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PRICE CYCLES AND BOOMS: DYNAMIC SEARCH EQUILIBRIUM

by

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Working Paper No.1-89

January, 1 9 8 9

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We are grateful to seminar participants at Tel-Aviv University, the Hebrew University of Jerusalem, the University of Toronto, the University of Western Ontario, Columbia University and the University of Pennsylvania. Financial assistance from the Foerder Institute of Economic Research is gratefully also acknowledged

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ABSTRACT

Search Theory has been extensively and successfully applied to explain the persistence of price dispersion. This paper presents an explicitly dynamic search model which is able to account for cyclical patterns of prices and demand over time. These cylical features of the model are the consequence of the dynamic strategic interaction between buyers and firms and do not require the presence of extraneous factors such as shocks or heterogeneity of agents in order to obtain. The model builds on earlier work by Burdett and Judd (1983) and may be interpreted as a dynamic extension of their model.

Price Cycles and Booms: Dynamic Search Equilibrium

1. Introduction

Stigler's celebrated seminal article (1961) focused economists' attention on the fact that if it is costly to secure a price quotation, buyers might consciously purchase at a price exceeding the lowest obtainable price in the market; with imperfect information, Jevons' 'law of one price' need not apply. This theoretical insight is in accord with empirical evidence substantiating the existence of significant price dispersion.

In Stigler's formulation, the buyer decides on the number of prices to be sampled prior to observing any price quotation. Subsequent authors (e.g. De Groot (1970), McCall (1970), Kohn and Shavell (1974)) pointed out that a sequential strategy, i.e., a search procedure in which the decision as to whether an additional price is to be sampled depends upon the realization of preceding samples, is generally superior to fixed sample size rules.

While the search literature showed that price dispersion is a viable prospect as far as the demand side of the market is concerned, its sustainability as an equilibrium phenomenon consistent with optimal behavior on **both** sides of the market was less clear cut as Rothschild (1973) argued forcefully. Moreover Diamond (1971) showed when all consumers have positive search costs, however small, all firms charge the monopoly price in equilibrium. A large number of equilibrium search models arose in response to this challenge. These include Axell (1977), Braverman (1980), Burdett and Judd (1983), MacMinn (1980), Carlson and McAffee (1983), Reinganum (1979), Rob (1985), Salop and Stiglitz (1977), Stiglitz (1987), Varian (1980) and Wilde and Schwartz (1979).

The preceding literature shows that search theory can successfully account for price dispersion as an equilibrium phenomenon. The objective of this paper is to demonstrate that in the context of an explicitly dynamic model, search theory is also capable of accounting for the existence of price cycles and periodic booms in demand, even in deterministic and stationary environments. Clearly the timing of booms and price cycles are interrelated. The incidence of boom periods as predicted by the model is useful in explaining why certain markets are periodically inactive.

The dynamics of buyers' search behavior in our model follows recent developments in the theory of optimal search. While the earlier search literature imposed a dichotomy between fixed sample size and sequential search procedures, several authors have more recently pointed out that in a dynamic framework the optimal procedure is generally a hybrid of the two (Benhabib and Bull (1983), Burdett and Judd (1983), Gal Landsberger and Levykson (1981), Manning and Morgan (1982, 1985)).[1] Specifically, at the beginning of each period the buyer may demand any

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[1] Among these references, only Burdett and Judd describe a market equilibrium.

number of price quotations at constant cost which are simultaneously received at the end of the period. At each period, the searcher first decides on the number of price quotations to purchase if search continues and then decides between stopping and purchasing at once or continuing search. If utility from future purchases is discounted at a positive rate, the exogenous lag between the time a price quotation is demanded and the time it is received might induce demand for additional price quotations at the present period. Thus, the search procedure is sequential but allows the number of prices included in a sample to be determined endogenously.

At no period are firms bound by previously quoted prices. This absence of commitment on the part of firms is found to affect the equilibrium price path in remarkable ways. In particular, if consumers are sufficiently patient, equilibria distinguished by the existence of a stationary (non-degenate) price distribution fail to exist. Moreover, the model is capable of producing equilibria in which the average price behaves cyclically. In each period at which a consumer is in the market, she has the option of not soliciting any prices. Thus, in principle, there could exist periods at which consumers choose to defer their search such that the accumulated unsatisfied demand generates future booms. We show that such effects can indeed characterize equilibria. While the existence of price and demand cycles is demonstrated analytically, simulation studies presented in section 4 indicate that these effects may be quite dramatic.

The model we present can be viewed as a dynamic extension of Burdett and Judd's (B-J) (1983) non-sequential model and our analysis draws heavily on their results.

2. Notation and Framework

There is a continuum of firms and buyers. Each firm can costlessly supply an unlimited quantity of a homogeneous product. Firms compete in prices. In each period a new cohort of identical buyers of finite measure $\mu > 0$ per firm enters the market. Each consumer has inelastic demand for exactly one unit for which she is willing to pay $p^* > 0$ at the most. Once a buyer has purchased a unit, she leaves the market forever. Each buyer discounts utility from future purchases at a positive rate $\delta > 1$, such that she is indifferent between purchasing at a cost of δp at the present period and purchasing at p at the next period. Buyers are risk-neutral, and minimize the cost of purchase, including search costs and the price paid, subject to their rate of impatience.

From the preceding description of buyers' behavior, it is clear that at any period $t > 1$, the measure of buyers in the market may exceed μ if at preceding periods buyers choose to delay their purchase. Accordingly we let $\mu_t \geq \mu$ be the total measure of consumers per firm at the outset of period t.

