land:

(8) $$y_1(r) = p_1(r)[f_1(\ell_1(r)) - \ell_1(r) f_1'(\ell_1(r))] > 0,$$

because of the assumed concavity of the production function f_1.

Total income of landowners is equal to aggregate land rent:

(9) $$Y = 2\pi \int_0^R y_1(r) r \, dr.$$

We shall show in the next section (see (26) below) that the labor/land ratio (ℓ_1) is diminishing as distance from the town increases. Thus, land is increasingly substituted for labor until some minimal labor/land ratio (say m) is reached. At this point, say R, all cultivation ceases:

(10) $$\ell_1(R) = m.$$

Denote the expenditure function by $E(\cdot)$, i.e., $E(p_0, p_1, u)$ is the (minimum) income required in order to attain the utility level u at prices p_0 and p_1 for cloth and corn, respectively. Since laborers are homogenous and perfectly mobile, they must all enjoy the same utility level, irrespective of where they choose to work and reside. This yields the fundamental **locational equilibrium** condition of the model:

(11) $$E(p_0(r), p_1(r), u) = w(r) \quad \text{for all} \quad r \geq 0.$$

For every r, the price of corn must be such that producers are just indifferent between selling the grain locally for $p_1(r)$ or shipping it to the town where the price is higher but having to deduct the costs of transport. Von Thuenen makes a series of careful calculations in which he reckons the costs of transport to consist largely of corn consumed on the way by the oxen pulling the load; thus, as a first approximation, we can assume transport costs in grain to be proportional to the quantity transported per mile. A bushel of grain in the countryside r miles from the town becomes $e^{-a_1 r}$ bushels in town. It follows that

$$(12) \qquad p_1(r) = p_1(0)e^{-a_1 r},$$

where a_1 is a parameter reflecting the costs of transport; the higher a_1, the greater the costs of transport. Although there is no argument for doing so other than symmetry and simplicity, assume that the costs of transporting cloth to the countryside also have the same proportionate form

$$(13) \qquad p_0(r) = p_0(0)e^{a_0 r} = e^{a_0 r}.^{5}$$

At every point r, the excess supply of corn consists of output minus consumption of laborers residing there, i.e.:

$$2\pi r[f_1(\ell_1(r)) - H_1(p_0(r), p_1(r), u)\ell_1(r)],$$

where $H_i(\cdot)$ is the Hicksian demand function for commodity $i = 0,1$. This excess supply is shipped to town. However, due to transport cost, a bushel of corn which is shipped from point r to town shrinks to $e^{-a_1 r}$ bushels in town. Thus, the corn market clearing equation (in town) requires that

$$(14) \quad 2\pi \int_0^R r[f_1(\ell_1(r)) - H_1(p_0(r), p_1(r), u)\ell_1(r)]e^{-a_1 r} dr = L_0H_1(p_0(0), p_1(0), u) + D_1(p_0(0), p_1(0), Y),$$

where L_0H_1 is the demand for corn by cloth laborers and $D_i(\cdot)$ is the Marshallian demand function of landowners for commodity $i = 0,1$.

Similarly, the cloth market clearing equation requires that:

$$(15) \quad 2\pi \int_0^R rH_0(p_0(r), p_1(r), u)\ell_1(r)e^{a_0 r} dr + L_0H_0(p_0(0), p_1(0), u) + D_0(p_0(0), p_1(0), Y) = Q_0.$$

Due to Walras's Law, one of the two market clearing equations, (14) or (15), is redundant. We henceforth drop equation (14). The 14 equations remaining determine the following endogenous variables:

total cloth output, Q_0;

corn output per unit area at location r, $q_1(r)$;

total corn output, Q_1;

labor/land ratio in corn production at location r, $\ell_1(r)$;

total labor input in corn production, L_1;

total labor input in cloth production, L_0;

wage rate in town, $w(0)$;

wage rate at location r, $w(r)$;

cloth price at location r, $p_0(r)$;

corn price at location r, $p_1(r)$;

land rent at location r, $y_1(r)$;

aggregate land rent, Y;

the location of the extensive margin of cultivation, R;

the laborers' common utility level, u;

3. Spatial Features of the von Thuenen Model

In this section we investigate the spatial properties of the endogenous variables. Specifically, we examine the gradients (namely, the derivatives with respect to r) of the wage, cloth and corn prices, rent, cloth and corn consumption and factor intensity (equivalently, population density).

Before proceeding further we will state some familiar results from demand theory that will be employed in our analysis. First, the envelope theorem implies the derivative property of the expenditure function:

$$(16)\qquad \frac{\partial E}{\partial p_i}(p_0,\ p_1,\ u) = H_i(p_0,\ p_1,\ u), \qquad i = 0,1.$$

Second, we have the following signs for the Hicks-Slutsky substitution terms $(\partial H_i/\partial p_j,\ \ i,j = 0,1)$:

$$\frac{\partial H_i}{\partial p_j}(p_0,\ p_1,\ u)\begin{cases} < \quad 0 & \text{if } i = j \\ > \quad 0 & \text{if } i \neq j. \end{cases}$$

A dot $(\cdot)$ over a variable will denote a derivative with respect to location r.

The price gradients are trivially obtained from (12)-(13):

$$(18)\qquad \dot{p}_0 = a_0p_0 > 0$$

and

$$(19)\qquad \dot{p}_1 = -a_1p_1 < 0.$$

That is, the price of cloth rises and the price of corn declines as one moves further away from the town.

The consumption gradients are derived by making use of (17):

$$(20)\qquad \dot{H}_0 = \frac{\partial H_0}{\partial p_0}\,\dot{p}_0 + \frac{\partial H_0}{\partial p_1}\,\dot{p}_1 < 0$$

and

$$(21) \qquad \dot{H}_1 = \frac{\partial H_1}{\partial p_0}\dot{p}_0 + \frac{\partial H_1}{\partial p_1}\dot{p}_1 > 0.$$

That is, as the price of cloth rises and the price of corn declines, the consumption of cloth declines and the consumption of corn rises further away from the town: laborers substitute the cheaper commodity (corn) for the more expensive commodity (cloth).

To obtain the wage gradient, differentiate the locational equilibrium condition (11) with respect to r to obtain:

$$\dot{w} = \frac{\partial E}{\partial p_0}\dot{p}_0 + \frac{\partial E}{\partial p_1}\dot{p}_1$$

$$= H_0\dot{p}_0 + H_1\dot{p}_1,$$

where use is made of the derivative property (16). Substituting (18) and (19) for $\dot{p}_0$ and $\dot{p}_1$, respectively, one obtains:

$$(22) \qquad \dot{w} = a_0p_0H_0 - a_1p_1H_1.$$

The interpretation of (22) is straightforward: as the laborer moves further away from the town, he has to be compensated by $a_0p_0H_0$ for the increase in transport cost (and price) of cloth and can give up $a_1p_1H_1$ due to the decline in the price of corn. Thus, the sign of the wage gradient is not generally determined without making further assumptions. Denote by $\alpha \equiv p_0H_0/w$ and by $1 - \alpha \equiv p_1H_1/w$ the budget shares of cloth and corn, respectively. Then, (22) can be rewritten as

$$(23) \qquad \dot{w} = [a_0\alpha - a_1(1-\alpha)]w.$$

Thus, assuming a symmetry in transport cost (namely, $a_0 = a_1$) and that, as is plausible in a low-income economy, corn's budget share is higher than cloth's budget share,[6] then $\dot{w} < 0$. That is, money wage declines further away from the town.