Let $F_t(\cdot)$ denote the (possibly degenerate) distribution of prices at period t. In equilibrium each buyer in the market is assumed to know the sequence of future price distributions, but is unaware of the price charged by any specific firm. In order to buy from a firm, a buyer must first solicit a price quotation from that firm. Any number of price quotations may be solicited at the beginning of period t at a constant cost of $c > 0$ per price.[2] All prices demanded at the onset of period t are simultaneously received at the end of that period. We denote by n_t the number of prices demanded at t.

We assume that firms are not committed to any price offer for more than one period. Thus, although consumers might have perfect recall, previously observed prices are rendered obsolete by the fact that the firms may have already changed their prices. This assumption implies that if a consumer buys the product at period t she may choose only from the set of firms sampled at that period. In deciding on the size of the sample at period t and in making their purchasing choice, consumers take into account the possibility of continuing search in the next period and, in particular, the changes that may occur in the price distribution over time.

In this section we assume that the horizon is finite, $T > 1$. This assumption is relaxed in section 5. At any period t, let q_t^n denote the probability that a randomly selected consumer observes n prices and let $\tilde{p}_t$ be the reservation price associated with that period such

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[2]It is assumed that buyers obtain positive utility from paying $p^* + c$ for the product. See B-J (1983).

that a unit is purchased only if its price does not exceed $\tilde{p}_t$. The existence of such a reservation price will be demonstrated in what follows.

Each firm optimally chooses its price, taking the price distribution at that period and the search beavior of buyers summarized by $\tilde{p}_t$ and the sequence $<q_t^n>_{n=0}^{\infty}$ as given. In equilibrium, individually optimal behavior of firms must be consistent with the price distribution to which it responds. This requires that all prices in the support of the distribution earn equal profits and that no price outside the distribution be associated with greater profit. More formally

Definition 1: Given the time horizon T, we define $<F_t(\cdot), \pi_t, \mu_t, <q_t^n>_{n=0}^{\infty}, \tilde{p}_t>_{t=1}^{T}$ as a **dynamic search equilibrium** if:

(i) For every period t given $(\mu_t, <q_t^n>_{n=0}^{\infty}, \tilde{p}_t)$ the profit of a firm is $\pi_t \geq 0$ if it asks a price in the support of $F_t(\cdot)$ and is less than or equal to π_t if its price lies outside the support.

(ii) For every period t, $(<q_t^n>_{n=0}^{\infty}, \tilde{p}_t)$ represents the optimal search behavior of consumers given the current and the future sequence of price distributions.

The following definitions characterize specific types of dynamic search equilibria:

Definition 2: A **stationary equilibrium** is a dynamic search equilibrium in which $F_{t_1}(P) = F_{t_2}(P)$ for every $t_1, t_2 \leq T$.

Definition 3: A **cyclic equilibrium** is a non-stationary dynamic search equilibrium for which there is an integer z, such that for every $t \leq T - z$, $F_t(p) = F_{t+z}(p)$.

At every period, consumers choose the number of price quotations they wish to sample. We do not restrict this number to be greater than zero. That is, at any period t, consumers might in principle abstain from sampling altogether. In this case they cannot buy the product at that period.

Definition 4: We say that **active demand** is positive at period t if a positive measure of buyers samples at least one price.

Note that in our model zero active demand is **not** due to a lack of potential consumers since a new cohort enters at each period. Rather, zero active demand characterizes a period at which all consumers, although impatient, **choose** not to sample and, as a consequence, not to buy. To be part of an equilibrium, this decision must, of course, be consistent with their aim of obtaining the product at the lowest possible discounted cost.

Definition 5: A dynamic search equilibrium is characterized by **endogenous booms** if there are periods at which active demand is zero and other periods at which active demand is greater than μ.

Obviously there is a relationship between the frequency of price cycles and the size of the booms. This relationship is clearly illustrated by simulation in section 4.

The model of B-J is identical to the above except that there is only one period. Their results for the one-period model are important for our analysis and are briefly summarized as follows. There are 1, 2, or 3 market equilibria with nonsequential search; one monopoly price equilibrium and zero, one or two dispersed price equilibria. Further, given p^*, there exists a $c^* > 0$ such that $c < c^*$ implies there are two dispersed price equilibria, $c = c^*$ implies there is one dispersed price equilibrium and $c > c^*$ implies there are no dispersed price equilibria.

In any dispersed price equilibrium, a proportion $1 > q > 0$ of buyers observe only one price while the proportion $1-q$ observe two prices.[3] At the equilibrium, q is determined as the solution of the equation:

$$(1) \quad V(p^*,q) = p^*\left[\frac{q}{2(1-q)^2} \ln\left[\frac{2-q}{q}\right] - \frac{q}{1-q}\right] = c$$

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[3]If all consumers observe only one price then all firms will charge the monopolistic price p^*. On the other hand, if all consumers observe more than one price, all firms charge the Bertrand price.

where $V(p^*,q)$ is the expected difference between the price paid by a consumer who observes two prices and a consumer who observes only one. Equation (1) states that in equilibrium this expected difference is equal to the cost of obtaining an additional price quotation.