In order to derive the factor intensity gradient differentiate the wage equation (7) with respect to r:

$$(24) \qquad \begin{aligned} \dot{w} &= \dot{p}_1f_1' + p_1f_1''\dot{\ell}_1 = -a_1p_1f_1' + p_1f_1'f_1''\dot{\ell}_1/f_1' \\ &= (-a_1 + f_1''\dot{\ell}_1/f_1')w, \end{aligned}$$

where use is made of (19) and (7). Compare (24) to (23) to conclude that

$$(25) \qquad f_1''\dot{\ell}_1/f_1' = (a_0 + a_1)\alpha > 0.$$

Since $\ddot{f}_1 < 0$, by the concavity of f_1, it follows from (25) that

$$(26) \qquad \dot{\ell}_1 < 0.$$

That is, the labor/land ratio diminishes further away from the town. Hence, even though the marginal product of labor increases, the (money) wage does not necessarily increase since the price of corn diminishes.

Nevertheless, as the factor intensity (ℓ_1) diminishes, the factor price ratio (w/y_1) must rise further away from the town:

$$(27) \qquad (w/\dot{y_1}) > 0.$$

The rent gradient must be downward sloping because both the marginal product of land and the price of corn diminish further away from the town:

$$(28) \qquad \dot{y}_1 < 0.$$

4. Comparative Statics

In this section we derive some comparative statics results for the von Thuenen's dual economy.[7] In order to keep the analysis tractable we simplify the model by specifying Cobb-Douglas preferences and technology. In this case equation (2) becomes

(2a) $$q_1(r) = f_1(\ell_1(r)) = \gamma \ell_1(r)^{\beta},$$

where $0 < \beta < 1$ is the labor's share in output and $\gamma > 0$ is a Hicks-neutral coefficient of technological change. Correspondingly, we have the following factor price equations:

(7a) $$w(r) = \gamma \beta p_1(r) \ell_1(r)^{\beta-1}$$

and

(8a) $$y_1(r) = \gamma(1-\beta) p_1(r) \ell_1(r)^{\beta}.$$

The aggregate land rent is

(9a) $$Y = 2\pi\gamma(1-\beta) \int_0^R p_1(r) \ell_1(r)^{\beta} r dr.$$

The expenditure function for the Cobb-Douglas preferences takes the following form:

(29) $$E(p_0, p_1, u) = p_0^{\alpha} p_1^{1-\alpha} u,$$

where $0 < \alpha < 1$ is the budget share of cloth. (Note that, not as in general as in equation (23), α is a constant which does not depend on prices and distance (r)).

The Hicksian demand functions are:

(30) $$H_0(p_0, p_1, u) = \alpha p_0^{\alpha-1} p_1^{1-\alpha} u,$$

and

(31) $$H_1(p_0, p_1, u) = (1-\alpha) p_0^{\alpha} p_1^{-\alpha} u.$$

The landowners' demands are:

(32) $$D_0(p_0(0), p_1(0), Y) = \alpha Y/p_0(0) = \alpha Y,$$

and

(33) $$D_1(p_0(0), p_1(0), Y) = (1-\alpha)Y/p_1(0).$$

Equation (23) may be solved for w:

(34) $$w(r) = w(0)e^{[a_0\alpha - a_1(1-\alpha)]r} = f_0 e^{[a_0\alpha - a_1(1-\alpha)]r},$$

where use is made of (6). On the other hand, equation (7a) implies that

(35) $$w(r) = \gamma\beta\, p_1(0)e^{-a_1 r} \ell_1(r)^{\beta-1}.$$

Thus, equating (34) and (35) we conclude that

(36) $$\ell_1(r) = \left[\frac{f_0 e^{(a_0\alpha + a_1\alpha)r}}{p_1(0)\gamma\beta}\right]^{\frac{1}{\beta-1}},$$

and, upon, substituting (10):

$$(37)\qquad m = \left[\frac{f_0 e^{(a_0\alpha + a_1\alpha)R}}{p_1(0)\gamma\beta}\right]^{\frac{1}{\beta-1}}.$$