$V(p^*,q)$ attains a unique maximum at some q^*, $0 < q^* < 1$, is strictly increasing (decreasing) if $q^* > q > 0$ (if $q^* < q < 1$) and $V(p^*,q) \to 0$ as $q \to 0$ or $q \to 1$. c^* satisfies:

$$V(p^*,q^*) = c^*.$$

The exogenous parameters p^* and c determine q which, in turn, determines the equilibrium price distribution $F^*(p)$:

$$(2) \qquad F^*(p) = \begin{cases} 0 & \text{if } p \le \underset{\sim}{p} \\ 1 - [\frac{p^* - p}{p}][\frac{q}{2(1 - q)}], & \text{if } \underset{\sim}{p} \le p \le p^* \\ 1 & \text{if } p > p^* \end{cases}$$

where:

$$(3) \qquad \underset{\sim}{p} = p^* q/(2 - q)$$

While in the nonsequential case, discussed by B-J, the reservation price is p^*, the exogenously determined monopoly price, this need not be the case in the multiperiod problem. Since, as will be shown in due course, the reservation price changes over time, we let $V(\tilde{p}, q)$ be defined similarily to $V(p^*, q)$ but for the reservation price $\tilde{p}$. Note that equation (1) implies that changing $\tilde{p}$ induces a parallel shift of the function $V(\ )$. Thus, while q^* is not affected by changes of $\tilde{p}$, the critical value $c(\tilde{p})$ which solves the equal profits conditions $V(\tilde{p}, q^*) = c(\tilde{p})$ depends on the consumers' reservation price $\tilde{p}$. As long as $\tilde{p}$ is sufficiently high that $V(\tilde{p}, q^*) \geq c$ there exists a price dispersion equilibrium. However, once $V(\tilde{p}, q^*) < c$ the only equilibrium is the monopoly price equilibrium. In the following section we derive the equilibrium properties of the multi-period problem.

3. Finite Time Dynamic Search

For any T period dynamic search problem, one can identify a Diamond-type equilibrium in which at every period all firms charge the monopolistic price p^* and consumers search only once. This is, however, not the only equilibrium that exists. We wish to characterize the other equilibria which are associated with non-trivial price paths.

For every T, the time path of prices is obtained by first deriving the equilibrium at T and then proceeding by backward induction.

First, observe that, by (2), the price distribution at the last period is independent of μ_T, the measure of buyers in the market.[4]

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[4]It is required that this measure be positive but this is ensured by the fact that $\mu_T \geq \mu$.

Thus, irrespective of previous events, $F_T(\cdot) = F^*(\cdot)$ provided that the condition necessary for the existence of price dispersion obtains, i.e., $V(p^*, q^*) \geq c$. When the reverse inequality holds, the only equilibrium is the monopoly price p^* at every period. Given the reservation price $\tilde{p}_T = p^*$ and $c < c^*$ there are two different dispersed price equilibria, each one of which is characterized by a different q_T. Our analysis will not be based on a specific choice between these two equilibria.

For a specific price dispersion equilibrium at period T let E_T be the expected overall cost of purchasing the product at period T. This cost consists of the average price plus the cost of sampling one firm:

$$(4) \qquad E_T = c + \int_{\underline{p}}^{p^*} p dF_T(p) = c + \int_{\underline{p}}^{p^*} p d\{1 - (\frac{p^* - p}{p})(\frac{q_T}{2(1-q_T)})\}$$

where q_T solves $V(p^*, q_T) = c$ and $\underline{p}$ is given by (3).

From the equilibrium condition $V(p^*, q_T) = c$ it is clear that a consumer who samples twice has the same expected overall purchasing cost. Thus E_T is well defined.

The following Lemma is used extensively in the subsequent analysis.

Lemma 1: $E_T < p^*$.

Proof: See Appendix 1.

Lemma 1 states that if there is a price dispersion equilibrium at period T then the average price in the market, plus the cost of obtaining one price quotation is below the reservation price.

Consider period $T-1$. A consumer in the market at this period buys at the lowest price observed as long as this price does not exceed δE_T, the discounted expected cost of purchasing at the following period. If the lowest price observed exceeds this price, consumers prefer to defer consumption. Note that although the consumer is not certain to get E_T, the assumption of risk neutrality implies that she considers only the expected price.

Since consumers will not buy at a price above δE_T at period $T-1$, it is obvious that no firms will charge more than this price at that period. Thus at period $T-1$ the new reservation price is:

$$\tilde{p}_{T-1} = \min\{p^*, \delta E_T\}. \tag{5}$$

Observe that if, at period $T-1$, there is a non-degenerate price distribution $F_{T-1}(\cdot)$, firm i's profits are:

$$\pi^i_{T-1}(p_i) = \mu_{T-1} p_i [q_{T-1} + 2(1-q_{T-1})(1-F_{T-1}(p_i))]. \tag{6}$$

As in B-J the equilibrium price distribution $F_{T-1}(\cdot)$ is derived from the equal profit condition: $\pi^i_{T-1}(p_i) = \pi^j_{T-1}(p_j)$, for every p_i and p_j in the support of $F_{T-1}(\cdot)$. Thus, from (6), μ_{T-1} plays no role in the derivation of $F_{T-1}(\ )$ and in analyzing period T-1 may be ignored.

When δ is sufficiently large such that $\delta E_T \geq p^*$, then by (5) the reservation price at T-1 is p^*, and thus $F_{T-1}(\cdot) = F^*(\cdot)$.

Proposition 1: For δ close enough to one, no stationary dispersed price equilibrium exists.

Proof: The equilibrium distribution at period T is $F_T = F^*$ and is derived for the reservation price p^*. Lemma 1 guarantees that $E_T < p^*$. Thus, for a δ close enough to one, $\delta E_T < p^*$. Using (5) implies that $\tilde{p}_{T-1} < p^*$, i.e., the reservation price at T-1 is below p^*. Since the derivation of $F_{T-1}(\cdot)$ is analogous to that of $F^*(\cdot)$ with $\tilde{p}_{T-1}$ replacing p^* it is clear that $F_{T-1}(\ )$ is different from F_T.

□

Note that at period T-1 a price dispersion equilibrium exists only if $V(\tilde{p}_{T-1}, q^*) \geq c$.

Let $A_t = E_t - c$ be the average price at t. Analogous arguments to those of lemma 1 establish that if there is a price dispersed equilibrium at period T-1, then $E_{T-1} < \tilde{p}_{T-1}$. Thus, for δ close enough to one, $E_{T-1} < E_T$ and $A_{T-1} < A_T$. That is, the average price at T-1 is less than the average price at T.

The possibility of zero active demand adds an additional complexity to our analysis. Consumers will decide not to sample at period t if the expected purchasing cost at period t exceeds the discounted expected purchasing cost at some later period (not necessarily t+1). Of course, zero active demand can be introduced in a trivial and artificial way if, at some periods, all firms "happen to" charge more than the reservation price.[5] If this occurs, consumers will obviously defer their search to later periods. This is **not** the type of effect we are after. What we are interested in is the existence of periods in which, although it is expected that firms will offer prices below (or equal to) the reservation price $\tilde{p}_t$ if solicited, consumers nevertheless prefer to defer search to later periods. To see how this can occur in our model, observe that the reservation price defines the most the consumer is willing to pay once the search cost has been incurred. Prior to incurring this cost, however, consumers compare the current expected cost of purchase, including the expected price and c, with discounted future purchasing costs. Thus a price which is acceptable once the search cost has been paid need not be before it has been paid. In what follows we demonstrate the existence of equilibria featuring zero active demand of this type.

Proposition 2: Consider two consecutive periods, $t, t+1 \leq T$ such that active demand is positive in each. If δ is close enough to

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[5]Note that even in a Diamond-type equilibrium one can define such zero active demand equilibria, if all firms charge more than p^*.

one, the fact that prices are dispersed at $t+1$ implies that they are dispersed at t as well. Moreover, the average price at $t+1$ exceeds that at t.

Proof: Suppose that all firms charge the same price at t. A buyer who has already paid for a price offer is willing to pay at most $\tilde{p}_t = \delta E_{t+1}$. Thus $\tilde{p}_t$ is the only possible equilibrium price at t. The expected cost of buying at t is therefore $c + \delta E_{t+1} > \delta E_{t+1}$, the discounted cost of buying at $t+1$. Thus active demand at t must be zero. This contradiction establishes the first part of the proposition.

Since active demand is positive in each period, the maximum price at t is δE_{t+1}. An analogous argument to that in Lemma 1 then establishes that $E_t < \delta E_{t+1}$. Thus, for a δ close enough to one, $E_t < E_{t+1}$ which implies that $A_t < A_{t+1}$. This completes the proof.

□

Proposition 3: For δ sufficiently close to one, if at a period t each firm's price is $p_t < p^*$, then at $t-1$ either prices are dispersed and $A_{t-1} < p_t$ or active demand is zero.

Proof: Suppose to the contrary that all firms charge the same price at $t-1$, say p_{t-1}. By an already familiar argument, $p_{t-1} = \delta(p_t + c)$. Since $p_{t-1} > p_t$, an analogous argument to that used in the proof of Proposition 2 establishes that active demand at $t-1$ is zero if δ is sufficiently close to one. This completes the proof. □

Proposition 4: When the time horizon T is sufficiently large and the discount factor sufficiently small there exists $t' < T$ such that at each $t > t'$ prices are non-degenerately dispersed and active demand is positive while at t' active demand is zero.

Proof: Suppose the proposition is incorrect. By Proposition 2, it must then be the case that prices are dispersed at each $1 \le t < T$ no matter how large T is. By already familiar reasoning $\tilde{p}_t < \tilde{p}_{t+1}$, $t = 1,2,\ldots,T$ where $\tilde{p}_t$ is both the reservation price and the maximum price assigned positive density at t. For a given c let us define $\hat{p}$ in the following way.

$$\text{(7)} \qquad \hat{p} \equiv \operatorname{Inf}\{p \in R_+ \mid \text{there is a } 0 \le q \le 1 \text{ for which } V(p,q) = c\}$$

For every $p < \hat{p}$ and $0 \le q \le 1$ we obtain $V(p,q) < c$. Thus, if at period t the reservation price is $\tilde{p}_t < \hat{p}$ there is no price dispersed equilibrium, while for $\tilde{p}_t \ge \hat{p}$ there is at least one.

We claim that if T is large enough there must be $t' < T$ such that $\tilde{p}_{t'} < \hat{p}$. This claim will complete the proof since for such t' only a single price equilibrium can exist and by using proposition 2 it is evident that at period t' there is no active demand.

If for every T there is no t' such that $\tilde{p}_{t'} < \hat{p}$ then let us define a sequence $\{\tilde{p}_\tau\}_{\tau=1}^{\infty}$ such that $\tilde{p}_\tau$ is the reservation price when there are τ periods remaining until the end of the game.[6] Clearly,

[6] Note that since the way we construct the search equilibrium is by starting from the last period and proceeding backward, the above sequence is properly defined.