Equations (11) and (34) imply that

$$(e^{a_0 r})^{\alpha}(p_1(0)e^{-a_1 r})^{1-\alpha} u = f_0 e^{[a_0\alpha - a_1(1-\alpha)]r},$$

so that:

$$(38)\qquad u = f_0\, p_1(0)^{\alpha-1}.$$

On substituting (30), (32), (36) and (38), equation (15) (the cloth market clearing equation) becomes:

$$2\pi\alpha\, f_0^{\frac{\beta}{\beta-1}}[p_1(0)\gamma\beta]^{\frac{1}{1-\beta}}\int_0^R e^{\delta_1 r}\, r dr$$

$$(39)\qquad + 2\pi\alpha(f_0\beta)^{\frac{\beta}{\beta-1}}[p_1(0)\gamma]^{\frac{1}{1-\beta}}(1-\beta)\int_0^R e^{\delta_1 r}\, r\, dr = (1-\alpha)\, f_0 L_0,$$

where

$$\delta_1 = \frac{a_0\alpha\beta + a_1[1-(1-\alpha)\beta]}{\beta-1} < 0,$$

because $\beta < 1$ (see appendix).

Similarly, on substituting (4) and (36), equation (5), the labor market clearing equation, becomes:

$$(40) \qquad L = L_0 + 2\pi f_0^{\frac{1}{\beta-1}}[p_1(0)\gamma\beta]^{\frac{1}{1-\beta}} \int_0^R e^{\delta_2 r} r\, dr,$$

where

$$\delta_2 = \frac{(a_0 + a_1)\alpha}{\beta - 1} < 0,$$

because $\beta < 1$. Recalling (37), we can rewrite (39) and (40) as:

$$(41) \qquad 2\alpha\pi m\beta^{-1}e^{-\delta_2 R}\int_0^R e^{\delta_1 r} r dr = (1-\alpha)L_0$$

and

$$(42) \qquad L = L_0 + 2\pi m e^{-\delta_2 R}\int_0^R e^{\delta_2 r} r\, dr.$$

Equations (37), (41) and (42) contain only three endogenous variables: $p_1(0)$, L_0 and R. We can use this set of three equations in order to study the comparative statics properties of the model. Specifically, we study the effects of changes in the exogenous variables (e.g., f_0, L and γ) on the endogenous variables $p_1(0)$, L_0 and R first and then we consider the effects on other endogenous variables (e.g., u, Y, Q_0, Q_1, etc.).

(a) **Population Growth**

In this subsection we examine the effect of a change in L. From (42) we conclude that

$$(43) \qquad 1 = \frac{\partial L_0}{\partial L} + 2\pi m \frac{\partial R}{\partial L}\left\{R - \delta_2 e^{-\delta_2 R} \int_0^R e^{\delta_2 r} r dr\right\}.$$

Since $\delta_2 < 0$, it follows from (41) that $\partial L_0/\partial L$ and $\partial R/\partial L$ must have the same sign (because an increase in R increases the left hand side of (41) and an increase in L_0 increases the right hand side of (41)). Hence, (43) implies that

$$(44) \qquad \frac{\partial L_0}{\partial L} > 0,$$

and

$$(45) \qquad \frac{\partial R}{\partial L} > 0.$$

That is, an increase in population increases the number of people living in the town and extends the extensive margin of cultivation.

Now, (37) implies that $\partial p_1(0)/\partial L > 0$ and hence

(46) $$\frac{\partial p_1(r)}{\partial L} > 0.$$

So, an increase in population increases the price of grain everywhere. Recalling that $\beta < 1$, it follows from (36) that

(47) $$\frac{\partial \ell_1(r)}{\partial L} > 0.$$

Thus, an increase in population increases the intensity of cultivation.