$\tilde{p}_\tau > \tilde{p}_{\tau+1}$ and as $\tilde{p}_\tau > \hat{p}$ for every τ there must be $\bar{p} \geq \hat{p}$ such that the sequence $\{\tilde{p}_\tau\}_{\tau=1}^{\infty}$ converges to $\bar{p}$. Let $E(p)$ be the expected purchasing cost in a dispersed price equilibrium when the consumers' reservation price is p. $E(p)$ is defined similarly to eq.(4) when we replace p^* by p. $E(p)$ is a continuous function for $p > \hat{p}$. Now observe the sequence $\{\tilde{p}_{\tau+1} - \tilde{p}_\tau\}_{\tau=1}^{\infty}$. Since $\tilde{p}_{\tau+1} = \delta E(\tilde{p}_\tau)$ we have that $\tilde{p}_{\tau+1} - \tilde{p}_\tau = \delta E(\tilde{p}_\tau) - \tilde{p}_\tau$ converges to $\delta E(\bar{p}) - \bar{p}$ as $\tau \to \infty$. Since $\bar{p} > \hat{p}$ and using lemma 1 there is an $\epsilon < 0$ such that $\delta E(\bar{p}) - \bar{p} < \epsilon$. Now, by analogy to proposition 1 we can conclude that for a δ close enough to one, $\lim_{\tau\to\infty} (\tilde{p}_{\tau+1} - \tilde{p}_\tau) < \epsilon < 0$ which implies a contradiction. □

Proposition 4 shows that any non-trivial equilibrium includes periods at which active demand is zero, provided that the horizon is sufficiently long and δ is sufficiently close to one.

Using the preceding propositions we can use backward induction to construct equilibria which exhibit cyclical patterns. Our construction is achieved by analyzing the evolution of the reservation price $\tilde{p}_t$ over time.

At period T the reservation price $\tilde{p}_T = p^*$, the equilibrium price distribution is $F_T(\cdot) = F^*(\cdot)$, and the expected purchasing cost is E_T defined formally by equation (4).

Now define the following sequence:

For every $t-1 < T$

$$(8)\quad \tilde{p}_{t-1} = \begin{cases} \delta\left(\int_{\underline{p}(\tilde{p}_t)}^{\tilde{p}_t} p\, d\left(1 - \frac{(\tilde{p}_t - p)}{p}\frac{q_t}{2(1-q_t)}\right) + c\right) & \text{if } \tilde{p}_t \geq \hat{p}. \\ \delta\, \tilde{p}_t & \text{if } \tilde{p}_t < \hat{p} \end{cases}$$

where

$\underline{p}(\tilde{p}_t) = \tilde{p}_t q_t/(2-q_t)$

$\hat{p}$ is defined by (7)

q_t is defined by $V(\tilde{p}_t, q_t) = c$.

Let

$$(9)\qquad \tilde{t} = \max\{t < T \mid \tilde{p}_t < \hat{p}\}.$$

Proposition 4 guarantees that for T large enough such $\tilde{t}$ exists.

For $T > t \geq \tilde{t}+1$ we have $\tilde{p}_t > \hat{p}$. Thus we can construct a price distribution $F_t(\cdot)$ analogously to the construction of $F^*(\ )$ with $\tilde{p}_t$ replacing p^*. It is immediately verifiable that the sequence $F_t(\cdot)$ of price distributions and the sequences $\tilde{p}_t$ and q_t constitute an equilibrium for the search problem that starts at period $\tilde{t}+1$ and ends at T. This equilibrium is characterized by increasing average prices and, as proposition 3 indicates, active demand is positive at each of these periods. Once period $\tilde{t}$ is reached the reservation price $\tilde{p}_{\tilde{t}}$ is sufficiently low such that no dispersed price equilibrium exists, i.e., $V(\tilde{p}_{\tilde{t}}, q^*) < c$. Thus there is a unique price equilibrium in which all firms charge the price $\tilde{p}_{\tilde{t}} = \delta(A_{\tilde{t}+1} + c)$ and by proposition 3 active demand at this period is zero.

At period $\tilde{t}-1$ the consumers' reservation price is $\delta\tilde{p}_{\tilde{t}}$. If firms ask prices above this, consumers are better off not buying and waiting for two periods until period $\tilde{t}+1$ in which their expected purchasing cost is, by construction, $\tilde{p}_{\tilde{t}}\delta^{-1}$.

Let

$$(10) \qquad \approx{t} = \max\{t < \tilde{t} \mid \delta^{(\tilde{t}-t)}\tilde{p}_{\tilde{t}} > \hat{p}\}.$$

For every $\approx{t} < t < \tilde{t}$ the consumers' reservation price is $\tilde{p}_t = \delta^{(\tilde{t}-t)}\tilde{p}_{\tilde{t}}$ and since it is below $\hat{p}$ the only equilibrium that can exist at this period is a single price equilibrium in which all firms charge the reservation price. During the periods between $\approx{t}+1$ until $\tilde{t}$ there is zero active demand and prices are declining.

The construction of the equilibrium price pattern prior to $\approx{t}$ replicates the preceding derivation.

Using the above construction and the examples we provide in the next section we conclude the following:

Proposition 5: (i) Cyclical dynamic search equilibria with endogenous booms do exist; (ii) for δ sufficiently close to one, the only equilibria are the Diamond-type equilibrium and cyclical equilibria.