Since $\alpha < 1$, it follows from (38) that

(48) $$\frac{\partial u}{\partial L} < 0.$$

That is, population increase leads to a fall in the welfare (real wage) of laborers but not to a fall in the nominal wage anywhere, since equation (34) implies that $w(r)$ does not change:

(49) $$\frac{\partial w(r)}{\partial L} = 0.$$

Since $\partial \ell_1(r)/\partial L > 0$ (and, consequently, $\partial q_1(r)/\partial L > 0$), $\partial R/\partial L > 0$ and $\partial p_1(r)/\partial L > 0$, it follows from (3) and (9a) that

(50) $$\frac{\partial Y}{\partial L} > 0,$$

and

(51) $$\frac{\partial Q_1}{\partial L} > 0.$$

Rents rise and total grain production rises with an increase in population, and so does total cloth output, since $\partial L_0/\partial L > 0$ and $Q_0 = f_0 L_0$, so it follows that

(52) $$\frac{\partial Q_0}{\partial L} > 0.$$

Summarizing: Population growth increases cloth and corn output and the labor/land ratio, extends the extensive margin of cultivation, increases the aggregate land rent, and the (relative) price of corn, but leaves the wage unchanged, and lowers the utility (real wage) of laborers.

(b) **A Hicks-Neutral technologocal improvement in corn production**

In this subsection we examine the effect of a change in γ. It follows from (42) that

(53) $$0 = \frac{\partial L_0}{\partial \gamma} + 2\pi m \frac{\partial R}{\partial \gamma}\left\{R - \delta_2 e^{-\delta_2 R} \int_0^R e^{\delta_2 r} r \, dr\right\}.$$

Hence, $\partial L_0/\partial \gamma$ and $\partial R/\partial \gamma$ must have opposite signs. However, as in the preceding subsection, (41) implies that they must have the same sign. Therefore:

(54) $$\frac{\partial L_0}{\partial \gamma} = \frac{\partial R}{\partial \gamma} = 0.$$

Thus, a neutral technological change in agriculture leaves the labor force in town and in the country and the extensive margin of cultivation unchanged.

Since $\partial R/\partial \gamma = 0$, it follows from (37) that

(55) $$\frac{\partial}{\partial \gamma}(p_1(0)\gamma) = 0,$$

so that

(56) $$\frac{\partial p_1(r)}{\partial \gamma} < 0.$$

That is, the price of grain must fall everywhere.

The wage, $w(r)$, does not change (see (34)):

(57) $$\frac{\partial w(r)}{\partial \gamma} = 0$$

and it follows from (36) and (55) that,

(58) $$\frac{\partial \ell_1(r)}{\partial \gamma} = 0.$$

Therefore, corn output which is equal to $2\pi\gamma\int_0^R \ell_1(r)^\beta r dr$ increases in proportion to γ:

(59) $$\frac{\partial Q_1}{\partial \gamma} \cdot \frac{\gamma}{Q_1} = 1.$$

Since $\gamma p_1(0)$, R and $\ell_1(r)$ do not change, it follows that the aggregate land rent does not change either:

(60) $$\frac{\partial Y}{\partial \gamma} = 0.$$

Since $\alpha < 1$, it follows from (56) and (38) that

(61) $$\frac{\partial u}{\partial \gamma} > 0.$$

Technical change in agriculture increases the utility (real wage) of laborers.

Summarizing: a Hicks-neutral technological improvement in corn production increases corn output and laborers' utility (real wage); it does not change cloth output, the ratio of city to farm workers, the location of the extensive margin of cultivation, the nominal wage, factor intensity in agriculture or aggregate land rate; and it lowers the price of grain relative to cloth.

(c) **A technological improvement in cloth production**

In this subsection we examine the effect of a change in f_0. As in the preceding subsection, it follows from (42) and (41) that:

(62) $$\frac{\partial L_0}{\partial f_0} = \frac{\partial R}{\partial f_0} = 0.$$

Neither the distribution of the labor force nor the extensive margin of cultivation are changed by a technical change in manufacturing. Hence, it follows from (37) that $f_0/p_1(0)$ must stay constant and, consequently,

(63) $$\frac{\partial p_1(r)}{\partial f_0} \cdot \frac{f_0}{p_1(r)} = 1.$$

That is, the price of grain rises everywhere exactly in proportion to the improvement of manufacturing productivity.