Each cycle in the construction consists of two phases. During the first phase active demand is positive, prices are non-degenerately dispersed and the average price is increasing. During the second phase, active demand is zero, prices are declining, and buyers "accumulate". Thus the equilibrium describes both price cycles and active demand cycles. Proposition 4 ensures that these cycles are not an artifact of some particular equilibrium but must occur in any non-trivial equilibrium.

4. Simulation of Cycles and Booms

In the following simulations we have generated a search equilibrium with a very simple cyclical price and demand pattern. It is important to note, however, that we do not suggest that all equilibria exhibit similar patterns. Consider a dynamic search model in which the consumers' reservation price is $p^* = 10$, the sampling cost is $c = 1$ and there is a common discount factor $\delta = 1.2$. Given these parameters we can calculate the equilibrium time path and the active demands. This information is summarized in Table 1 and in Figure 1. The cycles are of length 5 and a boom occurs at every fifth period, i.e., all sales are made at that period.

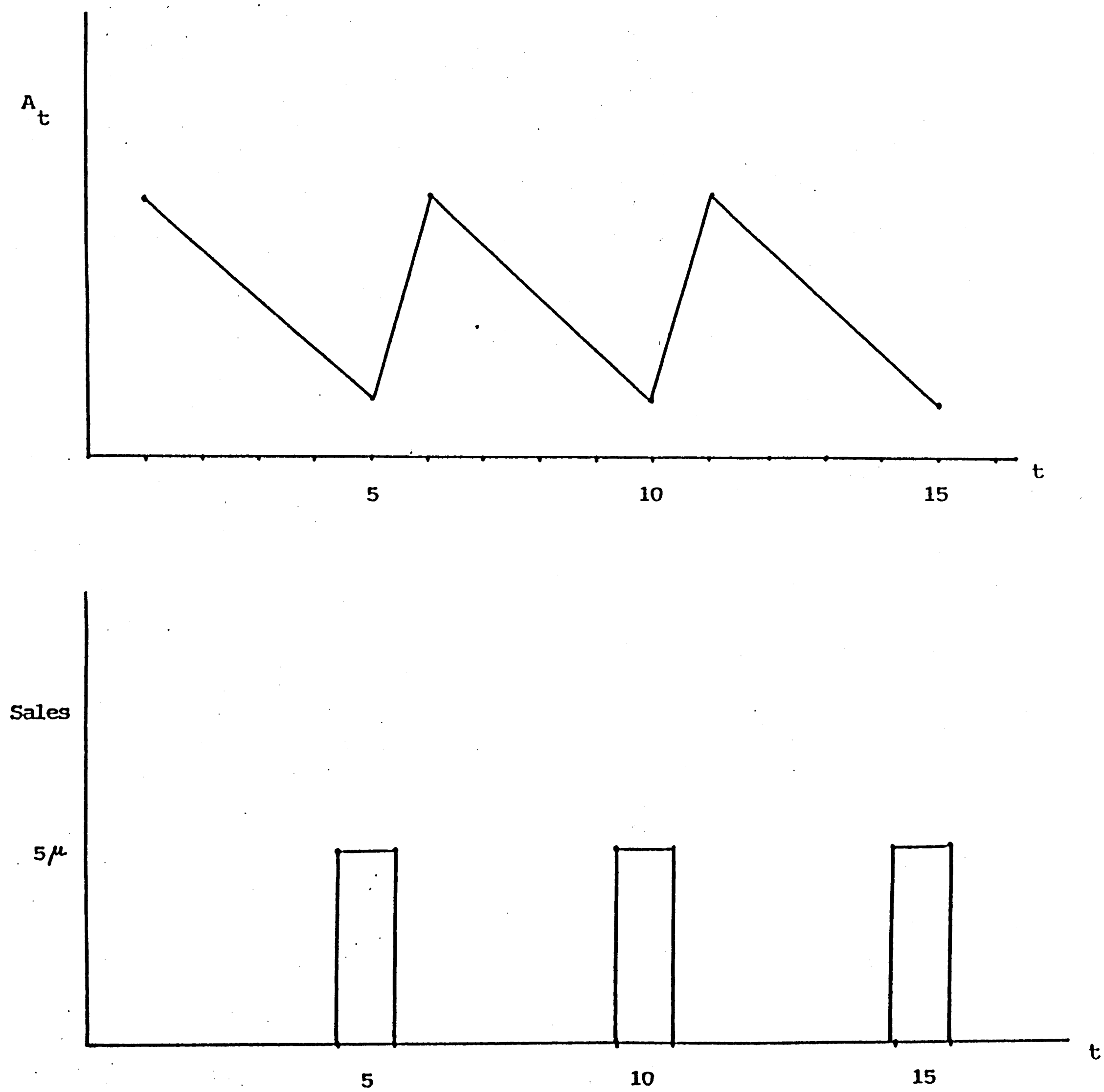

Figure 1

This is not, however, a unique equilibrium. Given the same parameters one can generate an equilibrium that has a cycle length of 3 periods and a boom occurs every third period. This boom is weaker than the boom in the previous equilibrium. This equilibrium is illustrated in figure 2.