The wage rate is seen from (34) to rise proportionately with f_0 as well:

(64) $$\frac{\partial w(r)}{\partial f_0} \cdot \frac{f_0}{w(r)} = 1.$$

Since one can rewrite (38) as

$$u = p_1(0)^{\alpha} f_0/p_1(0)$$

and since $f_0/p_1(0)$ remains constant, it follows that

(65) $$\frac{\partial u}{\partial f_0} > 0.$$

The welfare (real wage) of laborers is increased by a technical improvement in manufacturing.

Again, since $f_0/p_1(0)$ does not change, it follows from (36) that

(66) $$\frac{\partial \ell_1(r)}{\partial f_0} = 0$$

and, consequently,

(67) $$\frac{\partial Q_1}{\partial f_0} = 0.$$

So, total grain output and labor intensity in agriculture do not change.

Since $p_1(r)$ increases, while $\ell_1(r)$ and R stay constant, it follows from (9a) that

(68) $$\frac{\partial Y}{\partial f_0} > 0.$$

That is, total land rents are increased.

Summarizing: a technological improvement in cloth production increases cloth output, wages, corn prices and aggregate land rent proportionately; such change also raises the utility of labor, but does not change the location of the extensive margin of cultivation, the distribution of the labor force between city and farms, factor intensity in agriculture or total corn output.

5. Concluding Remarks

We have not analyzed either the effects of falling transport costs or changing preferences in any systematic way. It is clear from the fact that a_0, a_1 and α enter the model through δ_1 and δ_2 that the consequences of changing transport costs depend in a complicated way on the interaction of these costs with preferences and technology. The analysis of this interaction is a subject for further research. However, some intuitive remarks are in order.

First, transport costs are incurred in moving goods to and from the countryside where agricultural production takes place. To a first approximation, falling transport costs impact primarily on agriculture and, to this first approximation, should affect the equilibrium primarily as does neutral technical change in agriculture. As we have seen, the assumption of unit elasticity of substitution in preferences rules out any effect, but some effect may be expected when preferences differ from the Cobb-Douglas form. The sign of these effects depends on how the elasticity of substitution differs from one.

Second, preferences in the Cobb-Douglas model are completely reflected in the parameter α. A small α is equivalent to a small share of cloth in total consumption, i.e., a large share of food in the total. Clearly, to a first approximation, the greater the share of food the larger will be the labor force in agriculture. The effect of a fall in the share of food, however, does not automatically translate into migration from the rural sector into the town because prices may change in such a way that a lower share of food in the total does not necessarily reflect a smaller physical consumption and therefore output. If the price of grain falls more than proportionately in equilibrium, labor will move from the town out to the countryside.

FOOTNOTES

1 A notable exception is the model of Kelley, Williamson and Cheetham (1972). For a fine survey incorporating many of his own original contributions, see Dixit (1973).

2 The discussion concerning the marginal productivity theory of wages is continued in Dorfman (1986) and Samuelson (1986).

3 Not so by T.W.Schultz (1953, especially Ch.9, 146-151). See also Katzman (1977), who gives an extended discussin of the spatial aspects of economic development in Brazil. Of course, many economists have dealt with the problems of urbanization, urban labor markets, and rural-urban migration in the course of economic development. See Kelley, **et al.** (1972, Ch.7, 234-255). But space itself seems largely incidental to the models developed.

4 It does not matter whether we assume that the laborers rent the land from the landowners or the landowners hire the laborers.

5 Any form of transportation costs, monotonically increasing with distance, which caused corn prices to fall the further from the town, and cloth prices to rise, would suffice.