Table 1

T =	1	2	3	4	5	6	7	8	9	10	11	12	13
A_t	9.01	7.51	6.26	5.22	3.34	9.01	7.51	6.26	5.22	3.34	9.01	7.51	6.26
$\tilde{p}_t$	9.01	7.51	6.26	5.22	10.00	9.01	7.51	6.26	5.22	10.00	9.01	7.51	6.26
q_t	-	-	-	-	.26	-	-	-	-	.26	-	-	-
μ_t	μ	2μ	3μ	4μ	5μ	μ	2μ	3μ	4μ	5μ	μ	2μ	3μ
AD_t	0	0	0	0	5μ	0	0	0	0	5μ	0	0	0

Table 2

T =	1	2	3	4	5	6	7	8	9	10	11	12	13
A_t	9.10	7.58	5.32	9.10	7.58	5.32	9.10	7.58	5.32	9.10	7.58	5.32	9.10
$\tilde{p}_t$	9.10	7.58	10.00	9.10	7.58	10.00	9.10	7.58	10.00	9.10	7.58	10.00	9.10
q_t	-	-	.48	-	-	.48	-	-	.48	-	-	.48	-
μ_t	μ	2μ	3μ	μ	2μ	3μ	μ	2μ	3μ	μ	2μ	3μ	μ
AD_t	0	0	3μ	0	0	3μ	0	0	3μ	0	0	3μ	0

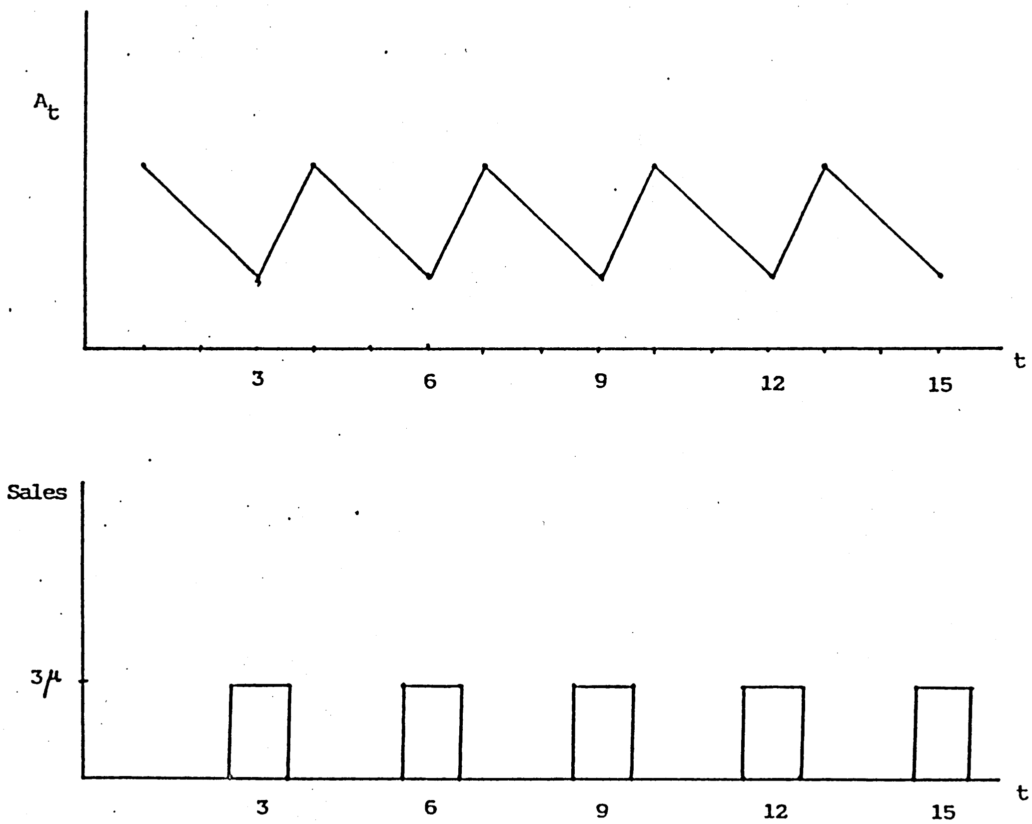

Figure 2

An Example of a A Stationary Equilibrium

Let us now change the parameters and assume that $c = 0.3$. In this case there is a stationary dispersed price equilibrium:

$$A_t = 8.34; \quad q = .90, \quad \underline{p} = 8.206 \quad \tilde{p}_t = 10.$$

It is important to note that this stationary equilibrium is for the discount factor 1.2. Once the discount factor falls below 1.07 we will once again have a cyclical equilibrium.

5. Dynamic Search with an Infinite Horizon

The analysis of the previous section was carried out under the assumption that the horizon is finite. In this section we argue that the essential characteristics of that analysis are not a consequence of that assumption but are an inherent feature of dynamic search equilibrium. Accordingly the following proposition extends our main results to the infinite horizon case.

Proposition 6: Consider a dynamic search problem with infinite horizon. For δ sufficiently close to one, a stationary price dispersed equilibrium does not exist.

Proof: Suppose the contrary and let $\tilde{F}(\cdot)$ denote the stationary equilibrium price distribution. It is easy to check that the arguments of B-J apply to the effect that $\tilde{F}(\cdot)$ must be absolutely continuous with connected support, say $[\underline{p}, \bar{p} \leq p^*]$ (since there is no terminal period, $\bar{p}$ may be less than p^*). From the analysis of B-J we know that prices are dispersed at any date t only if a proportion $1-q > 0$ of buyers sample two prices at that period, incurring a search cost of $2c$.

Consider the following sequential search rule: sample once at t and sample again at $t+1$ if and only if the outcome of the first sample exceeds some $p' > \underline{p}$. It is obvious that for every $c > 0$ there exists $p' > \underline{p}$ such that the expected cost of purchase from following this rule is less than the cost associated with sampling twice **before** obtaining any price quotations. Therefore a sufficiently patient individual will never sample twice in the same period, contradicting the existence of a stationary price equilibrium. □

Note how the preceding proof depends on the assumption that the horizon is infinite so that to **any** period t there corresponds a following period in which a second sample may be drawn, contingent upon the outcome at t. When the horizon is finite, this argument, of course, fails at the last period and it is necessary to resort to the more complex arguments presented earlier. It is also instructive to note that the argument in the above proof only applies to a stationary price distribution. For example, if prices are dispersed according to $F^*(\cdot)$ at t and concentrated at p^* in future periods, the sequential procedure does not dominate the fixed sample procedure. Indeed, cyclical equilibria of the type derived in the previous section exist for the infinite horizon case as well. One may immediately verify, for example, that in the case of an infinite horizon, an infinite repetition of the cycles described in tables 1 and 2 describe dynamic search equilibria for the relevant parameters.

Concluding Remarks

Search theory has been used successfully to explain the persistence of price dispersion. In this paper we have used an explicitly dynamic search model to account for cyclical patterns of prices and demand. From the vantage point of firms, the dynamics of the model refer to the absence of commitment to previously announced prices; new prices may be posted at any time. From the point of view of consumers, the dynamics of the model expresses itself in two ways. First, buyers are able to defer consumption to future periods. Second, there is a lag between the time a price is solicited and the time it is received. If buyers are impatient to consume, this lag may induce a demand for the simultaneous solicitation of more than one price. Interestingly, our analysis shows that even when this exogenous impatience tends to vanish, i.e., as $\delta \to 1$, an **endogenous** cause for impatience emerges: future prices may be less favorable than those presently available. Thus even buyers who are very patient simultaneously solicit several prices even if the horizon is infinite.

Perhaps the most striking feature of these results is that it has been unnecessary to appeal to exogenous factors such as shocks or heterogeneity of agents to obtain them. This contrasts with other microeconomic models of price cycles, e.g., Conlisk, Gertner and Sobel (1984), Sobel (1984) and Sheshinski and Weiss (1977, 1983). Certainly one can expect that the inclusion of shocks and differences among agents would only tend to reinforce our results. Indeed one may conceive of our work as an exploration into the minimum requirements for the generation of cyclical effects. It is our hope that models of this typewill prove useful in improvng our understanding of the micro foundations of business cycles.

APPENDIX 1: PROOF OF OF LEMMA 1

Integrating E_T by parts and letting $F(\underline{p}) = 0$ and $F(p^*) = 1$ yields that

$$(A.1) \qquad E_T = p* - \int_{\underline{p}}^{p*} [1 - (\frac{p* - p}{p}) \frac{q}{2(1-q)}] dp + c.$$

Solving the above integral yields that

$$(A.2) \qquad E_T = \underline{p} - \frac{q}{2(1-q)}(p* - \underline{p}) + \frac{qp*}{2(1-q)} \ln p \Big|_{\underline{p}}^{p*} + c.$$

Using (3) let us substitute $\underline{p} = p*q/(2-q)$

$$(A.3) \qquad E_T = \frac{q}{2(1-q)} p* \ln(\frac{2 - q}{q}) + c.$$

$c = V(q)$ implies that

$$(A.4) \qquad c = p*[\frac{q}{2(1-q)^2} \ln(\frac{2-q}{q}) - \frac{q}{1-q}].$$

Substituting (A.4) into (A.3) yields

$$(A.5) \qquad E_T = p* \frac{q(2-q)}{2(1-q)^2} \ln (\frac{2-q}{q}) - p* \frac{q}{1-q}.$$

Therefore $E_T - p^* < 0 \iff$

(A.6) $$p^* \frac{q}{2(1-q)} \ln\left(\frac{2-q}{q}\right) + p^*\left\{\frac{q}{2(1-q)^2} \ln\left(\frac{2-q}{2}\right) - \frac{q}{1-q}\right\} - p^* < 0$$

which for $0 < q < 1$ reduces to

(A.7) $$\ln \frac{2-q}{q} < \frac{2(1-q)}{q(2-q)}.$$

Let $\phi(q) = \frac{2(1-q)}{q(2-q)}$ and $\psi(q) = \ln\left(\frac{2-q}{q}\right)$.

Observe that $\phi(1) = \psi(1) = 0$.

Differentiate $\phi(\cdot)$ and $\psi(\cdot)$:

(A.8) $$\phi'(q) = -\frac{2[2-q(2-q)]}{q^2(2-q)^2}.$$

(A.9) $$\psi'(q) = \frac{-2}{(2-q)q}.$$

For $0 < q < 1$, $\psi(\cdot)$ and $\phi(\cdot)$ are monotonic decreasing. Also observe that, since $q(2-q) < 1$ for $0 < q < 1$, the absolute value of $\phi'(\cdot)$'s numerator exceeds the absolute value of $\psi'(\cdot)$'s numerator while $\psi'(\cdot)$'s denominator exceeds $\phi'(\cdot)$'s denominator. Therefore $|\phi'(q)| > |\psi'(q)|$ for $0 < q < 1$. It follows that $\phi(q) > \psi(q)$ for $\sigma < q < 1$.

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