6 These assumptions are not needed except to demonstrate $\dot{w} < 0$.

7 A similar comparative statics analysis for a purely urban model can be found in Pines and Sadka (1986).

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APPENDIX

I. We first show (39):

Substituting (12), (13), (30) and (32) into the cloth market clearing equation (15) yields:

$$\text{(A1)}\quad 2\pi\alpha\ p_1(0)^{1-\alpha}\ u \int_0^R e^{a_0 r(\alpha-1)}\ e^{-a_1 r(1-\alpha)}\ r\ell_1(r) e^{a_0 r} dr$$

$$+\ \alpha L_0 p_1(0)^{1-\alpha}\ u + \alpha Y = f_0 L_0 .$$

Substituting (38) into (A1):

$$\text{(A2)}\qquad 2\pi\alpha f_0 \int_0^R e^{[a_0\alpha\ -\ a_1(1-\alpha)]r}\ r\ell_1(r) dr + \alpha L_0 f_0 + \alpha Y = f_0 L_0 .$$

Substituting (9a) and (36) into (A2) yields:

$$\text{(A3)}\quad 2\pi\alpha\ f_0^{\frac{\beta}{\beta-1}}\ p_1(0)^{\frac{1}{1-\beta}}\ \gamma^{\frac{1}{1-\beta}}\ \beta^{\frac{1}{1-\beta}} \int_0^R e^{[a_0\alpha\ -\ a_1(1-\alpha)]r}\ r\ e^{\frac{a_0\alpha\ +\ a_1\alpha}{\beta-1} r} dr$$

$$+\ \alpha L_0 f_0 + 2\pi\alpha f_0^{\frac{\beta}{\beta-1}}\ p_1(0)^{\frac{1}{1-\beta}}\ \gamma^{\frac{1}{1-\beta}}\ \beta^{\frac{\beta}{1-\beta}}(1-\beta) \int_0^R e^{-a_1 r}\ e^{\frac{(a_0\alpha\ +\ a_1\alpha)\beta}{\beta-1} r} dr$$

$$= f_0 L_0 .$$

Rearranging terms yields:

$$\text{(A4)} \qquad 2\pi\alpha\ f_0^{\frac{\beta}{\beta-1}}\ p_1(0)^{\frac{1}{1-\beta}}\ \gamma^{\frac{1}{1-\beta}}\ \beta^{\frac{1}{1-\beta}} \int_0^R e^{\delta_1 r} r dr$$

$$+\ 2\pi\alpha f_0^{\frac{\beta}{\beta-1}}\ p_1(0)^{\frac{1}{1-\beta}}\ \gamma^{\frac{1}{1-\beta}}\ \beta^{\frac{\beta}{1-\beta}}\ (1-\beta) \int_0^R e^{\delta_1 r} r\ dr$$

$$= (1-\alpha) f_0 L_0,$$

where:

$$\text{(A5)} \qquad \delta_1 = \frac{a_0\alpha\beta + a_1[1-(1-\alpha)\beta]}{\beta-1} < 0,$$

because $\beta < 1$. This proves (39).

II. We next show (40):

Substituting (36) into (4) yields:

(A6) $$L_1 = 2\pi\, f_0^{\frac{1}{\beta-1}}\, p_1(0)^{\frac{1}{1-\beta}}\, \gamma^{\frac{1}{1-\beta}}\, \beta^{\frac{1}{1-\beta}} \int_0^R e^{\frac{(a_0\alpha + a_1\alpha)r}{\beta-1}}\, r\, dr$$

$$= 2\pi\, f_0^{\frac{1}{\beta-1}}\, [p_1(0)\gamma\beta]^{\frac{1}{1-\beta}} \int_0^R e^{\delta_2 r}\, r\, dr,$$

where

(A7) $$\delta_2 = \frac{a_0\alpha + a_1\alpha}{\beta-1} < 0,$$

because $\beta < 0$. Substituting (A6) into (5) yields

(A8) $$L = L_0 + 2\pi\, f_0^{\frac{1}{\beta-1}}\, [p_1(0)\gamma\beta]^{\frac{1}{1-\beta}} \int_0^R e^{\delta_2 r}\, r\, dr.$$

This proves (40).